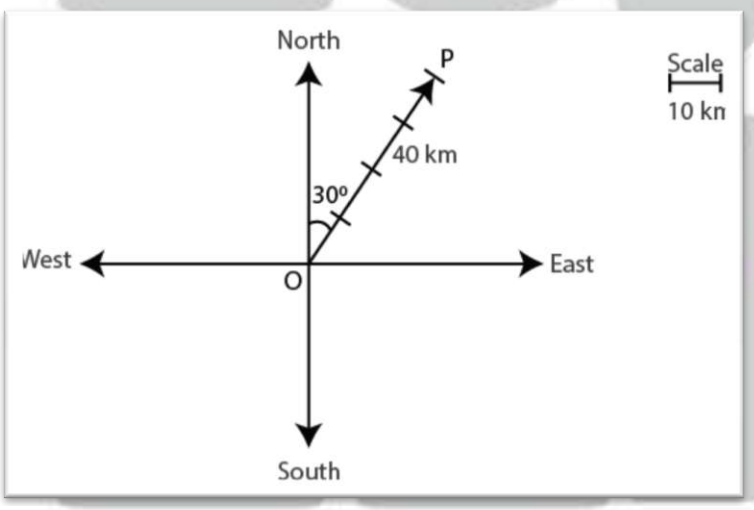
Chapter 10
Vector Algebra
Exercise 10.1

Question 1

Represent graphically a displacement of 40 km, 30° east of north.

Solution:



The vector $\overline{OP}$ represents the displacement of 40 km, 30° east of north.

Question 2

Classify the following measures as scalars and vectors.
(i) 10 kg (ii) 2 meters north-west (iii) 40°
(iv) 40 watt (v) 10–19 coulomb (vi) 20 m/s^2

Solution:

(i) 10 kg is a scalar quantity because it has only magnitude.
(ii) 2 meters north-west is a vector quantity as it has both magnitude and direction.
(iii) 40° is a scalar quantity as it has only magnitude.
(iv) 40 watts is a scalar quantity as it has only magnitude.
(v) 10–19 coulomb is a scalar quantity as it has only magnitude.
(vi) 20 m/s2 is a vector quantity as it has both magnitude and direction.

Question 3

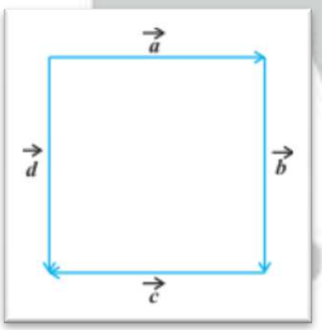
Classify the following as scalar and vector quantities.
(i) Time period (ii) distance (iii) force
(iv)Velocity (v) work done

Solution:

(i) Time period is a scalar quantity as it has only magnitude.
(ii) Distance is a scalar quantity as it has only magnitude.
(iii) Force is a vector quantity as it has both magnitude and direction.
(iv) Velocity is a vector quantity as it has both magnitude as well as direction.
(v) Work done is a scalar quantity as it has only magnitude.

Question 4

In Figure, identify the following vectors.



(i) Coinitial (ii) Equal (iii) Collinear but not equal

Solution:

I. Vectors $\bar{a}$ and $\bar{d}$ are cointial because they have the same initial point
II. Vectors $\bar{d}$ and $\bar{d}$ are equal because they have the same magnitude and direction.
III. Vectors $\bar{a}$ and $\bar{c}$ are collinear but not equal. This is because although they are parallel their directions are not the same.

Question 5

Answer the following as true or false.
(i) $\bar{a}$ and - $\bar{a}$ are collinear.
(ii) Two collinear vectors are always equal in magnitude.
(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Solution:

(i) True.
Vectors $\bar{a}$ and - $\bar{a}$ are parallel to the same line.
(ii) False.
Collinear vectors are those vectors that are parallel to the same line.
(iii) False.
Two vectors having the same magnitude need not necessarily be parallel to the same line.
(iv) False.
Only if the magnitude and direction of two vectors are the same, regardless of the positions of their initial points the two vectors are said to be equal.

Exercise 10.2

Question 1

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $\bar{b} = 2\,\hat{\imath} - 7\,\hat{\jmath} - 3\,\hat{k}$; $\bar{c} = \frac{1}{\sqrt{3}}\,\hat{\imath} + \frac{1}{\sqrt{3}}\,\hat{\jmath} - \frac{1}{\sqrt{3}}\,\hat{k}$

Solution:

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $\bar{b} = 2\,\hat{\imath} - 7\,\hat{\jmath} - 3\,\hat{k}$; $\bar{c} = \frac{1}{\sqrt{3}}\,\hat{\imath} + \frac{1}{\sqrt{3}}\,\hat{\jmath} - \frac{1}{\sqrt{3}}\,\hat{k}$

$|\bar{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

$|\bar{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$

$= \sqrt{4 + 49 + 9}$

$= \sqrt{62}$

$|\bar{c}|\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

Question 2

Write two different vectors having same magnitude.

Solution:

Consider $\bar{a} = \left(\hat{\imath} - 2\,\hat{\jmath} + 4\,\hat{k}\right)$ and $\bar{b} = \left(2\hat{\imath} + \hat{\jmath} - 4\,\hat{k}\right)$.
It can be observed that $|\bar{a}| = \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{1 + 4 + 6} = \sqrt{21}$ and
$|\bar{b}| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$

Thus, $\bar{a}$ and $\bar{b}$ are two different vectors having the same magnitude. Here the vectors are different as they have different directors.

Question 3

Write two different vectors having same direction

Solution:

Two different vectors having same directors are:
Let us
Consider of $\bar{P} = \left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ and $\bar{q}\left(2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$.
The directors cosines of $\bar{P}$ are given by,
$i = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}\ m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$ *and* $n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$
There direction section of $\bar{q}$ are given by
$i = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} m = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$
and $n = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$.

Question 4

Find the values of x and y so that vectors $2\hat{\imath} + 3\,\hat{\jmath}$ and x $\hat{\imath} + y\,\hat{\jmath}$ are equal

Solution:

Given vectors $2\hat{\imath} + 3\,\hat{\jmath}$ and x $\hat{\imath} + y\,\hat{\jmath}$ will be equal only if their corresponding components are equal. Thus, the required values of x and y are 2 and 3 respectively.

Question 5

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (–5, 7).

Solution:

The scalar and vector components are:
The vector with initial point P (2, 1) and terminal point Q (−5, 7) can be shown as,
$\overline{PQ} = (-5-2)\hat{\imath} + (7-1)\hat{\jmath}$
$\overline{PQ} = -7\hat{\imath} + 6\hat{\jmath}$
Thus, the required scalar components are –7 and 6 while the vector components are $-7\hat{\imath}$ *and* $6\hat{\jmath}$

Question 6

Find the some of the vectors $\bar{a} = \hat{\imath} - 2\,\hat{\jmath} + \hat{k}$, $\bar{b}$ =-2 $\hat{\imath} + 4\,\hat{\jmath} +\ 5\,\hat{k}$ and $\bar{c} =\ \hat{\imath} - 6\,\hat{\jmath} - 7\,\hat{k}$

Solution:

Let us find the sum of the vectors:
The given vectors are $\bar{a} = \hat{\imath} - 2\,\hat{\jmath} + \hat{k}$, $\bar{b}$ =-2 $\hat{\imath} + 4\,\hat{\jmath} +\ 5\,\hat{k}$ *and* $\bar{c} =\ \hat{\imath} - 6\,\hat{\jmath} - 7\,\hat{k}$
Hence
$\bar{a} + \bar{b} +\ \bar{c} = (\ 1 - 2 + 1)\hat{\imath} + (-2 + 4 - 6)\hat{\jmath} + (1 + 5 - 7)\hat{k}$
$= 0.\hat{\imath} -\ 4\hat{\jmath} -\ 1.\hat{k}$
$=\ -4\hat{\jmath} - \hat{k}$

Question 7

Find the unit vector in the directors of the vector $\bar{a} = \hat{\imath} +\ \hat{\jmath} + 2\hat{k}$

Solution:

We Know that
The unit vector $\hat{a}$ in the directors of vector $\bar{a} = \hat{\imath} +\ \hat{\jmath} + 2\hat{k}$ is given by $\hat{a} = \frac{\bar{a}}{|a|}$
So,
$|\bar{a}| =\ \sqrt{1^2 +\ 1^2 +\ 1^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$
Thus
$\hat{a} = \frac{\bar{a}}{|a|} = \frac{\hat{\imath} + \hat{\jmath} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\ \hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath}\ \frac{2}{\sqrt{6}}\ \hat{k}$

Question 8

Find the unit vector in the directors of vector$\overline{PQ}$, Where P and Q are the points (1, 2, 3) and (4,5,6), respectively

Solution:

Give points are P (1, 2, 3) and Q (4, 5, 6)
So $\overline{PQ} = (4 - 1)\ \hat{\imath} +\ (5 - 2)\hat{\jmath}\ + (6 - 3)\hat{k} = 3\hat{\imath}\ 3\hat{\jmath} +\ 3\hat{k}$
$|\overline{PQ}| =\ \sqrt{3^2 +\ 3^2 +\ 3^2} =\ \sqrt{9 + 9 + 9} =\ \sqrt{27} = 3\sqrt{3}$
Thus the unit vector in the direction of $\overline{PQ}$ is
$\frac{\overline{PQ}}{|\overline{PQ}|} = \frac{3\hat{\imath} + 3 + \hat{\jmath} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

Question 9

For given vectors $\bar{a}$ =2 $\hat{\imath} -\ \hat{\jmath} + 2\hat{k}$ and $\bar{b}$ =- $\hat{\imath} +\ \hat{\jmath} - \hat{k}$, find the unit vector in the direction of the vector $\bar{a} +\ \bar{b}$

Solution:

We know that
Give vector are $\bar{a}$ =2 $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\bar{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$
$\bar{a}$ =2 $\hat{\imath} - \hat{\jmath} + 2\hat{k}$
$\bar{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$
$\therefore \bar{a} + \bar{b}\ (2-1)\hat{\imath} + (-1+1)\hat{\jmath} + (2-1)\hat{k} = 1\hat{\imath} + 0\hat{\jmath} + 1\,\hat{k} = \hat{\imath} + \hat{k}$
$|\bar{a} + \bar{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$
Thus the unit vector in the direction of $(\bar{a} + \bar{b})$is
$\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = \frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}$

Question 10

Find a vector in the director of vector 5 $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8 units

Solution:

$\bar{a} = 5\,\hat{\imath} - \hat{\jmath} + 2\hat{k}$
So,
$|\bar{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}$
$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{5\,\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}$
Thus, the vector in the directors of vector 5 $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8 units is given by,
$8\hat{a} = 8\left(\frac{5\,\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$
$= 8\left(\frac{5\,\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}\right)$
$= \frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$

Question 11

Show that the vectors 2 $\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k}$ and - 4 $\hat{\imath} + 6\,\hat{\jmath} - 8\,\hat{k}$ are collinear.

Solution:

Let $\bar{a} = 2\,\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k}$ and $\bar{b}\ -\ 4\,\hat{\imath} + 6\,\hat{\jmath} - 8\,\hat{k}$
It is seen that $\bar{b} = -4\,\hat{\imath} + 6\,\hat{\jmath} - 8\,\hat{k} = -2(2\,\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k}) = -2\bar{a}$
$\therefore \bar{b} = \lambda\bar{a}$
$\lambda = -2$
Therefore we can say that the given vectors are collinear.

Question 12

Find the direction cosines of the vector $\hat{\imath} - 2\,\hat{\jmath} + 3\hat{k}$

Solution:

Let $\bar{a} = \hat{\imath} - 2\,\hat{\jmath} + 3\hat{k}$.
The modules is given by,
$|\bar{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$
Thus the direction cosines of $\bar{a}$ are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Question 13

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (- 1, -2, 1) directed from A to B

Solution:

Given points are A (1, 2, -3) and B (-1, -2, 1)
Now,
$\overline{\mathrm{AB}} = (-1-1)\hat{\imath} + (-2-2)\hat{\jmath}\,\{1-(-3)\}\hat{k}$
$|\overline{\mathrm{AB}}| = 2\hat{\imath} - 4\,\hat{\jmath} + 4\hat{k}$.
$|\overline{\mathrm{AB}}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$
Therefore the direction cosines of $|\overline{\mathrm{AB}}|$ *are* $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Question 14

Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$. is equally inclined to the axes OX and OY and OZ

Solution:

Firstly,
Let $\bar{a}\;\hat{\imath} + \hat{\jmath} + \hat{k}$.
Then,
$|\bar{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$
Hence the direction cosines of $\bar{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.
Now Let $\alpha.\beta.$ any $y$be the angels formed by $\bar{a}$ with the positive directors of x, y and z axes.
So, we have $\cos\alpha = \frac{1}{\sqrt{3}}\ cos\ \beta\,\frac{1}{\sqrt{3}} \cos y = \frac{1}{\sqrt{3}}$.
Therefore the given vector is equally inclined to axes OX, OY and OZ.

Question 15

Find the position vector of a point R which divides the line joining two points P and Q whose position vector are $\hat{\imath}+2\hat{\jmath}-\hat{k}$. and $-\ \hat{\imath}+\hat{\jmath}+\hat{k}$. respectively in the ratio 2:1

I. **internally**
II. **externally**

Solution:

We know that
The position vector of point R dividing the line joining two points
P and Q in the ratio m: n is given by:

I. internally: $\frac{m\bar{b}+n\bar{a}}{m+n}$
II. externally: $\frac{m\bar{b}-n\bar{a}}{m-n}$

$\overline{OP}=\hat{\imath}+2\hat{\jmath}-\ \hat{k}$ and $\overline{OQ}\ -\hat{\jmath}+\hat{\jmath}+\ \hat{k}$

I. The position vector of point R which divides the joining two point P and Q internally in the ratio

$\overline{OR}=\frac{2\,(-\hat{\imath}+\hat{\jmath}+\hat{K})+1\,(-2\hat{\imath}+2\hat{\jmath}+2\,\hat{K})}{2+1}=\frac{(-\,2\hat{\imath}+\hat{\jmath}+\hat{K})+1\,(\hat{\imath}+2\,\hat{\jmath}-\hat{K})}{3}$

$=\frac{-\hat{I}+4\,\hat{\jmath}+\hat{K}}{3}=-\frac{1}{3}\hat{\boldsymbol{\imath}}+\ \frac{4}{3}+\hat{\jmath}\ \frac{1}{3}\ \hat{K}$

II. The position vector of point R which divides the joining two point P and Q internally in the ratio

$\overline{OR}=\frac{2\,(-\hat{\imath}+\hat{\jmath}+\hat{k})-1\,(\hat{\imath}+2\,\hat{\jmath}-\hat{k})}{2-1}=\left(-\,2\hat{\imath}+\ \hat{\jmath}+\ \hat{k}\right)-\ \left(\hat{\imath}\ +2\,\hat{\jmath}-\ \hat{k}\right)$

$=-\,3\hat{\imath}+\ 3\,\hat{K}$

Question 16

Find the position vector of the midpoint of the joining the points P (2, 3,4) and Q (4, 1,-,2).

Solution:

The position vector of midpoint R of the vector joining points P (2, 3, 4) and Q (4, 1, -2) is given by

$\overline{OR}=\ \frac{(2\hat{\imath}+3\hat{\jmath}+4\,)+(\,4\hat{\imath}+\hat{\jmath}-2\,)}{2}=\ \frac{(2+4)\hat{\imath}+(3+1)\hat{\jmath}+(4-2)\hat{K}}{2}$

$=\frac{6\,\hat{\imath}+4\,\hat{\jmath}+\,2\hat{K}}{2}=3\hat{\imath}+2\,\hat{\jmath}+\hat{k}$

Question 17

Shoe that the points A, B and C with position vectors, $\bar{a}=3\hat{\imath}-4\,\hat{\jmath}-\ 4\hat{K},\bar{b}=2\hat{\imath}-\ \hat{\jmath}+\hat{K},\bar{c}=\hat{\imath}-3\,\hat{\jmath}-\ 5\hat{K},$

Solution:

We know
Give position vectors of points A,B and C are:

$\bar{a} = 3\hat{i} - 4\hat{j} - 4\hat{K}, \bar{b} = 2\hat{i} - \hat{j} + \hat{K}, \bar{c} = \hat{i} - 3\hat{j} - 5\hat{K},$

$\therefore\ \overline{ab} = \bar{b} - \bar{a} = (2 - 3)\,\hat{i} + (-1 + 4)\,\hat{j} + (1 + 4)\,\hat{K} = -\hat{i} + 3\hat{j} + 5\,\hat{K}$

$\overline{ba} = \bar{c} - \bar{b} = (1 - 2)\,\hat{i} + (-3 + 1)\,\hat{j} + (-5 + -1)\,\hat{K} = -\hat{i} + 2\hat{j} + 6\,\hat{K}$

$\overline{ca} = \bar{a} - \bar{c} = (3 - 1)\,\hat{i} + (-4 + 3)\,\hat{j} + (-4 + -5)\,\hat{K} = 2\,\hat{i} - \hat{j} + \hat{K}$

Now,

$|\overline{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

$|\overline{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

$|\overline{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

Hence

$|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 35 + 6 = 41 = |\overrightarrow{BC}|^2$

Hence, proved that the given points from the vertices of a right angled triangle

Question 18

In triangle ABC (Fig 10.18) which of the following is not true:

(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(B) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(C) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(D) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

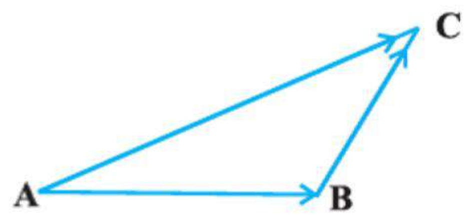


Fig 10.18

Applying the triangle law of addition in the triangle we get.

$\overline{AB} + \overline{BC} = \overline{AC}$ ……….(1)

$\overline{AB} + \overline{BC} = -\overline{CA}$

$\overline{AB} + \overline{BC} = +\overline{CA} = \bar{0}$ ………… (2)

$\therefore$The equation given the alternative A is true

$\overline{AB} + \overline{BC} = \overline{AC}$

$\Rightarrow \overline{AB} + \overline{BC} - \overline{AC} = \bar{0}$

$\therefore$The equation given the alternative B is true.

From equation (2) we have:

$\overline{AB} - \overline{CB} + \overline{CA} = \bar{0}$

$\therefore$The equation given the alternative D is true.

Now consider the equation given alternative C:

$\overline{AB} + \overline{CB} - \overline{CA} = \bar{0}$

$\Rightarrow \overline{AB} + \overline{CB} - \overline{CA} = \bar{0}$ ………….(3)

From equation (1) and (3) we get

$\overline{AC} = \overline{CA}$

$\overline{AC} = -\overline{AC}$
$\overline{AC} + AC = \overline{0}$
$2\,\overline{AC} = \overline{0}$
$\overline{AC} = \overline{0}$ Which is not true?
Thus, the equation given in alternative C is incorrect
The correct answer is C.

Question 19

If $\bar{a}$ and $\bar{b}$ are two collinear vectors then which of the following are incorrect:
A. $\bar{b} = \lambda\bar{a}$, for some scalar λ
B. $\bar{a} = \pm\,\bar{b}$
C. the respective components of $\bar{a}$ and $\bar{b}$ are proportional
D. the respective the vectors $\bar{a}$ and $\bar{b}$ have same direction, but different magnitudes

Solution:

We know
If $\bar{a}$ and $\bar{b}$ are to collinear vectors, then they are parallel
So we have
$\bar{b} = \lambda\bar{a}$ (For same scalar λ)
If $\lambda = \pm 1$, *then* $\bar{a} = \pm\,\bar{b}$
If$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{K}$ *and* $= \bar{b}\ b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{K}$ then
$\bar{b} = \lambda\bar{a}$
$b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = \lambda\left(a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\right)$
$b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath}\ (\lambda a_2)k$
$b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$
$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_2}{a_3} = \lambda$
Hence the respective components of $\bar{a}$ and $\bar{b}$ are proportional.
But vectors $\bar{a}$ and $\bar{b}$ can have different directions.
Thus the statement given in D is incorrect
The correct answer is D.

Exercise10. 3

Question 1

If the angle between two vectors $\bar{a}$ and $\bar{b}$ with magnitude $\sqrt{3}$ and 2, respectively having $\bar{a}, \bar{b} = \sqrt{6}$

Solution:

$|\bar{a}| = \sqrt{3}, \ |\bar{b}| = 2 \ and \ \bar{a}.\bar{b} = \sqrt{6}$

Now we know that $\bar{a}.\bar{b} = |\bar{a}|\ |\bar{b}|\ \cos\theta$

$\therefore \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\cos\theta = \frac{\sqrt{6}}{\sqrt{3}\times 2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{x}{4}$

Thus, the angle between the given vectors ā and b̄ $is\ \frac{x}{4}$

Question 2

Find the angle between the vectors $\hat{\imath} - 2\hat{\jmath} + \hat{k}.$ and $3\ \hat{\imath} - 2\hat{\jmath} + \hat{k}.$

Solution:

Give vectors are: ā $\hat{\imath} - 2\hat{\jmath} + \hat{k}.$ and b̄ $3\ \hat{\imath} - 2\hat{\jmath} + \hat{k}.$

$|\bar{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|\bar{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Now, ā . b̄ = $(\hat{\imath} - 2\hat{\jmath} + \hat{k}.)(3\hat{\imath} - 2\hat{\jmath} + \hat{k}.)$

= 1.3 + (-2) (-2) + 3.1

= 3 + 4 + 3

= 10

Also we know that ā . b̄ = $|\bar{a}||\bar{b}|\cos\theta$

$\therefore 10 = \sqrt{14}\,\sqrt{14}\cos\theta$

$\cos\theta = \frac{10}{14}$

$\theta = cos^{-1}\left(\frac{5}{7}\right)$

Hence the angle between the vectors is cos^{-1} (5/7)

Question 3

Find the projection of the vector $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$

Solution:

Firstly

Let ā = $\hat{\imath}$ - $\hat{\jmath}$ and b̄ = $\hat{\imath} + \hat{\jmath}$.

Now projection of vector ā on b̄ is given by,

$\left|\frac{1}{\bar{b}}\right| (\bar{a}, \bar{b}) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1 - 1) = 0$

Thus the projection of vector $\bar{a}$ on b is 0.

Question 4

Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$. on the vector7 $\hat{\imath} - \hat{\jmath} + 8\hat{k}$.

Solution:

Firstly

Let $\bar{a}$ $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$. and $\bar{b}$ $7\,\hat{\imath} - \hat{\jmath} + 8\hat{k}$.
Now projection of vector $\bar{a}$ on $\bar{b}$ is given by
$\frac{1}{|\bar{b}|}(\bar{a}, \bar{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}}\{1\,(7) + 3\,(-1) + 7\,(8)\} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$
Hence the projection is $60/\sqrt{114}$

Question 5

Show that each of the given three vectors is a unit vector:
$\frac{1}{7}(2\,\hat{\imath} + 3\hat{\jmath} + 6\hat{k}.), \frac{1}{7}(3\,\hat{\imath} - 6\hat{\jmath} + 2\hat{k}.), \frac{1}{7}(6\,\hat{\imath} + 2\hat{\jmath} - 3\hat{k}.)$

Solution:

Let $\bar{a} = \frac{1}{7}(2\,\hat{\imath} + 3\hat{\jmath} + 6\hat{k}.) = \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$
$\bar{b} = \frac{1}{7}(3\,\hat{\imath} - 6\hat{\jmath} + 2\hat{k}.) = \frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$
$\bar{c} = \frac{1}{7}(6\,\hat{\imath} - 2\hat{\jmath} - 3\hat{k}.) = \frac{6}{7}\hat{\imath} + \frac{2}{7}\hat{\jmath} - \frac{3}{7}\hat{k}$
$|\bar{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$
$|b| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$
$|\bar{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$
Hence each of the given three vectors is a unit vector.
$\bar{a}\,.\,\bar{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(\frac{-6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$
$\bar{b}\,.\,\bar{c} = \frac{3}{7} \times \frac{3}{7} + \left(\frac{-6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(\frac{-3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$
$\bar{c}.\,a = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(\frac{-3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$
Therefore the given three vectors are mutually perpendicular to each other

Question 6

Find $|\bar{a}|$ *and* $|\bar{b}|$, if $(\bar{a} + \bar{b}), (\bar{a} - \bar{b}) = 8\,|\bar{a}| = 8\,|\bar{b}|$

Solution:

Let us consider

$(\bar{a}.\bar{b}),(\bar{a}-\bar{b}) = 8$

$\bar{a}.\bar{a}$ - $\bar{a}.\bar{b} + \bar{b}.\bar{a} - \bar{b}.\bar{b} = 8$

$|a|^2 - |\bar{b}|^2 = 8$

$(8\,|\bar{b}|)^1 - |\bar{b}|^2 = 8$ $[|\bar{a}| = 8|\bar{b}|]$

$64|\bar{b}|^2 - |\bar{b}|^2 = 8$

$64|\bar{b}|^2 = 8$

$|\bar{b}|^2 = \frac{8}{63}$

$|\bar{b}| = \sqrt{\frac{8}{63}}$ [Magnitude of a vector is non- negative]

$|\bar{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$

And

$|\bar{a}| = 8|\bar{b}| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

Question 7

Evaluate the product $(3\,\bar{a} - 5\,\bar{b}).(2\,\bar{a} - 7\,\bar{b})$

Solution:

Let us consider the given expression

$(3\,\bar{a} - 5\,\bar{b}).(2\,\bar{a} - 7\,\bar{b})$

$= 3\,\bar{a}.2\,\bar{a} + 3\,\bar{a}.7\,\bar{b} - 5\,\bar{b}.2\,\bar{a} - 5\,\bar{b}.7\,\bar{b}$

$= 6\,\bar{a}.\,\bar{a} + 21\bar{a}.\bar{b} - 10\,\bar{a}.\bar{b} - 35\,\bar{b}.\bar{b}$

$= 6\,|\bar{a}|^2 + 11\bar{a}..\bar{b} - 35\,|\bar{b}|^2$

Question 8

Find the magnitude of two vectors $\bar{a}$ and $\bar{b}$ having the same magnitude and such that the angle between then is 60^0 and their scalar product is ½.

Solution:

Firstly,

Let θ be the angle between the vectors $\bar{a}$ and $\bar{b}$

it is given that $|\bar{a}| = |\bar{b}|, \bar{a}.\bar{b} = \frac{1}{2}$ $and\ \theta = 60^0$ ………….. (1)

We know that $\bar{a}.\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

$\therefore \frac{1}{2} = |\bar{a}||\bar{b}|\cos\theta\ 60^0$ [Using (1)]

$\frac{1}{2}|\bar{a}|^2 \times \frac{1}{2}$

$|\bar{a}|^2 = 1$

$|\bar{a}| = |\bar{b}| = 1$

Hence the magnitude of two vectors is 1

Question 9

Find $|\bar{x}|$, if for a unit vector $\bar{a}$ $(\bar{x} - \bar{a}).(\bar{x} + \bar{a}) = 12$

Solution:

$(\bar{x} - \bar{a})(\bar{x} + \bar{a}) = 12$

$\bar{x}.\bar{x} + \bar{x}.\bar{a} - \bar{a}.\bar{x}. - \bar{a}.\bar{a} = 12$

$|\bar{x}|^2 - |\bar{a}|^2 = 12$

$|\bar{x}|^2 - 1 = 12$ $[|\bar{a}|] = 1$ as $\bar{a}$ is a unit vector]

$|\bar{x}|^2 = 13$

$\therefore \overline{|x|} = \sqrt{13}$

Hence the value is $\sqrt{13}$

Question 10

If $\bar{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k},\ and\ \bar{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\bar{a} + \lambda\bar{b}$ find the value of λ

Solution:

We know that the

Give vectors are $\bar{a} = 2\hat{\imath} + 2\hat{\jmath}\,3\hat{k}, \bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k},\ and\ \bar{c} = 3\hat{\imath} + \hat{\jmath}$

Now

$\bar{a} + \lambda\bar{b} = (2\hat{\imath} + 2\hat{\jmath}\,3\hat{k}) + \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) = (2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}$

If $(\bar{a} + \lambda\bar{b}).\bar{c} = 0$

$[(2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}].(3\hat{\imath} + \hat{\jmath}) = 0$

$(2-\lambda)3 + (2+3\lambda)1 + (3+\lambda)0 = 0$

$6 - 3\lambda + 2 + 2\lambda = 0$

$-\lambda + 8 = 0$

$\lambda = 8$

Question 11

Show that $|\bar{a}|\bar{b} + |\bar{b}|\bar{a}$ is perpendicular to $|\bar{a}|\bar{b} - [b]\bar{a}$ for any two nonzero vectors $\bar{a}$ and $\bar{b}$

Solution:

$(|\bar{a}|\bar{b} + |\bar{b}|\,\bar{a}).(|\bar{a}|\bar{b} - |\bar{b}|\,\bar{a})$

$= |\bar{a}|^2\bar{b}.\bar{b} - |\bar{a}|\,|\bar{b}||\bar{b}\,.\bar{a}||b||\bar{a}|\bar{a}.\bar{b} - |\bar{b}|^2\bar{a}.\bar{a}$

$= |\bar{a}|^2|\bar{b}|^2 - |\bar{b}|^2|\bar{a}|^2$

$= 0$

Therefore $|\bar{a}|\bar{b} + |\bar{b}|\,\bar{a}$ and $|\bar{a}|\bar{b} - |\bar{b}|\bar{a}$ are perpendicular to each other.

Question 12

If $\bar{a}.\bar{a} = 0$ and $\bar{a}.\bar{b}$ then what can be concluded about the vector $\bar{b}$?

Solution:

We know

Give $\bar{a}.\bar{a} = 0$ and $\bar{a}.\bar{b} = 0$

Now,

$\bar{a}.\bar{a} = 0 \Rightarrow |\bar{a}|^2 = 0 \Rightarrow |\bar{a}| = 0$

$\therefore$ $\bar{a}$ is a zero vector.

Thus vector $\bar{b}$ satisfying $\bar{a}.\bar{b} = 0$ can be any vector

Question 13

If $\bar{a}, \bar{b}$ and $\bar{c}$ are unit vectors such that $\bar{a} + \bar{b} + \bar{c} = 0$ find the value of $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}\ \ a$

Solution:

Consider the given vectors

Given $\bar{a} + \bar{b} + \bar{c} = 0$

So,

$\bar{a}\,.(\bar{a} + \bar{b} + \bar{c}) = \bar{a}.\bar{0}$

$\bar{a}\,.\,\bar{a} + \bar{a}.\bar{b} + \bar{a}.\bar{c} = \bar{a}\,.\bar{0}$ [Distributivity of scalar product over addition]

$1 + \bar{a}.\bar{b} + \bar{a}.\bar{c} = 0$ …….. (1) $\left[\begin{smallmatrix} \bar{a}\,.\bar{a} = |\bar{a}|.|\bar{a}|\ \cos 0^0 \\ (\bar{a}\ \text{is unit vector}\ \Rightarrow |\bar{a}| = 1) \end{smallmatrix}\right]$

Next,

$\bar{b}\,(\bar{a} + \bar{b} + \bar{c}) = \bar{b}.\bar{0}$

$\bar{b}\,.\,\bar{a}\,\bar{b}.\bar{b} + \bar{b}.\bar{c} = \bar{b}\,.\bar{0}$

$\bar{b}.\bar{a} + 1\,\bar{b}.\bar{c} = 0$ …… (2) $[\bar{b}.\bar{b} = 1]$

And

$\bar{c}\,(\bar{a} + \bar{b} + \bar{c}) = \bar{c}.\bar{0}$

$\bar{c}\,.\,\bar{a} + \bar{c}.\bar{b} + \bar{c}.\bar{c} = \bar{c}\,.\bar{0}$

$\bar{c} + \bar{a} + \bar{c} + \bar{b}. + 1 = 0$ ……… (3) $[\bar{c}.\bar{c} = 1]$

From (1_, (2) and (3),

$(1 + \bar{a}.\bar{b} + \bar{a}.\bar{c})(\bar{b}.\bar{a} + 1 + \bar{b}.\bar{c}) + (\bar{c}.\bar{a} + \bar{c}.\bar{b} + 1) = 0 + 0 + 0$
$(3 + \bar{a}.\bar{b} + \bar{c}.\bar{a})(\bar{a}.\bar{b} + \bar{b}.\bar{c}) + (\bar{c}.\bar{a} + \bar{b}.\bar{c}) = 0$ [Scalar product is commutative]
$3 + 2(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}) = 0$
$\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a} = \frac{-3}{2}$
Hence the value is 3/2

Question 14

If either vector $\bar{a} = \bar{0}$ or $\bar{b}$, $\bar{0}$ then $\bar{a}.\bar{b} = 0$ But the converse need not be true. Justify your answer with an example

Solution:

Consider $\bar{a} = 2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ and $\bar{b} = 3\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$
Then, their dot product is given by.
$\bar{a}.\bar{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$
Now it`s seen that
$|\bar{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$
$\therefore \bar{a} \neq \bar{0}$
$|\bar{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$
$\therefore \bar{b} \neq \bar{0}$
Therefore the converse of the give statement need not be true.

Question 15

If the vertices A, B, C of a triangle ABC are (1, 2, 3) (-1, 0, 0), (0, 1, 2), respectively, then find ∠ABC [$\angle ABC$ is the agle between the vectors $\overline{BA}$ and $\overline{BC}$]

Solution:

We know
The vertices of ΔABC are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2).
Also given ∠ABC is the angle between the vectors $\overline{BA}$ and $\overline{BC}$
$\overline{BA} = \{1 - (-1)\}\hat{\imath} + (2 - 1)\hat{\jmath} + (3 - 0)\hat{k} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$
$\overline{BC} = \{0 - (-1)\}\hat{\imath} + (1 - 0)\hat{\jmath} + (2 - 0)\hat{k} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
$\therefore \overline{BA}.\overline{BC} = (2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}).(\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 6 = 10$
$|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$
$|\overline{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$
Now we know that
$\overline{BA}.\overline{BA} = |\overline{BA}|\,|\overline{BA}|\cos(\angle ABC)$
$\therefore 10 = \sqrt{17} \times \sqrt{6}\cos(\angle ABC)$

$\cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$

$\angle ABC = cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

Hence the angle is cos^{-1} (10/ √102)

Question 16

Show that the point A (1, 2, 7) B (2, 6, 3) and C (3, 10, - 1) are collinear.

Solution:

$\overline{AB}$ = (2 – 1) $\hat{\imath}$ + (6 – 2) $\hat{\jmath}$ + (3 – 7) $\hat{k}$ = $\hat{\imath}$ + 4$\hat{\jmath}$ - 4$\hat{k}$
$\overline{BC}$ = (3 – 1) $\hat{\imath}$ + (10 – 6) $\hat{\jmath}$ + (-1 – 3) $\hat{k}$ = $\hat{\imath}$ + 4$\hat{\jmath}$ - 4$\hat{k}$
$\overline{AC}$ = (3 – 1) $\hat{\imath}$ + (10 – 6) $\hat{\jmath}$ + (-1 – 3) $\hat{k}$ = $\hat{\imath}$ + 4$\hat{\jmath}$ - 4$\hat{k}$
Now,
$|\overline{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$
$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$
$|\overline{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$
$\therefore |\overline{AC}| = |\overline{AB}| + |\overline{BC}|$
Therefore, the given points A, B and C are collinear

Question 17

Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ *and* $\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ from the vertices of a right angled triangle

Solution:

Let vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, be position vectors of points A, B and C respectively
$\overline{OA}$ $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\overline{OA} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ And $\overline{OA}$ $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$
Now vectors $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respect the side ofΔABC.
Hence,
$\overline{AB}$ = (1 – 2) $\hat{\imath}$ + (-3+1) $\hat{\jmath}$ + (-5 – 1) $\hat{k}$ = $-\hat{\imath}$ - 2$\hat{\jmath}$ - 6$\hat{k}$
$\overline{BC}$ = (3 – 1) $\hat{\imath}$ + (-4 + 3) $\hat{\jmath}$ + (-4 +5) $\hat{k}$ =2 $\hat{\imath}$ + -$\hat{\jmath}$ +$\hat{k}$
$\overline{AC}$ = (2 - 3) $\hat{\imath}$ + (- 1 + 4) $\hat{\jmath}$ + (1 + 4) $\hat{k}$ = $\hat{\imath}$ + 43 - 5$\hat{k}$
$|\overline{AB}| = \sqrt{(-1)^2 + (-2) + (-6)^2} = \sqrt{1 + 4 + 36} = \sqrt{41}$
$|\overline{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$
$|\overline{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$
$\therefore |\overline{BC}|^2 + |\overline{AC}|^2 = 6 + 35 = 41 = |\overline{AB}|^2$
Therefore ABC is a right – angled triangle.

Question 18

If $\bar{a}$ is a nonzero vector of magnitude `a' and λ a nonzero scalar, then $\lambda\,\bar{a}$ is unit vector.
(A) $\lambda = 1$ (B) $\lambda = -1$ (C) A = $|\lambda|$ (D) 1/ $|\lambda|$

Solution:

Vector $\bar{\lambda}\lambda$ is unit vector if $|\lambda\bar{a}\,| = 1$
Now,
$|\lambda\bar{a}\,| = 1$
$|\lambda||\bar{a}| = 1$
$|\bar{a}| = \frac{1}{|\lambda|}$ $[\lambda \neq 0]$
$a = \frac{1}{|\lambda|}$ $[|\bar{a}| = a]$
Therefore, vector $\bar{\lambda}\bar{a}$ is a unit vector it $a = \frac{1}{|\lambda|}$
The correct answer is D

Exercise10. 4

Question 1

Find $|\bar{a} \times \bar{b}|$if $\bar{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\bar{b} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 2\hat{k}$

Solution:

$\bar{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\bar{b} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 2\hat{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{\imath}(-14 + 14) - \hat{\jmath}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{\jmath} + 19\hat{k}$$

Therefore,

$$|\bar{a} \times \bar{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

Question 2

Find a unit vector perpendicular to each of the vector $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$, where $\bar{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\bar{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.

Solution:

It given that,
$\bar{\boldsymbol{a}} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\bar{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
So we have
$\bar{a} + \bar{b} = 4\hat{\imath} + 4\hat{\jmath}, \bar{a} - \bar{b} = 2\hat{\imath} + 4\hat{k}$

$$(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b}) = \begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{\imath}\,(16) - \hat{\jmath}\,(16) + \hat{k}\,(-8) = 16\hat{\boldsymbol{\imath}} - 16\hat{\jmath} - 8\hat{k}$$

Thus,

$$|(\bar{a} + \bar{b})| \times |(\bar{a} + \bar{b})| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$
$$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$$
$$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$$

Therefore the unit vector perpendicular to each of the vectors $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ is given by the

$$= \frac{(\bar{a}+\bar{b}) \times (\bar{a}-\bar{b})}{|(\bar{a}+\bar{b}) \times (\bar{a}-\bar{b})|} = \pm \frac{16\,\hat{\imath} - 16\,\hat{\jmath} - 8\hat{k}}{24}$$
$$= \pm \frac{2\,\hat{\imath} - 2\,\hat{\jmath} - \hat{k}}{3} = \pm \frac{2}{3}\,\hat{\imath} \mp \frac{2}{3}\hat{\jmath} \mp \frac{1}{3}\,\hat{k}$$

Question 3

If a unit vector $\bar{a}$ makes an angles $\frac{\pi}{3}$ with $\hat{\imath}$ $\frac{\pi}{4}$ with $\hat{\jmath}$ and an angles θ with$\hat{k}$, then find θ and hence, the compounds of $\bar{a}$

Solution:

Let unit vector $\bar{a}$ have (a_1, a_2, a_3)components.
$\Rightarrow \bar{a}\ a_1\hat{\imath}, a_2\hat{\jmath}\ , a_3\hat{k}$
As $\bar{a}$ is a unit victor, $|\bar{a}| = 1$
Also given that $\bar{a}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$ $\frac{\pi}{4}$ with$\hat{\jmath}$, and an acute angle θ $with$ $\hat{k}$
Then we have

$\cos\frac{\pi}{3} = \frac{a_1}{|\bar{a}|}$

$\Rightarrow \frac{1}{2} = a_1 \qquad [|\bar{a}| = 1]$

$\cos\frac{\pi}{4} = \frac{a_2}{|\bar{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \qquad [|\bar{a}| = 1]$

Also $\cos\theta = \frac{a_3}{|\bar{a}|}$

$\Rightarrow a_3 = \cos\theta$
Now,
$|\bar{a}| = 1$

$\sqrt{a_1^{\mathbf{2}} + a_2^{\mathbf{2}} + a_3^{\mathbf{2}}} = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$
$\frac{3}{4} + \cos^2\theta = 1$
$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$
$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$
$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$
Thus, $\theta = \frac{\pi}{3}$ and the components of $\bar{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

Question 4

$(\bar{a} - \bar{b}) \times (\bar{a} - \bar{b}) = 2\,(\bar{a} \times \bar{b})$

Solution:

Firstly consider the LHS,
We have,
$(\bar{a} - \bar{b}) \times (\bar{a} + \bar{b})$
$= (\bar{a} - \bar{b}) \times \bar{a} + (\bar{a} - \bar{b}) \times \bar{b}$ [By Distributivity of vector product addition]
$= \bar{a} \times \bar{a} - \bar{b} \times \bar{a} + \bar{a} \times \bar{b} - \bar{b} \times \bar{b}$ [Again by Distributivity of vector product over addition]
$= \bar{0} + \bar{a} \times \bar{b} + \bar{a} \times \bar{b} - \bar{0}$
$= 2\,(\bar{a} \times \bar{b})$

Question 5

Find λ and μ if $(2\,\hat{\imath} + 6\,\hat{\jmath} + 27\,\hat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\,\hat{k}) = 0$

Solution:

It is given that,
Given
$(2\,\hat{\imath} + 6\,\hat{\jmath} + 27\,\hat{k}) \times (\hat{\imath} + \lambda\,\hat{\jmath} + \mu\,\hat{k}) = 0$
$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$
$\hat{\imath}\,(6\mu - 27\lambda) - \hat{\jmath}\,(2\mu - 27) + \hat{k}\,(2\lambda - 6) = 0\,\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$
On comparing the corresponding components we have,
$6\mu - 27\lambda = 0$
$2\mu - 27\lambda = 0$
$2\mu - 6\lambda = 0$
Now
$2\lambda - 6 = 0 \Rightarrow \lambda = 3$

$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$
Thus, $\lambda = 3$ and $\mu = \frac{27}{2}$

Question 6

Give that $\bar{a}.\bar{b} = 0$ and $\bar{a} \times \bar{b} = 0$ what can you conclude about the vectors $\bar{a}$ and $\bar{b}$?

Solution:

It is given that,
$\bar{a}.\bar{b} = 0$

I. Either $|\bar{a}| = 0$ or $|\bar{b}| = 0$, or $\bar{a}\ 1\bar{b}$ (in case $\bar{a}$ and $\bar{b}$ are non – zero) $\bar{a} \times \bar{b} = 0$
II. Either $|\bar{a}| = 0$ or $|\bar{b}| = 0$, or $\bar{a} \parallel \bar{b}$ (in case $\bar{a}$ and $\bar{b}$ are non – zero) But $\bar{a}$ and $\bar{b}$ cannot be perpendicular and parallel simultaneously Therefore, $|\bar{a}| = 0$ or $\bar{b} = 0$

Question 7

Let the vectors $\bar{a}, \bar{b}, \bar{c}$ give as $a_1\ \hat{\imath} + a_2 + \hat{\jmath} + a_3 + \hat{k}, b_1\ \hat{\imath} + b_2 + \hat{\jmath}\ b_3 + \hat{k}, c_1 + \hat{\imath}\ c_2 + \hat{\jmath}\ c_3 + \hat{k}$

Solution:

It is given that,
$a_1\ \hat{\imath} + a_2 + \hat{\jmath} + a_3 + \hat{k}, b_1\ \hat{\imath} + b_2 + \hat{\jmath}\ b_3 + \hat{k}, c_1 + \hat{\imath}\ c_2 + \hat{\jmath}\ c_3 + \hat{k}$
$(\bar{b} + \bar{c}) = (b_1 + c_1)\ \hat{\imath} + (b_2 + c_2)\hat{\jmath} + (b_3 + c_3)\hat{k}$

Now, $\bar{a} \times (\bar{b} + \bar{c}) \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$

$= \hat{\imath}\ [a_2\ (b_3 + c_3) - a_3(b_2 + c_2) - \hat{\jmath}\ ][a_1\ (b_3 + c_3) - a_3\ (b_1 + c_1)] + \hat{k}\ [a_1(b_2 + c_2) - a_3(b_1 + c_1)]$
$= \hat{\imath}\ [a_2\ b_3 + a_2c_3 - a_3b_2\ ] + \hat{\jmath}\ [-a_1b_3 - a_1c_3 + a_3c_1] + \hat{k}\ [a_1b_2 + a_1\ c_2 - a_2\ b_1 - a_2\ c_1]$ .... (1)
And,

$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$= \hat{\imath}\ [a_2\ b_3 - a_3b_2] + \hat{\jmath}\ [b_1\ a_3 - a_1b_3] + \hat{k}[a_1\ b_2 - a_2b_1]$ (2)

$\bar{a} \times \bar{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= \hat{\imath}\ [a_2\ b_3 - a_3c_2] + \hat{\jmath}\ [a_3\ c_1 - a_1c_3] + \hat{k}[a_1\ c_2 - a_2c_1]$ (3)
On adding (2) and (3, we get
$(\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c}) = \hat{\imath}[a_2\ b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{\jmath}\ [b_1a_3 - a_3c_1 - a_1b_3 - a_1c_3] + \hat{k}\ [a_1b_2 + a_1\ c_2 - a2\ b1 - a2\ c1$ (4)

From (1) and (4) we obtain
$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$
- Hence proved

Question 8

If either
$\bar{a} = \bar{0}$ or $\bar{b} = \bar{0}$ then $\bar{a} \times \bar{b} = \bar{0}$ is the converse true? Justify your answer with an example

Solution:

Take any parallel non-zero vectors to that $\bar{a} \times \bar{b} = \bar{0}$
Let $\bar{a} = 2\,\hat{\imath} + 3\hat{\jmath} + 4\,\hat{k}, \bar{\mathbf{b}} = 4\,\hat{\imath} + 6\hat{\jmath} + 8\,\hat{k}$
Then
$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{\imath}\,(24 - 24) - \hat{\jmath}(16 - 16) + \hat{k}\,(12 - 12) = 0\,\hat{\imath} + 0\hat{\jmath} + 0\,\hat{k} = 0$$
Now it`s seen that
$|\bar{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$
$\therefore \bar{a} \neq \bar{0}$
$|\bar{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$
$\therefore \bar{b} \neq \bar{0}$
Thus the converse of the given need not be true.

Question 9

Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5)

Solution:

We know
Given A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5) are the vertices of triangle ABC.
The adjacent sides $\overline{AB}$ *and* $\overline{BC}$ OF ΔABC are given as
$\overline{AB} = (2 - 1)\,\hat{\imath} + (3 - 1)\hat{\jmath} + (5 - 2)\,\hat{k} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$
$\overline{BC} = (1 - 2)\,\hat{\imath} + (5 - 3)\hat{\jmath} + (5 - 5)\,\hat{k} = -\hat{\imath} + 2\hat{\jmath}$
Now
Area of $\Delta\, ABC = \frac{1}{2}\,|\overline{AB} \times \overline{BC}|$
$$\overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{\imath}\,(-6) - \hat{\jmath}\,(3) + \hat{k}\,(2 + 2) = -6\,\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$
$\therefore |\overline{AB} \times \overline{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$
Therefore, the area of ΔABC is $\frac{\sqrt{61}}{2}$square units.

Question 10

Find the area of the parallelogram whose adjacent sides are determined by the vector $\bar{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and vector $\bar{b} = 2\,\hat{\imath} - 7\,\hat{\jmath} + \hat{k}$

Solution:

Let area of the parallelogram whose adjacent sides are $\bar{a}$ and $\bar{b}$ is $|\bar{a} \times \bar{b}|$
Now the adjacent sides are given as:
$\bar{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ And $\bar{b} = 2\,\hat{\imath} - 7\,\hat{\jmath} + \hat{k}$
$\therefore \bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{\imath}\,(-1 + 21) - \hat{\jmath}\,(1 - 6) + \hat{k}\,(-7 + 2) = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$
$|\bar{a} \times \bar{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$
Therefore the area of given parallelogram is $5\sqrt{2}$ square units.

Question 11

Let the vectors $\bar{a}$ and $\bar{b}$ be such that $|\bar{a}| = 3$ and $|\bar{b}| = \frac{\sqrt{2}}{3}$, then $\bar{a} \times \bar{b}$ is unit vector, if the angle between $\bar{a}$ and $\bar{b}$ is

Solution:

Given, $|\bar{a}| = 3$ and $|\bar{b}| = \frac{\sqrt{2}}{3}$.
We know that $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta\hat{\pi}$, where $\hat{\pi}$ is a unit vector perpendicular to both $\bar{a}$ and $\bar{b}$ and θ is the angle between $\bar{a}$ *and* $\bar{b}$
Now, $\bar{a} \times \bar{b}$ is a unit vector if $|\bar{a} \times \bar{b}| = 1$
$|\bar{a} \times \bar{b}| = 1$
$|\bar{a}||\bar{b}||\sin\theta\hat{\pi}| = 1$
$|\bar{a}||\bar{b}||\sin\theta| = 1$
$3 \times \frac{\sqrt{2}}{3} \times sin\theta = 1$
$\sin\theta = \frac{1}{\sqrt{2}}$
$\theta = \frac{\pi}{4}$
Thus $\bar{a} \times \bar{b}$is a unit vector if the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{4}$
So correct answer is B

Question 12

Area of a rectangle having vertices A, B, C and D with position vector- $\hat{\imath}+\frac{1}{2}\hat{\jmath}+4\hat{k}$, = $\hat{\imath}+\frac{1}{2}\hat{\jmath}+4\hat{k}$, = $\hat{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$, = $\bar{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$ and $\hat{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$ respective is

(A) $\frac{1}{2}$ **(B) 1**

(C) 2 **(D) 4**

Solution:

The position vectors of vertices A, B, C and D of rectangle ABCD are given as:

$\overline{OA}$ =- $\hat{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$, $\overline{OB}=\hat{\imath}+\frac{1}{2}\hat{\jmath}+4\hat{k}$, $\overline{OC}=\hat{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$, $\overline{OD}$ =- $\hat{\imath}-\frac{1}{2}\hat{\jmath}+4\hat{k}$,

The adjacent side $\overline{AB}$ and $\overline{BC}$ of the given rectangle are given as:

$\overline{AB}=(1+1)\hat{\imath}+\left(\frac{1}{2}-\frac{1}{2}\right)\hat{\jmath}+(4-4)\hat{k}=2\hat{\imath}$

$\overline{BC}=(1-1)\hat{\imath}+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{\jmath}+(4-4)\hat{k}=-\hat{\jmath}$

$\therefore \overline{AB}\times\overline{AB}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 0 & 0\\ 0 & -1 & 0\end{vmatrix}=\hat{k}(-2)=-2\hat{k}$

$\Rightarrow |\overrightarrow{AB}\times\overrightarrow{BC}|=2$

We know that the of parallelogram whose adjacent sides are $\bar{a}$ and $\bar{b}$ is$|\bar{a}\times\bar{b}|$.

Thus, the area of the given rectangle is$|\overline{AC}\times\overline{BC}|$= 2 square units.

So the correct answer is C.

Miscellaneous Exercise

Question 1

Write down a unit in XY – plane, making an angle of 30^0 with the positive direction of $x-$ axis.

Solution:

If $\bar{r}$ is a unit in the XY – plane, then $\bar{r}=\cos\theta\,\hat{\imath}+\sin\theta\,\hat{\jmath}$

Here, θ is the angle made by the unit vector with the positive direction of the x – axis

Hence, for θ = 30^0 we have:

$\bar{r}=\cos 30^0\,\hat{\imath}+\sin 30^0\,\hat{\jmath}=\frac{\sqrt{3}}{2}\hat{\imath}+\frac{1}{2}\hat{\jmath}$

Therefore, the required unit vector is $\frac{\sqrt{3}}{2}\hat{\imath}+\frac{1}{2}\hat{\jmath}$

Question 2

Find the scalar components and magnitude of the vector joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2)

Solution:

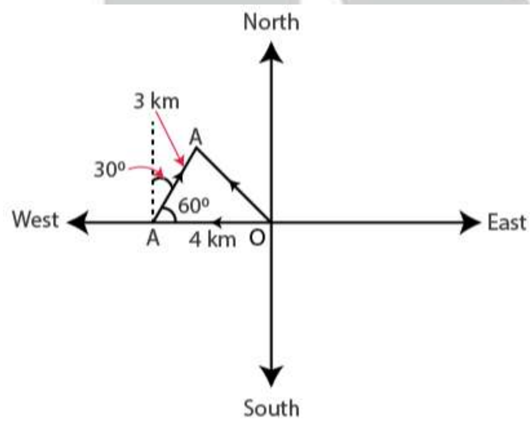
The vector joining the point P (x_1, y_1, z_1) Q (x_2, y_2, z_2) can be formed out by.
$\overline{PQ}$ = Position vector of Q – Position vector of P
$= (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$
$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Therefore, the scalar components and the magnitude of the vector joining the given points are respectively. $\{(x_2 - x_1, (y_2 - y_1), (z_2 - z_1)),\}$ And $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Question 3

A girl walk 4 km towards west, then she walks 3 km in a direction 30^{0}east of north and stops. Determine the girl`s displacement from her initial point of departure.

Solution:

It is given that,
Let O and B the initial and final position of the girl respectively
Then the girl`s position can be shown as:



$\overrightarrow{OA}$ = -4$\hat{\imath}$
$|\overrightarrow{AB}|$ $\hat{\imath}$ $|\overrightarrow{AB}|$ cos 60^0 + $\hat{\jmath}$ $|\overrightarrow{AB}|$ sin 60^0
$= \hat{\imath}\, 3 \times \frac{1}{2} + \hat{\jmath}\, 3 \times \frac{\sqrt{3}}{2}$
$= \frac{3}{2}\,\hat{\imath} + \frac{3\sqrt{3}}{2}\,\hat{\jmath}$
By the Triangle law of vector addition we have
$= (-4\hat{\imath}) + \left(\frac{3}{2}\,\hat{\imath} + \frac{3\sqrt{3}}{2}\,\hat{\jmath}\right)$
$= \left(-4 + \frac{3}{2}\right)\hat{\imath} + \frac{3\sqrt{3}}{2}\,\hat{\jmath}$

$= \left(\frac{-8+3}{2}\right)\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

$= \frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

Therefore the girl`s displacement from her initial point of departure is

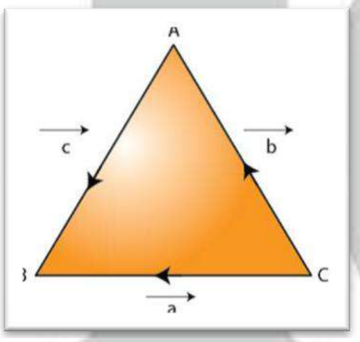
$\frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

Question 4

If $\bar{a} = \bar{b} + \bar{c}$, then is it true that $|\bar{a}| = |\bar{b}| + |\bar{c}|$? Justify your answer.

Solution:

In ΔABC, Let $\overline{CB} = \bar{a}, \overline{CA} = \bar{b}$, and $\overline{AB} = \bar{c}$ (as shown in the following figure).



So by the Triangle law of vector addition we have $\bar{a} = \bar{b} + \bar{c}$.

And we know that $|\bar{a}|, |\bar{b}|$ *and* $|\bar{c}|$ represent the sides of ΔABC.

Also it is know that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\bar{a}| < |\bar{b}| + |\bar{c}|$

Therefore. it is not true that $|\bar{a}| = |\bar{b}| + |\bar{c}|$

Question 5

Find the value of x for which $x\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ is a unit vector.

Solution:

We know

Given $x\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ is a unit vector

So, $\left|x\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)\right| = 1$

Now

$\left|x\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)\right| = 1$

$\sqrt{x^2 + x^2 + x^2} = 1$
$\sqrt{3x^2} = 1$
$\sqrt{3}\, x = 1$
$x = \pm \frac{1}{\sqrt{3}}$
Therefore the requirement value of x is $\pm \frac{1}{\sqrt{3}}$

Question 6

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\bar{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\bar{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

Solution:

Let us consider the,
Given vectors,
$\bar{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\bar{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$
Let $\bar{c}$ be the resultant of $\bar{a}$ and $\bar{b}$
Then
$\bar{c} = \bar{a} + \bar{b} = (2+1)\hat{\imath} + (3-2)\hat{\jmath} + (-1+1)\hat{k} = 3\hat{\imath} + \hat{\jmath}$
$|\bar{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
$\therefore \hat{c} = \frac{\bar{c}}{|\bar{c}|} = \frac{(3\hat{\imath} + \hat{\jmath})}{\sqrt{10}}$
Therefore the vector of magnitude 5 unit and parallel to the resultant of vector $\bar{a}$ and $\bar{b}$ is $\pm 5.\hat{c} = \pm 5.\frac{1}{\sqrt{10}}(3\hat{\imath} + \hat{\jmath}) = \pm \frac{3\sqrt{10}\,\hat{\imath}}{2} \pm \frac{\sqrt{10}}{2}\hat{\jmath}$

Question 7

If $\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\bar{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\bar{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a unit vector parallel to the vector $2\bar{a} - \bar{b} + 3\bar{c}$

Solution:

Let us consider the given vectors
Given,
$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\bar{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\bar{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$
$2\bar{a} - \bar{b} + 3\bar{c} = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$
$= 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k} - 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + 3\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$
$= 3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$|2\bar{a} - \bar{b} + 3\bar{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$
Therefore the unit vector along $2\bar{a} - \bar{b} + 3\bar{c}$ is

$\frac{2\,\bar{a}-\bar{b}+3\bar{c}}{|2\,\bar{a}-\bar{b}+3\bar{c}|} = \frac{3\hat{\imath}-3\,\hat{\jmath}+2\,\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{\imath} - \frac{3}{\sqrt{22}}\hat{\jmath} + \frac{2}{\sqrt{22}}\,\hat{k}$

Question 8

Show that the point A (1, -2, -8),B (5, 0, -2) and C, (11, 3,7) are collinear, and find the ratio in which B divides AC.

Solution:

Firstly let us consider
Given points are A (1, -2, -8), B (5, 0, -2) and C, (11. 3.7)
$\therefore\ \overline{\mathrm{AB}} = (5-1)\hat{\imath} + (0+2)\hat{\jmath} + (-2+8)\hat{k} = 4\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$
$\overline{\mathrm{BC}} = (11-5)\hat{\imath} + (3-0)\hat{\jmath} + (7+2)\hat{k} = 6\hat{\imath} + 3\hat{\jmath} + 9\hat{k}$
$\overline{\mathrm{AC}} = (11-1)\hat{\imath} + (3+2)\hat{\jmath} + (7+8)\hat{k} = 10\hat{\imath} + 5\hat{\jmath} + 15\hat{k}$
$|\overline{\mathrm{AB}}| = \sqrt{4^2+2^2+6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$
$|\overline{\mathrm{BC}}| = \sqrt{6^2+3^2+9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$
$|\overline{\mathrm{AB}}| = \sqrt{10^2+5^2+15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$
$\therefore |\overline{\mathrm{AB}}| = |\overline{\mathrm{BC}}| + |\overline{\mathrm{BC}}|$
Therefore the given point A, B and C are collinear
Now let point B divide AC in the ratio λ:1 So, we have:
$\overline{OB} = \frac{\lambda\overline{OC}+\overline{OA}}{\lambda+1}$
$5\,\hat{\imath} - 2\hat{k} = \dfrac{\lambda(11\,\hat{\imath} + 3\hat{\jmath} + 7\hat{k}) + \lfloor \hat{\imath} - 2\hat{\jmath} - 8\hat{k} \rfloor}{\lambda+1}$
$(\lambda+1)(5\,\hat{\imath} - 2\hat{k}) = 11\,\lambda\hat{\imath} + 3\,\lambda\hat{\jmath} + 7\,\lambda\hat{k}\,\hat{\imath} - 2\hat{\jmath} - 8\hat{k}$
$5(\lambda+1)\hat{\imath} - 2\,(\lambda+1)\hat{k} = (11\,\lambda+1)\hat{\imath} + (3\lambda-2)\hat{\jmath} + (7\,\lambda-8)\hat{k}$
On equating the corresponding components we have
$5(\lambda+1) = 11\,\lambda + \hat{\imath}$
$5\,\lambda + 5 = 11\,\lambda + 1$
$6\,\lambda = 4$
$\lambda = \frac{4}{6} = \frac{2}{3}$

Question 9

Find the position vector of a point R which divides the line joining two points P and Q whose position vector are $(2\,\bar{a} + \bar{b})$ and $(2\,\bar{a} - \bar{b})$ externally in the ratio 1: 2 Also show that P is the midpoint of the line segment RQ

Solution:

We know

Given $\overline{OP} = 2\bar{a} + \bar{b}, \overline{OQ} = \bar{a} - 3\bar{b}$

Also given that point R divides a line segment joining two points P and Q externally in the Ratio 1: 2 So on using the section formula we have,

$\overline{OR} = \frac{2\,(2\,\bar{a}+\bar{b})-(\bar{a}-3\bar{b})}{2-1} = \frac{4\,\bar{a}+2\,\bar{b}-\bar{a}+3\,\bar{b}}{1}\ 3\,\bar{a} + 5\,\bar{b}$

Hence the position vector of point R is $3\,\bar{a} + 5\,\bar{b}$

Now,

Position vector of the mid-point of= $\frac{\overline{OQ}+\overline{OR}}{2}$

$= \frac{(\bar{a}-3\,\bar{b})-(3\bar{a}-5\bar{b})}{2}$

$= 2\,\bar{a} + \bar{b}$

$= \overline{op}$

Therefore, P is the mid- point of the line segment RQ.

Question 10

The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\,\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\,\hat{k}$
Find the unit vector parallel to its diagonal also find its area.

Solution:

Firstly let us consider

Adjacent sides of a parallelogram are given as: $\bar{a} = 2\hat{\imath} - 4\hat{\jmath} + 5\,\hat{k}$ and $\bar{b} = \hat{\imath} - 2\hat{\jmath} - 3\,\hat{k}$

We know that the diagonal of a parallelogram is given by $\bar{a} + \bar{b}$

$\bar{a} + \bar{b} = (2+1)\hat{\imath} + (-4-2)\hat{\jmath}\,(5-3)\hat{k} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$

Hence the unit vector parallel to the diagonal is

$$\frac{\bar{a}+\bar{b}}{|\bar{a}+\bar{b}|} = \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{7} = \frac{3}{7}\,\hat{\imath} - \frac{6}{7}\,\hat{\jmath} + \frac{2}{7}\,\hat{k}$$

So, the area of parallelogram ABCD = $|\bar{a} \times \bar{b}|$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$= \hat{\imath}\,(12+10) - \hat{\jmath}\,(-6-5) + \hat{k}\,(-4+4)$

$= 22\hat{\imath} + 11\hat{\jmath}$

$= 11\,(2\hat{\imath} + \hat{\jmath})$

$\therefore\ |\bar{a} \times \bar{b}| = 11\sqrt{2^2 + 1^2} = 11\sqrt{5}$

Therefore, the area of the parallelogram is $11\sqrt{5}$ square units

Question 11

Show that the direction cosines of a vector equally to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},$

Solution:

Firstly,
Let`s assume a vector to be equally inclined to OX, OY and OZ at angle a.
Then the directors cosines of the vector are cos a, and cos a,
Now we know that
$cos^2 a + cos^2 a + cos^2 a = 1$
$3 cos^2 a = 1$
cos a = $\frac{1}{\sqrt{3}}$
Therefore the direction cosines of the vector which are equally inclined to the axes are
$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},$
Hence proved

Question 12

Let $\bar{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \bar{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\bar{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\bar{d}$ which is perpendicular to both $\bar{a}$ and $\bar{b}$, and $\bar{a}.\bar{d} = 15$

Solution:

Assume
Let $\bar{d} = d_1\hat{\imath} + d_2\hat{\jmath} + d_3\hat{k}$
As $\bar{d}$ is perpendicular to both $\bar{a}$ and $\bar{b}$ we have
$\bar{d}.\bar{a} = 0$
$d_1 + 4 d_2 + 2 d_3 = 0$ …….. (i)
And
$\bar{d}.\bar{b} = 0$
$3d_1 - 2d_2 + 7 d_3 = 0$ …….. (ii)
Also, give that
$\bar{c}.\bar{d} = 15$
$2d_1 - 2 d_2 + 4 d_3 = 15$ …….. (iii)
On solving (i), (ii), and (iii) we obtain
$d_1 = \frac{160}{3}, d_2 = \frac{5}{3}$, And $d_3 = \frac{70}{3}$,
$\therefore \bar{d} = \frac{160}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{\imath} - 5\hat{\jmath} - 70\hat{k})$
Therefore, the required vector is $\frac{1}{3}(160\hat{\imath} - 5\hat{\jmath} - 70\hat{k})$

Question 13

The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ.

Solution:

Let`s consider the
Sum of the given vectors is given by.
$(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}) + (\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$
$= (2 + \lambda)\,\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$
Hence, unit vector along $(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}) + (\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is given as:
$\frac{(2+\lambda)\,\hat{\imath} + 6\hat{\jmath} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + (-2)^2}} = \frac{(2+\lambda)\,\hat{\imath} + 6\hat{\jmath} - 2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}} = \frac{(2+\lambda)\,\hat{\imath} + 6\hat{\jmath} - 2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$
Scalar product of $(\hat{\imath} + \hat{\jmath} + \hat{k})$ with this unit vector is 1
$(\hat{\imath} + \hat{\jmath} + \hat{k}).\frac{(2+\lambda)\,\hat{\imath} + 6\hat{\jmath} - 2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} = 1$
$\frac{(2+\lambda)\,\hat{\imath} + 6\hat{\jmath} - 2}{\sqrt{\lambda^2+4\lambda+44}} = 1$
$\sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$
$\lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$
$\lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$
$8\lambda = 8$
$\lambda = 1$
Therefore, the value of A is 1.

Question 14

If $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors of equal magnitude, shows that the vectors $\bar{a} + \bar{b} + \bar{c}$ is equally inclined $\bar{a}, \bar{b}$ and $\bar{c}$

Solution:

As $\bar{a}, \bar{b}$ and $\bar{c}$ are mutually perpendicular to vectors, we have
$\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{c}.\bar{a} = 0$
Given that
$|\bar{a}| = |\bar{b}| = |\bar{c}|$
Let vector $\bar{a} + \bar{b} + \bar{c}$ be included to $\bar{a}.\bar{b}$ and $\bar{c}$ at angles θ_1, θ_2 and θ_3 respectively
So we have
$\cos\theta_1 = \frac{(\bar{a}+\bar{b}+\bar{c}).\bar{a}}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|} = \frac{\bar{a}.\bar{a}+\bar{b}.\bar{a}+\bar{c}.\bar{a}}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|}$
$= \frac{|\bar{a}|^2}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|}$ $[\bar{b}.\bar{a} + \bar{c}.\bar{a} = 0]$
$= \frac{|\bar{a}|}{|\bar{a}+\bar{b}+\bar{c}|}$
$\cos\theta_2 = \frac{(\bar{a}+\bar{b}+\bar{c}).\bar{b}}{|\bar{a}+\bar{b}+\bar{c}||\bar{b}|} = \frac{\bar{a}.\bar{a}+\bar{b}.\bar{b}+\bar{c}.\bar{b}}{|\bar{a}+\bar{b}+\bar{c}||\bar{b}|}$
$= \frac{|\bar{b}|^2}{|\bar{a}+\bar{b}+\bar{c}||\bar{b}|}$ $[\bar{a}.\bar{b} + \bar{c}.\bar{b} = 0]$
$= \frac{|\bar{b}|}{|\bar{a}+\bar{b}+\bar{c}|}$

$\cos\theta_1 = \frac{(\bar{a}+\bar{b}+\bar{c}).\bar{c}}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|} = \frac{\bar{a}.\bar{a}+\bar{b}.\bar{a}+\bar{c}.\bar{c}}{|\bar{a}+\bar{b}+\bar{c}||\bar{c}|}$

$= \frac{|\bar{c}|^2}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|}$ $[\bar{a}.\bar{c} + \bar{b}.\bar{c} = 0]$

$= \frac{|\bar{c}|}{|\bar{a}+\bar{b}+\bar{c}|}$

Now as $|\bar{a}| = |\bar{b}| = |\bar{c}|, \cos\theta_1 = \cos\theta_2 = \cos\theta_3$

$\therefore \theta_1 = \theta_2 = \theta_3$

Therefore, the vector $\left((\bar{a}+\bar{b}+\bar{c})\right)$ is equally inclined to $\bar{a}, \bar{b}$ and $\bar{c}$

Hence proved

Question 15

Proved that $(\bar{a}+\bar{b}).(\bar{a}+\bar{b}) = |\bar{a}|^2 + |\bar{b}|^2$, if and only if $\bar{a}, \bar{b}$ are perpendicular, given $\bar{a} \neq \bar{0}, \bar{b} \neq \bar{0}$.

Solution:

$(\bar{a}+\bar{b}).(\bar{a}+\bar{b}) = |\bar{a}|^2 + |\bar{b}|^2$

$\bar{a}.\bar{a} + \bar{a}.\bar{b} + \bar{b}.\bar{a} + \bar{b}.\bar{b} = |\bar{a}|^2 + |\bar{b}|^2$ [Distributivity of scalar products over addition]

$|\bar{a}|^2 + 2\bar{a}.\bar{b} + |\bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2$ [$\bar{a}.\bar{b} = \bar{b}.\bar{a}$ (Scalar product is commutative)]

$2\bar{a}.\bar{b} = 0$

$\bar{a}.\bar{b} = 0$

Therefore, $\bar{a}$ and $\bar{b}$ are perpendicular [$\bar{a} \neq \bar{0} = \bar{b} \neq \bar{0}$ (Given)]

Hence proved

Question 16

If θ is the angle between two vectors $\bar{a}$ and $\bar{b}$ then $\bar{a}.\bar{b} =\geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ **(B) $0 \leq \theta \leq \frac{\pi}{2}$**

(C) $0 < \theta < \pi$ **(D) $0 \leq \theta \leq \pi$**

Solution:

Explanation:

Let`s assume θ to be the angle between two vector $\bar{a}$ and $\bar{b}$.

Then without loss of generally $\bar{a}$ and $\bar{b}$ are non – zero vectors so that $|\bar{a}|$ and $|\bar{b}|$ are positive we also know, $\bar{a}.\bar{b} = |\bar{a}|\,|\bar{b}|\,\cos\theta$

$\bar{a}.\bar{b} \geq 0$

$|\bar{a}|\,|\bar{b}|\,\cos\theta \geq 0$

$\cos\theta \geq 0$ [$|\bar{a}|$ And $|\bar{b}|$ are positive]

$0 \leq \theta \leq \frac{\pi}{2}$

Therefore $\bar{a}.\bar{b} \geq 0$ When $0 \leq \theta \leq \frac{\pi}{2}$
The correct answer is B.

Question 17

Let $\bar{a}$ and $\bar{b}$ be two units vectors and θ is the angle between them. Then $\bar{a} + \bar{b}$ is a unit vector of
(A) $\theta = \frac{\pi}{4}$ **(B) $\theta = \frac{\pi}{3}$**
(C) $\theta = \frac{\pi}{2}$ **(D) $\theta = \frac{2\pi}{3}$**

Solution:

Explanation:
Let $\bar{a}$ and $\bar{b}$ be two unit vectors and θ is the angle between them.
Then, $|\bar{a}| = |\bar{b}| = 1$
Now $\bar{a} + \bar{b}$ is a unit vector if $|\bar{a} + b| = 1$
$|\bar{a} + \bar{b}| = 1$
$(\bar{a} + \bar{b})^2 = 1$
$(\bar{a} + \bar{b}).(\bar{a} + \bar{b}) = 1$
$\bar{a}.\bar{a} + \bar{a}.\bar{b} + \bar{b}.\bar{a} + \bar{b}.\bar{b} = 1$
$|\bar{a}|^2 + 2\bar{a}.\bar{b}\,|\bar{b}|^2 = 1$
$1^2 + 2\,|\bar{a}||\bar{b}|\,cos\theta + 1^2 = 1$
$1 + 2.1.\cos\theta + 1 = 1$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = \frac{2\pi}{3}$$
Therefore $\bar{a} + \bar{b}$ is a unit vector if $\theta = \frac{2\pi}{3}$
Hence the correct answer is D.

Question 18

The value $\hat{\imath}\,(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}\,(\hat{\imath} \times \hat{\jmath})$ is
(A) 0 **(B) -1**
(C) 1 **(D) 3**

Solution:

It is given that,
$\hat{\imath}\,(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}\,(\hat{\imath} \times \hat{\jmath})$
$= \hat{\imath}.\hat{\imath} + \hat{\jmath}\,(-\hat{\jmath}) + \hat{k}.\hat{k}$

$= 1 - \hat{j}.\hat{j} + 1$
$= 1 - 1 + 1$
$= 1$
Hence the correct answer is C.

Question 19

If θ is the angle between any two vectors $\bar{a}$ and $\bar{b}$ then $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$ when θ is equal to

(A) 0 **(B) $\frac{\pi}{4}$**

(C) $\frac{\pi}{2}$ **(D) π**

Solution:

Let θ be the angle between two vectors $\bar{a}$ and $\bar{b}$
Then without loss of generally $\bar{a}$ and $\bar{b}$ are non – zero vectors so that $|\bar{a}|$ and $|\bar{b}|$ are positive
$|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$
$|\bar{a}|\,|\bar{b}|$ Cos $\theta = |\bar{a}|\,|\bar{b}| \sin\theta$
$\cos\theta = \sin\theta$ [$|\bar{a}|$ And $|\bar{b}|$ are positive]
$\tan\theta = 1$
$\theta = \dfrac{\pi}{4}$
Thus, $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$ when θ is equal to $\frac{\pi}{4}$
So the correct answer is B.