Chapter 3
Trigonometric Functions

Exercise 3.1

Question 1

Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) – 47° 30' (iii) 240° (iv) 520°

Answer:

I. 25^0

Here $180^0 = \pi$ radian

It can be written as

$25^0 = \frac{5\pi}{180} \times 25$ radian

So we get

$= \frac{5\pi}{36}$ radian

II. $-47^0 30'$

Here $1^0 = 60'$

It can be written as

$-47^0 30' = -47\frac{1}{2}$ degree

So we get

$= \frac{-95}{2}$ degree

Here $180^0 = \pi$ radian

$\frac{-95}{2}$ degree $= \frac{\pi}{180} \times \left(\frac{-95}{2}\right)$radian

It can be written as

$= \left(\frac{-19}{36 \times 2}\right) \pi$ radian $= \frac{-19}{72} \pi$ radian

we get

$-47^0 30' = \frac{-19}{72} \pi$ radian

III. 240^0

Here $180^0 =$ radian

it can be written as

$240^0 = \frac{\pi}{180} \times 240$ radian

So we get

$= \frac{4}{3}$ π radian

IV. 520^0

Here 180^0 = π radian

It can be written as

$520^0 = \frac{\pi}{180} \times 520$ radian

So we get

$= \frac{26\pi}{9}$ radain

Question 2

Find the degree measures corresponding to the following radian measures (Use π = 22/7)

I. 11/16
II. -4
III. 5π/3
IV. 7π/6

Answer:

I. 11/ 16

Here π radian = 180^0

$\frac{11}{16}$ radian $= \frac{180}{\pi} \times \frac{11}{16}$ degree

We can write it as

$= \frac{45 \times 22}{\pi \times 4}$ degree

So we get

$= \frac{45 \times 11 \times}{22 \times 4}$ degree

$= \frac{315}{8}$ degree

$= 39 \frac{315}{8}$ degree

Take $1^0 = 60'$

$= 39^0 + + \frac{3 \times 60}{8}$ minutes

We get

$= 39^0 + 22' + \frac{1}{2}$ minutes

Consider 1' = 60''

$= 39^0\ 22'\ 30''$

II. -4

Here π radian = 180^0

-4 radian $= \frac{180}{\pi} \times (-4)$ degree

We can write it as

$= \frac{180 \times 7\ (-4)}{22}$ degree

By further calculation

$= \frac{-2520}{11}$ degree = - 219 $\frac{1}{11}$ degree

Take $1^0 = 60^0$

$= 229^0 + \frac{1 \times 60}{11}$ minutes

So we get that

$= - 299^0 + 5 + \frac{5}{11}$ minutes

Again 1’ = 60’’

= - 229’ 5’ 27’

III. 5π/3

Here π radian = 180^0

$\frac{5\pi}{3}$ radian = $\frac{180}{\pi} \times \frac{5\pi}{3}$ degree

we get

= 300^0

IV. 7π/6

Here π radian = 180^0

$\frac{7\pi}{6}$ radian = $\times \frac{7\pi}{6}$

we get

= 210^0

Question 3

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer:

It is given that

No. of revolutions made by the wheel in

1 minute = 360

1 second = 360/6 = 60

We know that

The wheel turns an angle of 2π radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of 6 × 2π radian = 12 π radian

Therefore, in one second, the wheel turns an angle of 12π radian.

Question 4

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use π = 22/7).

Answer:

Consider a circle of radius r unit with 1 unit as they are length which subtends an angle θ radian at the centre

$\theta = 1/r$

Here r = 100 cm, 1 = 22 cm

$\theta = \frac{22}{100}$ radian $= \frac{180}{\pi} \times \frac{22}{100}$ degree

It can be written as

$= \frac{180 \times 7 \times 22}{22 \times 100}$ degree

$= \frac{126}{10}$ degree

So we get

$= 12\frac{3}{5}$ degree

Here $1^0 = 60'$

$= 12^0\, 36'$

Therefore the required angle is $12^0\, 30'$

Question 5

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

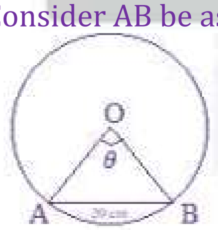
Answer:

The dimensions of the circle are

Diameter = 40 cm

Radius = 40/2 = 20 cm

Consider AB be as the chord of the circle i.e. length = 20 cm



In ΔOAB,

Radius of circle = OA = OB = 20 cm

Similarly AB = 20 cm

Hence, ΔOAB is an equilateral triangle.

$\theta = 60° = \pi/3$ radian

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre

We get $\theta = 1/r$

$\frac{\pi}{3}\frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\,\pi}{3}$ cm

Question 6

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer:

Consider r_1 and r_2 as the radii of the two circles
Let an arc of length 1 subtend an angle of 60^0 at the centre of the circle of radius r_1 and an arc of length I subtend an angle of 75^0 at the centre of radius r_2
Here $60^0 = \pi/3$ radian and $75^0 = 5\pi/\ 12$ radian
In a circle of radius r unit if an arc of length 1 unit subtends an angle θ radian at the centre we get
$\theta = 1/r$ or $1 = r\,\theta$
We know that
$l = \frac{r_1\,\pi}{3}$ and $l = \frac{r_2\,5\pi}{12}$
By equating both we get
$\frac{r_1\,\pi}{3} = \frac{r_2\,5\pi}{12}$
on further calculation
$r_1 = \frac{r_2\,5\pi}{4}$
So we get
$\frac{r_1}{r_2} = \frac{5}{4}$
Therefore the ratio of the radii is 5:4

Question 7

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length
(i) 10 cm (ii) 15 cm (iii) 21 cm

Answer:

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = 1/r$
We know that r = 75 cm
(i) l = 10 cm
So we get
$\theta = 10/75$ radian
By further simplification
$\theta = 2/15$ radian
(ii) l = 15 cm
So we get
$\theta = 15/75$ radian
By further simplification

θ = 1/5 radian
(iii) l = 21 cm
So we get
θ = 21/75 radian
By further simplification
θ = 7/25 radian

Exercise 3.2

Question 1

Find the values of other five trigonometric functions in Exercises 1 to 5. 1. cos x = -1/2, x lies in third quadrant.

Answer:

It is given that
cos x = - 12
sec x – 1/cos x Substituting the values
$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$
Consider
$\sin^2 x + \cos^2 x = 1$
We can write it as
$\sin^2 x = 1 - (-\frac{1}{2})^2$
$\sin^2 x = 1 - 1/4 = 3/4$
$\sin^2 x = \pm \sqrt{3}/2$
Here x lies in the third quadrant so the value of sin x will be negative sin x = $-\sqrt{3}/2$
We can write it as
$\cos ecx = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$
So we get
$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$
Here
$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

Question 2

sin x = 3/5, x lies in second quadrant.

Answer:

It is given that
sin x = 3/5
We can write it as
$\text{cosec } x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$
We know that
$\sin^2 x + \cos^2 x = 1$
We can write it as
$\cos^2 x = 1 - \sin^2 x$
Substituting the value
$\cos^2 x = 1 - (3/5)^2$
$\cos^2 x = 1 - 9/25$
$\cos^2 x = 16/25$
$\cos x = \pm 4/5$
Here x lines in the second quadrant so the value of cos x will be negative
$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$
so we get
$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$
Here
$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$

Question 3

cot x = 3/4, x lies in third quadrant.

Answer:

It is given that
cot x = 3/4
We can write it as
$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$
We know that
$1 + \tan^2 x = \sec^2 x$
We can write it as
$1 + (4/3)^2 = \sec^2 x$
Substituting the values
$1 + 16/9 = \sec^2 x$
$\cos^2 x = 25/9$
$\sec x = \pm 5/3$
Here x lies in the third quadrant so the value of sec x will be negative
sec x = - 5/3

We can write it as

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$

So we get

$\tan x = \frac{\sin x}{\cos x}$

$\frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$

by further calculation

$\sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$

Here

$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$

Question 4

sec x = 13/5, x lies in fourth quadrant.

Answer:

It is given that

sec x = 13/5

We can write it as

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$

We know that

$\sin2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 - \cos^2 x$

Substituting the values

$\sin^2 x = 1 - (5/13)^2$

$\sin^2 x = 1 - 25/169 = 144/169$

$\sin^2 x = \pm 12/13$

Here x lies in the fourth quadrant so the value of sin x will be negative

sin x = - 12/13

We can write it as

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{12}{13}\right)} = \frac{13}{12}$

So we get

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$

Here

$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$

Question 5

tan x = -5/12, x lies in second quadrant.

Answer:

It is given that
tan x = - 5/12
We can write it as
$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$
We know that
$1 + \tan^2 x = \sec^2 x$
We can write it as
1 + (-5/12)2 = sec2 x
Substituting the values
$1 + 25/144 = \sec^2 x$
$\sec^2 x = 169/144$
sec x = ± 13/12
Here x lies in the second quadrant so the value of sec x will be negative
sec x = - 13/12
We can write it as
$\cot x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$
So we get
$\tan x = \frac{\sin x}{\cos x}$
$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$
By further calculation
$\text{six} = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$
Here

$\operatorname{cosec} x = \frac{1}{\sec x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$

Question 6

Find the values of the trigonometric functions in Exercises 6 to 10. sin 765°

Answer:

We know that values of sin x repeat after an interval of 2π or 360°
So we get

$\sin 765^0 = \sin (2 \times 360^0 + 45^0)$
By further calculation
= sin 45o
$= 1/\sqrt{2}$

Question 7

cosec (–1410°)

Answer:

We know that values of cosec x repeat after an interval of 2π or 360°
So we get
$\text{cosec}(-1410^0) = \text{cosec}(-1410^0 + 4 \times 360^0)$
By further calculation
$= \text{cosec}(-1410^0 + 1440^0)$
$= \text{cosec } 30^0 = 2$

Question 8

$\tan \frac{19\pi}{3}$

Answer:

We know that values of tan x repeat after an interval of π or 180°
So we get
$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$
By further calculation
$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3}$
We get
$= \tan 60^0$
$= \sqrt{3}$

Question 9

$\sin\left(-\frac{11\pi}{3}\right)$

Answer:

We know that values of sin x repeat after an interval of 2π or 360°
So we get

$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right)$

By further calculation

$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Question 10

$\cot\left(-\frac{15\pi}{4}\right)$

Answer:

We know that values of tan x repeat after an interval of π or 180^0

So we get

$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$

By the further calculation

$= \cot\frac{\pi}{4} = 1$

Exercise 3.3

Question 1

$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

Answer:

L.H.S. = $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

So we get

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$

By further calculation

= 1/4 + 1 /4 – 1

= - ½

= RHS

Question 2

$2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

Answer:

Consider
$L.H.S = 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$
By further calculation
$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$
It can be written as
$= 2 \times \frac{1}{4} + \left(\operatorname{cosec}\frac{\pi}{6}\right)^2 + \left(\frac{1}{4}\right)$
So we get

$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$

Here

= 1 /2 + 4/ 4

= 1 /2 + 1

= 3 /2

=RHS

Question 3

$\cot^2\frac{\pi}{6}\operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Answer:

L.HS. = $\cot^2\frac{\pi}{6}\operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

So we get

$= \left(\sqrt{3}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$

By further calculation
$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3 \times \frac{1}{3}$
We get
= 3 + 2 + 1
= 6
R.H.S

Question 4

$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Answer:

L.H.S. = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

So we get

= $2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$

By further calculation

= $2\left\{\sin\frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8$

It can be written as

= $2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$

= 1+ 1 +8

-10

=RHS

Question 5

Find the value of:
(i) $\sin 75^0$
(ii) $\tan 15^0$

Answer:

(i) $\sin 75^0$
It can be written as
= $\sin(45^0 + 30^0)$
Using the formula $[\sin(x + y) = \sin x \cos y + \cos x \sin y]$
= $\sin 45^0 \cos 30^0 + \cos 45^0 \sin 30^0$
Substituting the values
= $\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$
By further calculation

= $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$
= $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) tan 15^0
It can be written as
= sin (45^0 - 30^0)
Using the formula

$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

$= \frac{\tan 45^0 - \tan 30^0}{1+\tan 45^0 \tan 30^0}$

Substituting values

$= \frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

By further calculation

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

So we get

$\frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$

$= \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$

Question 6

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Answer:

$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

We can write it as

$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)+\frac{1}{2}\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$

By further simplification

$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$

$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$

Using the formula

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

- $2\sin A\sin B = \cos(A+B)-\cos(A-B)$

- $= 2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$

We get

$= \cos\left[\frac{\pi}{2}-(x+y)\right]$

$= \sin(x+y)$

= RHS

Question 7

$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan X}{1-\tan x}\right)^2$

Answer:

Consider

LHS $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

By using formula

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A+B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$

So we get

$= \frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{\tan -\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$

It can be written as

$= \frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$

$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$

= R.H.S

Question 8

$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Answer:

Consider

L.H.S. = $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$

= $\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$

So we get

= $\frac{-\cos^2 x}{-\sin^2 x}$

= $\cot^2 x$

= RHS

Question 9

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Answer:

LHS = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

It can be written as

= sin x cos x (tan x + cot x)

So we get

= $\sin x \cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

= $(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

=1

RHS

Question 10

$$\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$$

Answer:

LHS = $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)$

By multiplying and dividing by 2

$= \frac{1}{2}$ [2 sin(n + 1)x sin(n + 1)x + 2 cos(n + 1) x cos(n + 2)x]

Using formula

-2 sin A sin B = cos(A + B) − cos(A − B)

2 cos A cos B = cos(A + B) + cos(A − B)

$= \frac{1}{2}\begin{bmatrix} \cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\} \\ +\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\} \end{bmatrix}$

By further calculation

$= \frac{1}{2} \times 2\cos\{(n+1)x-(n+2)x\}$

= cos(-x)

= cos x

= RHS

Question 11

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}+x\right)=-\sqrt{2}\ \sin x$$

Answer:

LHS = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}+x\right)$

Using the formula

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$= -2\sin\left[\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right]\sin\left[\frac{\left(\frac{3\pi}{4}-x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right]$

So we get

$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$

It can be written as

$= -2\sin\left(\pi\frac{\pi}{4}\right)\sin x$

By further calculation

$-2\sin\frac{\pi}{4}\sin x$

Substituting the values

$= -2\times\frac{1}{\sqrt{2}}\times\sin x$

$= -\sqrt{2}\sin x$

RHS

Question 12

$\sin^2 6x - \sin^2 4x = \sin 2 \sin 10x$

Answer:

Consider
LHS = $\sin^2 6x - \sin^2 4x$
Using the formula
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
So we get
= $(\sin 6x + \sin x)(\sin 6x - \sin 4x)$
By further calculation
= $\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$
We get
= $(2\sin 5x \cos x)(2\cos 5x \sin x)$
= $\sin 10x \sin 2x$
=RHS

Question 13

$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer:

LHS = $\cos^2 2x - \cos^2 6x$
Using the formula
$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
So we get
=$(\cos 2x + \cos 6x)(\cos 2x - 6x)$
By further calculation
= $\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$
We get
= $[2\cos 4x \cos(-2x)][-2\sin 4x \sin(-2x)]$
It can be written as
= $[2\cos 4x \cos 2x][-2\sin 4x(-\sin 2x)]$
So we get
= $(2\sin 4x \cos 4x)(2\sin 2x \cos 2x)$
= $\sin 8x \sin 4x$

= RHS

Question 14

$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Answer:

LHS $\sin 2x + 2\sin 4x + \sin 6x$
= $[\sin 2x + \sin 6x] + 2\sin 4x$
Using the formula
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
= $\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$
By further simplification
= 2 sin 4x cos (– 2x) + 2 sin 4x
It can be written as
= 2 sin 4x cos 2x + 2 sin 4x
Taking common terms
= 2 sin 4x (cos 2x + 1)
Using the formula
= 2 sin 4x (2 $\cos^2$ x – 1 + 1)
We get
= 2 sin 4x (2 $\cos^2$ x)
= 4cos2 x sin 4x
= R.H.S.

Question 15

cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Answer:

Consider
LHS = cot 4x (sin 5x + sin 3x)
It can be written as
= $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5X-3X}{2}\right)\right]$
Using formula
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
= $\left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x \cos x]$
So we get

= 2cos 4x cos x
Similarly
R.H.S. = cot x (sin 5x – sin 3x)
It can be written as
$= \frac{\cos 4x}{\sin 4x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5X-3X}{2}\right)\right]$
Using formula
$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
$= \left(\frac{\cos x}{\sin x}\right)[2\cos 4x \sin x]$
So we get
= 2cos 4x cos x
Hence, LHS = RHS

Question 16

$\frac{\cos 9X - \cos 5X}{\sin 17X - \sin 3X} = -\frac{\sin 2X}{\cos 10X}$

Answer:

LHS $\frac{\cos 9X - \cos 5X}{\sin 17X - \sin 3X}$
Using the formula
$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
$\frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$
By further calculation
$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$
So we get
$= -\frac{\sin 2x}{\cos 10x}$
= RHS

Question 17

$\frac{\sin 5x+\sin 3x}{\cos 5x.+\cos 3x} = \tan 4x$

Answer:

Consider
LHS $\frac{\sin 5x+\sin 3x}{\cos 5x.+\cos 3x}$

Using the formula

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$

By further calculation

$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$

So we get

$\frac{\sin 4x}{\cos 4x}$

= $\tan 4x$

= RHS

Question 18

$\frac{\sin x - \sin y}{\cos x + \cos y}\ \tan\frac{x-y}{2}$

Answer:

Consider

LHS $\frac{\sin x - \sin y}{\cos x + \cos y}$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$

By further calculation

$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$

So we get

$= \tan\left(\frac{x-y}{2}\right)$

= RHS

Question 19

$\frac{\sin x - \sin 3x}{\cos x + \cos 3x}\ \tan 2x$

Answer:

Consider

LHS $\frac{\sin x - \sin 3x}{\cos x + \cos 3x}$

Using the formula

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right).\cos\left(\frac{x-3x}{2}\right)}$

By further calculation

$= \frac{\sin 2x}{\cos 2x}$

So we get

= tan 2x

= RHS

Question 20

$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ **2sin x**

Answer:

Consider

LHS $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

Using the formula

$\sin A + \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos^2 A - \sin^2 A = \cos 2A$

$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$

So we get

=- 2(-sin x)

= 2 sin x

= RHS

Question 21

$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$ **= cot 3x**

Answer:

LHS $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

It can be written as

$= \frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$

Using the formula

$\cos A - \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x+2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x+2x}{2}\right)+\sin 3x}$

By further calculation

$= \frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x+\sin 3x}$

So we get

$= \frac{\cos 3x\,(2\cos x+1)}{2\sin 3x\cos x+\sin 3x}$

= cot 3x

= RHS

Question 22

cot x cot 2x – cot 2x cot 3x – cot 3x cot x = 1

Answer:

Consider

LHS cot x cot 2x – cot 2x cot 3x – cot 3x cot x

It can be written as

$= \cot x\cot 2x - \cot 3x\,(\cot 2x + \cot x)$

$= \cot x\cot 2x - \cot(2x + x)(\cot 2x + \cot x)$

Using the formula

$\cot(A + B) = \frac{\cot\cot\ \cot B-1}{\cot A+\cot B}$

$= \cot x\cot 2x - \left[\frac{\cot 2x\ \cot x-1}{\cot x+\cot 2x}\right](\cot 2x + \cot x)$

So we get

$= \cot x\cot 2x - (\cot 2x + \cot x - 1)$

= 1

= RHS

Question 23

$\tan 4x = \frac{4\tan x\,(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Answer:

Consider

LHS = tan 4x = tan 2(2x)

By using the formula

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= \frac{2\tan 2x}{1-\tan^2 (2x)}$

It can be written as

$= \frac{2\left(\frac{2\tan x}{1-\tan 2}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$

$= \frac{\left(\frac{4\tan x}{1-\tan 2}\right)}{\left[1-\frac{4\tan^2}{(1-\tan^2 x)^2}\right]}$

Taking LCM

$\frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$

On further simplification

$= \frac{4\tan x\,(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$

We get

$= \frac{4\,\mathrm{tsn}\,x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$

It can be written as

$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

= RHS

Question 24

cos 4x = 1 – 8sin² x cos² x

Answer:

Consider

LHS = cos 4x

We can write it as

= cos 2(2x)

Using the formula $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\sin^2 2x$

Again by using the formula $\sin^2 A = 2\sin A \cos A$

$= 1 - 2(2\sin x \cos x)^2$

So we get

$= 1 - 8\sin^2 x \cos^2 x$

= R.H.S

Question 25

$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Answer:

Consider
L.H.S. = cos 6x
It can be written as
= cos 3(2x)
Using the formula $\cos 3A = 4 \cos^3 A - 3 \cos A$
$= 4 \cos^3 2x - 3 \cos 2x$
Again by using formula $\cos 2x = 2 \cos^2 x - 1$
$= 4 [(2 \cos^2 x - 1)3 - 3 (2 \cos^2 x - 1)$
By further simplification
$= 4 [(2 \cos^2 x) 3 - (1)3 - 3 (2 \cos^2 x) 2 + 3 (2 \cos^2 x)] - 6\cos^2 x + 3$
We get
$= 4 [^{8\cos 6x} - 1 - 12 \cos 4x + 6 \cos^2 x] - 6 \cos {}^2x + 3$
By multiplication
$= 32 \cos {}^6x - 4 - 48 \cos {}^2x + 24 \cos^2 x - 6 \cos {}^2x + 3$
On further calculation
$= 32 \cos {}^6x - 48 \cos {}^4x + 18 \cos {}^2x - 1$
= R.H.S.

Exercise 3.4

Question 1

Find the principal and general solutions of the following equations:
1. $\tan x = \sqrt{3}$

Answer:

It is given that
tax $x = \sqrt{3}$
We know that
$\tan\frac{\pi}{3} = \sqrt{3}$
it can be written as
$\tan\left(\frac{4\pi}{3}\right) \tan = \left(\pi + \frac{\pi}{3}\right)$
So we get
$=\tan\frac{\pi}{3} = \sqrt{3}$

Hence the principal solution are x = π /3and 4π / 3
tan x tan$\frac{\pi}{3}$
we get
$x = n\pi + \frac{\pi}{3}$ where $n \in z$
Hence the general solution is
$x = n\pi + \frac{\pi}{3}$ where $n \in z$

Question 2

sec x =2

Answer:

It is given that
sec x =2
We know that
$\sec\frac{\pi}{3} = 2$
it can be written as
$\sec\frac{5\pi}{3} = \sec\left(2\pi\frac{\pi}{3}\right)$
So we get
$\sec\frac{\pi}{3} = 2$
Hence the principal solution are x = π /3and 5π / 3
$\sec x = \sec\frac{\pi}{3}$
We know that sec x = 1 /cos x
$\cos x = \cos\frac{\pi}{3}$
So we get
$x = 2n\pi \pm \frac{\pi}{3}$, where $n \in z$
Hence the general solution is
$x = 2n\pi \pm \frac{\pi}{3}$, where $n \in z$

Question 3

cot x = $-\sqrt{3}$

Answer:

It is given that
$\cot x = -\sqrt{3}$
We know that
$\cot\frac{\pi}{6} = \sqrt{3}$

It can be written as

$\cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

And

$\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

So we get

$\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Hence the principal solution are $x = 5\pi/6$ and $11\pi/6$

$\cot x = \cot\frac{5\pi}{6}$

We know that $\cot x = 1/\tan x$

$\tan x = \tan\frac{5\pi}{6}$

So we get

$x = n\pi + \frac{5\pi}{6}$, where $n \in z$

Hence the general solution is

$x = n\pi + \frac{5\pi}{6}$, where $n \in z$

Question 4

cosec x = −2

Answer:

It is given that

$\operatorname{cosec} x = -2$

We know that

$\operatorname{cosec}\frac{\pi}{6} = 2$

It can be written as

$\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

And

$\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

So we get

$\operatorname{cosec}\frac{7\pi}{6} = -2$ and $\operatorname{cosec}\frac{11\pi}{6} = -2$

Hence the principal solution are $x = 7\pi/6$ and $11\pi/6$

$\operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$

We know that $\operatorname{cosec} x = 1/\sin x$

$\sin x = \sin\frac{7\pi}{6}$

So we get

$x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in z$

Hence the general solution is

$x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in z$

Question 5

Find the general solution for each of the following equations:
cos 4x = cos 2x

Answer:

It is given that
cos 4x = cos 2x
It can be written as
cos 4x = cos 2x = 0
Using the formula
$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
We get that
-2 $\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$
By further simplification
sin 3x sin x = 0
We can be written as
sin 3x or sin x = 0
by equation the value
$3x = n\pi/3$ or $x = n\pi$ where $n \in z$
We get that
$x = n\pi/3$ or $x = n\pi$ where $n \in z$

Question 6

cos 3x + cos x – cos 2x = 0

Answer:

cos 3x + cos x – cos 2x = 0

It is given that
cos 3x + cos x – cos 2x = 0

We can be written as
$2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) - \cos 2x = 0$
Using the formula
$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

We get that
2 cos 2x cos x – cos 2x =0
By further simplification
2 cos 2x (2cos x – 1)
We can be written as
cos 2x =0 or 2cos x – 1 = 0
cos 2x = 0 or cos x = 1/2
by equation the value
$2x = (2n +1)\frac{\pi}{2}$ or $\cos x = \cos\frac{\pi}{3}$ Where n ∈ z
We get
$x = (2n +1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, Where n ∈ z

Question 7

sin 2x + cos x = 0

Answer:

It is given that
sin 2x + cos x = 0
We can write it as
2 sin x cos x + cos x = 0
cos x (2 sin x + 1) = 0
cos x = 0 or 2 sin x + 1 = 0
Let cos x = 0
$\cos x = (2n +1)\frac{\pi}{2}$,Where n ∈ z
2 sin x + 1 = 0
So we get
$\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6}$
We can be written as
$= \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$
We get
$x = n\pi + (-1)^n \frac{7\pi}{6}$ where n ∈ z
$(2n + 1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$ n ∈ z

Question 8

$\sec^2 2x = 1 - \tan 2x$

Answer:

It is given that
$\sec^2 2x = 1 - \tan 2x$
We can write it as
$1 + \tan2\ 2x = 1 - \tan 2x$
$\tan^2 2x + \tan 2x = 0$
Taking common terms
$\tan 2x (\tan 2x + 1) = 0$
Here
$\tan 2x = 0$ or $\tan 2x + 1 = 0$
If $\tan 2x = 0$
$\tan 2x = \tan 0$
We get
$2x = n\pi + 0$, where $n \in Z$
$x = n\pi/2$, where $n \in Z$
$\tan 2x + 1 = 0$
We can write it as
$\tan 2x = -1$
So we get
$= -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right)$
$= \tan\frac{3\pi}{4}$
Here
$2x = n\pi + 3\pi/4$, where $n \in Z$
$x = n\pi/2 + 3\pi/8$, where $n \in Z$
Hence, the general solution is $n\frac{\pi}{2} + \frac{3\pi}{8}, n\epsilon Z$

Question 9

$\sin x + \sin 3x + \sin 5x = 0$

Answer:

It is given that
$\sin x + \sin 3x + \sin 5x = 0$
We can write it as
$(\sin x + \sin 5x) + \sin 3x = 0$
Using the formula
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
$\left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0$
By further calculation
$2\sin 3x \cos(-2x) + \sin 3x = 0$
It can be written as

2 sin 3x cos 2x + sin 3x = 0
By taking out the common terms
sin 3x (2 cos 2x + 1) = 0
Here
sin 3x = 0 or 2 cos 2x + 1 = 0
If sin 3x = 0
3x = nπ, where n ∈ Z
We get
x = nπ/3, where n ∈ Z
If 2 cos 2x + 1 = 0
cos 2x = - 1/2
By further simplification
= - cos π/3
= cos (π – π/3)
So we get
cos 2x = cos 2π/3
Here
$2x = 2n\pi \pm \frac{2\pi}{2}$, where n ∈ z
Dividing by 2
$x = 2n\pi \pm \frac{\pi}{3}$, where n ∈ z
Hence the general solution is
$\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3}$, n ∈ z

Miscellaneous Exercise

Question 1

Prove that
$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

Answer:

L.H.S. $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$
Using the formula
$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
So we get
$2\cos\frac{\pi}{13}2\cos\frac{9\pi}{13}+2\cos\left[\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right]\cos\left[\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right]$
By further calculation
$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}+\cos\left(\frac{-\pi}{13}\right)$
We get

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13} + \cos\left(\frac{-\pi}{13}\right)$

Taking out the common terms

$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$

It can be written as

$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$

on further calculation

$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$

We get

$= 2\cos\frac{\pi}{13} \times 2 \times 0 \times \cos\frac{5\pi}{26}$

= 0

= RHS

Question 2

(sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

Answer:

Consider

LHS = (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

By further calculation

= sin 3x sin x + sin2 x + cos 3x cos x – cos2 x

Taking out the common terms

= cos 3x cos x + sin 3x sin x – (cos2 x – sin2 x)

Using the formula

cos (A – B) = cos A cos B + sin A sin B

= cos (3x – x) – cos 2x

So we get

= cos 2x – cos 2x

= 0

= RHS

Question 3

$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$

Answer:

Consider

LHS = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

By expanding using formula we get

= $\cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$
Grouping the terms
= $(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2 (\cos x \cos y - \sin x \sin y)$
Using the formula $\cos (A + B) = (\cos A \cos B - \sin A \sin B)$
= $1 + 1 + 2 \cos (x + y)$
By further calculation
= $2 + 2 \cos (x + y)$
Taking 2 as common
= $2 [1 + \cos (x + y)]$
From the formula $\cos 2A = 2 \cos^2 A - 1$
= $1\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right]$
We get

= $4\cos^2\left(\frac{x+y}{2}\right)$

= RHS

Question 4

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x+y}{2}$$

Answer:

$(\cos x + \cos y)^2 + (\sin x - \sin y)^2$
By expanding using formula
= $\cos^2 x + \cos^2 y - 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$
Grouping the terms
= $(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2 (\cos x \cos y + \sin x \sin y)$
Using the formula $\cos (A + B) = (\cos A \cos B - \sin A \sin B)$
= $1 + 1 - 2 \cos (x - y)$
By further calculation
= $2 [1 - \cos (x - y)]$
Form formula $\cos 2A = 1 - 2 \sin^2 A$
= $2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right]$
We get
= $4\sin^2\left(\frac{x-y}{2}\right)$

Question 5

sin x + sin 3x + sin 5x + sin 7x = 4 cos x cos 2x sin 4x

Answer:

Consider
L.H.S. sin x + sin 3x + sin 5x + sin 7x
Grouping the terms
= (sin x + sin 5x)+(sin 3x + sin 7x)
Using the formula
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
So we get
$= 2\sin\left(\frac{x+5x}{2}\right).\cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$
By further calculation
= 2 sin 3 x cos 2x (-2x) +2 sin 5x cos (-2x)
We get
= 2 sin 3x cos 2x + 2sin 5x cos 2x
Taking out the common terms
= 2 cos 2x [sin 3x + sin 5x]
Using the formula we can write it as
$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right).\cos\left(\frac{3x-5x}{2}\right)\right]$
We get
= 2cos 2x [2 sin 4x. cos(-x)]
= 4 cos 2x sin 4x cos x
= RHS

Question 6

$\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

Answer:

LHS $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{79x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right]}$

$= \frac{[2\sin 6x.\cos x]+[2\sin 6x.\cos 3x]}{[2\cos 6x.\cos x]+[2\cos 6x.\cos 3x]}$
Taking out the common terms
$\frac{2\sin 6x[\cos x+\cos 3x]}{2\cos 6x[[\cos x+\cos 3x]]}$
We get
= tan 6x

= RHS

Question 7

$\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$

Answer:

L. H. S. $\sin 3x + \sin 2x - \sin x$
It can be written as
= sin 3x + (sin 2x – sin x)
Using the formula
$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$
$= \sin 3x + \left[2 \cos\left(\frac{2x+x}{2}\right) \sin\left(\frac{2x-x}{2}\right)\right]$
By further simplification
$= \sin 3x + \left[2 \cos\left(\frac{3x}{2}\right) \sin\left(\frac{x}{2}\right)\right]$
$= \sin 3x + 2 \cos \frac{3x}{2} \sin \frac{x}{2}$
Using formula sin 2A = 2 sin A cos B
$= 2\sin \frac{3x}{2} . \cos \frac{3x}{2} + 2 \cos \frac{3x}{2} \sin \frac{x}{2}$
Taking out the common terms
$= 2\cos\left(\frac{3x}{2}\right) \left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$
From the formula
$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$
$= 2 \cos\left(\frac{3x}{2}\right) \left|2 \sin\left[\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right] \cos\left[\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right]\right|$
By further simplification
$= 2\cos\left(\frac{3x}{2}\right) . 2 \sin x \cos\left(\frac{x}{2}\right)$
We get
$= 4 \sin x \cos\left(\frac{x}{2}\right) \cos\left(\frac{3x}{2}\right)$
= RHS

Question 8

Find sin x/2, cos x/2 and tan x/2 in each of the following:

$\tan x = -\frac{4}{3}$, in quadrant ||

Answer:

It is given that

x is in quadrant ||

$\frac{\pi}{2} < x < \pi$

dividing by 2

$\frac{\pi}{2} < \frac{x}{2} < \frac{x}{2}$

Hence, sin x/2 , and tan x/2 are all positive

$\tan x = -\frac{4}{3}$

From the following $\sec^2 x = 1 + \tan^2 x$

Substituting the value

$\sec^2 x = 1 + (-4/3)^2$

We get

= 1 + 16/9 = 25/9

Here

$\cos^2 x = \frac{9}{25}$

$\cos x = \pm \frac{3}{5}$

Here x is in quadrant ||, cos x is negative

cos x = -3/5

From the formula

$\cos x = 2\cos^2\frac{x}{2} - 1$

Substituting the values

$\frac{-3}{2} = 2\cos^2\frac{x}{2} - 1$

By further calculation

$2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$

$2\cos^2\frac{x}{2} = \frac{2}{5}$

$\cos^2\frac{x}{2} = \frac{1}{5}$

We get

$\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

$\cos\frac{x}{2} = \frac{\sqrt{5}}{5}$

From the following

$\sin^2\frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$

We get

$\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

$\sin\frac{x}{2} = \frac{2\sqrt{5}}{5}$

Here

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$

Hence, the respective values of sin x/2, cos x/2 and tan x/2 are

$\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{2}$, and 2

Question 9

cos x = - 1/3, x in quadrant III

Answer:

It is given that
x is in quadrant |||
$<\pi < x < \frac{3\pi}{2}$
dividing by 2
$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$
Hence, cos x/2 and tan x/2 are negative where sin x/2 is positive
$\cos x = -\frac{1}{3}$
From the formula $x = 1 - 2\sin^2 x/2$
We get
$\sin^2 x/2 = (1 - \cos x)/2$
Substituting the value
$\sin^2 \frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2}$
We get
$= \frac{\frac{4}{3}}{2} = \frac{2}{3}$
Here
$\sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}}$
$\sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
Using the formula
$\cos x = 2\cos^2 \frac{x}{2} - 1$
Substituting value
$= \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2}$
$= \frac{\left(\left(\frac{2}{3}\right)\right)}{2} = \frac{1}{3}$
We get
$\cos\frac{x}{2} = \frac{1}{\sqrt{3}}$
By further calculation
$\cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)}$

Therefore the, respective values of sin x/2, cos x/2 and tan x/2 are

$\frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and - $\sqrt{2}$

Question 10

sin x = -1/4, x in quadrant III

Answer:

It is given that

x is in quadrant III

$\frac{\pi}{2} < < x\ \pi$

dividing by 2

$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Hence, sin x/2, cos x/2 and tan x/2 is positive

$\sin x = -\frac{1}{4}$

From the formula $\cos^2 x = 1 - \sin^2 x$

We get

$\cos^2 x = 1 - (1/4)^2$

Substituting the value

$\cos^2 x = 1 - 1/16 = 15/16$

We get

$\cos x = -\frac{\sqrt{15}}{4}$

Here

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

Substituting the value

$= \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$

$\sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}}$

Multiplying and dividing by 2

$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$

By further calculation

$= \sqrt{\frac{8+2\sqrt{15}}{16}}$

$= \sqrt{\frac{8+2\sqrt{15}}{4}}$

Here

$os^2\frac{x}{2} = \frac{1+\cos x}{2}$

Substituting the value

$= \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$

We get

$\cos^2\frac{x}{2}\sqrt{\frac{4-\sqrt{15}}{8}}$

Multiplying and dividing by 2

$=\sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$

It can be written as

$= \sqrt{\frac{8-2\sqrt{15}}{16}}$

$=\sqrt{\frac{8-2\sqrt{15}}{4}}$

We know that

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

Substituting the value

$= \frac{\left(\sqrt{\frac{8+2\sqrt{15}}{4}}\right)}{\left(\sqrt{\frac{8-2\sqrt{15}}{4}}\right)} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$

By multiplying and dividing the terms

$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$

We get

$= \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2}$

$= 4 + \sqrt{15}$

Therefore the respective values of sin x/2 , cos x/2 and tan x/2 are

$\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8+2\sqrt{15}}}{4}$ and $4 + \sqrt{15}$