# <u>Chapter 6</u> <u>Linear Inequalities</u>

# <u>Exercise 6.1</u>

# **Question 1**

Solve 24x < 100, when (i) x is a natural number. (ii) x is an integer.

#### **Answer:**

(i) Given that 24x < 100</li>
Now we have to divide the inequality by 24 then we get x < 25/6</li>
Now when x is a natural integer then
It is clear that the only natural number less than 25/6 is 1, 2, 3, 4.
Thus, 1, 2, 3, 4 will be the solution of the given inequality when x is a natural number.
Hence {1, 2, 3, 4} is the solution set.
(ii) Given that 24x < 100</li>
Now we have to divide the inequality by 24 then we get x < 25/6</li>
now when x is an integer then
It is clear that the integer number less than 25/6 is...-1, 0, 1, 2, 3, 4.
Thus, solution of 24 x < 100 are...,-1, 0, 1, 2, 3, 4, when x is an integer.</li>
Hence {..., -1, 0, 1, 2, 3, 4} is the solution set.

# **Question 2**

**Solve - 12x > 30, when** (i) x is a natural number. (ii) x is an integer.

#### Answer:

(i) Given that, -12x > 30Now by dividing the inequality by -12 on both sides we get, x < -5/2When x is a natural integer then It is clear that there is no natural number less than -2/5 because -5/2 is a negative number and natural numbers are positive numbers. Therefore there would be no solution of the given inequality when x is a natural number. (ii) Given that, -12x > 30

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Now by dividing the inequality by -12 on both sides we get, x < -5/2When x is an integer then It is clear that the integer number less than -5/2 are..., -5, -4, - 3 Thus, solution of – 12x > 30 is ...,-5, -4, -3, when x is an integer. Therefore the solution set is {..., -5, -4, -3}

# **Question 3**

**Solve 5x – 3 < 7, when** (i) x is an integer (ii) x is a real number

#### Answer:

(i) Given that, 5x - 3 < 7Now by adding 3 both side we get, 5x - 3 + 3 < 7 + 3Above inequality becomes 5x < 10 Again by dividing both sides by 5 we get, 5x/5 < 10/5 x < 2 When x is an integer then It is clear that the integer number less than 2 are..., -2, -1, 0, 1. Thus, solution of 5x - 3 < 7 is ..., -2, -1, 0, 1, when x is an integer. Therefore the solution set is {..., -2, -1, 0, 1} (ii) Given that, 5x - 3 < 7Now by adding 3 both side we get, 5x - 3 + 3 < 7 + 3Above inequality becomes 5x < 10 Again by dividing both sides by 5 we get, 5x/5 < 10/5 x < 2 When x is a real number then It is clear that the solutions of 5x - 3 < 7 will be given by x < 2 which states that all the real numbers that are less than 2. Hence the solution set is  $x \in (-\infty, 2)$ 

#### **Question 4**

**Solve 3x + 8 >2, when** (i) x is an integer.

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(ii) x is a real number.

#### Answer:



#### **Question 5**

4x + 3 < 5x + 7

#### Answer:

Given that, 4x + 3 < 5x + 7Now by subtracting 7 from both the sides, we get 4x + 3 - 7 < 5x + 7 - 7The above inequality becomes, 4x - 4 < 5xAgain by subtracting 4x from both the sides, 4x - 4 - 4x < 5x - 4xx > -4

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The solutions of the given inequality are defined by all the real numbers greater than -4. Required solution set is  $(-4, \infty)$ 

# **Ouestion 6**

3x - 7 > 5x - 1

#### **Answer:**

Given that. 3x - 7 > 5x - 1Now by adding 7 to both the sides, we get 3x - 7 + 7 > 5x - 1 + 73x > 5x + 6Again by subtracting 5x from both the sides, 3x - 5x > 5x + 6 - 5x-2x > 6Dividing both sides by -2 to simplify we get -2x/-2 < 6/-2x < -3 : The solutions of the given inequality are defined by all the real numbers less than -3. Hence the required solution set is  $(-\infty, -3)$ 

# **Ouestion 7**

 $3(x-1) \le 2(x-3)$ 

#### **Answer:**

```
Given that, 3(x - 1) \le 2(x - 3)
Given that, 3(x - 1) \le 2(x - 3)
By multiplying above inequality can be written as
3x - 3 \le 2x - 6
Now by adding 3 to both the sides, we get
3x - 3 + 3 \le 2x - 6 + 3
3x \leq 2x - 3
Again by subtracting 2x from both the sides,
3x - 2x \le 2x - 3 - 2x
x \leq -3
Therefore the solutions of the given inequality are defined by all the real numbers less than or equal to
-3.
```

Hence the required solution set is  $(-\infty, -3]$ 

# **Ouestion 8**

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# $3(2-x) \ge 2(1-x)$

#### Answer:

Given that,  $3(2 - x) \ge 2(1 - x)$ By multiplying we get  $6 - 3x \ge 2 - 2x$ Now by adding 2x to both the sides,  $6 - 3x + 2x \ge 2 - 2x + 2x$  $6 - x \ge 2$ Again by subtracting 6 from both the sides, we get  $6 - x - 6 \ge 2 - 6$  $-x \ge -4$ Multiplying throughout inequality by negative sign we get x ≤ 4 : The solutions of the given inequality are defined by all the real numbers greater than or equal to 4. Hence the required solution set is  $(-\infty, 4]$ **Question 9** x + x/2 + x/3 < 11**Answer:** Given that  $x + \frac{x}{2} + \frac{x}{2} < 11$ By taking x as common then we get  $x\left(1+\frac{1}{2}+\frac{1}{3}\right) < 11$ By taking LCM  $x\left(\frac{6+3+2}{2}\right) < 11$  $\frac{11\,\tilde{x}}{6} < 11$ Dividing by both side  $\frac{11 x}{6 \times 11} < 11$ 

 $\frac{x}{6} < 1$ x < 6

The solutions of the given inequality are defined by all the real number less than 6. Hence the solution set is  $(-\infty, 6)$ 

# **Question 10**

x/3 > x/2 + 1

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#### Answer:

```
Given that

\frac{x}{3} > \frac{x}{2} + 1

On rearranging and by taking LCM we get

\left(\frac{2x-3x}{6}\right) > 1

-x/6 > 1
```

- x > 6

x < - 6

: the solutions of the given inequality are defined by all the real numbers less than - 6. Hence the required solution set is  $(-\infty, -6)$ 

# **Question 11**

 $3(x-2)/5 \le 5(2-x)/3$ 

#### **Answer:**

Given that.  $\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$ Now by cross - multiplying the denominators, we get  $9(x-2) \le 25(2-x)$  $9x - 18 \le 50 - 25x$ Now adding 25x both the sides,  $9x - 18 + 25 x \le 50 - 25 x + 25 x$ plete KIT of Education  $34x - 18 \le 50$ Adding 25x both the sides,  $34x - 18 + 18 \le 50 + 18$  $34x \le 68$ Dividing both sides by 34,  $34x/34 \le 68/34$ x ≤ 2 The solutions of the given inequality are defined by all the real numbers less than or equal to 2. Required solution set is  $(-\infty, 2]$ 

#### **Question 12**

 $\frac{1}{2}\left(\frac{3x}{5}+4\right)\geq\frac{1}{3}(x-6)$ 

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#### **Answer:**

Given that  $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$ Now cross multiplying the denominators we get  $3\left(\frac{3x}{5}+4\right) \ge 2x(x-6)$ Multiplying by 3 we get  $12+12 \ge 2x-\frac{9x}{5}$   $24 \ge \frac{10x-9x}{5}$   $24 \ge \frac{x}{5}$  $\therefore$  The solutions of the given inequality are defined by all the real numbers less than or equal to 120.

Thus,  $(-\infty, 120]$  is the required solution set.

#### **Question 13**

2(2x+3) - 10 < 6(x-2)

#### Answer:

```
Given that,

2(2x + 3) - 10 < 6(x - 2)

By multiplying we get

4x + 6 - 10 < 6x - 12

On simplifying we get

4x - 4 < 6x - 12

-4 + 12 < 6x - 4x

8 < 2x

4 < x
```

 $\therefore$  The solutions of the given inequality are defined by all the real numbers greater than or equal to 4. Hence the required solution set is (4, - $\infty$ )

## **Question 14**

 $37 - (3x + 5) \ge 9x - 8(x - 3)$ 

#### Answer:

Given that,  $37 - (3x + 5) \ge 9x - 8 (x - 3)$ On simplifying we get =  $37 - 3x - 5 \ge 9x - 8x + 24$ 

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=  $32 - 3x \ge x + 24$ On rearranging =  $32 - 24 \ge x + 3x$ =  $8 \ge 4x$ =  $2 \ge x$ All the real numbers of x which are less than or equal to 2 are the solutions of the given **Question 15**  $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ 

# 4 3 5

#### Answer:

$$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5} = \frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$$
$$= \frac{x}{4} < \frac{25x - 10 - 21x + 9}{15}$$
$$= \frac{x}{4} < \frac{4x - 1}{5}$$
$$= 15x < 4 (4x - 1)$$
$$= 15x < 16x - 4$$
$$= 4 < x$$

All the real numbers of x which are greater than 4 are the solutions of the given inequality Hence,  $(4, \infty)$  will be the solution for the given inequality

# **Question 16**

$$\frac{(2x-1)}{3} \ge \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$

Answer:

Given,

$$\frac{(2x-1)}{3} \ge \frac{(3x-2)}{4} - \frac{(2-x)}{5} = \frac{(2x-1)}{3} \ge \frac{5(3x-2) - 4(2-x)}{20}$$
$$= \frac{(2x-1)}{3} \ge \frac{15x - 10 - 8 + 4x}{20}$$
$$= \frac{(2x-1)}{3} \ge \frac{19x - 18x}{20}$$
$$= 20 (2x - 1) \ge 3 (19x - 18)$$
$$= 40x - 20 \ge 57x - 54$$
$$= -20 + 54 \ge 57x - 40x$$
$$= 34 \ge 17x$$
$$= 2 \ge x$$
$$\therefore All the real numbers of x which are less than or equivalent times that the real numbers of x where the times that the real numbers of x where the times that times the times that times the times that the times times the times times that the times times the times times that times times the times ti$$

ll the real numbers of x which are less than or equal to 2 are the solutions of the given inequality

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Hence,  $(-\infty, 2]$  will be the solution for the given inequality

# **Question 17**

3x - 2 < 2x + 1

#### Answer:

Given. 3x - 2 < 2x + 1Solving the given inequality, we get 3x - 2 < 2x + 1= 3x - 2x < 1 + 2= x < 3Now, the graphical representation of the solution is as follows: x < 3 0 -3 -2 -1 2 **Ouestion 18**  $5x - 3 \ge 3x - 5$ **Answer:** We have,  $5x - 3 \ge 3x - 5$ Solving the given inequality, we get  $5x - 3 \ge 3x - 5$ On rearranging we get  $= 5x - 3x \ge -5 + 3$ On simplifying  $= 2x \ge -2$ Now divide by 2 on both sides we get = x ≥ -1 The graphical representation of the solution is as follows: -1 <u><</u> x -1 0 1 2 -2 3 4 -3 -4 **Ouestion 19** 

3(1-x) < 2(x+4)

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#### Answer:

Given, 3(1-x) < 2(x + 4)Solving the given inequality, we get 3(1-x) < 2(x + 4)Multiplying we get = 3 - 3x < 2x + 8On rearranging we get = 3 - 8 < 2x + 3x = -5 < 5xNow by dividing 5 on both sides we get -5/5 < 5x/5 = -1 < xNow, the graphical representation of the solution is as follows:

-1 < x

-4 -3 -2 -1 0 1 2 3 4

## **Question 20**

 $\frac{x}{2} \ge \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ 

#### **Answer:**

Given  $\frac{x}{2} \ge \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ Solving the given inequality we get  $\frac{x}{x} \ge \frac{5(5x-2)-3(7x-3)}{10}$  $\frac{x}{2} \ge \frac{5(5x-2)-3(7x-3)}{15}$ On computing we get  $\geq \frac{25x-10-21x+9}{2}$  $=\frac{x}{2}$ 15  $\geq \frac{4\,x-1}{15}$  $=\frac{\overline{x}}{2}$  $= 15x \ge 2(4x - 1)$  $= 15x \ge 8x - 2$  $= 15x - 8x \ge 8x - 2 - 8x$  $=7x \ge -2$  $= x \ge -2/7$ Now, the graphical representation of the solution is as follows:

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#### **Question 21**

Ravi obtained 70 and 75 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

#### **Answer:**

Let us assume, x be the marks obtained by Ravi in his third unit test According to question, the entire students should have an average of at least 60 marks  $(70 + 75 + x)/3 \ge 60$ = 145 + x ≥ 180 = x ≥ 180 - 145 = x ≥ 35 Hence, all the students must obtain 35 marks in order to have an average of at least 60 marks

### **Question 22**

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

#### Answer:

Let us assume Sunita scored x marks in her fifth examination Now, according to the question in order to receive A grade in the course she must have to obtain average 90 marks or more in her five examinations  $(87 + 92 + 94 + 95 + x)/5 \ge 90$ =  $(368 + x)/5 \ge 90$ =  $368 + x \ge 450$ =  $x \ge 450 - 368$ =  $x \ge 82$ Hence, she must have to obtain 82 or more marks in her fifth examination

#### **Question 23**

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

#### **Answer:**

Let us assume x be the smaller of the two consecutive odd positive integers  $\therefore$  other integer is = x + 2 It is also given in the question that, both the integers are smaller than 10  $\therefore$  x + 2 < 10 x < 8 ... (i) Also, it is given in the question that sum off two integers is more than 11  $\therefore$  x + (x + 2) > 11 2x + 2 > 11 x > 9/2 x > 4.5 ... (ii) Thus, from (i) and (ii) we have x is an odd integer and it can take values 5 and 7 Hence, possible pairs are (5, 7) and (7, 9)

# **Question 24**

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

#### **Answer:**

Let us assume x be the smaller of the two consecutive even positive integers  $\therefore$  other integer = x + 2 It is also given in the question that, both the integers are larger than 5  $\therefore$  x > 5 ....(i) Also, it is given in the question that the sum of two integers is less than 23.  $\therefore$  x + (x + 2) < 23 2x + 2 < 23 x < 21/2 x < 10.5 .... (ii) Thus, from (i) and (ii) we have x is an even number and it can take values 6, 8 and 10 Hence, possible pairs are (6, 8), (8, 10) and (10, 12).

#### **Question 25**

The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

#### **Answer:**

Let us assume the length of the shortest side of the triangle be x cm  $\therefore$  According to the question, length of the longest side = 3x cm And, length of third side = (3x - 2) cm

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As, the least perimeter of the triangle = 61 cm Thus, x + 3x + (3x - 2) cm  $\ge 61$  cm =  $7x - 2 \ge 61$ =  $7x \ge 63$ Now divide by 7 we get =  $7x/7 \ge 63/7$ =  $x \ge 9$ Hence, the minimum length of the shortest side will be 9 cm

### **Question 26**

A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

#### Answer:

Let us assume the length of the shortest piece be x cm  $\therefore$  According to the question, length of the second piece = (x + 3) cm And, length of third piece = 2x cm As all the three lengths are to be cut from a single piece of board having a length of 91 cm  $\therefore x + (x + 3) + 2x \le 91 \text{ cm}$  $= 4x + 3 \le 91$  $= 4x \le 88$  $= 4x/4 \le 88/4$  $= x \le 22 \dots (i)$ Also, it is given in the question that, the third piece is at least 5 cm longer than the second piece  $\therefore 2x \ge (x+3) + 5$  $2x \ge x + 8$  $x \ge 8 \dots (ii)$ Thus, from equation (i) and (ii) we have:  $8 \le x \le 22$ Hence, it is clear that the length of the shortest board is greater than or equal to 8 cm and less than or equal to 22 cm

# Exercise 6.2

#### **Question 1**

Solve the following inequalities graphically in two-dimensional plane:

#### $\mathbf{x} + \mathbf{y} < \mathbf{5}$

#### Answer:

Given x + y < 5<br/>Considerx05y50Now draw a dotted line x + y = 5 in the graph ( $\because x + y = 5$  is excluded in the given question)Now Consider x + y < 5Select a point (0, 0) $\Rightarrow 0 + 0 < 5$  $\Rightarrow 0 < 5$  (this is true) $\therefore$  Solution region of the given inequality is below the line x + y = 5. (That is origin is included in the

region) The graph is as follows:



# **Question 2**

#### $2x + y \ge 6$

#### Answer:

Given  $2x + y \ge 6$ Now draw a solid line 2x + y = 6 in the graph ( $\because 2x + y = 6$  is included in the given question) Now Consider  $2x + y \ge 6$ Select a point (0, 0)  $\Rightarrow 2 \times (0) + 0 \ge 6$  $\Rightarrow 0 \ge 5$  (this is false)

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: Solution region of the given inequality is above the line 2x + y = 6. (Away from the origin) The graph is as follows:



# **Question 3**

 $3x + 4y \le 12$ 

#### Answer:

Given  $3x + 4y \le 12$ Now draw a solid line 3x + 4y = 12 in the graph ( $\because 3x + 4y = 12$  is included in the given question) Now Consider  $3x + 4y \le 12$ Select a point (0, 0)  $\Rightarrow 3 \times (0) + 4 \times (0) \le 12$  $\Rightarrow 0 \le 12$  (this is true)  $\therefore$  Solution region of the given inequality is below the line 3x + 4y = 12. (That is origin is included in the region) The graph is as follows:



# **Question 4**

 $y + 8 \ge 2x$ 

#### Answer:

Given  $y + 8 \ge 2x$ Now draw a solid line y + 8 = 2x in the graph ( $\because y + 8 = 2x$  is included in the given question) Now Consider  $y + 8 \ge 2x$ Select a point (0, 0)  $\Rightarrow$  (0) + 8  $\ge 2 \times$  (0)  $\Rightarrow$  0  $\le 8$  (this is true)  $\therefore$  Solution region of the given inequality is above the line y + 8 = 2x. (That is origin is included in the region) The graph is as follows:



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#### **Question 5**

#### $x - y \le 2$

#### **Answer:**



# **Question 6**

#### 2x - 3y > 6

#### **Answer:**

Given 2x - 3y > 6Now draw a dotted line 2x - 3y = 6 in the graph ( $\because 2x - 3y = 6$  is excluded in the given question) Now Consider 2x - 3y > 6Select a point (0, 0)  $\Rightarrow 2 \times (0) - 3 \times (0) > 6$  $\Rightarrow 0 > 5$  (this is false)  $\therefore$  Solution region of the given inequality is below the line 2x - 3y > 6. (Away from the origin)

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#### The graph is as follows:



#### **Question 7**

## $-3x + 2y \ge -6$

#### Answer:

Given  $-3x + 2y \ge -6$ Now draw a solid line -3x + 2y = -6 in the graph ( $\because -3x + 2y = -6$  is included in the given question) Now Consider  $-3x + 2y \ge -6$ Select a point (0, 0)  $\Rightarrow -3 \times (0) + 2 \times (0) \ge -6$  $\Rightarrow 0 \ge -6$  (this is true)  $\therefore$  Solution region of the given inequality is above the line  $-3x + 2y \ge -6$ . (That is origin is included in the region) The graph is as follows:





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# **Question 1**

Solve the following system of inequalities graphically:  $x \geq 3, \, y \geq 2$ 

### Answer:

Given  $x \ge 3$ .......(i)  $y \ge 2$ .....(ii) Since  $x \ge 3$  means for any value of y the equation will be unaffected so similarly for  $y \ge 2$ , for any value of x the equation will be unaffected. Now putting x = 0 in the (i)  $0 \ge 3$  which is not true Putting y = 0 in (ii)  $0 \ge 2$  which is not true again This implies the origin doesn't satisfy in the given inequalities. The region to be included will be on the right side of the two equalities drawn on the graphs. The shaded region is the desired region.

# For Enquiry - 6262969604 6262969699 **Ouestion 2** $3x + 2y \le 12, x \ge 1, y \ge 2$ **Answer**: Given $3x + 2y \le 12$ Solving for the value of x and y by putting x = 0 and y = 0 one by one We get y = 6 and x = 4So the points are (0, 6) and (4, 0)Now checking for (0, 0) $0 \le 12$ which is also true. Hence the origin lies in the plane and the required area is toward the left of the equation. Now checking for $x \ge 1$ , The value of x would be unaffected by any value of y The origin would not lie on the plane $\Rightarrow 0 \ge 1$ which is not true The required area to be included would be on the left of the graph $x \ge 1$ Similarly, for $y \ge 2$ Value of y will be unaffected by any value of x in the given equality. Also, the origin doesn't satisfy the given inequality. $\Rightarrow 0 \ge 2$ which is not true, hence origin is not included in the solution of the inequality. The region to be included in the solution would be towards the left of the equality $y \ge 2$ The shaded region in the graph will give the answer to the required inequalities as it is the region which is covered by all the given three inequalities at the same time satisfying all the given conditions.



**Question 3** 

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#### $2x + y \ge 6$ , $3x + 4y \le 12$

#### **Answer:**

Given  $2x + y \ge 6$ .....(i)  $3x + 4y \le 12$  ......(ii)  $2x + y \ge 6$ Putting value of x = 0 and y = 0 in equation one by one, we get value of v = 6 and x = 3So the point for the (0, 6) and (3, 0)Now checking for (0, 0) $0 \ge 6$  which is not true, hence the origin does not lies in the solution of the equality. The required region is on the right side of the graph. Checking for  $3x + 4y \le 12$ Putting value of x = 0 and y = 0 one by one in equation We get v = 3, x = 4The points are (0, 3), (4, 0)Now checking for origin (0, 0)  $0 \le 12$  which is true. So the origin lies in solution of the equation. The region on the right of the equation is the region required. The solution is the region which is common to the graphs of both the inequalities. The shaded region is the required region.



#### **Question 4**

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x + y \ge 4, 2x - y < 0
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#### Answer:

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Given x + y \ge 4
Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 4 and x = 4
The points for the line are (0, 4) and (4, 0)
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Checking for the origin (0, 0) $0 \ge 4$ This is not true. So the origin would not lie in the solution area. The required region would be on the right of line's graph. 2x - y < 0Putting value of x = 0 and y = 0 in equation one by one, we get value of v = 0 and x = 0Putting x = 1 we get y = 2So the points for the given inequality are (0, 0) and (1, 2)Now that the origin lies on the given equation we will check for (4, 0) point to check which side of the line's graph will be included in the solution.  $\Rightarrow$  8 < 0 which is not true, hence the required region would be on the left side of the line 2x-y < 0The shaded region is the required solution of the inequalities.

#### **Ouestion 5**

2x - y > 1, x - 2y < -1Complete KIT of Education

# Answer:

Given 2x - y > 1.....(i) Putting value of x = 0 and y = 0 in equation one by one, we get value of y = -1 and x = 1/2 = 0.5The points are (0,-1) and (0.5, 0) Checking for the origin, putting (0, 0)0 > 1, which is false Hence the origin does not lie in the solution region. The required region would be on the right of the line's graph. x - 2y < -1.....(ii) Putting value of x = 0 and y = 0 in equation one by one, we get value of  $y = \frac{1}{2} = 0.5$  and x = -1

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The required points are (0, 0.5) and (-1, 0)

Now checking for the origin, (0, 0)

0 < -1 which is false

Hence the origin does not lies in the solution area, the required area would be on the left side of the line's graph.

 $\therefore$  the shaded area is the required solution of the given inequalities.



# For Enquiry - 6262969604 6262969699 **Ouestion 7** $2x + y \ge 8, x + 2y \ge 10$ **Answer:** Given $2x + y \ge 8$ Putting value of x = 0 and y = 0 in equation one by one, we get value of v = 8 and x = 4The required points are (0, 8) and (4, 0)Checking if the origin is included in the line's graph (0, 0) $0 \ge 8$ , which is false Hence the origin is not included in the solution area and the requires area would be the area to the right of line's graph. $x + 2y \ge 10$ Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 5 and x = 10The required points are (0, 5) and (10, 0)Checking for the origin (0, 0) $0 \ge 10$ which is false, Hence the origin would not lie in the required solution area. The required area would be to the left of the line graph. The shaded area in the graph is the required solution of the given inequalities.



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#### Question 8

#### $x + y \le 9, y > x, x \ge 0$

#### Answer:

Given  $x + y \le 9$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 9 and x = 9The required points are (0, 9) and (9, 0)Checking if the origin is included in the line's graph (0, 0) $0 \le 9$ Which is true, so the required area would be including the origin and hence will lie on the left side of the line's graph. y > x, Solving for v = xWe get x = 0, y = 0 so the origin lies on the line's graph. The other points would be (0, 0) and (2, 2)Checking for (9, 0) in y > x, We get 0 > 9 which is false, since the area would not include the area below the line's graph and hence would be on the left side of the line. We have  $x \ge 0$ The area of the required line's graph would be on the right side of the line's graph. Therefore the shaded are is the required solution of the given inequalities.



#### **Question 9**

 $5x + 4y \le 20, x \ge 1, y \ge 2$ 

#### Answer:

Given  $5x + 4y \le 20$ ,

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Putting value of x = 0 and y = 0 in equation one by one, we get value of v = 5 and x = 4The required points are (0, 5) and (4, 0)Checking if the origin lies in the solution area (0, 0) $0 \leq 20$ Which is true, hence the origin would lie in the solution area. The required area of the line's graph is on the left side of the graph. We have  $x \ge 1$ . For all the values of y, x would be 1, The required points would be (1, 0), (1, 2) and so on. Checking for origin (0, 0) $0 \ge 1$ , which is not true So the origin would not lie in the required area. The required area on the graph will be on the right side of the line's graph. Consider  $y \ge 2$ Similarly for all the values of x, y would be 2. The required points would be (0, 2), (1, 2) and so on. Checking for origin (0, 0) $0 \ge 2$ , this is no true

Hence the required area would be on the right side of the line's graph. The shaded area on the graph shows the required solution of the given inequalities.



#### **Question 10**

#### $3x + 4y \le 60, x + 3y \le 30, x \ge 0, y \ge 0$

#### **Answer:**

Given  $3x + 4y \le 60$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 15 and x = 20The required points are (0, 15) and (20, 0)

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Checking if the origin lies in the required solution area (0, 0)  $0 \le 60$ , this is true. Hence the origin would lie in the solution area of the line's graph. The required solution area would be on the left of the line's graph. We have  $x + 3y \le 30$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 10 and x = 30The required points are (0, 10) and (30, 0)Checking for the origin (0, 0)  $0 \le 30$ , this is true. Hence the origin lies in the solution area which is given by the left side of the line's graph. Consider  $x \ge 0$ ,  $y \ge 0$ , The given inequalities imply the solution lies in the first quadrant only. Hence the solution of the inequalities is given by the shaded region in the graph.

## **Question 11**

#### $2x + y \ge 4$ , $x + y \le 3$ , $2x - 3y \le 6$

#### Answer:

```
Given 2x + y \ge 4,
Putting value of x = 0 and y = 0 in equation one by one, we get value of
y = 4 and x = 2
The required points are (0, 4) and (2, 0)
Checking for origin (0, 0)
0 \ge 4, this is not true
Hence the origin doesn't lies in the solution area of the line's graph. The solution area
would be given by the right side of the line's graph.
x + y \le 3,
Putting value of x = 0 and y = 0 in equation one by one, we get value of
y = 3 and x = 3
The required points are (0, 3) and (3, 0)
Checking for the origin (0, 0)
```

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 $0 \le 3$ , this is true Hence the solution area would include the origin and hence would be on the left side of the line's graph.  $2x - 3y \le 6$ Putting value of x = 0 and y = 0 in equation one by one, we get value of y = -2 and x = 3The required points are (0, -2), (3, 0)Checking for the origin (0, 0) $0 \le 6$  this is true So the origin lies in the solution area and the area would be on the left of the line's graph. Hence the shaded area in the graph is the required solution area for the given inequalities.



#### Answer:

Given,  $x - 2y \le 3$ Putting value of x = 0 and y = 0 in equation one by one, we get value of y = -3/2 = -1.5 and x = 3The required points are (0, -1.5) and (3, 0)Checking for the origin (0, 0)  $0 \le 3$ , this is true. Hence the solution area would be on the left of the line's graph  $3x + 4y \ge 12$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 3 and x = 4The required points are (0, 3) and (4, 0)Checking for the origin (0, 0)  $0 \ge 12$ , this is not true So the solution area would of include the origin and the required solution area would be on the right side of the line's graph.

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We have  $x \ge 0$ ,

For all the values of y, the value of x would be same in the given inequality, which would be the region above the x axis on the graph.

Consider,  $y \ge 1$ 

For all the values of x, the value of y would be same in the given inequality.

The solution area of the line would be not include origin as  $0 \ge 1$  is not true.

The solution area would be on the left side of the line's graph.

The shaded area in the graph is the required solution area which satisfies all the given inequalities at the same time



# **Question 13**

 $4x + 3y \le 60, y \ge 2x, x \ge 3, x, y \ge 0$ 



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the right of the line's graph. We have x,  $y \ge 0$ Since given both x and y are greater than 0  $\therefore$  the solution area would be in the first Ist quadrant only. The shaded area in the graph shows the solution area for the given inequalities



#### Answer:

Given,  $3x + 2y \le 150$ Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 75 and x = 50The required points are (0, 75) and (50, 0)Checking for the origin (0, 0)  $0 \le 150$ , this is true Hence the solution area for the line would be on the left side of the line's graph which would be including the origin too. We have  $x + 4y \le 80$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 20 and x = 80The required points are (0, 20) and (80, 0)Checking for the origin (0, 0)  $0 \le 80$ , this is also true so the origin lies in the solution area. The required solution area would be toward the left of the line's graph.

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Given  $x \le 15$ ,

For all the values of y, x would be same

Checking for the origin (0, 0)

 $0 \le 15$ , this is true so the origin would be included in the solution area. The required

solution area would be towards the left of the line's graph.

Consider  $y \ge 0, x \ge 0$ 

Since x and y are greater than 0, the solution would lie in the 1st quadrant.

The shaded area in the graph satisfies all the given inequalities and hence is the solution area for given inequalities.



# **<u>Question 15</u>**

```
x + 2y \le 10, x + y \ge 1, x - y \le 0, x \ge 0, y \ge 0
```

#### Answer:

Given,  $x + 2y \le 10$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 5 and x = 10The required points are (0, 5) and (10, 0)Checking for the origin (0, 0) $0 \le 10$ , this is true. Hence the solution area would be toward origin including the same. The solution area would be toward the left of the line's graph. We have  $x + y \ge 1$ , Putting value of x = 0 and y = 0 in equation one by one, we get value of y = 1 and x = 1

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The required points are (0, 1) and (1, 0)

Checking for the origin (0, 0)

 $0 \ge 1$ , this is not true.

Hence the origin would not be included into the solution area. The required solution area would be toward right of the line's graph.

Consider  $x - y \le 0$ ,

Putting value of x = 0 and y = 0 in equation one by one, we get value of

y = 0 and x = 0

Hence the origin would lie on the line.

To check which side of the line graph would be included in the solution area we would check for the (10, 0)

 $10 \le 0$  which is not true hence the solution area would be on the left side of the line's graph. Again we have  $x \ge 0$ ,  $y \ge 0$ 

Since both x and y are greater than 0, the solution area would be in the 1st quadrant. Hence, the solution area for the given inequalities would be the shaded area of the graph satisfying all the given inequalities.







# Miscellaneous Exercise

# Solve the inequalities in Exercises 1 to 6

Question 1

 $2 \le 3x - 4 \le 5$ 

#### Answer:

According to the question, The inequality given is,

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 $2 \le 3x - 4 \le 5$   $\Rightarrow 2 \le 3x - 4 \le 5$   $\Rightarrow 2 + 4 \le 3x - 4 + 4 \le 5 + 4$   $\Rightarrow 6 \le 3x \le 9$   $\Rightarrow 6/3 \le 3x/3 \le 9/3$   $\Rightarrow 2 \le x \le 3$ Hence, all real numbers x greater than or equal to 2 but less than or equal to 3 are solution of given equality.  $x \in [2, 3]$ 

#### <u>Question 2</u> $6 \le -3 (2x - 4) < 12$

#### **Answer:**

According to the question, The inequality given is,  $6 \le -3 (2x - 4) < 12$   $\Rightarrow 6 \le -3 (2x - 4) < 12$ Dividing the inequality by 3 we get.  $\Rightarrow 2 \le -(2x - 4) < 4$ Multiplying the inequality by -1.  $\Rightarrow -2 \ge 2x - 4 > -4$  [multiplying the inequality with -1 changes the inequality sign.]  $\Rightarrow -2 + 4 \ge 2x - 4 + 4 > -4 + 4$   $\Rightarrow 2 \ge 2x > 0$ Dividing the inequality by 2  $\Rightarrow 0 < x \le 1$ Hence, all real numbers x greater than 0 but less than or equal to 1 are solution of given equality.  $x \in (0, 1]$ 

# **Question 3**

#### $-3 \leq 4 - 7x/2 \leq 18$

#### Answer:

According to the question, The inequality given is,  $-3 \le 4 - 7x/2 \le 18$   $\Rightarrow -3 - 4 \le 4 - 7x/2 - 4 \le 18 - 4$   $\Rightarrow -7 \le -7x/2 \le 18 - 14$ Multiplying the inequality by -2.  $\Rightarrow 14 \ge 7x \ge -28$ 

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⇒  $-28 \le 7x \le 14$ Dividing the inequality by 7 ⇒  $-4 \le x \le 2$ Hence, all real numbers x greater than or equal to -4 but less than or equal to 2 are solution of given equality.  $x \in [-4, 2]$ 

#### **Question 4**

#### $-15 \leq 3(x-2)/5 \leq 0$

#### **Answer:**

According to the question, The inequality given is,  $-15 \le 3(x-2)/5 \le 0$   $\Rightarrow -15 < 3(x-2)/5 \le 0$ Multiplying the inequality by 5.  $\Rightarrow -75 < 3(x-2) \le 0$ Dividing the inequality by 3 we get  $\Rightarrow -25 < x - 2 \le 0$   $\Rightarrow -25 + 2 < x - 2 + 2 \le 0 + 2$   $\Rightarrow -23 < x \le 2$ Hence, all real numbers x greater than -23 but less than or equal to 2 are solution of given equality.  $x \in (-23, 2]$ 

# **Question 5**

- 12 < 4 - 3x/ (-2) ≤ 2

#### Answer:

According to the question, The inequality given is,  $-12 < 4 - \frac{3x}{-5} \le 2$   $\Rightarrow -12 < 4 - \frac{3x}{-5} \le 2$   $\Rightarrow -12 < 4 - \frac{3x}{-5} - 4 \le 2 - 4$   $\Rightarrow -16 < \frac{-3x}{-5} - 5 - 2$   $\Rightarrow -16 < \frac{3x}{5} - 5 - 2$ Multiplying the inequality by 5

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 $\Rightarrow -16 \times 5 < \frac{-3x}{5} \times 5 \le -2 \times 5$  $\Rightarrow -80 < 3x \le -10$  $\Rightarrow -\frac{80}{3} < \times \le -\frac{10}{3}$ Hence, all real numbers x greater than -80/3 but less than or equal to -10/3 are solution of given equality.

 $x \in (-80/3, -10/3]$ 

# **Question 6**

 $7 \le (3x + 11)/2 \le 11$ 

#### **Answer:**

According to the question, The inequality given is,  $7 \le \frac{(3x+11)}{2} \le 11$   $\Rightarrow 7 \le \frac{(3x+11)}{2} \le 11$ Multiplying the inequality by 2  $\Rightarrow 7 \times 2 \le \frac{(3x+11)}{2} \times 2 \le 11$   $\Rightarrow 14 \le 3x + 11 \le 22$   $\Rightarrow 14 - 11 \le 3x + 11 - 11 \le 22 - 11$   $\Rightarrow 3 \le 3x \le 11$   $\Rightarrow 1 \le x \le 11/3$ Hence, all real numbers x greater than or equal to -4 but less than or equal to 2 are solution of given equality.  $x \in [1, 11/3]$ Solve the inequalities in Exercises 7 to 11 and represent the solution graphically on number line.

# Question 7

# 5x + 1 > - 24, 5x - 1 < 24

#### **Answer:**

According to the question, The inequalities given are, 5x + 1 > -24 and 5x - 1 < 24 $\Rightarrow 5x > -24 = 1$  $\Rightarrow 5x > -25$  $\Rightarrow x > -5$  ...... (i) 5x - 1 < 24 $\Rightarrow 5x < 24 + 1$ 

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 $\Rightarrow 5x < 25$   $\Rightarrow x < 5 \dots (ii)$ From equations (i) and (ii), We can infer that the solution of given inequalities is (-5, 5).



#### **Question 8**

2(x-1) < x + 5, 3(x+2) > 2 - x

#### Answer:

According to the question, The inequalities given are, 2(x-1) < x + 5 and 3(x+2) > 2 - x2(x-1) < x + 5 $\Rightarrow 2x - 2 < x + 5$  $\Rightarrow$  2x - x < 5 + 2  $\Rightarrow x < 7$  ......(i) 3(x+2) > 2 - x $\Rightarrow$  3x + 6 > 2 - x  $\Rightarrow$  3x + x > 2 - 6  $\Rightarrow 4x > -4$  $\Rightarrow$  x > -1 ..... (ii) From equations (i) and (ii), We can infer that the solution of given inequalities is (-1, 7).  $-\infty$ 8 (-1,7)

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#### **Question 9**

3x - 7 > 2(x - 6), 6 - x > 11 - 2x

#### **Answer:**

According to the question, The inequalities given are, 3x - 7 > 2(x - 6) and 6 - x > 11 - 2x 3x - 7 > 2(x - 6)  $\Rightarrow 3x - 7 > 2x - 12$  $\Rightarrow 3x - 2x > 7 - 12$ 

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#### **Question 11**

A solution is to be kept between  $68^{\circ}$  F and  $77^{\circ}$  F. What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by F = (9/5) C + 32?

#### Answer:

According to the question, The solution has to be kept between 68° F and 77° F

So, we get,  $68^{\circ} < F < 77^{\circ}$ Substituting,  $F = \frac{9}{5}C + 32$   $\Rightarrow 68 = \frac{9}{5}C + 32 < 77$   $\Rightarrow 68 - 32 < \frac{9}{5}C + 32 - 32 < 77 - 32$   $\Rightarrow 36 = \frac{5}{9}C < 45$   $\Rightarrow 36 \times \frac{5}{9} < \frac{9}{5}c \frac{5}{9} < 45 \times \frac{5}{9}$   $\Rightarrow 20 < C < 25$ Hence, we get, The range of temperature in degree Celsius is between 20° C to 25° C.

### **Question 12**

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

#### **Answer:**

```
According to the question,
8% of solution of boric acid = 640 litres
Let the amount of 2\% boric acid solution added = x litres
Then we have.
Total mixture = x + 640 litres
We know that,
The resulting mixture has to be more than 4% but less than 6% boric acid.
\therefore 2% of x + 8% of 640 > 4% of (x + 640) and
2\% \text{ of } x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)
2\% of x + 8% of 640 > 4% of (x + 640)
\Rightarrow (2/100) × x + (8/100) × 640 > (4/100) × (x + 640)
\Rightarrow 2x + 5120 > 4x + 2560
\Rightarrow 5120 - 2560 > 4x - 2x
\Rightarrow 2560 > 2x
\Rightarrow x < 1280 ....(i)
2\% of x + 8% of 640 < 6% of (x + 640)
\Rightarrow (2/100) × x + (8/100) × 640 < (6/100) × (x + 640)
\Rightarrow 2x + 5120 < 6x + 3840
\Rightarrow 6x - 2x > 5120 - 3840
\Rightarrow 4x > 1280
\Rightarrow x > 320 .....(i)
From (i) and (ii)
320 < x < 1280
```

Therefore, the number of litres of 2% of boric acid solution that has to be added will be more than 320 litres but less than 1280 litres.

## **Question 13**

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

#### Answer:

According to the question, 45% of solution of acid = 1125 litres Let the amount of water added = x litres Resulting mixture = x + 1125 litres We know that. The resulting mixture has to be more than 25% but less than 30% acid content. Amount of acid in resulting mixture = 45% of 1125 litres.  $\therefore$  45% of 1125 < 30% of (x + 1125) and 45% of 1125 > 25% of (x + 1125) 45% of 1125 < 30% of (x + 1125) $\Rightarrow 45 \times 1125 < 30x + 30 \times 1125$  $\Rightarrow$  (45 - 30) × 1125 < 30x  $\Rightarrow 15 \times 1125 < 30x$  $\Rightarrow$  x > 562.5 .....(i) 45% of 1125 > 25% of (x + 1125) $\Rightarrow 45 \times 1125 > 25x + 25 \times 1125$  $\Rightarrow$  (45 – 25) × 1125 > 25x  $\Rightarrow 25x < 20 \times 1125$  $\Rightarrow$  x < 900 .....(ii) ∴ 562.5 < x < 900

Therefore, the number of litres of water that has to be added will have to be more than 562.5 litres but less than 900 litres.

#### **Question 14**

IQ of a person is given by the formula  $IQ = \frac{MA}{CA} \times 100$ , Where MA is mental age and CA is chronological age. If  $80 \le IQ \le 140$  for a group of 12 years old children, find the range of their mental age.

#### Answer:

According to the question, Chronological age = CA = 12 years

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IQ for age group of 12 is  $80 \le IQ \le 140$ . We get that,  $80 \le IQ \le 140$ Substituting,  $IQ = \frac{MA}{CA} \times 100$ We get  $\Rightarrow 80 \le \frac{MA}{CA} \times 100 \le 140$   $\Rightarrow 80 \le \frac{MA}{12} \times 100 \le 140 \times \frac{12}{100}$   $\Rightarrow 9.6 \le MA \le 16.8$  $\therefore$  Range of mental age of the group of 12 years old children is  $9.6 \le MA \le 16.8$ 

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