Chapter 10
Straight Lines

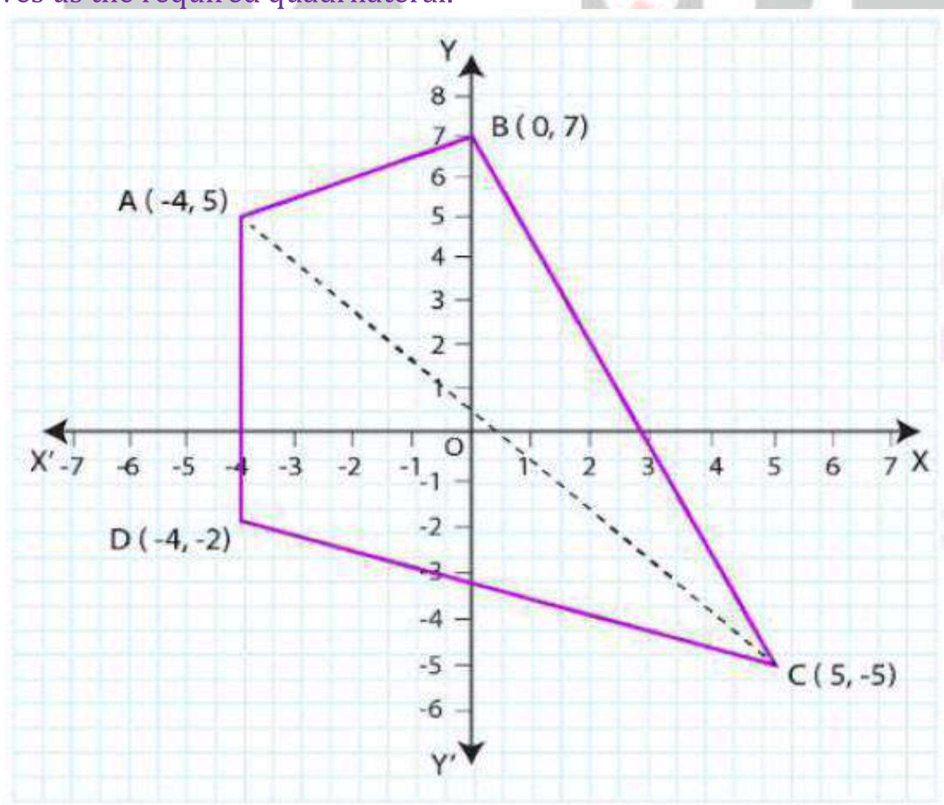
Exercise 10.1

Question 1

Draw a quadrilateral in the Cartesian plane, whose vertices are (– 4, 5), (0, 7), (5, – 5) And (– 4, –2). Also, find its area.

Answer:

Let ABCD be the given quadrilateral with vertices A (-4, 5), B (0, 7), C (5.-5) and D (-4,-2).
Now let us plot the points on the Cartesian plane by joining the points AB, BC, CD, and AD which gives us the required quadrilateral.



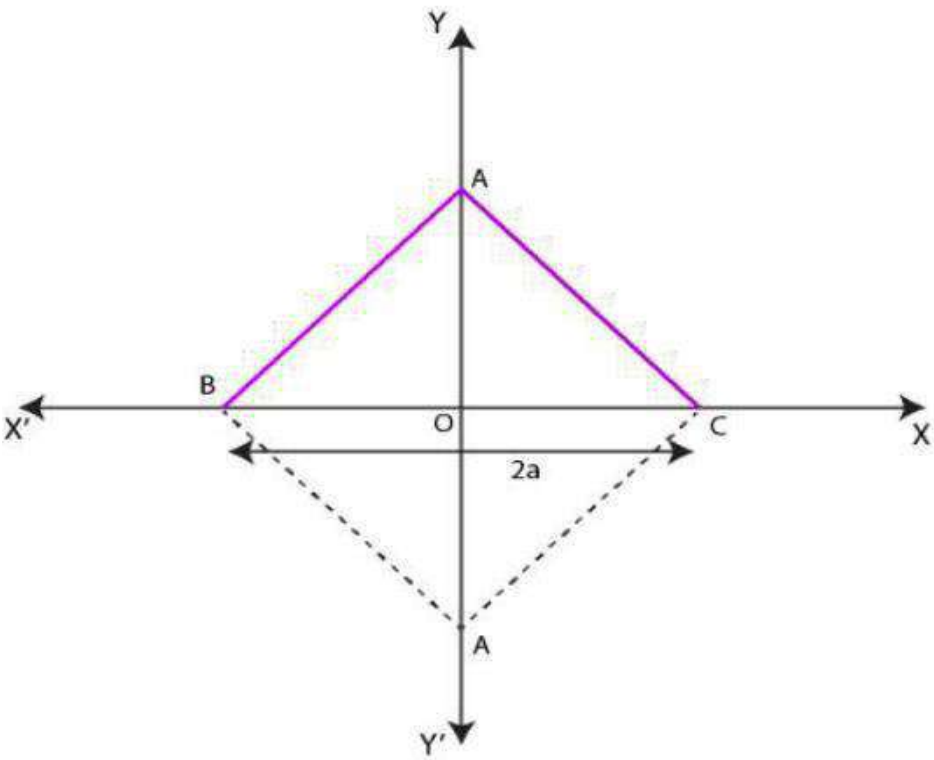
To find the area, draw diagonal AC
So, area (ABCD) = area (ΔABC) + area (ΔADC)
Then, area of triangle with vertices (x_1,y_1) , (x_2, y_2) and (x_3,y_3) is
Are of Δ ABC = $\frac{1}{2}[x_1 (y_2 - y_3) + x_3 (y_3 - y_2) + x_3 (y_1 - y_2)]$
= $\frac{1}{2}[-4 (7 + 5) + 0 (-5 - 5) + 5 (5 - 7)]$ unit^2

= ½ [-4 (12) + 5 (-2)] $unit^2$
= ½ (58) $unit^2$
= 29 $unit^2$
Are of Δ ACD = ½ [x1 $(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$]
= ½ [-4 (-5 + 2) + 5 (-2 - 5) + (-4) (5 – (-5))] $unit^2$
= ½ [-4 (-3) + 5 (-7) – 4 (10)] $unit^2$
= ½ (-63) $unit^2$
= -63/2 $unit^2$
Since area cannot be negative area Δ ACD = 63/2 $unit^2$
Area (ABCD) = 29 + 63/2
= 121/2 $unit^2$

Question 2

The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

Answer:



Let us consider ABC be the given equilateral triangle with side 2a.
Where, AB = BC = AC = 2a
In the above figure, by assuming that the base BC lies on the x axis such that the midpoint of BC is at the origin i.e. BO = OC = a, where O is the origin.
The co-ordinates of point C are (0, a) and that of B are (0,-a)

Since the line joining a vertex of an equilateral Δ with the mid-point of its opposite side is perpendicular.
So, vertex A lies on the y –axis
By applying Pythagoras theorem
$(AC)^2 = OA^2 + OC^2$
$(2a)^2 = a^2 + OC^2$
$4a^2 - a^2 = OC^2$
$3a^2 = OC^2$
$OC = \sqrt{3}a$
Co-ordinates of point C = $\pm \sqrt{3}a, 0$
∴ The vertices of the given equilateral triangle are (0, a), (0, -a), ($\sqrt{3}$a, 0) Or (0, a), (0, -a) and (-$\sqrt{3}$a, 0)

Question 3

Find the distance between P (x1, y1) and Q (x2, y2) when:
(i) PQ is parallel to the yaxis,
(ii) PQ is parallel to the x-axis.

Answer:

Points P (x_1, y_1) and $Q(x_2, y_2)$
(i) When PQ is parallel to y axis then $x_1 = x_2$
So, the distance between P and Q is given by $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(y_2 - y_1)^2}$
$= |y_2 - y_1|$

(ii) When PQ is parallel to the x-axis then $y_1 = y_2$
So, the distance between P and Q is given by $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(x_2 - x_1)^2}$
$= | x_2 - x_1 |$

Question 4

Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

Answer:

Let us consider (a, 0) be the point on the x-axis that is equidistant from the point (7, 6) and (3, 4).
So,
Now, let
$\sqrt{(7-a)^2 + \ (6+0)^2} \ = \sqrt{(3-a)^2 + \ (4-0)^2}$
$\sqrt{49 + a^2 \ 14a + 36} = \sqrt{9 + a^2 \ 6a + 16}$

$\sqrt{a^2 + 14a + 58} = \sqrt{a^2 + 6a + 15}$
Now, let us square on both the sides we get,
a2 – 14a + 85 = a2 – 6a + 25
-8a = -60
a = 60/8
= 15/2
∴ The required point is (15/2, 0)

Question 5

Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, – 4) and B (8, 0).

Answer:

The co-ordinates of mid-point of the line segment joining the points P (0, – 4) and B (8, 0) are (0+8)/2, (-4+0)/2 = (4, -2)
The slope 'm' of the line non-vertical line passing through the point (x_1 , y_1) and (x_2, y_2) is given by m = (y_2 – y_1)/(x_2 – x_1) where, x ≠ x1
The slope of the line passing through (0, 0) and (4, -2) is (-2-0)/(4-0) = -1/2
∴ The required slope is -1/2.

Question 6

Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-angled triangle.

Answer:

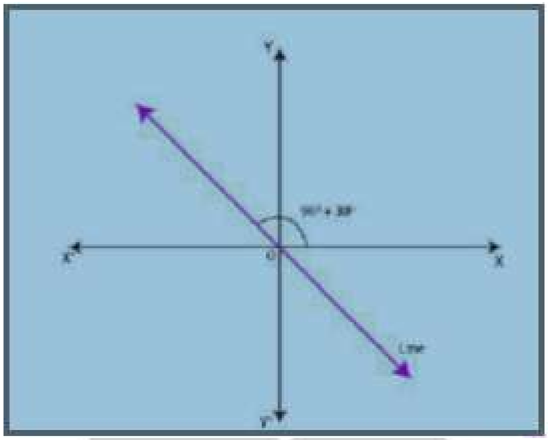
The vertices of the given triangle are (4, 4), (3, 5) and (–1, –1).
The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and (x_2, y_2) is given by m = (y_2 – y_1)/(x_2 – x_1) where, x ≠ x1
So, the slope of the line AB (m_1) = (5-4)/ (3-4) = 1/-1 = -1
the slope of the line BC (m_2) = (-1-5)/(-1-3) = -6/-4 = 3/2
the slope of the line CA (m_3) = (4+1)/(4+1) = 5/5 = 1
It is observed that, $m_1.m_3$ = -1.1 = -1
Hence, the lines AB and CA are perpendicular to each other
∴ given triangle is right-angled at A (4, 4)
And the vertices of the right-angled Δ are (4, 4), (3, 5) and (-1, -1)

Question 7

Find the slope of the line, which makes an angle of 30° with the positive direction of y-axis measured anticlockwise.

Answer:

We know that, if a line makes an angle of 30° with the positive direction of y-axis measured anti-clock-wise , then the angle made by the line with the positive direction of x- axis measure anti-clock-wise is 90° + 30° = 120°



∴ The slope of the given line is tan 120° = tan (180° - 60°)
= - tan 60°
= $-\sqrt{3}$

Question 8

Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Answer:

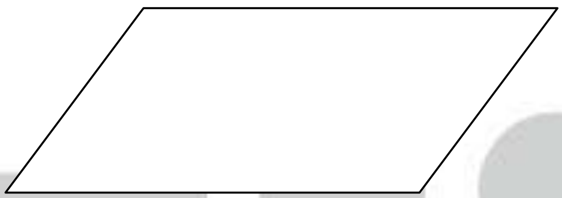
If the points (x, – 1), (2, 1) and (4, 5) are collinear, then Slope of AB = Slope of BC
Then, (1+1)/(2-x) = (5-1)/(4-2)
2/(2-x) = 4/2
2/(2-x) = 2
2 = 2(2-x)
2 = 4 – 2x
2x = 4 – 2
2x = 2
x = 2/2
= 1
∴ The required value of x is 1.

Question 9

Without using distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) are the vertices of a parallelogram.

Answer:

Let the given point be A (-2, -1) , B (4, 0) , C (3, 3) and D (-3, 2)



So now, The slope of AB = (0+1)/(4+2) = 1/6
The slope of CD = (3-2)/ (3+3) = 1/6
Hence, slope of AB = Slope of CD
∴ AB || CD
Now,
The slope of BC = (3-0)/ (3-4) = 3/-1 = -3
The slope of CD = (2+1)/ (-3+2) = 3/-1 = -3
Hence, slope of BC = Slope of CD
∴ BC || CD
Thus the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.
Hence the given vertices, A (-2, -1), B (4, 0), C (3, 3) and D (-3, 2) are vertices of a parallelogram.

Question 10

Find the angle between the x-axis and the line joining the points (3, –1) and (4, – 2).

Answer:

The Slope of the line joining the points (3, -1) and (4, -2) is given by
$m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x1$
m = (-2 –(-1))/(4-3)
= (-2+1)/(4-3)
= -1/1
= -1
The angle of inclination of line joining the points (3, -1) and (4, -2) is given by tan θ = -1
θ = (90° + 45°) = 135°
∴ The angle between the x-axis and the line joining the points (3, –1) and (4, –2) is 135°.

Question 11

The slope of a line is double of the slope of another line. If tangent of the angle between them is 1/3, find the slopes of the lines.

Answer:

Let us consider 'm1' and 'm' be the slope of the two given lines such that m1 = 2m
We know that if θ is the angle between the lines l1 and l2 with slope m1 and m2, then
$\tan\theta = \left|\frac{m_2 - m_2}{1+m_1\,m_2}\right|$
Given here the tangent of the angle between the two lines is 1/3
so
$\frac{1}{3} = \left|\frac{m-2\,m}{1+m_1\,m_2}\right| = \left|\frac{-m}{1+2\,m^2}\right|$
$\frac{1}{3} = \frac{-m}{1+2\,m^2}$

Now case 1:

$1+2m^2 = -3m$
$2m^2 + 1 + 3m = 0$
2m (m+1) + 1(m+1) = 0
(2m+1) (m+1)= 0
m = -1 or -1/2
If m = -1, then the slope of the lines are -1 and -2
If m = -1/2, then the slope of the lines are -1/2 and -1

Case 2:

$\frac{1}{3} = \frac{-m}{1+2\,m^2}$
$2m^2 - 3m + 1 = 0$
$2m^2 - 2m - m + 1 = 0$
2m (m - 1) - 1(m - 1) = 0
m = 1 or 1/2
If m = 1, then the slope of the lines are 1 and 2
If m = 1/2, then the slope of the lines are 1/2 and 1
∴ The slope of the lines are [-1 and -2] or [-1/2 and -1] or [1 and 2] or [1/2 and 1]

Question 12

A line passes through (x1, y1) and (h, k). If slope of the line is m, show that k – y1 = m (h – x1).

Answer:

Given: the slope of the line is 'm'
The slope of the line passing through (x1, y1) and (h, k) is (k – y1)/ (h – x1)
So,
$(k - y_1)/(h - x_1) = m$

$(k - y_1) = m\ (h - x_1)$
Hence proved

Question 13

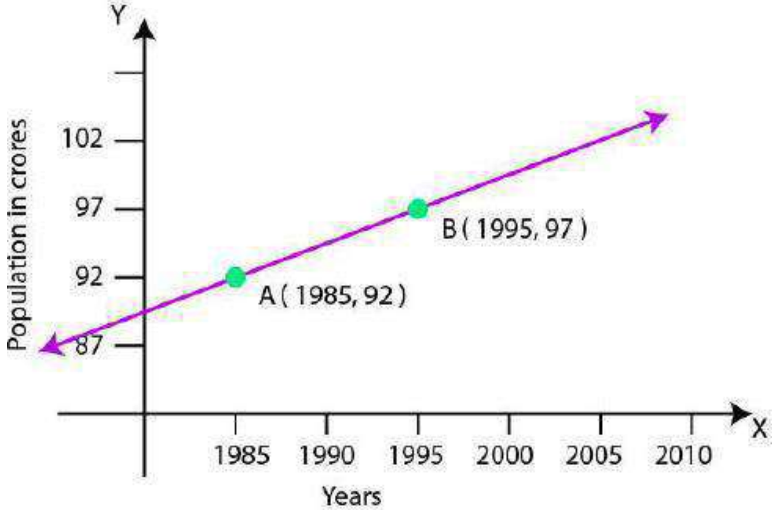
If three points (h, 0), (a, b) and (0, k) lie on a line, show that a/h + b/k = 1

Answer:

Let us consider if the given points A (h, 0), B (a, b) and C (0, k) lie on a line
Then, slope of AB = slope of BC
(b - 0)/(a - h) = (k - b)/(0 - a)
let us simplify we get,
-ab = (k-b) (a-h)
-ab = ka- kh –ab +bh
ka +bh = kh
Divide both the sides by kh we get,
ka/kh + bh/kh = kh/kh
a/h + b/k = 1
Hence proved.

Question 14

Consider the following population and year graph (Fig 10.10), find the slope of the line AB and using it, find what will be the population in the year 2010?



Answer:

We know that, the line AB passes through points A (1985, 92) and B (1995, 97),
Its slope will be (97 - 92)/ (1995 - 1985) = 5/10 = ½

Let 'y' be the population in the year 2010. Then, according to the given graph, AB must pass through point C (2010, y)
So now, slope of AB = slope of BC
$\frac{1}{2} = \frac{y-97}{2010-1995}$
15/2 = y - 97
y = 7.5 + 97 = 104.5
∴ The slope of the line AB is 1/2, while in the year 2010 the population will be 104.5 crores.

Exercise 10.2

In Exercises 1 to 8, find the equation of the line which satisfy the given conditions:

Question 1

Write the equations for the x-and y-axes.

Answer:

The y-coordinate of every point on x-axis is 0.
∴ Equation of x-axis is y = 0.
The x-coordinate of every point on y-axis is 0.
∴ Equation of y-axis is x = 0.

Question 2

Passing through the point (– 4, 3) with slope 1/2

Answer:

Given:
Point (-4, 3) and slope, m = 1/2
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
So, y – 3 = 1/2 (x – (-4))
y – 3 = 1/2 (x + 4)
2(y – 3) = x + 4
2y – 6 = x + 4
x + 4 – (2y – 6) = 0
x + 4 – 2y + 6 = 0
x – 2y + 10 = 0
∴ The equation of the line is x – 2y + 10 = 0.

Question 3

Passing through (0, 0) with slope m.

Answer:

Given:
Point (0, 0) and slope, m = m
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m(x - x_0)$
So, $y - 0 = m(x - 0)$
$y = mx$
$y - mx = 0$
∴ The equation of the line is $y - mx = 0$.

Question 4

Passing through (2, 2√3) and inclined with the x-axis at an angle of 75^0

Answer:

Given: point (2, 2√3) and θ = 75°
Equation of line: $(y - y_1) = m(x - x_1)$
Where, m = slope of line = tan θ and (x_1, y_1) are the points through which line passes
∴ m = tan 75°
75° = 45° + 30°
Applying the formula:
$\tan(A+B) = \frac{\tan A + B \tan B}{1 - \tan A . \tan B}$
$\tan(45+30) = \frac{\tan 45^0 + B \tan 30^0}{1 - \tan 45^0 \tan .30^0} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$
$\tan \mathbf{75^0} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$
Lets we get $\mathbf{75^0 \frac{3+1+2\sqrt{3}}{3-1} = 2 + \sqrt{3}}$
We know that the point (x, y) lies on the line with slope m through the fixed point (x_1, y_1), if and only if, its coordinates satisfy the equation $y - y_1 = m(x - x_1)$
Then, $y - 2\sqrt{3} = (2 + \sqrt{3})(x - 2)$
$y - 2\sqrt{3} = 2x - 4 + \sqrt{3}x - 2\sqrt{3}$
$y = 2x - 4 + \sqrt{3}x$
$(2 + \sqrt{3})x - y - 4 = 0$
∴ The equation of the line is $(2 + \sqrt{3})x - y - 4 = 0$.

Question 5

Intersecting the x-axis at a distance of 3 units to the left of origin with slope –2.

Answer:

Given:
Slope, m = -2
We know that if a line L with slope m makes x-intercept d, then equation of L is
y = m(x – d).
If the distance is 3 units to the left of origin then d = -3
So, y = (-2) (x – (-3))
y = (-2) (x + 3)
y = -2x – 6
2x + y + 6 = 0
∴ The equation of the line is 2x + y + 6 = 0.

Question 6

Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30o with positive direction of the x-axis.

Answer:

Given: θ = 30°
We know that slope, m = tan θ
m = tan30° = (1/√3)
We know that the point (x, y) on the line with slope m and y-intercept c lies on the line if and only if
y = mx + c.
If distance is 2 units above the origin, c = +2
So, y = (1/√3)x + 2
y = (x + 2√3) / √3
√3 y = x + 2√3
x - √3 y + 2√3 = 0
∴ The equation of the line is x - √3 y + 2√3 = 0.

Question 7

Passing through the points (–1, 1) and (2, – 4).

Answer:

Given:
Points (-1, 1) and (2, -4)
We know that the equation of the line passing through the points (x1, y1) and (x2, y2) is given by
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$y - y_1 = \frac{-4-1}{2-1}(x - (-1)$

y – 1 = -5/3 (x + 1)
3 (y – 1) = (-5) (x + 1)
3y – 3 = -5x – 5
3y – 3 + 5x + 5 = 0
5x + 3y + 2 = 0
∴ The equation of the line is 5x + 3y + 2 = 0.

Question 8

Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 30o.

Answer:

Given: p = 5 and ω = 30°
We know that the equation of the line having normal distance p from the origin and angle ω which the normal makes with the positive direction of x-axis is given by x cos ω + y sin ω = p.
Substituting the values in the equation, we get
x cos30° + y sin30° = 5
x(√3 / 2) + y(1/2) = 5
√3 x + y = 5(2) = 10
√3 x + y – 10 = 0
∴ The equation of the line is √3 x + y – 10 = 0.

Question 9

The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find equation of the median through the vertex R.

Answer:

Given:
Vertices of ΔPQR i.e. P (2, 1), Q (-2, 3) and R (4, 5)
Let RL be the median of vertex R.
So, L is a midpoint of PQ.
We know that the midpoint formula is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

∴ L $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$= (0,2)

We know that the equation of the line passing through the points (x1, y1) and (x2, y2) is given by y- y1 = $\frac{y_2 - y_1}{x_2 - x_1} = (x - x_1)$
∴ y – 5 = $\frac{2-5}{0-4}$ $(x - 4)$
y – 5 = -3/-4 (x-4)

(-4) (y – 5) = (-3) (x – 4)
-4y + 20 = -3x + 12
-4y + 20 + 3x – 12 = 0
3x – 4y + 8 = 0
∴ The equation of median through the vertex R is 3x – 4y + 8 = 0.

Question 10

Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

Answer:

Given:
Points are (2, 5) and (-3, 6).
We know that slope, m = (y2 – y1)/(x2 – x1)
= (6 - 5)/ (-3 - 2)
= 1/-5 = -1/5
We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.
Then, m = (-1/m)
= -1/ (-1/5)
= 5
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y0), if and only if, its coordinates satisfy the equation y – y0 = m (x – x_0)
Then, y – 5 = 5(x – (-3))
y – 5 = 5x + 15
5x + 15 – y + 5 = 0
5x – y + 20 = 0
∴ The equation of the line is 5x – y + 20 = 0

Question 11

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.

Answer:

We know that the coordinates of a point dividing the line segment joining the points (x1, y1) and (x2, y2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\left(\frac{1\,(2)+n\,(1)}{1+n}, \frac{1(3)+n\,(0)}{1+n}\right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

We know that slope, m = (y2 – y1)/(x2 – x1)

= (3 - 0)/ (2 - 1)
= 3/1
= 3

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.
Then, m = (-1/m) = -1/3
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_2 = m (x - x_0)$
Here, the point is
$\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$
$(y - \frac{3}{1+n}) = \frac{-1}{3} (x - \frac{2+n}{1+n}$
3((1 + n) y – 3) = (-(1 + n) x + 2 + n)
3(1 + n) y – 9 = - (1 + n) x + 2 + n
(1 + n) x + 3(1 + n) y – n – 9 – 2 = 0
(1 + n) x + 3(1 + n) y – n – 11 = 0
∴ The equation of the line is (1 + n) x + 3(1 + n) y – n – 11 = 0.

Question 12

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Answer:

Given: the line cuts off equal intercepts on the coordinate axes i.e. a = b.
We know that equation of the line intercepts a and b on x-and y-axis, respectively, which is
x/a + y/b = 1
So, x/a + y/a = 1
x + y = a … (1)
Given: point (2, 3)
2 + 3 = a
a = 5
Substitute value of 'a' in (1), we get
x + y = 5
x + y – 5 = 0
∴ The equation of the line is x + y – 5 = 0.

Question 13

Find equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Answer:

We know that equation of the line making intercepts a and b on x-and y-axis, respectively, is x/a + y/b = 1. … (1)
Given: sum of intercepts = 9
a + b = 9
b = 9 - a
Now, substitute value of b in the above equation, we get
x/a + y/(9 - a) = 1
Given: the line passes through the point (2, 2),
So, 2/a + 2/(9 - a) = 1
[2(9 - a) + 2a] / a(9 - a) = 1
[18 – 2a + 2a] / a(9 - a) = 1
18/a(9 - a) = 1
18 = a (9 – a)
18 = 9a – a2
a2 – 9a + 18 = 0
Upon factorizing, we get
a2 – 3a – 6a + 18 = 0
a (a – 3) – 6 (a – 3) = 0
(a – 3) (a – 6) = 0
a = 3 or a = 6
Let us substitute in (1),
Case 1 (a = 3):
Then b = 9 – 3 = 6
x/3 + y/6 = 1
2x + y = 6
2x + y – 6 = 0
Case 2 (a = 6):
Then b = 9 – 6 = 3
x/6 + y/3 = 1
x + 2y = 6
x + 2y – 6 = 0
∴ The equation of the line is 2x + y – 6 = 0 or x + 2y – 6 = 0.

Question 14

Find equation of the line through the point (0, 2) making an angle 2π/3 with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Answer:

Given:
Point (0, 2) and θ = 2π/3
We know that m = tan θ

$m = \tan(2\pi/3) = -\sqrt{3}$

We know that the point (x, y) lies on the line with slope m through the fixed point (x0, y0), if and only if, its coordinates satisfy the equation y – y0 = m (x – x0)

$y - 2 = -\sqrt{3}(x - 0)$

$y - 2 = -\sqrt{3}x$

$\sqrt{3}x + y - 2 = 0$

Given, equation of line parallel to above obtained equation crosses the y-axis at a distance of 2 units below the origin.

So, the point = (0, -2) and $m = -\sqrt{3}$

From point slope form equation,

$y - (-2) = -\sqrt{3}(x - 0)$

$y + 2 = -\sqrt{3}x$

$\sqrt{3}x + y + 2 = 0$

∴ The equation of line is $\sqrt{3}x + y - 2 = 0$ and the line parallel to it is $\sqrt{3}x + y + 2 = 0$.

Question 15

The perpendicular from the origin to a line meets it at the point (–2, 9), find the equation of the line.

Answer:

Given:

Points are origin (0, 0) and (-2, 9).

We know that slope, m = (y2 – y1)/(x2 – x1)

= (9 - 0)/(-2-0)

= -9/2

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

m = (-1/m) = -1/(-9/2) = 2/9

We know that the point (x, y) lies on the line with slope m through the fixed point (x0, y0), if and only if, its coordinates satisfy the equation y – y0 = m (x – x0)

$y - 9 = (2/9)(x - (-2))$

$9(y - 9) = 2(x + 2)$

$9y - 81 = 2x + 4$

$2x + 4 - 9y + 81 = 0$

$2x - 9y + 85 = 0$

∴ The equation of line is $2x - 9y + 85 = 0$.

Question 16

The length L (in centimeter) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L= 125.134 when C = 110, express L in terms of C.

Answer:

Let us assume 'L' along X-axis and 'C' along Y-axis, we have two points (124.942, 20) and (125.134, 110) in XY-plane.
We know that the equation of the line passing through the points (x1, y1) and (x2, y2) is given by
y- y_1 = $\frac{y_2 - y_1}{x_2 - x_1}$ $(x - x_1)$
C- 20 = $\frac{110-20}{025.034-124.942}$ (L- 124. 942)
C- 20 $\frac{90}{0.192}$ (L – 124. 942)
0. 192 (C – 20) = 90 (L – 124. 942)
L= $\frac{0.192}{90}$ (C- 20) + 124. 942
∴ the required relation is L = $\frac{0.192}{90}$ (C- 20) + 124. 942

Question 17

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs. 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs. 17/litre?

Answer:

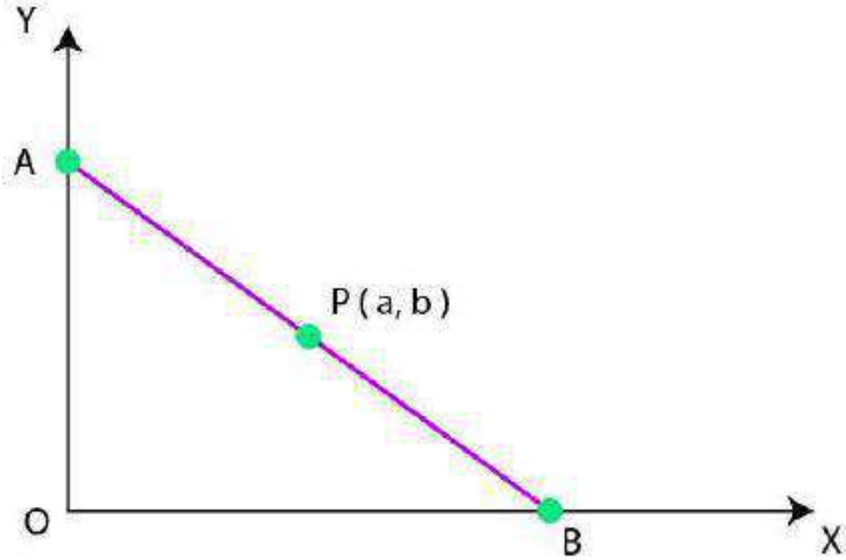
Assuming the relationship between selling price and demand is linear.
Let us assume selling price per litre along X-axis and demand along Y-axis, we have two points (14, 980) and (16, 1220) in XY-plane.
We know that the equation of the line passing through the points (x1, y1) and (x2, y2) is given by
y –y_1 = = $\frac{y_2 - y_1}{x_2 - x_1}$ $(x - x_1)$
y – 980 = $\frac{1220-980}{16-14}$ $(x - 14)$
y- 980 = $\frac{240}{2}$ (x- 14)
y – 980 = 120 (x – 14)
y = 120 (x – 14) + 980
When x = Rs 17/litre,
y = 120 (17 – 14) + 980
y = 120(3) + 980
y = 360 + 980 = 1340
∴ The owner can sell 1340 litres weekly at Rs. 17/litre.

Question 18

P (a, b) is the mid-point of a line segment between axes. Show that equation of the line is x/a + y/b = 2

Answer:

Let AB be a line segment whose midpoint is P (a, b).
Let the coordinates of A and B be (0, y) and (x, 0) respectively.



We know that the midpoint is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since P is the midpoint of (a,b)

$\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$ = (a,b)

$\left(\frac{x}{2}, \frac{y}{2}\right)$ = (a, b)

a= x/2 and b = y/2

x= 2_a and y = 2b

A= (0, 2b) and B = (2a, 0)

We know that the equation of the line passing through the point (x_1, x_1) and (x_2, y_2) is given by

y – y_1 = $\frac{y_2-y_1}{x_2-x_1}$ $(x - x_1)$

y – y 2b $\frac{0-2b}{za-0}$ $(x-0)$

y – y 2b $\frac{-2b}{za}$(x)

y – y 2b $\frac{-b}{a}$(x)

a (y – 2b) = -bx

ay – 2ab = -bx

bx + ay = 2ab

Divide both the sides with ab, then

$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2\,ab}{ab}$

$\frac{x}{a} + \frac{y}{b}$ = 2

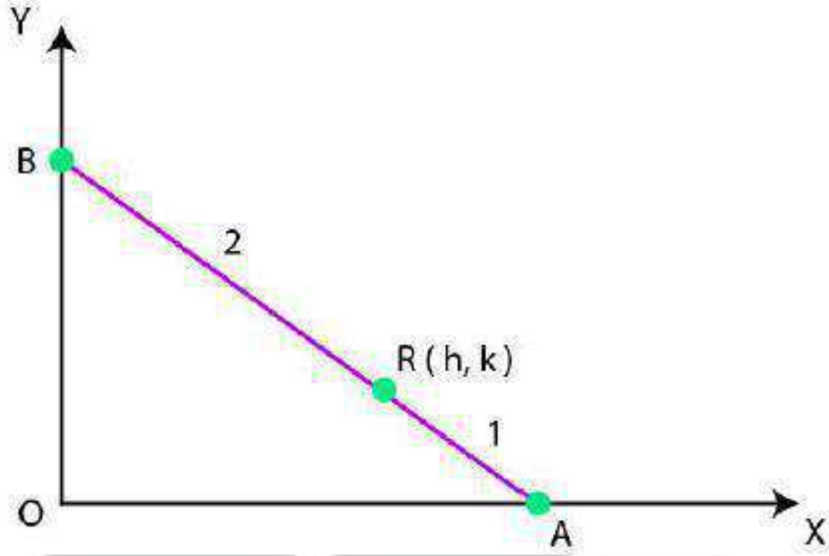
Hence proved.

Question 19

Point R (h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the line.

Answer:

Let us consider, AB be the line segment such that r (h, k) divides it in the ratio 1: 2.
So the coordinates of A and B be (0, y) and (x, 0) respectively.



We know that the coordinates of a point dividing the line segment joins the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n is

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$\left(\frac{1\,(0)+2\,(x)}{1+z}, \frac{1(y)+2\,(0)}{1+z}\right)$ = (h,k)

$\left(\frac{2x}{3}, \frac{y}{3}\right)$ = h, k

h = 2x/3 and k = y/3
x = 3h/2 and y = 3k
∴ A = (0, 3k) and B = (3h/2, 0)
We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$y - 3k = \frac{0-3k}{\frac{3h}{2}-0}(x-0)$
3h(y - 3k) = -6kx

3hy - 9hk = -6kx
6kx + 3hy = 9hk
Let us divide both the sides by 9hk, we get,
2x/3h + y/3k = 1
∴ The equation of the line is given by 2x/3h + y/3k = 1

Question 20

By using the concept of equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.

Answer:

According to the question,
If we have to prove that the given three points (3, 0), (– 2, – 2) and (8, 2) are collinear, then we have to also prove that the line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).
By using the formula,
The equation of the line passing through the points (x1, y1) and (x2, y2) is given by
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$y - 0 = \frac{-2-0}{-2-3}(x - 3)$
$y = \frac{-2}{-5}(x - 3)$
-5y = -2 (x – 3)
-5y = -2x + 6
2x – 5y = 6
If 2x – 5y = 6 passes through (8, 2),
2x – 5y = 2(8) – 5(2)
= 16 – 10
= 6
= RHS
The line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2). Hence proved. The given three points are collinear.

Exercise 10.3

Question 1

Reduce the following equations into slope - intercept form and find their slopes and they - intercepts.
(i) x + 7y = 0
(ii) 6x + 3y – 5 = 0
(iii) y = 0

Answer:

(i) x + 7y = 0

Given:

The equation is x + 7y = 0

Slope – intercept form is represented in the form 'y = mx + c', where m is the slope and c is the y intercept

So, the above equation can be expressed as

y = -1/7x + 0

∴ The above equation is of the form y = mx + c, where m = -1/7 and c = 0.

(ii) 6x + 3y – 5 = 0

Given:

The equation is 6x + 3y – 5 = 0

Slope – intercept form is represented in the form 'y = mx + c', where m is the slope and c is the y intercept

So, the above equation can be expressed as

3y = -6x + 5

y = -6/3x + 5/3

= -2x + 5/3

∴ The above equation is of the form y = mx + c, where m = -2 and c = 5/3.

(iii) y = 0

Given:

The equation is y = 0

Slope – intercept form is given by 'y = mx + c', where m is the slope and c is the y intercept

y = 0 × x + 0

∴ The above equation is of the form y = mx + c, where m = 0 and c = 0.

Question 2

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) 3x + 2y – 12 = 0

(ii) 4x – 3y = 6

(iii) 3y + 2 = 0

Answer:

(i) 3x + 2y – 12 = 0

Given:

The equation is 3x + 2y – 12 = 0

Equation of line in intercept form is given by x/a + y/b = 1, where 'a' and 'b' are intercepts on x axis and y – axis respectively.

So, 3x + 2y = 12

now let us divide both sides by 12, we get

3x/12 + 2y/12 = 12/12

$x/4 + y/6 = 1$
∴ The above equation is of the form $x/a + y/b = 1$, where $a = 4$, $b = 6$
Intercept on x – axis is 4
Intercept on y – axis is 6
(ii) 4x – 3y = 6
Given:
The equation is $4x – 3y = 6$
Equation of line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepts on x axis and y – axis respectively.
So, $4x – 3y = 6$
Now let us divide both sides by 6, we get
$4x/6 – 3y/6 = 6/6$
$2x/3 – y/2 = 1$
$x/(3/2) + y/(-2) = 1$
∴ The above equation is of the form $x/a + y/b = 1$, where $a = 3/2$, $b = -2$
Intercept on x – axis is 3/2
Intercept on y – axis is -2
(iii) 3y + 2 = 0
Given:
The equation is $3y + 2 = 0$
Equation of line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepts on x axis and y – axis respectively.
So, $3y = -2$
Now, let us divide both sides by -2, we get
$3y/-2 = -2/-2$
$3y/-2 = 1$
$y/(-2/3) = 1$
∴ The above equation is of the form $x/a + y/b = 1$, where $a = 0$, $b = -2/3$
Intercept on x – axis is 0
Intercept on y – axis is -2/3

Question 3

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.
(i) $x - \sqrt{3}y + 8 = 0$
(ii) $y – 2 = 0$
(iii) $x – y = 4$

Answer:

(i) $x - \sqrt{3}y + 8 = 0$
Given:
The equation is $x - \sqrt{3}y + 8 = 0$
Equation of line in normal form is given by $x \cos \theta + y \sin \theta = p$ where 'θ' is the angle

between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, $x - \sqrt{3}y + 8 = 0$
$x - \sqrt{3}y = -8$
Divide both the sides by $\sqrt{(12 + (\sqrt{3})2)} = \sqrt{(1 + 3)} = \sqrt{4} = 2$
$x/2 - \sqrt{3}y/2 = -8/2$
$(-1/2)x + \sqrt{3}/2y = 4$
This is in the form of: x cos 120o + y sin 120o = 4
∴ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 120°$ and $p = 4$.
Perpendicular distance of line from origin = 4
Angle between perpendicular and positive x – axis = 120°

(ii) y – 2 = 0

Given:
The equation is $y - 2 = 0$
Equation of line in normal form is given by $x \cos \theta + y \sin \theta = p$ where 'θ' is the angle between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, $0 \times x + 1 \times y = 2$
Divide both sides by $\sqrt{(02 + 12)} = \sqrt{1} = 1$
$0 (x) + 1 (y) = 2$
This is in the form of: x cos 90o + y sin 90o = 2
∴ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 90°$ and $p = 2$.
Perpendicular distance of line from origin = 2
Angle between perpendicular and positive x – axis = 90°

(iii) x – y = 4

Given:
The equation is $x - y + 4 = 0$
Equation of line in normal form is given by $x \cos \theta + y \sin \theta = p$ where 'θ' is the angle between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, $x - y = 4$
Divide both the sides by $\sqrt{(12 + 12)} = \sqrt{(1+1)} = \sqrt{2}$
$x/\sqrt{2} - y/\sqrt{2} = 4/\sqrt{2}$
$1/\sqrt{2}x + (-1/\sqrt{2})y = 2\sqrt{2}$
This is in the form: x cos 315o + y sin 315o = $2\sqrt{2}$
∴ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 315°$ and $p = 2\sqrt{2}$.
Perpendicular distance of line from origin = $2\sqrt{2}$
Angle between perpendicular and positive x – axis = 315°

Question 4

Find the distance of the point (–1, 1) from the line 12(x + 6) = 5(y – 2).

Answer:

Given:
The equation of the line is $12(x + 6) = 5(y - 2)$.
$12x + 72 = 5y - 10$

12x – 5y + 82 = 0 … (1)
Now, compare equation (1) with general equation of line Ax + By + C = 0, where A = 12, B = –5, and C = 82
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by
$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
Given point $(x_1, y_1) = (-1, 1)$
∴ Distance point (- 1,1) from the given line is
$d = \frac{|12 \times (-1) + (-5) \times 1 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{144+25}} = \frac{|65|}{\sqrt{169}} = \frac{65}{13}$ units
= 5 units
∴ The distance is 5units.

Question 5

Find the points on the x-axis, whose distances from the line x/3 + y/4 = 1 are 4 units.

Answer:

Given:
The equation of line is x/3 + y/4 = 1
4x + 3y = 12
4x + 3y – 12 = 0 …. (1)
Now, compare equation (1) with general equation of line Ax + By + C = 0, where A = 4, B = 3, and C = -12
Let (a, 0) be the point on the x-axis, whose distance from the given line is 4 units.
So, the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by
$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$
$4 = \frac{|4a - 12|}{\sqrt{16+9}} = \frac{4a - 12}{5}$
|4a – 12| = 4 × 5
± (4a – 12) = 20
4a – 12 = 20 or - (4a – 12) = 20
4a = 20 + 12 or 4a = -20 + 12
a = 32/4 or a = -8/4
a = 8 or a = -2
∴ The required points on the x – axis are (-2, 0) and (8, 0)

Question 6

Find the distance between parallel lines
(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
(ii) l(x + y) + p = 0 and l (x + y) – r = 0

Answer:

(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
Given:
The parallel lines are 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0.
By using the formula,
The distance (d) between parallel lines Ax + By + C1 = 0 and Ax + By + C2 = 0 is given by
d = $\frac{|c_1 - c_2|}{\sqrt{A^2 + B^2}}$
Where A = 15, B = 8 C_1 = - 34, C_2 = 3
Distance between parallel lines id
d= $\frac{|-34-31|}{\sqrt{15^2+8^2}}$
= $\frac{|-65|}{\sqrt{225+64}}$
= 65/√289
= 65/17
∴ The distance between parallel lines is 65/17
(ii) l(x + y) + p = 0 and l (x + y) – r = 0
Given:
The parallel lines are l (x + y) + p = 0 and l (x + y) – r = 0.
lx + ly + p = 0 and lx + ly – r = 0
by using the formula,
The distance (d) between parallel lines Ax + By + C1 = 0 and Ax + By + C2 = o is given

Question 7

Find equation of the line parallel to the line 3x – 4y + 2 = 0 and passing through the point (–2, 3).

Answer:

Given:
The line is 3x – 4y + 2 = 0
So, y = 3x/4 + 2/4
= 3x/4 + ½
Which is of the form y = mx + c, where m is the slope of the given line.
The slope of the given line is 3/4
We know that parallel line have same slope.
∴ Slope of other line = m = 3/4
Equation of line having slope m and passing through (x1, y1) is given by
y – y1 = m (x – x1)
∴ Equation of line having slope 3/4 and passing through (-2, 3) is
y – 3 = ¾ (x – (-2))

4y – 3 × 4 = 3x + 3 × 2
3x – 4y = 18
∴ The equation is 3x – 4y = 18

Question 8

Find equation of the line perpendicular to the line x – 7y + 5 = 0 and having x intercept 3.

Answer:

Given:
The equation of line is x – 7y + 5 = 0
So, y = 1/7x + 5/7 [which is of the form y = mx + c, where m is the slope of the given line.]
Slope of the given line is 1/7
Slope of the line perpendicular to the line having slope m is -1/m
Slope of the line perpendicular to the line having a slope of 1/7 is -1/(1/7) = -7
So, the equation of line with slope -7 and x intercept 3 is given by y = m(x – d)
y = -7 (x – 3)
y = -7x + 21
7x + y = 21
∴ The equation is 7x + y = 21

Question 9

Find angles between the lines √3x + y = 1 and x + √3y = 1.

Answer:

Given:
The lines are √3x + y = 1 and x + √3y = 1
So, y = -√3x + 1 … (1) and
y = -1/√3x + 1/√3 …. (2)
Slope of line (1) is m1 = -√3, while the slope of line (2) is m2 = -1/√3
Let θ be the angle between two lines

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)}{1 + (\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)}\right| = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$

= 1/ √3
$\theta = 30^0$
∴ The angle between the given lines is either 30° or 180°- 30° = 150°

Question 10

The line through the points (h, 3) and (4, 1) intersects the line 7x – 9y –19 = 0. At right angle. Find the value of h.

Answer:

Let the slope of the line passing through (h, 3) and (4, 1) be m1
Then, m1 = (1-3)/(4-h) = -2/(4-h)
Let the slope of line 7x – 9y – 19 = 0 be m2
7x – 9y – 19 = 0
So, y = 7/9x – 19/9
m2 = 7/9
Since, the given lines are perpendicular
m1 × m2 = -1
-2/(4-h) × 7/9 = -1
-14/(36-9h) = -1
-14 = -1 × (36 – 9h)
36 – 9h = 14
9h = 36 – 14
h = 22/9
∴ The value of h is 22/9

Question 11

Prove that the line through the point (x1, y1) and parallel to the line Ax + By + C = 0 is A (x – x1) + B (y – y1) = 0.

Answer:

Let the slope of line Ax + By + C = 0 be m
Ax + By + C = 0
So, y = -A/Bx – C/B
m = -A/B
By using the formula,
Equation of the line passing through point (x1, y1) and having slope m = -A/B is
y – y1 = m (x – x1)
= -A/B (x – x1)
B (y – y1) = -A (x – x1)
∴ A(x – x1) + B(y – y1) = 0
So, the line through point (x1, y1) and parallel to the line Ax + By + C = 0 is A (x – x1) + B (y – y1) = 0
Hence proved.

Question 12

Two lines passing through the point (2, 3) intersects each other at an angle of 60o. If slope of one line is 2, find equation of the other line.

Answer:

Given: m1 = 2
Let the slope of the first line be m1
And let the slope of the other line be m2.
Angle between the two lines is 6
0°.

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_2\, m_2}\right|$

$\tan 60^0 = \left|\frac{2 - m_2}{1 + 2\, m_2}\right|$

$\sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2\, m_2}\right)$

$\sqrt{3} = \frac{2 - m_2}{1 + 2\, m_2}$ or $\sqrt{3} = -\left(\frac{2 - m_2}{1 + 2\, m_2}\right)$

$\sqrt{3} + (1 + 2\, m_2) = 2 - m_2$ or $\sqrt{3} + (1 + 2\, m_2) = (-2 - m_2)$

$\sqrt{3} + 2\sqrt{3\, m_2} + m_2 = 2$ or $\sqrt{3} + 2\sqrt{3\, m_2} - m_2 = -2$

$m_2\,(2\sqrt{3} + 1) = 2 - (2\sqrt{3} - 1) = -(2 + \sqrt{3})$

$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)}$ or $m_2 = \frac{2 + \sqrt{3}}{(2\sqrt{3} - 1)}$

So now let us consider
Case 1: When

$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)}$

The equation of the line passing through point (2,3) and having a slope m_2 is

$y - 3 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)(x - 2)$

$(2\sqrt{3} + 1)\, y - 3\,(2\sqrt{3} + 1) = (2 - \sqrt{3})$

$(\sqrt{3} - 2)\, x + (2\sqrt{3} + 1) = 4 + 2\sqrt{3} + 6\sqrt{3} + 3$

$(\sqrt{3} - 2)\, x + (2\sqrt{3} + 1)\, y = 8\sqrt{3} - 1$

∴ Equation of the other line is $(\sqrt{3} - 2)\, x + (2\sqrt{3} + 1)\, y = 8\sqrt{3} - 1$

Case 2: When

$m_2 = \frac{2 + \sqrt{3}}{(2\sqrt{3} - 1)}$

The equation of the line passing through point (2,3) and having a slope m_2 is

$y - 3 = \left(\frac{2 + \sqrt{3}}{2\sqrt{3} - 1}\right)(x - 2)$

$(2\sqrt{3} - 1)\, y - 3\,(2\sqrt{3} - 1) = (2 - \sqrt{3})\, x + 2(2 + \sqrt{3})$

$(2\sqrt{3} - 1)\, y + (2 - \sqrt{3})\, x + 4 + 2\sqrt{3} + 6\sqrt{3} - 3$

$(2\sqrt{3} - 1)\, y + (2 - \sqrt{3})\, x = 8\sqrt{3} + 1$

∴ Equation of the other line is$(2\sqrt{3} -1) y + (2 - \sqrt{3}) x = 8\sqrt{3} + 1$

Question 13

Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Answer:

Given:
The right bisector of a line segment bisects the line segment at 90°.
End-points of the line segment AB are given as A (3, 4) and B (–1, 2).
Let mid-point of AB be (x, y)
x = (3+4)y/2
y = (-1+2)/2
(x, y) = (7/2, 1/2)
Let the slope of line AB be m1
m1 = (2 – 4)/(-1 – 3)
= -2/(-4)
= 1/2
And let the slope of the line perpendicular to AB be m2
m2 = -1/(1/2)
= -2
The equation of the line passing through (1, 3) and having a slope of –2 is
(y – 3) = -2 (x – 1)
y – 3 = - 2x + 2
2x + y = 5
∴ The required equation of the line is 2x + y = 5

Question 14

Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.

Answer:

Let us consider the co-ordinates of the foot of the perpendicular from (-1, 3) to the line 3x – 4y – 16 = 0 be (a, b)
So, let the slope of the line joining (-1, 3) and (a, b) be m1
m1 = (b-3)/(a+1)
And let the slope of the line 3x – 4y – 16 = 0 be m2
y = 3/4x – 4

$m2 = 3/4$
Since these two lines are perpendicular, $m1 \times m2 = -1$
$(b-3)/(a+1) \times (3/4) = -1$
$(3b-9)/(4a+4) = -1$
$3b - 9 = -4a - 4$
$4a + 3b = 5$ …….(1)
Point (a, b) lies on the line $3x - 4y = 16$
$3a - 4b = 16$ ……..(2)
Solving equations (1) and (2), we get
$a = 68/25$ and $b = -49/25$
∴ The co-ordinates of the foot of perpendicular is (68/25, -49/25)

Question 15

The perpendicular from the origin to the line y = mx + c meets it at the point (−1, 2). Find the values of m and c.

Answer:

Given:
The perpendicular from the origin meets the given line at (−1, 2).
The equation of line is $y = mx + c$
The line joining the points (0, 0) and (−1, 2) is perpendicular to the given line.
So, the slope of the line joining (0, 0) and (−1, 2) = $2/(-1) = -2$
Slope of the given line is m.
$m \times (-2) = -1$
$m = 1/2$
Since, point (-1, 2) lies on the given line,
$y = mx + c$
$2 = 1/2 \times (-1) + c$
$c = 2 + 1/2 = 5/2$
∴ The values of m and c are 1/2 and 5/2 respectively.

Question 16

If p and q are the lengths of perpendiculars from the origin to the lines x cos θ − y sin θ = k cos 2θ and x sec θ + y cosec θ = k, respectively, prove that p2 + 4q2 = k2

Answer:

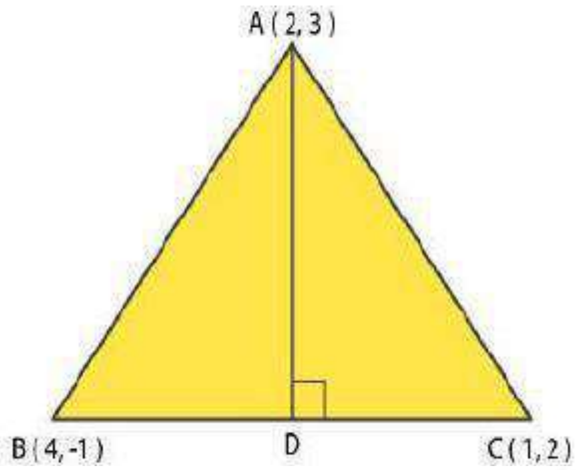
Given:
The equations of given lines are
$x \cos \theta - y \sin \theta = k \cos 2\theta$ …………………… (1)

x sec θ + y cosec θ = k ………………… (2)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by
d $=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$
So now compare equation (i) to the general equation of line i.e., Ax + By + c = 0, we get
A = $\cos\theta$, B = -$\sin\theta$ and C = -k $\cos2\theta$
I is given that p is the length of the particular from (0,0) o line (1)
p$=\frac{A\times0+B\times0+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-K\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}$ = K cos 2 θ
p= K cos 2 θ
Let us square on both side be get
$p^2 = k^2\cos^2 2\theta$ … … … … … …(3)
Now compare equation (2) to the general equation of line i.e. Ax + By + C =0, we get
A = secθ , B = cosec θ, and C = -K
It is given that q is that of the perpendicular from (0,0) to line (2)
p$=\frac{A\times0+B\times0+C|}{\sqrt{A^2+B^2}}$
$=\frac{|C|}{\sqrt{A^2+B^2}}$
$=\frac{|-K\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}$
$=\frac{|-K|}{\sqrt{\sec^2\theta+cosec^2\theta}}$
$\frac{k}{\sqrt{Sec^2\theta+cosec^2\theta}}$
$\frac{k}{\sqrt{\frac{1}{\cos^2\vartheta}+\frac{1}{cosec\ 2\theta}}}=\frac{k\cos\theta\sin\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}$ k= cosθ $sin\theta$
q = k cosθ sin θ
Multiply both sides by 2, we get
2q = 2k cos θ sin θ = k × 2sin θ cos θ
2q = k sin 2θ
Squaring both sides, we get
4q2 = k2 sin22θ …………………(4)
Now add (3) and (4) we get
p2 + 4q2 = k2 cos2 2θ + k2 sin2 2θ
p2 + 4q2 = k2 (cos2 2θ + sin2 2θ) [Since, cos2 2θ + sin2 2θ = 1]
∴ p2 + 4q2 = k2
Hence proved.

Question 17

In the triangle ABC with vertices A (2, 3), B (4, –1) and C (1, 2), find the equation and length of altitude from the vertex A.

Answer:



Let AD be the altitude of triangle ABC from vertex A.
So, AD is perpendicular to BC
Given:
Vertices A (2, 3), B (4, −1) and C (1, 2)
Let slope of line BC = m1
m1 = (- 1 - 2)/(4 - 1)
m1 = -1
Let slope of line AD be m2
AD is perpendicular to BC
m1 × m2 = -1
-1 × m2 = -1
m2 = 1
The equation of the line passing through point (2, 3) and having a slope of 1 is
y – 3 = 1 × (x – 2)
y – 3 = x – 2
y – x = 1
Equation of the altitude from vertex A = y – x = 1
Length of AD = Length of the perpendicular from A (2, 3) to BC
Equation of BC is
y + 1 = -1 × (x – 4)
y + 1 = -x + 4
x + y – 3 = 0 …………………(1)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by
d = $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
Now compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get
Length of AD = $\frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} = \frac{|2|}{\sqrt{2}} = \sqrt{2}$ units
[where, A = 1, B = 1 and C = -3]
∴ The equation and the length of the altitude from vertex A are y – x = 1 and
√2 units respectively.

Question 18

If p is the length of perpendicular from the origin to the line whose intercepts on the axes area and b, then show that 1/p2 = 1/a2 + 1/b2

Answer:

Equation of a line whose intercepts on the axes area and b is x/a + y/b = 1
bx + ay = ab
bx + ay – ab = 0 ……………….(1)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by
d =$\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
p = $\frac{|A \times 0 + B \times 0 - ab}{\sqrt{a^2 + b^2}}$
= $\frac{|-ab|}{\sqrt{a^2 + b^2}}$
Now compare equation (1) to the line general equation of line i.e. Ax +By + C = 0 we get
A =b , B = a and C= -ab
If p is the length of the perpendicular from point (x_1, y_1) = (0, 0) to line (1) we get

p = $\frac{|A \times 0 + B \times 0 - ab}{\sqrt{a^2 + b^2}}$
= $\frac{|-ab|}{\sqrt{a^2 + b^2}}$

Now square on both the sides we get
p^2 $\frac{(-ab)^2}{a^2 + b^2}$
$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2\, b^2}$
$\frac{1}{p^2} = \frac{a^2}{a^2\, b^2} + \frac{b^2}{a^2 b^2}$
∴ $1/p^2 = 1/a_2 + 1/b^2$
Hence proved.

Miscellaneous Exercise

Question 1

Find the values of k for which the line (k – 3) x – (4 – k^2) y + k2 – 7k + 6 = 0 is

a) Parallel to the x-axis,
b) Parallel to the y-axis,
c) Passing through the origin.

Answer:

It is given that
(k – 3) x – (4 – k^2) y + k^2 – 7k + 6 = 0 … (1)
(a) Here if the line is parallel to the x-axis
Slope of the line = Slope of the x-axis
It can be written as
(4 – k^2) y = (k – 3) x + k^2 – 7k + 6 = 0
We get
y= $\frac{(k-3)}{(4-k^2)}\ x + \frac{k^2-7k+6}{(4-k^2)}$

Which is of the from y = mx + c
Here the slope of the x –axis
y= $\frac{(k-3)}{(4-k^2)}$

Consider the slope of the given line
= $\frac{(k-3)}{(4-k^2)}$
By further calculation
k – 3 = 0
k = 3
Hence, if the given line is parallel to the x-axis, then the value of k is 3.

(b) Here if the line is parallel to the y-axis, it is vertical and the slope will be undefined. So the slope of the given line
Hence, if the given line is parallel to the y-axis, then the value of k is ± 2.
= $\frac{(k-3)}{(4-k^2)}$
Here
= $\frac{(k-3)}{(4-k^2)}$ is undefined at K^2 = 4
k^2 = 4
k =$\pm$ 2

(c) Here if the line is passing through (0, 0) which is the origin satisfies the given equation of line.
(k – 3) (0) – (4 – k^2) (0) + k^2 – 7k + 6 = 0
By further calculation
k^2 – 7k + 6 = 0
Separating the terms
k^2– 6k – k + 6 = 0
We get
(k – 6) (k – 1) = 0
k = 1 or 6
Hence, if the given line is passing through the origin, then the value of k is either 1 or 6.

Question 2

Find the values of θ and p, if the equation x cos θ + y sin θ = p is the normal form of the line $\sqrt{3}$x + y + 2 = 0.

Answer:

It is given that
$\sqrt{3}$ x + y + 2 = 0
It can be reduce as
$\sqrt{3}$ x + y + 2 = 0
- $\sqrt{3}$ x –y = 0
By divided both sides by $\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$ - 2we get
- $\frac{\sqrt{3}}{2}$ x + -$\frac{1}{2}$ y = $\frac{1}{2}$

It can be written as
$\left(\frac{\sqrt{3}}{2}\right) x + \left(-\frac{1}{2}\right)$ y = 1 ……..(1)

By comparing equation (1) to x cos$\theta + y$ sin $\theta = p$, we get
cos$\theta - \frac{\sqrt{3}}{2}$ sin θ = -$\frac{1}{2}$ and p = 1
Here the value of sin θ and cos θ are negative
$\theta = \pi = \frac{\pi}{6} = \frac{7\pi}{6}$

Question 3

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and –6, respectively.

Answer:

Consider the intercepts cut by the given lines on a and b axes.
a + b = 1 …… (1)
ab = - 6 …….. (2)
By solving both the equations we get
a = 3 and b = -2 or a = - 2 and b = 3
We know that the equation of the line whose intercepts on a and b axes is
$\frac{x}{a} + \frac{y}{b}$ = 1 or bx + ay –ab = 0
Case I – a = 3 and b = - 2
So the equation of the line is – 2x + 3y + 6 = 0, i.e. 2x – 3y = 6.
Case II – a = -2 and b = 3
So the equation of the line is 3x – 2y + 6 = 0, i.e. -3x + 2y = 6

Hence, the required equation of the lines are 2x – 3y = 6 and -3x + 2y = 6.

Question 4

What are the points on the y-axis whose distance from the line x/3 + y/4 = 1 is 4 units.

Answer:

Consider (0, b) as the point on the y-axis whose distance from line x/3 + y/4 = 1 is 4 units.
It can be written as 4x + 3y – 12 = 0 ……. (1)
By comparing equation (1) to the general equation of line Ax + By + C = 0, we get
A = 4, B = 3 and C = - 12
We know that the perpendicular distance (d) of a line Ax + By + C = 0 from (x1, y1) is written as
By cross multiplication
d = $\frac{|Ax_1 + by_1 + C|}{\sqrt{A^2 + B^2}}$
If (0, b) is the point on the y – axis whose distance from line x / 3 + y/4 is u unit then
4= $\frac{|4(0) + 3(b) - 12}{\sqrt{4^2 3^2}}$
by further calculation
4= $\frac{|3b - 12|}{5}$
20 = |3b – 12|
We get
20 = ± (3b – 12)
Here 20 = (3b – 12) or 20 = - (3b – 12)
It can be written as
3b = 20 + 12 or 3b = -20 + 12
So we get
b = 32/3 or b = -8/3
Hence, the required points are (0, 32/3) and (0, -8/3).

Question 5

Find the perpendicular distance from the origin to the line joining the point $(\boldsymbol{cos\theta}, \boldsymbol{sin\theta})$ and $(\boldsymbol{cos}\emptyset., \sin\emptyset)$

Answer:

Hence the equation of the line joing the poitns $(cos\theta, sin\theta)$ and$(cos\emptyset., \sin\emptyset)$
y –sinθ $= \frac{\sin\emptyset - \sin\emptyset}{\cos\emptyset - \cos\emptyset}\ (x - cos\theta)$
By cross multiplication
y $(\cos\cos\emptyset - cos\theta) - \sin\theta\ (cos\emptyset - \ cos\theta)$ $= x(sin\emptyset) - \ cos\theta\ (sin\emptyset - sin\theta)$
By multiplying the terms we get
$x(\sin\theta - sin\emptyset) = y\ (\cos\emptyset - \cos\theta)$ +cosθ sin ∅ - cosθ sin ∅ - sinθ cos∅ + sinθ cos θ = 0
On further simplification

$x(\sin\theta - sin\emptyset) = y\ (\cos\emptyset - \cos\theta)$ + sin $(\emptyset - \theta)$ = 0
So we get
A x + by + C= 0 where A = sinθ - - sin$\emptyset$ B = cos$\emptyset$ $-$ cos θ and C = sin ($\emptyset -\ \theta$)
We know that the perpendicular distance (d) of a line A $x + By + C =$ from (x_1, y_1) is written as
d = $\frac{|\ Ax+By_1+C}{\sqrt{A^2+B^2}}$
So the perpendicular distance (d) of the given line from (x_1, y_1) = (0,0) is
d= $\frac{|\ (\sin\theta - \sin\emptyset)(0)+(cos\emptyset - \cos\theta)\ (0)+\sin(\emptyset-\theta)|}{\sqrt{(\sin\theta - \sin\emptyset)^2+(\cos\emptyset - \cos\theta)^2}}$
By expanding using formula
= $\frac{|\sin(\emptyset+\theta)|}{\sqrt{sin^2\theta + sin^2\emptyset - 2\sin\theta\sin\emptyset\ cos^2\emptyset + cos^2\theta - 2\cos\emptyset\cos\theta}}$

Groping of terms

= $\frac{|\sin(\emptyset-\theta)|}{\sqrt{(sin^2+cos^2)+(sin^2\emptyset+cos^2\emptyset) - 2\sin\theta\sin\emptyset + \cos\theta\cos\emptyset}}$

By further simplification
= $\frac{|\sin(\emptyset-\theta)}{\sqrt{1+1-2\ (\cos(\emptyset-\theta))}}$
= $\frac{|\sin(\emptyset-\theta)}{\sqrt{2\ (1-\cos(\emptyset-\theta))}}$

Using the formula
= $\frac{|\sin(\emptyset-\theta)|}{\sqrt{2\left[2\ sin^2\left[\frac{\emptyset-\theta}{2}\right]\right]}}$
We get

$\frac{|\sin(\emptyset-\theta)|}{\left|2\sin\left[\frac{\emptyset-\theta}{2}\right]\right|}$

Question 6

Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines x – 7y + 5 = 0 and 3x + y = 0.

Answer:

Here the equation of any line parallel to the y-axis is of the form
x = a ……. (1)
Two given lines are
x – 7y + 5 = 0 …… (2)
3x + y = 0 …… (3)
By solving equations (2) and (3) we get
x = -5/22 and y = 15/22

(-5/ 22, 15/22) is the point of intersection of lines (2) and (3)
If the line x = a passes through point (-5/22, 15/22) we get a = -5/22
Hence, the required equation of the line is x = -5/22.

Question 7

Find the equation of a line drawn perpendicular to the line x/4 + y/6 = 1 through the point, where it meets the y-axis.

Answer:

It is given that
x/4 + y/6 = 1
We can write it as
3x + 2y – 12 = 0
So we get
y = -3/2 x + 6, which is of the form y = mx + c
Here the slope of the given line = -3/2
So the slope of line perpendicular to the given line = -1/ (-3/2) = 2/3
Consider the given line intersect the y-axis at (0, y)
By substituting x as zero in the equation of the given line
y/6 = 1
y = 6
Hence, the given line intersects the y-axis at (0, 6)
We know that the equation of the line that has a slope of 2/3 and passes through point (0, 6) is
(y – 6) = 2/3 (x – 0)
By further calculation
3y – 18 = 2x
So we get
2x – 3y + 18 = 0
Hence, the required equation of the line is 2x – 3y + 18 = 0.

Question 8

Find the area of the triangle formed by the lines y – x = 0, x + y = 0 and x – k = 0.

Answer:

It is given that
y – x = 0 …… (1)
x + y = 0 …… (2)
x – k = 0 ……. (3)

Here the point of intersection of
Lines (1) and (2) is

x = 0 and y = 0
Lines (2) and (3) is
x = k and y = - k
Lines (3) and (1) is
x = k and y = k
So the vertices of the triangle formed by the three given lines are (0, 0), (k, -k) and (k, k)
Here the area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is
$\frac{1}{2} |x1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$
So the area of triangle formed by the three given lines
= ½ |0 (-k – k) + k (k – 0) + k (0 + k)| square units
By further calculation
= ½ $|k^2 + k^2|$ square units
So we get
= ½ $|2k^2|$
= k^2 square units

Question 9

Find the value of p so that the three lines 3x + y – 2 = 0, px + 2y – 3 = 0 and 2x – y – 3 = 0 may intersect at one point.

Answer:

It is given that
3x + y – 2 = 0 …… (1)
px + 2y – 3 = 0 ….. (2)
2x – y – 3 = 0 …… (3)
By solving equations (1) and (3) we get
x = 1 and y = -1
Here the three lines intersect at one point and the point of intersection of lines (1) and (3) will also satisfy line (2)
p (1) + 2 (-1) – 3 = 0
By further calculation
p – 2 – 3 = 0
So we get
p = 5
Hence, the required value of p is 5.

Question 10

If three lines whose equations are y = m1x + c1, y = m2x + c2 and y = m3x + c3 are concurrent, then show that m1 (c2 – c3) + m2 (c3 – c1) + m3 (c1 – c2) = 0.

Answer:

It is given that

$y = m_1x + c_1$ ….. (1)

$y = m_2x + c_1$ ….. (2)

$y = m_3x + c_3$ ….. (3)

By subtracting equation (1) from (2) we get

$0 = (m_2 - m_1)\, x + (c_2 - c_1)$

$(m_1 - m_2)\, x = c_2 - c_1$

So we get

$$x = \frac{C_2 - C_1}{m_1 - m_2}$$

By substituting this value in equation (1) we get

$$y = m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$$

By multiplying terms

$$y = \frac{m_1\, c_2 - m_1\, c_1}{m_1 - m_2} + c_1$$

Taking LCM

$$y = \frac{m_1\, c_2 - m_1\, c_1 - m_1\, c_1 - m_2\, c_1}{m_1 - m_2}$$

on further simplification

$$y = \frac{m_1\, c_2 - m_2\, c_1}{m_1 - m_2}$$

Here

$\left[\frac{c_2 - C_1}{m_1 - m_2}, \frac{M_1\, C_2 - M_2\, C_1}{m_1 - m_2}\right]$ is the point of intersection of lines (1) and (2)

Lines (1), (2) and (3) are concurrent, so the point of interaction of lines (1) and (2) will satisfy equation (3)

$$\frac{m_1\, c_2 - m_2\, c_1}{m_1 - m_2} = m_3\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$$

By multiplication the terms and taking LCM

$$\frac{m_1\, c_2 - m_2\, c_1}{m_1 - m_2} = \frac{m_3\, c_2 - m_{3+}\, c_1\ - c_3\, m_2}{m_1 - m_2}$$

By cross multiplication

$m_1 c_2 - m_2 c_1 - m_3 c_1 + m_3 c_1 - c_3 m_1 + c_3 m_2 = 0$

Taking out the common terms

$m_1(c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2)$

Therefore $m_1(c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2)$

Question 11

Find the equation of the lines through the point (3, 2) which make an angle of 45° with the line x –2y = 3.

Answer:

Consider m1 as the slope of the required line
It can be written as
y = 1/2 x – 3/2 which is of the form y = mx + c
So the slope of the given line m2 = 1/2
We know that the angle between the required line and line x – 2y = 3 is 45o
If θ is the acute angle between lines l1 and l2 with slopes m_1 and m_2
$\tan\theta \left|\frac{m_2 - m_1}{1 + m_1 m \quad 2}\right|$
We get
$\tan 45^0 = \left|\frac{m_2 - m_1}{1 + m_1 m \quad 2}\right|$
substituting the values

$1 = \left|\frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}}\right|$

By taking LCM

$1 = \left|\frac{\left[\frac{1 - 2m_1}{2}\right]}{\frac{2 + m_1}{2}}\right|$

on further calculation

$1 = \left|\frac{1 - 2m_1}{2 + m_1}\right|$

We get
$1 = \pm \left[\frac{1 - 2m_1}{2 + m_1}\right]$

Here
$1 = \frac{1 - 2m_1}{2 + m_1}$ or $1 = \pm \left[\frac{1 - 2m_1}{2 + m_1}\right]$
It can be written as
$2 + m_1 = 1 - 2m_1$ or $2 + m_1 = -1 + 2m_1$
$m_1 = -1/3$ or $m_1 = 3$
Case I – $m_1 = 3$
Here the equation of the line passing through (3, 2) and having a slope 3 is
y – 2 = 3 (x – 3)
By further calculation
y – 2 = 3x – 9
So we get
3x – y = 7

Case II – m_1 = -1/3
Here the equation of the line passing through (3, 2) and having a slope -1/3 is
y – 2 = - 1/3 (x – 3)
By further calculation
3y – 6 = - x + 3
So we get
x + 3y = 9
Hence, the equations of the lines are 3x – y = 7 and x + 3y = 9.

Question 12

Find the equation of the line passing through the point of intersection of the lines 4x + 7y – 3 = 0 and 2x – 3y + 1 = 0 that has equal intercepts on the axes.

Answer:

Consider the equation of the line having equal intercepts on the axes as x/a + y/a = 1
It can be written as
x + y = a ..... (1)
By solving equations 4x + 7y – 3 = 0 and 2x – 3y + 1 = 0 we get
x = 1/13 and y = 5/13
(1/13, 5/13) is the point of intersection of two given lines
We know that equation (1) passes through point (1/13, 5/13)
1/13 + 5/13 = a
a = 6/13
So the equation (1) passes through (1/13, 5/13)
1/13 + 5/13 = a
We get
a = 6/13
Here the equation (1) becomes
x + y = 6/13
13x + 13y = 6
Hence, the required equation of the line is 13x + 13y = 6.

Question 13

Show that the equation of the line passing through the origin and making an angle θ with the line y = mx + c is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Answer:

Consider y = m1x as the equation of the line passing through the origin it is given that the line makes an angels θ with line y mx + c2 then angel θ written as tan θ $\left|\frac{m_1 - m}{1 + m_1\, m}\right|$

By substituting the value

tan θ $\left|\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}\right|$

We get

tanθ = $\pm$ $\left[\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}\right]$

Here

tan θ $\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$ or tanθ = $\pm$ $\left[\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}\right]$

case I –

tan θ $\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$

We can write it is

tan θ + $\frac{y}{x}$ m tan θ $-$ $\frac{y}{x}$ $-$ m

By further simplification

m + tan θ = $\frac{y}{x}$ (1 – m tan θ)

so we get

$\frac{y}{x} = \frac{m + tan\theta}{1 - m\ tan\theta}$

Case II-

tan θ = - $\left[\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}\right]$

We can write it as

tanθ + $\frac{y}{x}$ m tan θ = - $\frac{y}{x}$ + m

By further simplification

$\frac{y}{x}$ (1 + m tanθ)= m – tanθ

So we get

$\frac{y}{x} = \frac{m-tan\theta}{1+m\ tan\theta}$

Hence the required line is given by

$\frac{y}{x} = \frac{m\pm tan\theta}{1\mp m\ tan\theta}$

Question 14

In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line x + y = 4?

Answer:

We know that the equation of the line joining the points (-1, 1) and (5,7) is given by

y= 1 = $\frac{7-1}{5+1}(x+1)$
By further calculation
y – 1 = $\frac{6}{6}(x+1)$
So we get
x –y + 2 = 0 .... (1)
So the equation of the given line is
x + y – 4 = 0 .... (2)
Here the point of intersection of line (1) and (2) is given by
x = 1 and y = 3
Consider (1,3) divide the line segment joining (-1,1) and (5,7) in the ratio 1: k using the section formula
(1,3) = $\left[\frac{k\ (-1)+1\ (5)}{1\ +k},\ \frac{k\ (1)+1(7)}{1+k}\right]$
By further calculation
(1,3) $\left[\frac{-k+5}{1+k}, \frac{k+7}{1+k}\right]$

So we get
$\frac{-k+5}{1+k}$ = 1 $\frac{k+7}{1+k} = 3$
We can write it as
$\frac{-k+5}{1+k}$ = 1

By cross multiplication

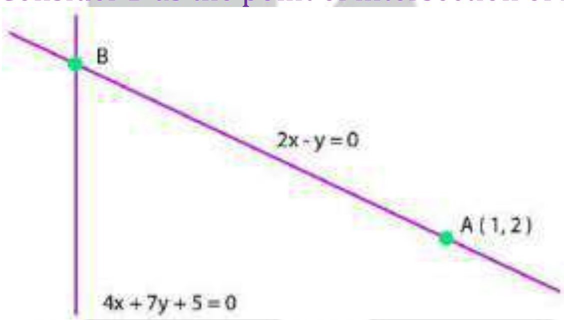
$-k + 5 = 1 + k$
We get
2k = 4
k = 2
Hence, the line joining the points (-1, 1) and (5, 7) is divided by the line x + y = 4 in the ratio 1: 2.

Question 15

Find the distance of the line 4x + 7y + 5 = 0 from the point (1, 2) along the line 2x – y = 0.

Answer:

It is given that
2x – y = 0 ..... (1)
4x + 7y + 5 = 0 ...... (2)
Here A (1, 2) is a point on the line (1)
Consider B as the point of intersection of lines (1) and (2)



By solving equations (1) and (2) we get x = -5/18 and y = - 5/9
So the coordinates of point B are (-5/18, -5/9)
From distance formula the distance between A and B

$$AB = \sqrt{\left(1+\frac{5}{18}\right)^2 + \left(2+\frac{5}{9}\right)^2} \text{ units}$$

By taking LCM

$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

It can be written as

$$= \sqrt{\left(\frac{23}{2 \times 9}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

So we get

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

By taking the common terms out

$= \sqrt{\left(\frac{23}{9}\right)^2 + \left(\frac{1}{4} + 1\right)^2}$ units

We get

$= \frac{23}{9}\sqrt{\frac{5}{4}}$ units

$= \frac{23}{9} \times \frac{\sqrt{5}}{2}$ units

$= \frac{23\sqrt{5}}{18}$ units

Here the required distance is $\frac{23\sqrt{5}}{18}$ units

Question 16

Find the direction in which a straight line must be drawn through the point (–1, 2) so that its point of intersection with the line x + y = 4 may be at a distance of 3 units from this point.

Answer:

Consider y = mx + c as the line passing through the point (-1, 2)
So we get
2 = m (-1) + c
By further calculation
2 = -m + c
c = m + 2
Substituting the value of c
y = mx + m + 2 …… (1)
So the given line is
x + y = 4 ……. (2)
By solving both the equations we get

$x = \frac{2-m}{m+1}$ *and* $y = \frac{5m+2}{m+1}$

$\left(\frac{2-m}{m+1}, \frac{5m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2)

Here the point at a distance of 3 units from (-1, 2)

From distance formula

$\sqrt{\left(\frac{2-m}{m+1} + 1\right)^2 + \left(\frac{5m+2}{m+1} - 2\right)^2} = 3$

Squaring on both side

$\left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 = 3^2$

By further calculation
$\frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$

Dividing the equation by 9
$\frac{1+m^2}{(m+1)^2} = 1$
By cross
By cross multiplication
$1 + m^2 = m^2 + 1 + 2m$
So we get
$2m = 0$
$m = 0$
Hence, the slope of the required line must be zero i.e. the line must be parallel to the x axis

Question 17

The hypotenuse of a right angled triangle has its ends at the points (1, 3) and (−4, 1). Find the equation of the legs (perpendicular sides) of the triangle.

Answer:

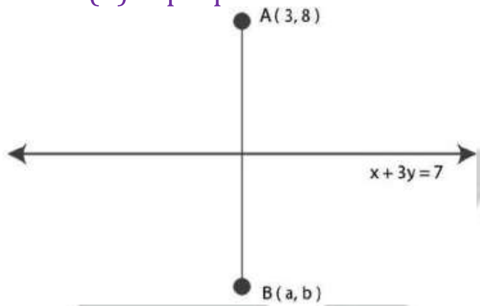
Consider ABC as the right angles triangle where ∠C = 90o
Here infinity such lines are present.
m is the slope of AC
So the slope of BC = -1/m
Equation of AC -
$y - 3 = m (x - 1)$
By cross multiplication
$x - 1 = 1/m (y - 3)$
Equation of BC –
$y - 1 = - 1/m (x + 4)$
By cross multiplication
$x + 4 = - m (y - 1)$
By considering values of m we get
If m = 0,
So we get
$y - 3 = 0, x + 4 = 0$
If $m = \infty$,
So we get
$x - 1 = 0, y - 1 = 0$ we get $x = 1, y = 1$

Question 18

Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

Answer:

It is given that
x + 3y = 7 ….. (1)
Consider B (a, b) as the image of point A (3, 8)
So line (1) is perpendicular bisector of AB.



Here

Slope of AB = $\frac{B-8}{A-3}$
Slope of line (1) = $\frac{1}{3}$
Line (I) is perpendicular to AB
$\left(\frac{b-8}{a-3}\right) \times \left(-\frac{1}{3}\right) = -1$
By further calculation
$\frac{b-8}{3a-9} = 1$
By cross multiplication
b – 8 = 3a – 9
3a – b = 1 ………. (2)
We know that
Midpoint of AB = $\left(\frac{a+3}{2}, \frac{b+8}{2}\right)$

So the midpoint of line segment AB will satisfy line (1)
From equation (1)
$\left(\frac{a+3}{2}\right) + 3\left(\frac{b+8}{2}\right) = 7$
By further calculation
a + 3+3b + 24 = 14
on further simplification
a + 3b = -13 ………. (3)
By solving equation (2) and (3) we get
a = -1 and b= -4

Question 19

If the lines y = 3x + 1 and 2y = x + 3 are equally inclined to the line y = mx + 4, find the value of m.

Answer:

It is given that
y = 3x + 1 …… (1)
2y = x + 3 …… (2)
y = mx + 4 …… (3)
Here the slopes of
Line (1), m1 = 3
Line (2), m2 = ½
Line (3), m3 = m
We know that the lines (1) and (2) are equally inclined to line (3) which means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\left|\frac{m_1 - m_2}{1 + m_1\ m_2}\right| = \left|\frac{m_2 - m_2}{1 + m_2\ m_3}\right|$$

Substituting the value we get

$$\left|\frac{3-m}{3+3m}\right| = \left|\frac{\frac{1}{2} - m}{1+\frac{1}{2}m}\right|$$

By taking LCM

$$\left|\frac{3-\mathrm{m}}{1+3\mathrm{m}}\right| = \left|\frac{1-2\mathrm{m}}{\mathrm{m}+2}\right|$$

It can we written as

$$\frac{3-\mathrm{m}}{1+3\mathrm{m}} = \pm\left(\frac{1-2m}{m+2}\right)$$

Here

$$\frac{3-\mathrm{m}}{1+3\mathrm{m}} = \frac{1-2\mathrm{m}}{\mathrm{m}+2} \text{ or } \frac{3-\mathrm{m}}{1+3\mathrm{m}} = -\left(\frac{1-2m}{m+2}\right)$$

If

$$\frac{3-\mathrm{m}}{1+3\mathrm{m}} = \frac{1-2\mathrm{m}}{\mathrm{m}+2}$$

By cross multiplication
(3-m) (m+2) = 1 (1-2m) (1+3m)

On further calculation

- $m^2 + m + 6 = 1 + m - 6m^2$

So we get
$5m^2 + 5 = 0$
Dividing the equation by 5
$m^2 + 1 = 0$
m =√-1, which is not real

Therefore this case is not possible

If

$\frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$

By crossing multiplication

(3-m) (m+2) = -1 (1-2m) (1+3m)

on further calculation

$-m^2 + m + 6 = -(1 + m - 6m^2)$

So we get

$7m^2 - 2m - 7 = 0$

Here we get

$m = \frac{2 \pm \sqrt{4-4(7)(-7)}}{2(7)}$

By further simplification

$m = \frac{2 \pm 2\sqrt{1+49}}{14}$

We can write it as

$m = \frac{1 \pm 5\sqrt{2}}{7}$

Hence the required value of m is

$\frac{1 \pm 5\sqrt{2}}{7}$

Question 20

If sum of the perpendicular distances of a variable point P (x, y) from the lines x + y – 5 = 0 and 3x – 2y + 7 = 0 is always 10. Show that P must move on a line.

Answer:

it is given that

x + y – 5 = 0 .....(1)

3x – 2y + 7 = 0 ...... (2)

Here the perpendicular distance of P(x,y) from line (1) and (2) are written as

$d_1 = \frac{|x+y-5|}{\sqrt{(1)^2+(1)^2}}$ *and* $d_1 \frac{|3x-2y+7|}{\sqrt{(3)^2+(-2)^2}}$

So we get that

$d_1 = \frac{|x+y-5|}{\sqrt{2}}$ and $d_2 = \frac{|3x-2y+7|}{\sqrt{13}}$

We know that $d_1 + d_2 = 10$

Substituting the values

$\frac{|x+y-5|}{\sqrt{2}} + \frac{|3x-2y+7|}{\sqrt{13}} = 10$

By further calculation

$\sqrt{13}\,|x+y-5| + \sqrt{2}\,|3x-2y+7| - 10\sqrt{26} = 0$

It can be written as

$\sqrt{13}\,(x+y-5) + \sqrt{2}\,(3x-2y+7) - 10\sqrt{26} = 0$

Now by assuming $(x+y-5)$ *and* $(3x-2y+7)$ are positive

$\sqrt{13}x + \sqrt{13}\,y - 5\sqrt{13} + 3\sqrt{2}\,x - 2\sqrt{2}\,y + 7\sqrt{2} - 10\sqrt{26} = 0$

Taking out the common terms

$x\left(\sqrt{13} + 3\sqrt{2}\right) + \left(\sqrt{13} - 2\sqrt{2}\right) + \left(7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26}\right) = 0$ which is the equation of line

In the same way we can find the equation of line for any signs of $(x+y-5)$ *and* $(3x-2y+7)$

Hence point P must move on a line

Question 21

Find equation of the line which is equidistant from parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.

Answer:

9x + 6y – 7 = 0 ………….. (1)

3x +2y + 6 = 0 ………….. (2)

Consider P (h,k) arbitrary point that is equidistant from lines (1) and (2)

Here the perpendicular distance of P (h,k) from line (2) is written as

$d_1 \frac{|9h+6k-7|}{(9)^2+(6)^2} = \frac{|9h+6k-7|}{\sqrt{117}} = \frac{9h+6k-7}{3\sqrt{13}}$

Similarly the perpendicular distance of P (h,k) from line 92) is written as

$d_1 \frac{|3h+2k-6|}{(3)^2+(2)^2} = \frac{|3h+2k-6|}{\sqrt{13}}$

We know that P (h,k) is equidistance from line (1) and (2) d_1 *or* d_2

Substituting the values

$\frac{|9h+6k-7|}{3\sqrt{13}} = \frac{|3h+2k+6|}{\sqrt{13}}$

By further calculation

$|9h+6k-7| = 3\,|3h+2k+6|$

It can be written as

$|9h+6k-7| = \pm\, 3\,(3h+2k+6)$

Here

9h + 6k – 7 = 3 (3h + 2k + 6) or 9h + 6k – 7 = - 3 (3h + 2k + 6)

9h + 6k – 7 = 3 (3h + 2k + 6) is not possible as

9h + 6k – 7 = 3 (3h + 2k + 6)

By further calculation

- 7 = 18

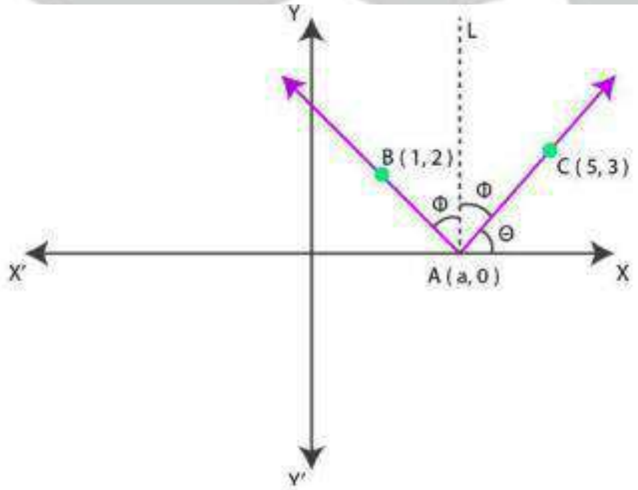
We know that

9h + 6k – 7 = -3 (3h + 2k + 6)
By multiplication
9h + 6k – 7 = -9h – 6k – 18
We get
18h + 12k + 11 = 0
Hence, the required equation of the line is 18x + 12y + 11 = 0.

Question 22

A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

Answer:



Consider the coordinates of point A as (a, 0)
Construct a line (AL) which is perpendicular to the x-axis
Here the angle of incidence is equal to angle of reflection
∠BAL = ∠CAL = Φ
∠CAX = θ
It can be written as
∠OAB = 180° – (θ + 2Φ) = 180° – [θ + 2(90° – θ)]
On further calculation
= 180° – θ – 180° + 2θ
= θ
So we get
∠BAX = 180° – θ
Slope of line AC = $\frac{3-0}{5-a}$
$\tan\theta = \frac{3}{5-a}$ …………..(1)

Slope of line AC = $\frac{2-0}{1-a}$

We get

tan $(180^0 - \theta) = \frac{2-0}{1-a}$

By further calculation

-tan$\theta = \frac{2}{1-a}$

tan$\theta = \frac{2}{a-1}$ ……………. (2)

From equation (1) and (2) we get

$\frac{3}{5-a} = \frac{2}{1-a}$

by cross multiplication

3a – 3 = 10 -2a

we get

a = 13/5

Hence the coordinates of point A are (13/5,0).

Question 23

Prove that the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2-b^2}, 0\right)$ *and* $\left(-\sqrt{a^2-b^2}, 0\right)$

Answer:

It is given that

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

We can write that

bx cos θ + ay sin θ - ab = 0 ………… (1)

Here the length of the perpendicular from point $\left(\sqrt{a^2-b^2}, 0\right)$ *to line* (1)

$p_1 \frac{|b\cos\theta\left(\sqrt{a^2-b^2}\right)+a\sin\theta\ (0)-ab|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}} = \frac{|b\cos\theta\sqrt{a^2-b^2}-ab|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$ ……. (2)

Similarly the length of the perpendicular from joint $\left(-\sqrt{a^2-b^2}, 0\right)$ *to line* (2)

$P_2 \frac{|b\cos\theta\left(\sqrt{a^2-b^2}\right)+a\sin\theta\ (0)-ab|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}} = \frac{|b\cos\theta\sqrt{a^2-b^2}+ab|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$ …………(3)

By multiplying equations (2) and (3) we get

$p_1\,p_2 = \frac{|b\cos\theta\sqrt{a^2-b^2}-ab\,||(b\cos\theta\sqrt{a^2-b^2}+ab\,)|}{\left(\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}\right)^2}$

We get

$\frac{|(b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab)(b\cos\theta\left(\sqrt{a^2-b^2}\right)+ab)|}{(b^2\cos^2\theta+a^2\sin^2\theta)}$

From the formula

$\frac{\left|\left(b\ cos\theta\ \sqrt{a^2-b^2}\ \right)^2-(ab)^2\right|}{(b^2\ cos^2\ \theta+a^2\ sin^2\ \theta)}$

By squaring the numerator we get

$=\frac{|b^2\ cos^2\theta\ (a^2-b^2)-a^2\ b^2|}{(b^2\ cos^2\ \theta+a^2\ sin^2\ \theta)}$

By expending using formula

$=\frac{|a^2\ b^2\ cos^2\ \theta-b^4\ cos^2\ \theta-a^2\ b^2|}{b^2\ cos^2\ \theta+a^2\ sin^2\ \theta}$

Taking out the common terms

$=\frac{b^2\ |a^2\ cos^2\ \theta-b^2\ cos^2\ \theta-a^2|}{b^2\ cos^2\ \theta+a^2\ sin^2\ \theta}$

We get

$=\frac{b^2\ |a^2\ cos^2\ \theta-b^2\ cos^2\ \theta-a^2\ sin^2\ \ \theta-a^2\ cos^2\ \theta|}{b^2\ cos^2\ \theta+a^2\ sin^2\ \theta}$

Here = $sin^2\ \theta+\ cos^2\ \theta$ = 1

$=\frac{b^2\ (-b^2cos^2\ \theta+a^2\ sin^2\ \theta)}{b^2\ cos^2\ \theta+a^2\ sin^2\theta}$

So we get

$\frac{b^2\ (b^2cos^2\ \theta+a^2\ sin^2\ \theta)}{(b^2\ cos^2\ \theta+a^2\ sin^2\theta)}$

= b^2

Therefore it is proved

Question 24

A person standing at the junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 wants to reach the path whose equation is 6x – 7y + 8 = 0 in the least time. Find equation of the path that he should follow.

Answer:

It is given that

2x – 3y + 4 = 0 …… (1)

3x + 4y – 5 = 0 ……. (2)

6x – 7y + 8 = 0 …… (3)
Here the person is standing at the junction of the paths represented by lines (1) and (2).
By solving equations (1) and (2) we get
x = - 1/17 and y = 22/17
Hence, the person is standing at point (-1/17, 22/17).
We know that the person can reach path (3) in the least time if he walks along the perpendicular line to (3) from point (-1/17, 22/17)
Here the slope of the line (3) = 6/7
We get the slope of the line perpendicular to line (3) = -1/ (6/7) = - 7/6
So the equation of line passing through (-1/17, 22/17) and having a slope of -7/6 is written as
$\left(y-\frac{22}{17}\right) = -\frac{7}{6}\left(x+\frac{1}{17}\right)$
By further calculation
6 (17y – 22) = - 7 (17x + 1)
By multiplication
102y – 132 = - 119x – 7
We get
1119x + 102y = 125
Therefore, the path that the person should follow is 119x + 102y = 125.