

PAPER 3

BUSINESS MATHEMATICS

AND

LOGICAL REASONING &

STATISTICS

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CHAPTER 1

RATIO AND PROPORTION, INDICES, LOGARITHMS

**Ratio and
Proportion**



UNIT I: RATIO

TYPES OF RATIO

Continued Ratio is the relation (or compassion) between the magnitudes of three or more Quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as $a : b : c$

A ratio compounded of itself is called Duplicate ratio $a^2 : b^2$ is the duplicate ratio of $a : b$

similarly the triplicate ratio $a : b$ is $a^3 : b^3$.

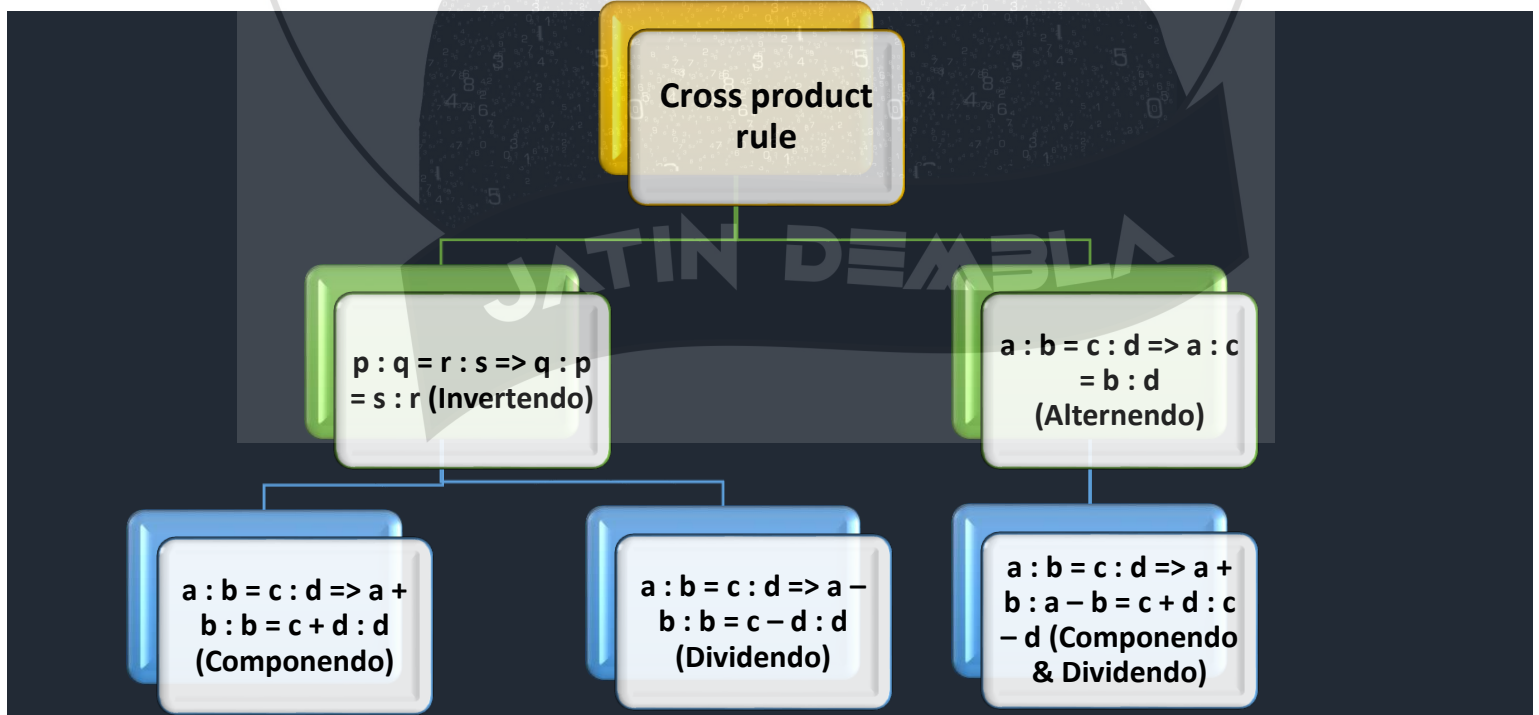
The sub-duplicate ratio of $a : b$ is $a : b$ and the sub-triplicate ratio of $a : b$ is $a^{1/3} : b^{1/3}$

Continued Ratio is the relation (or compassion) between the magnitudes of three or more Quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as $a : b : c$

RATIO	<p>A ratio is a comparison of the sizes of two or more quantities of the same kind by division.</p> <p>If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as $a : b$. Thus, the ratio of a to b = a/b or $a : b$. The quantities a and b are called the terms of the ratio, a is called the first term or antecedent and b is called the second term or consequent.</p>
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UNIT II: PROPORTIONS

MEANING	An equality of two ratios is called a proportion . Four quantities a, b, c, d are said to be in proportion if $a : b = c : d$.
TERMS OF PROPORTION.	The quantities a, b, c, d are called terms of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively . <u>First and fourth terms are called extremes (or extreme terms)</u> . <u>Second and third terms are called means (or middle term)</u> .



PROPERTIES.

This is called product rule.

Three quantities a,b,c of the same kind (in same units) are said to be in continuous proportion if $a:b = b:c$, i.e. $a/b = b/c$, i.e. $b^2 = ac$

If a,b,c are in continuous proportion, then the middle term b is called the mean proportion between a and c, a is the first proportional and c is the third proportion.

Thus, if b is mean proportional between a and c, then $b^2 = ac$ i.e. $b = \sqrt{ac}$

If $a : b = c : d$ then d is called fourth proportional.

If $a : b = c : d$ are in proportion then $a/b = c/d$ i.e. $ad = bc$
i.e. product of extremes = product of means.

UNIT 3 INDICES

Laws and Properties.

1.	$a^m \times a^n = a^{m+n}$, when m and n are positive integers (base must be same)
2.	$a^m/a^n = a^{m-n}$ when m and n are positive integers and $m > n$
3.	$(a^m)^n = a^{mn}$ where m and n are positive integers
4.	$(ab)^n = a^n \cdot b^n$ when n can take all of the values.
5.	$a^0 = 1$
6.	$a^{-m} = 1/a^m$ and $1/a^{-m} = a^m$

UNIT IV: LOGARITH**LOGARITHM.**

- The two equations $ax = n$ and $x = \log_a n$ are only transformations of each other and should be remembered to change one form of the relation into the other.
- The logarithm of 1 to any base is zero. This is because any number raised to the power zero is one.
- Since $a^0 = 1$, $\log_a 1 = 0$
- The logarithm of any quantity to the same base is unity. This is because any quantity raised to the power 1 is that quantity.
- Since $a^1 = a$, $\log_a a = 1$

Fundamental Laws of Logarithm.

<u>1.</u>	$\log_a mn = \log_a m + \log_a n$
<u>2.</u>	$\log_a \frac{m}{n} = \log_a m - \log_a n$
<u>3.</u>	$\log_a m^n = n \log_a m$
<u>4.</u>	$\log_a a = 1, a = 1$
<u>5.</u>	$\log_a 1 = 0$
<u>6.</u>	$\log_b a \times \log_a b = 1$
<u>7.</u>	$\log_b a \times \log_c b = \log_c a$
<u>8.</u>	$\log_b a = \log a / \log b$
<u>9.</u>	$\log_b a = 1 / \log_a b$

Then 2.4 kg, 9.6 kg and x kg are in continued proportion since

$$b^2 = ac \text{ So, } 2.4/9.6 = 9.6/x \text{ or, } x = (9.6 \times 9.6)/2.4 = 38.4$$

7. The inverse ratio of 11 : 15 is

- a. 15:11
- b. 11:11
- c. 15:15
- d. $\sqrt{11}:\sqrt{15}$

ANSWER: a

SOLUTION:

One ratio is the inverse of another if their product is 1. Thus a : b is the inverse of b : a and vice-versa.

8. If $a : b = c : d = e : f = \dots\dots\dots$, then each of these ratios is equal

- a. $(a + c + e + \dots\dots) : (b + d + f + \dots\dots)$ is equal to each ratio
- b. $(a + c + e + \dots\dots) : (b + d + f + \dots\dots)$ is greater to each ratio
- c. $(a + c + e + \dots\dots) : (b + d + f + \dots\dots)$ is zero ratio
- d. None

ANSWER: a

SOLUTION:

Due to addendo property.

9. If $a : b = c : d = 2.5 : 1.5$, what are the values of $ad : bc$ and $a + c : b + d$?

- a. $ad : bc$ and $a + c : b + d$ are 2 : 1 and 8 : 3
- b. $ad : bc$ and $a + c : b + d$ are 1 : 1 and 5 : 3
- c. $ad : bc$ and $a + c : b + d$ are 1 : 1 and 5 : 5
- d. None

ANSWER b

SOLUTION:

we have $a/b = c/d = 2.5/1.5 \dots\dots\dots(1)$

From (1) $ad = bc$, or $ad/bc = 1$, $ad : bc = 1 : 1$

$$\frac{a+c}{b+d} = \frac{2.5}{1.5} = \frac{25}{15} = \frac{5}{3}$$

i.e., $a + c : b + d = 5 : 3$

Hence, the values of $ad : bc$ and $a + c : b + d$ are $1 : 1$ and $5 : 3$ respectively.

10. Simplify $2x^{1/2} 3x^{-1}$ if $x = 4$

- a. 3
- b. 6
- c. 0.3
- d. 30

Answer a

SOLUTION:

$$\begin{aligned} &\text{we have } 2x^{1/2} 3x^{-1} \\ &= 6x^{1/2} x^{-1} = 6x^{1/2-1} \\ &= 6x^{-1/2} \\ &= 3 \end{aligned}$$

11. Find the value of k from $(\sqrt{9})^{-7} \times (\sqrt{3})^{-5} = 3^k$

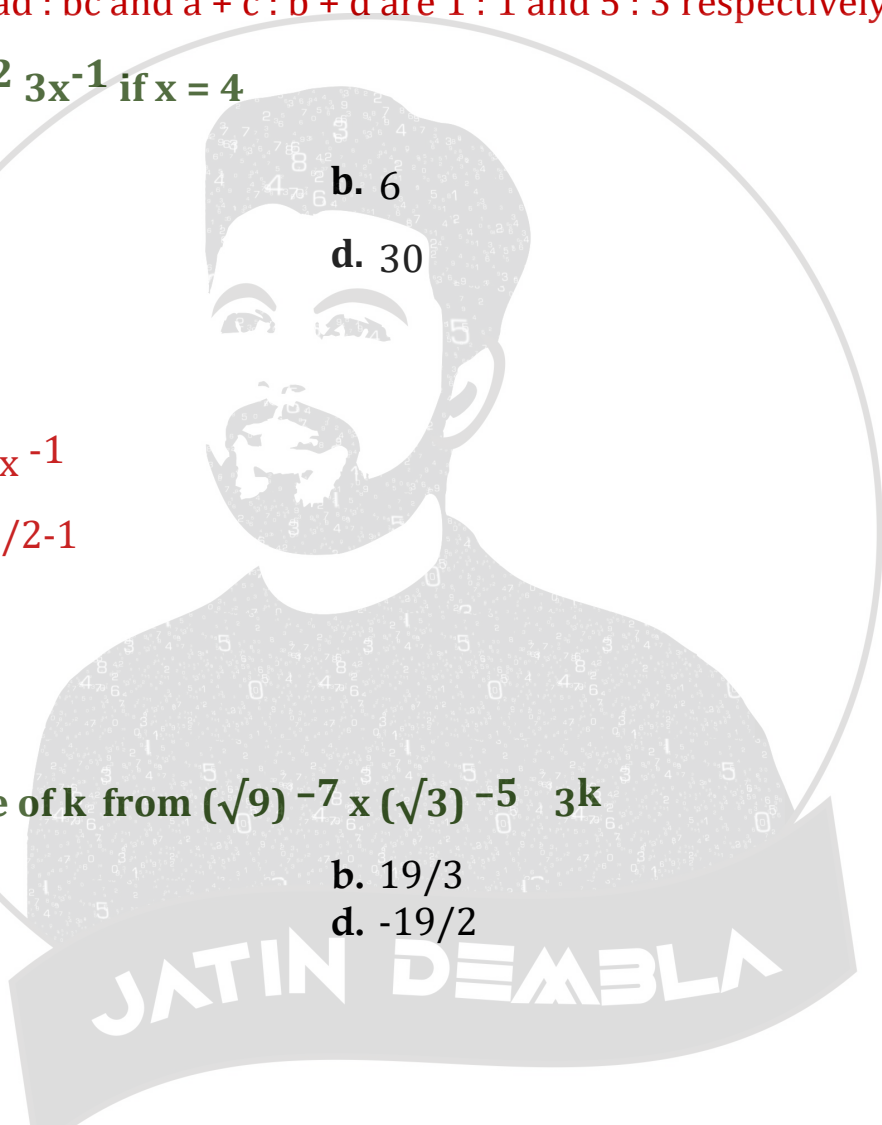
- a. $19/2$
- b. $19/3$
- c. $-19/3$
- d. $-19/2$

ANSWER: d

SOLUTION:

$$\begin{aligned} &(3^2 \times 1/2)^{-7} \times (3^{1/2})^{-5} = 3^k \\ &3^{-19/2} = 3^k \\ &k = -19/2 \end{aligned}$$

12. $\log_2 1 = ?$



- a. 0
- b. 1
- c. x
- d. m

ANSWER a

SOLUTION:

According to properties of logarithm $\log_a 1 = 0$

13. $\log 6 + \log 8$ is expressed as

- a. $\log 11$
- b. $\log 48$
- c. $\log 6/8$
- d. $\log 14$

ANSWER b

SOLUTION:

According to properties of logarithm i.e., $\log_a m + \log_a n = \log_a mn$

14. $a^{\log_a x} = x$

- a. Inverse logarithm Property
- b. proportionate logarithm Property
- c. either a or b
- d. none

ANSWER a

SOLUTION:

According to properties of logarithm i.e. **Inverse logarithm Property** is the base elevated and power is be answer

15. $2^4 = 16$ $\log_2 16 = 4$ is correct or not ?

- a. correct
- b. not correct
- c. partial correct
- d. not sure

ANSWER a

SOLUTION:

The logarithm of 16 to the base 2 is equal to 4

$$\frac{4}{15} A = \frac{2}{5} B$$

$$\Rightarrow A = \left(\frac{2}{5} \times \frac{15}{4} \right) B$$

$$\Rightarrow A = \underline{3} B$$

$$\Rightarrow \frac{A}{B} = \frac{3}{2}$$

$$A : B = 3 : 2.$$

$$\therefore \text{B's share} = \text{Rs.} \left[1210 \times \frac{2}{5} \right]$$

16. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?

A. Rs. 460

Rs. 484

C. Rs. 550

Rs. 664

Answer: Option B

Explanation:

Rs. 484.

17. A sum of Rs.312 was divided among 100 boys and girls in such a way that the boy gets Rs.3.60 and each girl Rs. 2.40 the number of girls is

a. 35

b. 40

c. 45

d. 50

Answer: Option b

Explanation:

Step (i): Let x be the number of boys and y be the number of girls.

Given total number of boys and girls = 100

$$x+y=100 \text{ ----- (i)}$$

Step (ii): A boy gets Rs. 3.60 and a girl gets Rs. 2.40

The amount given to 100 boys and girls = Rs. 312

$$3.60x + 2.40y = 312 \text{ ----- (ii)}$$

Step (iii):

Solving (i) and (ii)

$$3.60x + 3.60y = 360 \text{ ----- Multiply (i) by 3.60}$$

$$3.60x + 2.40y = 312 \text{ ----- (ii)}$$

$$1.20y = 48$$

$$y = 48 / 1.20$$

$$= 40$$

⇒ Number of girls = 40

18. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

a. 2 : 5

b. 3 : 5

c. 4 : 5

d. 6 : 7

Answer: Option C

Explanation:

Let the third number be x .

$$\text{Then, first number} = 120\% \text{ of } x = \frac{120x}{100} = \frac{6x}{5}$$

$$\text{Second number} = 150\% \text{ of } x = \frac{150x}{100} = \frac{3x}{2}$$

$$\therefore \text{Ratio of first two numbers} = \left(\frac{6x}{5} : \frac{3x}{2} \right) = 12x : 15x = 4 : 5.$$

19. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

a. 2 : 3 : 4

b. 6 : 7 : 8

c. 6 : 8 : 9

d. None of these

Answer: a

Explanation:

Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

a. $3 : 3 : 10$

b. $10 : 11 : 20$

c. $23 : 33 : 60$

d. None of these

Answer: Option C**Explanation:**Let $A = 2k$, $B = 3k$ and $C = 5k$.

$$\text{A's new salary} = \frac{115}{100} \text{ of } 2k = \left(\frac{115}{100} \times 2k \right) = \frac{23k}{10}$$

$$\text{B's new salary} = \frac{110}{100} \text{ of } 3k = \left(\frac{110}{100} \times 3k \right) = \frac{33k}{10}$$

$$\text{C's new salary} = \frac{120}{100} \text{ of } 5k = \left(\frac{120}{100} \times 5k \right) = 6k$$

$$\therefore \text{New ratio} = \left(\frac{23k}{10} : \frac{33k}{10} : 6k \right) = 23 : 33 : 60$$

27. If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number?

a. $2 : 5$

b. $3 : 7$

c. $5 : 3$

d. $7 : 3$

Answer: Option C**Explanation:**

$$\text{Let } 40\% \text{ of } A = \frac{2}{3} B$$

$$\text{Then, } \frac{40A}{100} = \frac{2B}{3}$$

$$\Rightarrow \frac{2A}{5} = \frac{2B}{3}$$

$$\Rightarrow \frac{A}{B} = \left(\frac{2}{3} \times \frac{5}{2} \right) = \frac{5}{3}$$

$$\therefore A : B = 5 : 3.$$

$$\text{Let } \log a = \frac{1}{2} \log b = \frac{1}{5} \log c = k$$

$$\text{then } \log a = k \Rightarrow a = e^k$$

$$\frac{1}{2} \log b = k \Rightarrow \log b = 2k$$

$$\Rightarrow b = e^{2k}$$

$$\frac{1}{5} \log c = k \Rightarrow \log c = 5k$$

$$\Rightarrow c = e^{5k}$$

$$\therefore a^4 b^3 c^{-2} = e^{4k} \cdot e^{6k} \cdot e^{-10k} \\ = e^0 = 1$$

33. The ratio of market prices of wheat and paddy is 2:3 and the ratio of quantities consumed in a family is 5:4 . find the ratio of expenditure of wheat and paddy.

a. 6:5

b. 5:6

c. 1:1

d. 8:15

Answer: b

Explanation:

Expenditure = Price x Quantity

$$\frac{\text{Wheat Price}}{\text{Paddy price}} = \frac{2}{3} \quad \text{and} \quad \frac{\text{Wheat Quantity consumed}}{\text{Paddy Quantity consumed}} = \frac{5}{4}$$

Multiplying both ratios

$$\frac{\text{Wheat Price} \times \text{Wheat Quantity consumed}}{\text{Paddy Price} \times \text{Paddy Quantity consumed}} = \frac{2 \times 5}{3 \times 4}$$

$$\frac{\text{Wheat Expenditure}}{\text{Paddy Expenditure}} = \frac{5}{6}$$

34. If A:B = 2:3, B:C = 4:5 and C:D = 6:7 , then find the value of A:B:C:D

a. 15:24:30:35

b. 16:24:30:35

c. 17:24:30:35

d. 18:24:30:35

Answer: b

c. 3^{1000}

d. 3^{10000}

Answer: c**Explanation:**

$$\log_3 x = 10$$

$$\text{Hence, } x = 3^{10}$$

$$\log_x y = 100$$

$$y = x^{100} = (3^{10})^{100} = y = 3^{1000}$$

42. The third proportional between $a^2 - b^2$ and $(a + b)^2$ is

a. $\frac{a+b}{a-b}$

b. $\frac{a-b}{a+b}$

c. $\frac{(a+b)^3}{a-b}$

d. $\frac{(a+b)^3}{(a-b)^3}$

Answer: c**Explanation:**

Let x be the required third proportional, then

$$(a^2 - b^2) : (a + b)^2 :: (a + b)^2 : x$$

$$\Rightarrow \frac{a^2 - b^2}{(a + b)^2} = \frac{(a + b)^2}{x}$$

$$\Rightarrow x(a^2 - b^2) = (a + b)^4 \text{ i.e. } x(a - b)(a + b) = (a + b)^4$$

$$\Rightarrow x = \frac{(a + b)^3}{a - b}$$

43. A sum of Rs. 53 is divided in such a way that A gets Rs. 7 more than what B gets and B gets Rs. 8 more than what C gets. The ratio of their share is. _____

a. 25:18:10

b. 25:18:1

c. 2:18:10

d. 25:8:10

Answer: a**Explanation:**

Let the share of c = Rs. x,

then share of B = Rs. (x+8) and share of A = Rs. (x+8+7)

Therefore $x + (x+8) + (x+15) = 53$

$$\Rightarrow 3x = 30 \text{ i.e. } x = 10$$

Hence ratio

$$A : B : C = 25 : 18 : 10$$

46. The ratio of numbers of girls and boys participating in sports of a school is 4:5. If the number of girls is 212, determine the number of boys participating in the sports.

- a. 256
c. 251
- b. 265
d. 263

Answer: b

Explanation:

As per given condition, $\frac{\text{Number of girls}}{\text{Number of boys}} = \frac{4}{5}$

$$\therefore \frac{212}{\text{Number of boys}} = \frac{4}{5}$$
$$\therefore \text{Number of boys} = 265$$

47. Income ratio of Ramesh and Suresh is 5:6. Their spending ratio is 7:9. Ramesh saves 4000 and Suresh saves 3000. Income and spending respectively of Ramesh and Suresh are

- a. Ramesh - 25000, 21000; Suresh - 30000, 27000
- b. Ramesh - 36000, 32000; Suresh - 30000, 27000
- c. Ramesh - 30000, 27000; Suresh - 36000, 32000
- d. None of the above

Answer: a

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Explanation:

$$\text{Income ratio} = \text{Ramesh: Suresh} = 5:6 = \frac{5}{6};$$

Common factor helps in finding actual values easily

So, take 'A' as common factor.

$$\text{Income of Ramesh} = 5A; \text{Income of Suresh} = 6A$$

$$\frac{\text{Spending of Ramesh}}{\text{Spending of Suresh}} = \frac{\text{Ramesh Income} - \text{Ramesh Saving}}{\text{Suresh Income} - \text{Suresh Saving}} = \frac{7}{9}$$

$$\therefore \frac{5A - 4000}{6A - 3000} = \frac{7}{9}$$

$$\therefore 9(5A - 4000) = 7(6A - 3000)$$

$$\therefore A = 5000$$

$$\text{Income of Ramesh} = 5A = 25000;$$

$$\text{Income of Suresh} = 6A = 30000$$

$$\text{Spending of Ramesh} = 25000 - 4000 = 21000$$

$$\text{Spending of Suresh} = 30000 - 3000 = 27000$$

Ramesh - 25000, 21000; Suresh - 30000, 27000

48. Find A:B:C:D when A:B = 2:3 ; B:C = 7:9 ; C:D = 5:7

a. 70 : 105 : 135 : 189

b. 105 : 115 : 236 : 189

c. 70 : 124 : 155 : 201

d. 12 : 78 : 256 : 189

Answer: a

Explanation:

$$A:B = 2:3 ; B:C = 7:9 ; C:D = 5:7$$

$$a = 2$$

$$b = 3$$

$$c = 7$$

$$d = 9$$

$$e = 5$$

$$f = 7$$

$$A:B:C:D = [2 \times 7 \times 5] : [3 \times 7 \times 5] : [3 \times 9 \times 5] : [3 \times 9 \times 7]$$

$$A:B:C:D = 70 : 105 : 135 : 189$$

49. Find the mean proportional between 7 and 63?

a. 35

b. 21

c. 27

d. 30

Answer: b**Explanation:****In a: b: c, mean proportion = b****a: b: c can be written as a: b::b c**

$$a: b::b c = \frac{a}{b} = \frac{b}{c} = b^2 = ac$$

Here, a= 7 ; c=63

$$\therefore b = \sqrt{7 \times 63} = 21$$

50. It was intended that Rs. 585 be divided among P, Q and R in the ratio of 4 : 3 : 2, but by mistake the distribution was made in the proportion of 1/4 : 1/3 : 1/2. How much does 'R' gain by the error?

a. Rs. 99

b. Rs. 126

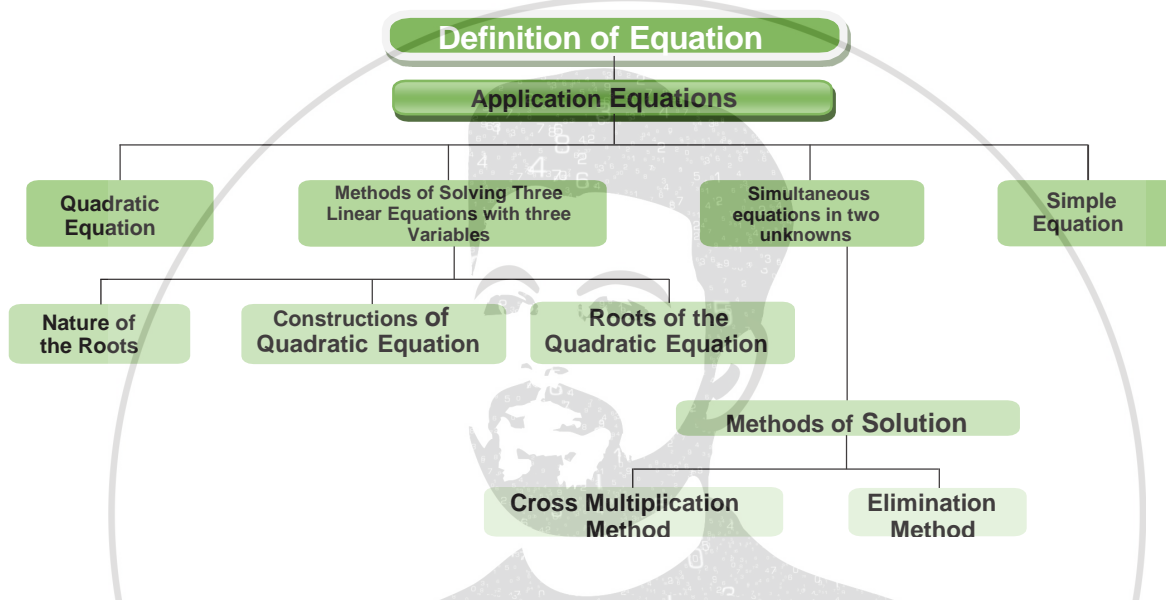
c. Rs. 140

d. Rs. 152

Answer: c**Explanation:****Total amount = Rs. 585****on dividing it in the ratio of 4 : 3 : 2****Share of P = $\frac{4}{9} * 585 = \text{Rs. } 260$** **Share of Q = $\frac{3}{9} * 585 = \text{Rs. } 195$** **Share of R = $\frac{2}{9} * 585 = \text{Rs. } 130$** **But the amount has been divided in the proportion of 1/4 : 1/3 : 1/2 i.e. 3 : 4 : 6****Share of P = $\frac{3}{13} * 585 = \text{Rs. } 135$** **Share of Q = $\frac{4}{13} * 585 = \text{Rs. } 180$** **Share of R = $\frac{6}{13} * 585 = \text{Rs. } 270$** **Therefore, gain for R by virtue of error = Rs. 270 – Rs. 130 = Rs. 140**

CHAPTER 2

UNIT I: EQUATIONS



EQUATION	Equation is defined to be a mathematical statement of equality. If the equality is true for certain value of the variable involved, the equation is often called a conditional equation and equality sign '=' is used; while if the equality is true for all values of the variable involved, the equation is called an identity.
TYPES OF EQUATION	<p>Simultaneous Linear Equations: Two or more linear equations involving two or more variables.</p> <p>Quadratic equation: An equation of degree 2 (highest Power of the variable is 2)</p> <p>Cubic Equation: The equation of degree 3</p>
SIMPLE EQUATION	A simple equation in one unknown x is in the form $ax + b = 0$. Where a, b are known constants and $a \neq 0$

<p>SIMULTANEOUS LINEAR EQUATIONS IN TWO UNKNOWNNS</p>	<p>The general form of a linear equations in two unknowns x and y is $ax + by + c = 0$ where a, b are non-zero coefficients and c is a constant. Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of simultaneous equations in x and y. A value for each unknown which satisfies simultaneously both the equations will give the roots of the equations.</p>
<p>ELIMINATION METHOD</p>	<p>In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.</p>
<p>CROSS MULTIPLICATION METHOD</p>	<p>Let two equations be: $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$ $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $x = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$</p>
<p>QUADRATIC EQUATION</p>	<p>An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree. When $b=0$ the equation is called a pure quadratic equation; when $b \neq 0$ the equation is called an affected quadratic. The roots of a quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p>
<p>CONSTRUCT A QUADRATIC EQUATION</p>	<p>$x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$</p>



1. If one root of a equation is $2 + \sqrt{5}$, then the quadratic equation is;

- a. $x^2 + 4x - 1 = 0$
- b. $x^2 - 4x - 1 = 0$
- c. $x^2 + 4x + 1 = 0$
- d. $x^2 + 4x + 1 = 0$

ANSWER: b

SOLUTION:

If one root is $2 + \sqrt{5}$, then other root will be $2 - \sqrt{5}$. because irrational roots always occur in pairs. Now, equation will be:

$$[x - (2 + \sqrt{5})] [x - (2 - \sqrt{5})] = 0$$

$$x^2 - (2 + \sqrt{5})x - (2 - \sqrt{5})x + (2 + \sqrt{5})(2 - \sqrt{5}) = 0$$

$$x^2 - 2x - \sqrt{5}x - 2x + \sqrt{5}x + (4 - 5) = 0$$

$$x^2 - 4x - 1 = 0$$

2. The equation of a line which is perpendicular to $5x - 2y = 7$ and passes through the mid-point of the line joining $(2, 7)$ and $(-4, 1)$ is :

- a. $2x - 5y - 18 = 0$
- b. $2x + 5y + 18 = 0$
- c. $2x + 5y - 18 = 0$
- d. None of these.

ANSWER: C

SOLUTION:

The equation of a line perpendicular to $5x - 2y - 7 = 0$ is $2x + 5y + k = 0$ -----(1)

mid - point of the line joining $(2, 7)$ & $(-4, 1)$

$$= \left(\frac{2 + (-4)}{2}, \frac{7 + 1}{2} \right) = (-1, 4)$$

Equation - (1) Passes through the mid-point

$$2x + 5y + k = 0$$

$$2(-1) + 5(4) + k = 0$$

c. 1500 , 30

d. None

ANSWER: a**SOLUTION:**

Let the starting salary be x and the annual increment be y . Then, $x + 4y = 1500$

$$X + 10y = 1800$$

Subtracting (1) from (2)

$$X + 10y = 1800$$

$$X + 4y = 1500$$

$$6y = 300$$

$$Y = 50$$

Substituting $y = 50$ in (1), we get $x = 1,300$

Therefore, starting salary = $x = 1,300$

Annual increment = $y = 50$.

5. The value of k for which the points $(k, 1)$, $(5, 5)$ and $(10, 7)$ may be collinear is:

a. $k = -5$ b. $K=7$ c. $k = 9$ d. $K=1$ **ANSWER: a****SOLUTION:**

If the point are collinear, then Area = 0.

$$15k + 35 + 10 - 5 - 50 - 7k = 0 \quad | -2k - 10 = 0$$

$$1 - (2k + 10) = 0$$

$$2k + 10 = 0$$

$$k = \frac{-10}{2} = -5$$

6. A man went to the Reserve Bank of India with 1,000. He asked the cashier

to give him .5 and 10 notes only in return. The man got 175 notes in all. Find how many notes of 5 and f 10 did he receive?

- a. (2 , 150)
- b. (40. 110)
- c. (150, 25)
- d. None

ANSWER : c

SOLUTION:

Let the number of notes of, 5 be x and notes of 10 be y.

Then, $x + y = 175$ (1)

$5x + 10y = 1000$ (2)

Solving (1) and (2) simultaneously. we get

$x + 5y = 875$

$5x + 10y = 1000$

$(-)$ $(-)$ $(-)$

$-5y = -125$

$y = 25$

7. If $(2 + \sqrt{3})$ is a root of a quadratic equation $x^2 + Px + q = 0$, then find the value of p and q.

- a. (4,-1)
- b. (4,1)
- c. (-4,1}
- d. (2,3)

ANSWER: b

SOLUTION:

If one of the roots of the equation is $2 + \sqrt{3}$, then other root is $2 - \sqrt{3}$

\therefore Sum of roots = $2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Product of roots = $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

\therefore Required equation is :

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

Or $x^2 - 4x + 1 = 0$

Now comparing with $x^2 + px + q = 0$

we get, $p = -4$ and $q = 1$

Required answer is (4.1)

8. If the length of a rectangle is 5 cm more than the breadth and if the perimeter of the rectangle is 40 cm, then the length & breadth of the rectangle will be :

a. 7.5 cm, 2.5 cm

b. 10 cm, 5 cm

c. 12.5 cm, 7.5 cm

d. 15.5 cm, 10.5 cm

ANSWER: C

SOLUTION: Let the breadth of the rectangle be x cm.

Therefore length; $(x + 5)$ cm.

Now, Perimeter = $2(l + b)$

$$40 = 2[(x + 5) + X]$$

$$20 = X + 5 + X -$$

$$20 - 2X + 5$$

$$2X = 20 - 5$$

$$2X = 15$$

$$X = 7.5$$

So, breadth = $x = 7.5$ cm and length = $x + 5 = 7.5 + 5 = 12.5$ cm.

9. A straight line of $x = 15$ is

a. Parallel to Y axis

b. Parallel to X axis

c. A diagonal line

d. Passes through origin

ANSWER : a

SOLUTION :

A straight line $x = 15$ is parallel to Y axis.

The equation clearly depicts that the line passes through the point P (15,0).

10. The point of intersection of the lines $2x - 5y = 6$ and $X + y = 3$

a. (0, 3)

b. (3, 0)

c. (3, 3)

d. (0, 0)

ANSWER b

SOLUTION

$$2x - 5y = 6 \dots\dots (1)$$

$$X + Y = 3 \dots\dots (2)$$

Multiplying eq. (2) by 5 to make the co-efficients of y in eq.(1) and eq. (2) same, we get:-

$$5x + 5Y = 15 \dots\dots\dots(3)$$

Adding eq.(1) and eq. (3)

$$\begin{array}{r} 2x - 5y = 6 \cdot \\ \underline{5x + 5y = 15} \\ 7x = 21 \end{array}$$

$$X = 21/7$$

$$X = 3$$

Substituting the value of x in eq (1)

$$2x - 5y = 6$$

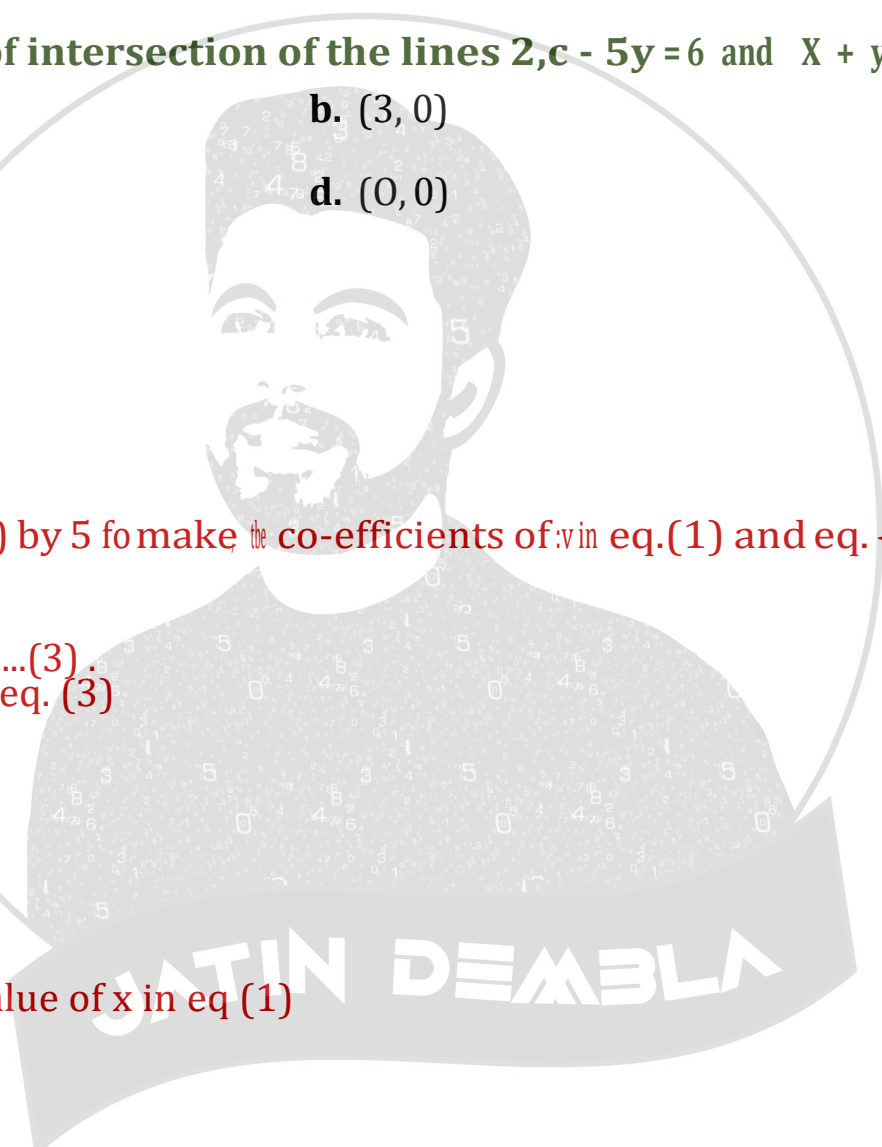
$$2 \times 3 - 5y = 6$$

$$6 - 5y = 6$$

$$5y = 6 - 6$$

$$Y = 0.$$

Point of intersection is (3,0).



$$2x = 12 \Rightarrow X = 6$$

14. If thrice of A's age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

- a. 9
b. 10
c. 11
d. 12

ANSWER: a

SOLUTION:

Let x years be A's present age. By the question $2x -$

$$3(x-6) = x$$

$$\text{or } 2x - 3x + 18 = x$$

$$\text{or } -x + 18 = x$$

$$\text{or } 2x = 18$$

$$\text{or } x = 9$$

A's present age is 9 years.

15. A number consists of two digits the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.

- a. 40
b. 42
c. 39
d. 21
- a. 40
b. 42
c. 39
d. 21

ANSWER: b

SOLUTION:

Let x be the digit in the unit's place. So the digit in the ten's place is 2x. Thus the number becomes $10(2x) + x$. By the question

$$20x + x - 18 = 10x + 2x$$

$$\text{or } 21x - 18 = 12x$$

$$\text{or } 9x = 18$$

$$\text{or } x = 2$$

So the required number is $10(2 \times 2) + 2 = 42$.

16. For a certain commodity the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg. is $d = 100(10 - p)$. The supply equation giving the supply s in kg. for a price p in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

- a. 230
b. 300
c. 600
d. 390

ANSWER: b

SOLUTION:

Given $d = 100(10 - p)$ and $s = 75(p - 3)$.

Since the market price is such that demand (d) = supply (s)

we have $100(10 - p) = 75(p - 3)$

or $1000 - 100p = 75p - 22$

or $-175p = -1022$.

$$p = \frac{-1022}{-175} \times 7$$

So market price of the commodity is ` 7 per kg.

the required quantity bought = $100(10 - 7) = 300$ kg.

and the quantity sold = $75(7 - 3) = 300$ kg.

17. The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction.

- a. 11/17
b. 12/17
c. 13/17
d. 14/18

ANSWER: b

SOLUTION:

Let x be the numerator and the fraction be $\frac{x}{x+5}$

By the question $\frac{x+3}{x+5+3} = \frac{3}{4}$ or

$$4x + 12 = 3x + 24 \text{ or } x = 12$$

The required fraction is $12/17$

18. Solve: $2x + 5y = 9$ and $3x - y = 5$.

a. $x = 2, y = 1.$

c. $x = 1, y = 1$

ANSWER: a.

SOLUTION:

$$2x + 5y = 9 \dots\dots\dots (i)$$

$$3x - y = 5 \dots\dots\dots(ii)$$

By making (i) $\times 1, 2x + 5y = 9$
and by making (ii) $\times 5, 15x - 5y = 25$

Adding $17x = 34$ or $x = 2$. Substituting this values of x in (i) i.e. $5y = 9 - 2x$ we find;

$$5y = 9 - 4 = 5$$

$$y = 1$$

$$x = 2, y = 1.$$

19. The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

a. 40YEARS

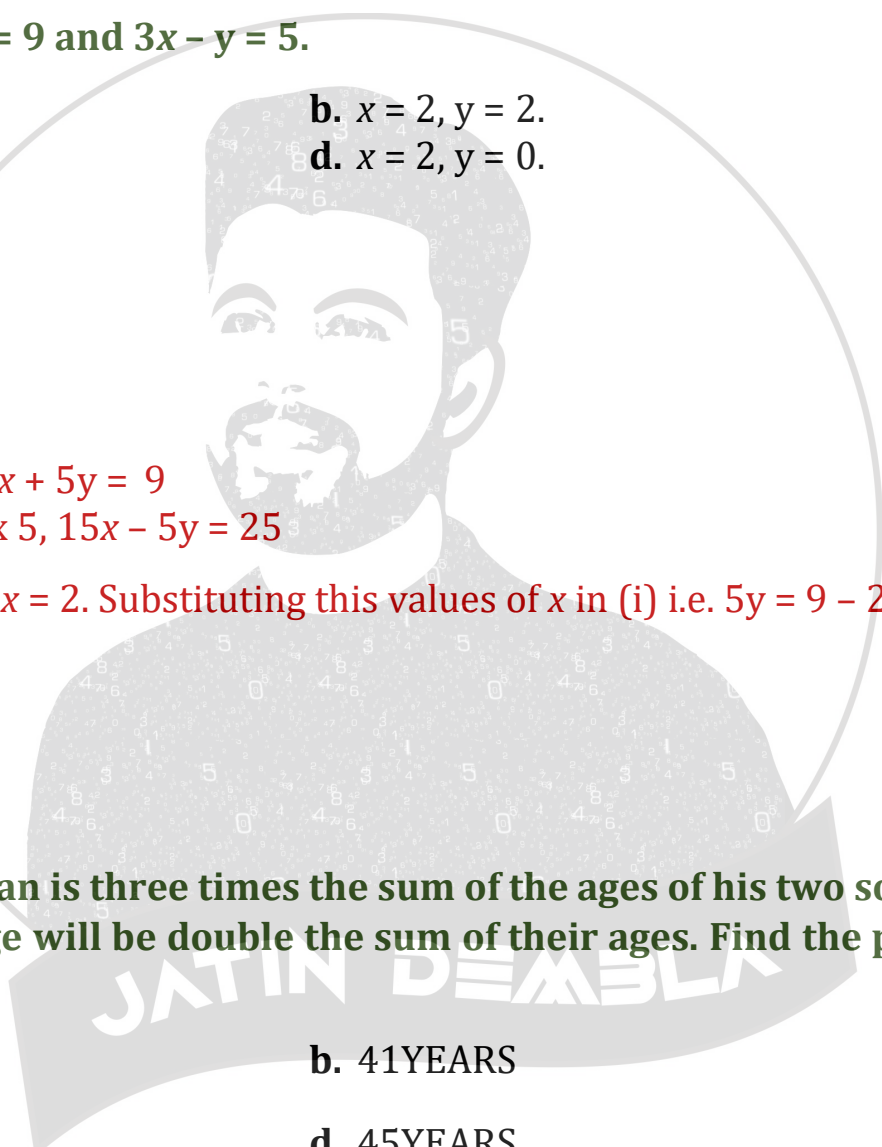
b. 41YEARS

c. 55YEARS

d. 45YEARS

ANSWER: d

SOLUTION:



Let x years be the present age of the man and sum of the present ages of the two sons be y years.

By the condition $x = 3y$ (i)

And $x + 5 = 2(y + 5 + 5)$(ii)

From (i) & (ii) $3y + 5 = 2(y + 10)$

or $3y + 5 = 2y + 20$

or $3y - 2y = 20 - 5$

or $y = 15$

$x = 3 \times y = 3 \times 15 =$

45

Hence the present age of the main is 45 years

20. Examine the nature of the roots of the following equation $x^2 - 8x + 16 = 0$

- a. roots are real and equal b. roots are real, rational and unequal
 c. roots are imaginary and unequal d. roots are real irrational and unequal

ANSWER a

SOLUTION:

$a = 1, b = -8, c = 16$

$b^2 - 4ac = (-8)^2 - 4.1.16 = 64 - 64 = 0$

The roots are real and equal.

22 Two times a number, decreased by 12 equals three times the number, decreased by 15. Which is the number?

- a. -6 b. -62
 c. 2 d. 6

Answer: a

Solution:

$$2x = 5x + 18 \quad 2x = 5x + 18$$

$$3x = -18 \quad 3x = -18$$

$$x = -6$$

23. The roots of a quadratic equation:

a. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

b. $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

c. Either a or b

d. None

ANSWER: a

SOLUTION:

The nature of **the roots** α and β of **equation** $ax^2 + bx + c = 0$ depends on the quantity or expression $(b^2 - 4ac)$ under the square **root sign**. ... Hence, the expression $(b^2 - 4ac)$ is called the discriminant of the **quadratic equation** $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

24. Which of the following is correct ?

- I. If $b^2 - 4ac = 0$ the roots are real and equal;
- II. If $b^2 - 4ac > 0$ then the roots are imaginary;
- III. If $b^2 - 4ac < 0$ then the roots are equal;
- IV. If $b^2 - 4ac$ is a perfect square (> 0) the roots are real, rational and unequal
- V. If $b^2 - 4ac > 0$ but not a perfect square the roots are real, irrational and unequal.

a. All are correct

b. ii & iii

c. all are correct except ii & iii

d. i & iii & iv is correct

ANSWER: C

SOLUTION:

- I. If $b^2 - 4ac = 0$ the roots are real and equal;
- II. If $b^2 - 4ac > 0$ then the roots are real and unequal (or distinct);
- III. If $b^2 - 4ac < 0$ then the roots are imaginary;

IV. If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

V. If $b^2 - 4ac > 0$ but not a perfect square the roots are real, irrational and unequal

Since $b^2 - 4ac$ discriminates the roots $b^2 - 4ac$ is called the discriminant in the equation $ax^2 + bx + c = 0$ as it actually discriminates between the roots.

25. Find the roots of the quadratic equation: $x^2 + 2x - 15 = 0$?

a. 5, 3

c. -3, 5

b. 3, -5

d. -3, -5

ANSWER: b

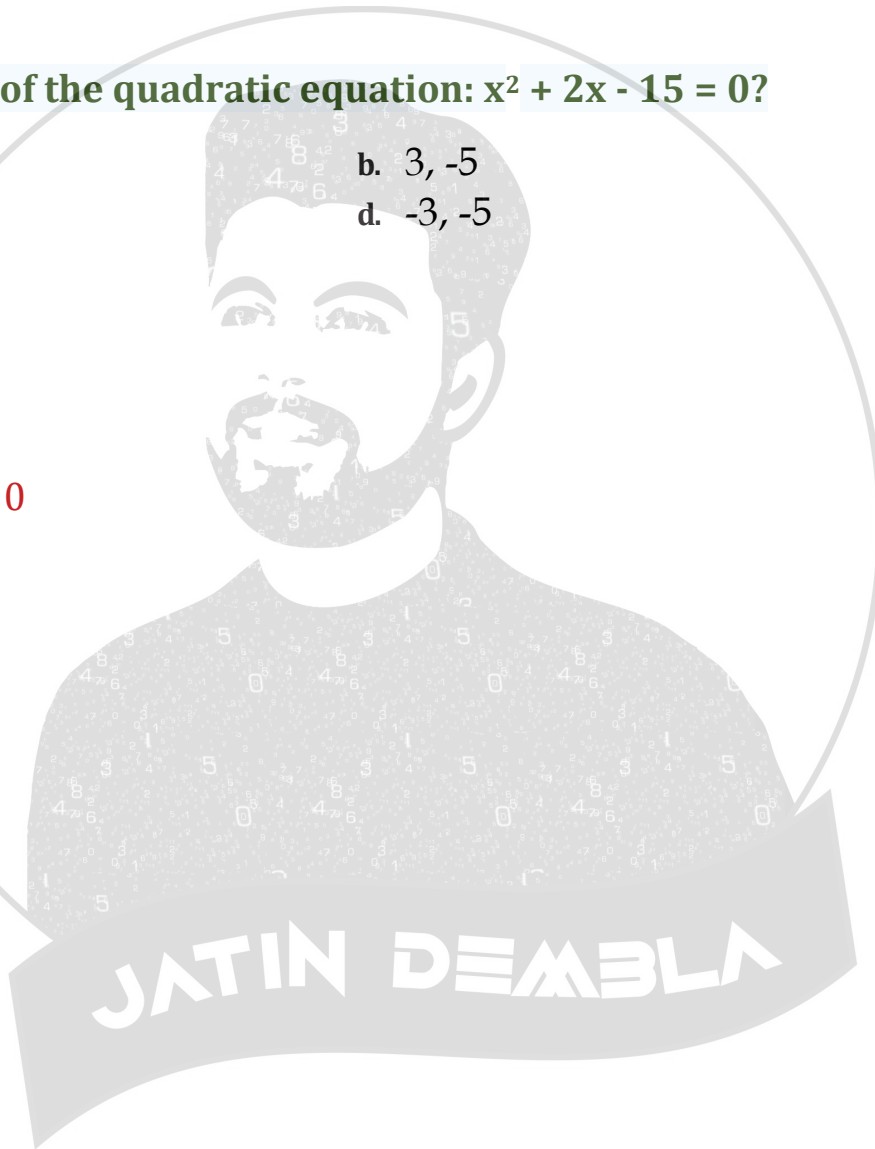
SOLUTION:

$$x^2 + 5x - 3x - 15 = 0$$

$$x(x + 5) - 3(x + 5) = 0$$

$$(x - 3)(x + 5) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -5.$$



ANSWER: A**SOLUTION:**

Squaring the both sides,

$$3-2x+7+2x+2\sqrt{(3-2x)(7x-2x)} = 16$$

$$\sqrt{21} - 8x - 4x^2 = 3$$

Squaring both sides,

$$21 - 8x - 4x^2 = 9 = 4(x^2+2x-3) = 0$$

$$4(x(x+3)-1(x+3))=0$$

$$4((x+3)-1(x-1))=0$$

$$\Rightarrow X=1 \text{ or } x=-3$$

Both these values satisfy the original equation.

$$30. \text{ I. } a^2 + 11a + 30 = 0,$$

II. $b^2 + 6b + 5 = 0$ to solve both the equations to find the values of a and b?

- | | |
|--|------------------|
| a. If $a < b$ | b. If $a \leq b$ |
| c. If the relationship between a and b cannot be established | d. If $a > b$ |

ANSWER: b**SOLUTION:**

$$\text{I. } (a + 6)(a + 5) = 0$$

$$\Rightarrow a = -6, -5$$

$$\text{II. } (b + 5)(b + 1) = 0$$

$$\Rightarrow b = -5, -1 \Rightarrow a \leq b$$

31. A number is equal to 7 times itself minus 18. Which is the number?

- | | |
|-------|-------|
| a. -3 | b. 3 |
| c. 2 | d. -2 |

ANSWER: B**SOLUTION:**

The statement is equivalent to the following equation:

$$x=7x-18 \Rightarrow x=7x-18$$

$$18=7x-x \Rightarrow 18=7x-x$$

$$6x=18 \Rightarrow 6x=18$$

$$x=3$$

32. If a and b are the roots of the equation $x^2 - 9x + 20 = 0$, find the value of $a^2 + b^2 + ab$?

a. -21

b. 1

c. 61

d. 21

ANSWER: C.**SOLUTION:**

$$a^2 + b^2 + ab = a^2 + b^2 + 2ab - ab$$

$$\text{i.e., } (a + b)^2 - ab$$

from $x^2 - 9x + 20 = 0$, we have

$a + b = 9$ and $ab = 20$. Hence the value of required expression $(9)^2 - 20 = 61$.

33. If $a + b = 29$, $b + c = 45$, $a + c = 44$. Find $a + b + c = ?$

a. -21

b. 1

c. 59

d. 118

ANSWER: C.**SOLUTION: .**

$$(a + b) + (b + c) + (a + c) = 29 + 45 + 44$$

$$a + b + b + c + a + c = 118$$

$$2a + 2b + 2c = 118$$

$$2(a + b + c) = 118$$

$$a + b + c = 59$$

34. A simple equation in one unknown x is in the form $ax + b = 0$. Is true or not?

- a. True
- b. False
- c. Not sure
- d. Partial

ANSWER : a

SOLUTION:

A simple equation in one unknown x is in the form $ax + b = 0$. Where a, b are known constants and $a \neq 0$

35 If both the roots of $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common then $2r - p$ is equal to

- a. -1
- b. 0
- c. 1
- d. 2

ANSWER : b

SOLUTION:

Given equation can be written as $(6k + 2)x^2 + rx + 3k - 1 = 0$ (i) and

$2(6k + 2)x^2 + px + 2(3k - 1) = 0$ (ii)

Given equation can be written as $(6k + 2)x^2 + rx + 3k - 1 = 0$ (i) and
 $2(6k + 2)x^2 + px + 2(3k - 1) = 0$ (ii) Condition for common roots is $\frac{12k + 4}{6k + 2}$
 $= \frac{p}{r} = \frac{6k - 2}{3k - 1} = 2$ or $2r - p = 0$

36. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$)

Condition for common roots is $\frac{12k + 4}{6k + 2} = \frac{p}{e}$

ANSWER: b

SOLUTION:

Expressions are $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, then

$$\Rightarrow \frac{x^2}{-22a + 14a} = \frac{x}{a - 2a} = \frac{1}{-14 + 11} \quad \text{or} \quad \frac{x^2}{-8a} = \frac{x}{-a} = \frac{1}{-3} \quad \text{or} \quad x^2 = \frac{8a}{3} \text{ and } x = \frac{a}{3}$$

$\left(\frac{a}{3}\right)^2 = \frac{8a}{3} \Rightarrow \frac{a^2}{9} = \frac{8a}{3} \Rightarrow a = 0, 24$. Trick: We can check by putting the values of a from the options.

If x be real, then the minimum value of $x^2 - 8x + 17$ is **43**.

- a. -1
- b. 0
- c. 1
- d. 2

ANSWER: c

SOLUTION:

$\frac{-4x}{p + q + r}$ Since x is real, so $(x - 4)^2$ is always positive and its least value is 0 and so the minimum value of given expression is 1.

44. Solve the equation $8 + 2(x - 4) = 16$.

- a. -1
- b. 8
- c. 10
- d. 2

ANSWER : b

SOLUTION:

First, we remove the parentheses and get $8 + 2x - 2 \cdot 4 = 16$, or $8 + 2x - 8 = 16$, which gives us $2x = 16$. We divide by 2 in order to get $x = 8$.

45. Solve the equation: $x^3 + 10 = 2x^3 + 10 = 2x$. A) -1

- a. 6
- b. 8
- c. 10
- d. 2

ANSWER: a

SOLUTION:

We first find the lowest common multiple of 4 and 3. It is 12. Multiplying both sides by 12 gives us $x \cdot 3 \cdot 12 - x \cdot 4 \cdot 12 = 2 \cdot 12 \cdot 3 \cdot 12 - x \cdot 4 \cdot 12 = 2 \cdot 12$, or $4x - 3x = 24$, which means that $x = 24$.

49. A number, multiplied by 5, equals itself minus 48. Which is the number?

- a. 6
b. -5
c. 0
d. 12

ANSWER: d

SOLUTION:

$$5x = x - 48$$

$$4x = -48$$

$$x = -12$$

- a. 6
b. -5
c. 0
d. 10

50. Find the solution y to the equation $5y + 49 = 2 + 2y + 46$.

ANSWER: d

SOLUTION:

First, we find the LCM of the denominators (6 and 9). It is 18. Multiplying both sides by 18 yields $18 \cdot 5y + 49 = 2 \cdot 18 + 18 \cdot 2y + 46$, which can be also written as $2(5y + 4) = 36 + 3(2y + 4)$. Removing the parentheses, we get $10y + 8 = 36 + 6y + 12$. By subtracting $6y$ from both sides, we get $4y + 8 = 48$, or $4y = 40$. Dividing by four gives us $y = 10$.

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UNIT 2: MATRICES

MATRICES	<p>Matrices applications are used in Business, Finance and Economics. Matrices applications are helpful to solve the linear equations with the help of this cost estimation, sales projection, etc., can be predicted</p>	
CRAMER'S RULE	<p>In this unit basic applications to matrices and determinates has been studied. Matrix is defined. Some special types of matrices are mentioned. Operations of matrices dealt with. Determinants are defined and their properties are discussed. The methods Cramer's rule.</p>	
MATRIX	<p>A is rectangular matrix with m rows and n columns. The numbers a_{ij}, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$ of this array are called its elements a_{ij}, is associated. We shall denote a matrix either using by using brackets $[]$; or $()$.</p> <p>Order of a Matrix: A matrix A with m rows and n columns is called a matrix of order (m, n) or $m \times n$ (read as m by n).</p>	
TYPES OF MATRICES	Row Matrix	<p>A matrix which has only one row is called a row matrix or row vector.</p> <p>The matrices of the type $[a_1, a_2, a_3, \dots, a_n]$; $[1, 2, 5]$ are examples of row matrices.</p>
	Column Matrix:	<p>A matrix which has only one column is called a column matrix or a column vector.</p>
	Zero Matrix or	<p>If every element of a $m \times n$ matrix is zero, the matrix is called zero matrix or null matrix of order (m, n) and it is denoted by : 0</p>

Null Matrix:	
Square Matrix and Rectangular Matrix	If the number of rows and columns in a matrix are same, such a matrix is called a square matrix; otherwise it is called a rectangular matrix
Scalar Matrix:	A diagonal matrix whose leading diagonal elements are all equal is called a scalar matrix,
Unit Matrix	A scalar matrix whose diagonal elements are equal to unity is called unit matrix and it is denoted by $I_{n \times n}$, if it is order of order
Upper triangle matrix:	A matrix is known as upper triangular matrix if all the elements below leading diagonal are zero. For example. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 7 & 8 \\ 2 & 5 & 4 & 6 \\ 5 & 0 & / & 8 \end{bmatrix}$ the are zero.
Lower Triangular Matrix	A matrix is known as lower triangular matrix if all the elements above the leading diagonal are zero
Sub Matrix:	The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.
Equal Matrices:	Two matrices $A=[a_{ij}]$ and $B=[b_{ij}]$ are said to be equal if they satisfy the following two conditions

- i. The order of both the matrices is same;
- ii. Corresponding elements in both the matrices are equal

Addition and Subtraction of matrices: Let A and B be two matrices of the same order. Then the addition of A and B, denoted by $A+B$, is the matrix obtained by adding corresponding entries of A and similarly to subtract two matrices we just subtract their corresponding elements

Property: If A, B, C are matrices of same order, then

- (i) $A + B = B + A$ (Commutative Law)
- (ii) $(A + B) + C = A + (B + C)$ (Associative Law)
- (iii) $K (A + B) = k.A + m.B$, where m is constant.

Multiplication of two matrices.

The product $A B$ of two matrices A and B defined only if the number of columns in Matrix A is equal to the number of rows in Matrix B.

Properties of matrix Multiplication

- (i) **Matrix multiplication is not commutative in general, i.e. $AB \neq BA$.**
- (ii) **Matrix multiplication is associative $(AB) C = A(BC)$, where both sides are defined.**
- (iii) **Multiplication distributes over addition of Matrices i.e.,**
 - (a) $A (B + C) = AB + AC$
 - (b) $(A + B) C = AC + BC$
- (iv) **If A, B and C are three matrices such that $AB = AC$, then the general $B \neq C$.**

(v) If A is $m \times n$ matrix and O is an $n \times p$ null matrix, then $AO = O, A= O$

(vi) If A is a square matrix and I is a unit matrix of the same order, then $AI = IA = O$

Product of the two no-zero matrices is non zero matrix

Transpose of Matrix: The matrix is obtained by interchanging rows and columns of a matrix A is called its transpose. Transpose of a matrix by A^T or A' .

Properties of transpose of a Matrix:

- (1) A matrix is transpose of its matrix i.e. $A = (A')'$.
- (2) The transpose of the sum of the two matrices is the sum of their transpose matrices, i.e. $(A + B)' = A' + B'$
- (3) Transpose of a multiplication of a matrix and constant number is equal to the multiplication of the constant number by the transpose of matrix, i.e. $(KA)' = K.A'$
- (4) The transpose of the two matrices are equal to the product of

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their transpose in reverse order, i.e., $(AB)' = B'. A'$

1.5 Determinants

Determinant of order 2

Consider a 2×2 matrix: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

▪ Determinant of A , denoted $|A|$, is a number and can be evaluated by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

1.5 Determinants of order 3

Consider an example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Its determinant can be obtained by:

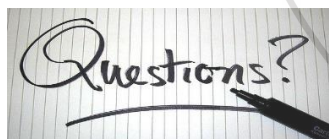
$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} - 6 \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} + 9 \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \\ &= 3(-3) - 6(-6) + 9(-3) = 0 \end{aligned}$$

You are encouraged to find the determinant by using other rows or columns

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JATIN DEMBLA

PROPERTIES OF DETERMINANTS	<ol style="list-style-type: none"> 1) The value of determinant remains unaltered interchanged if its rows or columns interchanged. 2) The value of determinant change signs if any two rows (or columns) interchanges. 3) The value of determinant is zero if any two rows (any columns) then value of determinant is equal to zero. 4) The value determinant becomes k times (where k is constant) if any row or columns multiplied by k the value of determinant also multiplied by k. 5) The value of determinant is zero if any two rows (or column) are proportional then the value of determinant is equal to zero. 6) If each element of any row (or column) is a sum of two numbers, the determinant can be expressed as the sum of the determinants. 7) The value of determinant remains same if to any (or column) multiple of row (or column) is added or subtracted
ADJOINT OF MATRIX	Adjoint of matrix A is transpose of the co-factor matrix of A,



1. If the order of matrix A is $m \times p$. And the order of B is $p \times n$. Then the order of matrix AB is ?

- a. $n \times p$
c. $n \times p$

- b. $m \times n$
d. $n \times m$

ANSWER: b

EXPLANATION:

By definition, the order of a matrix is number of rows X number of columns, generally denoted by $m \times n$ (not compulsory)

2. Select a suitable figure from the four alternatives that would complete the

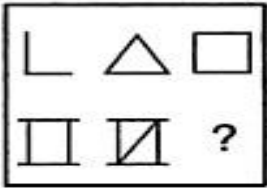
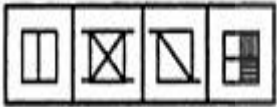


figure matrix.



(1) (2) (3) (4)

a. 1

c. 3

b. 4

d. 2

ANSWER d

EXPLANATION:

One line is increasing in each subsequent figure.

3 What is the order of a matrix?

a. Number of rows X number of columns

c. number of rows X number of rows

b. number of columns X number of rows

d. number of columns X number of columns

Answer: a

Explanation:

By definition, the order of a matrix is *number of rows X number of columns*, generally denoted by $m \times n$ (not compulsory).

4 How do you allocate a matrix using a single pointer in C? (r and c are the number of rows and columns respectively)

a. `int *arr = malloc(r * c * sizeof(int));`

c. `int *arr = (int *)malloc(r + c * sizeof(int));`

b. `int *arr = (int *)malloc(r * c * sizeof(int));`

d. `int *arr = (int *)malloc(r * c * sizeof(arr));`

Answer: b

Explanation:

Total number of elements in the matrix will be $r \times c$

5 Which of the following are the uses of matrices?

- a. In solving linear equations
- b. Image processing
- c. Graph theory
- d. All of the mentioned

Answer: d

Explanation:

Solving linear equations is a separate field in Mathematics involving matrices, Image processing stores the pixels in the form of matrices, and the graphs are represented with the help of matrices to indicate the nodes and edges.

6. What is the disadvantage of matrices?

- a. Internal complexity
- b. Searching through a matrix is complex
- c. Not space efficient
- d. All of the mentioned

Answer: d

Explanation:

Time complexity of a matrix is $O(n^2)$ and sometimes the internal organization becomes tedious.

7 Matrix A when multiplied with Matrix C gives the Identity matrix I, what is C?

- a. Identity matrix
- b. Inverse of A
- c. Square of A
- d. Transpose of A

Answer: b

Explanation:

Any square matrix when multiplied with its inverse gives the identity matrix. Note that non square matrices are not invertible.

8. 2×3 matrix can be multiplied by a 3×4 matrix. The order of resulting matrix will be

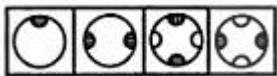
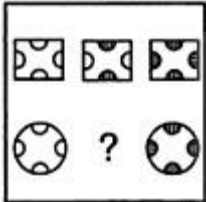
- a. 3×4
- b. 4×3
- c. 2×3
- d. 3×2

Answer: a

Explanation:

Two matrices can be multiplied only if the number of columns of the first is the same as the number of rows of the second

9. Select a figure from the four alternatives that would complete the Figure



Matrix.

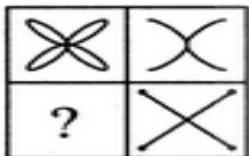
- (1) (2) (3) (4)
- a. 4 b. 3
- c. 2 d. 1

Answer: a

Explanation:

Similar small figures are increasing by one in each of the bigger figure.

10. Select a suitable figure from the four alternatives that would complete the figure matrix.



- (1) (2) (3) (4)

- a. 4 b. 3
- c. 2 d. 1

Answer: b

Explanation:

In each row, the third figure is a collection of the common elements (line segments) of the first and the second figures.

11. For a non-trivial solution $|A|$ is

a. $|A| > 0$

b. $|A| < 0$

c. $|A| \neq 0$

d. $|A| = 0$

Answer: D

Explanation:

If A, B and C are three matrices such that $AB = AC$, then the general $B \neq C$

Hence, $|A| = 0$

12. If A is a symmetric matrix, then $A^t =$

a. 0

b. A

c. $|A|$

d. diagonal matrix

Answer: b

Explanation:

Symmetric matrix is a square matrix that is equal to its transpose.

Hence, A is symmetric matrix

13. Additive inverse of a matrix A is

a. $\text{adi } A/|A|$

b. A^2

c. $|A|$

d. A

Answer: A

Explanation:

The **additive inverse** of a number a is the number that, when added to a, yields zero. This number is also known as the opposite (number), sign change, and negation

14. Two matrices A and B are multiplied to get BA if

a. no of rows of A is equal to no

b. no of columns of A is equal to

of columns of B

columns of B

c. both are rectangular

d. both have same order

Answer: a

Explanation:

It is the reverse, the **number of columns** of the first matrix should match the **number of rows** of the second matrix.

15. A matrix having m rows and n columns with $m \neq n$ is said to be a

a. scalar matrix

b. identity matrix

c. square matrix

d. rectangular matrix

Answer: A

Explanation:

A square diagonal **matrix** with all its main diagonal entries equal is a **scalar matrix**, that is, a **scalar** multiple λI of the identity **matrix**

16. $[a \ b \ c]$ is a

a. zero matrix

b. row matrix

c. column matrix

d. diagonal matrix

Answer: B

Explanation:

Row matrix consisting of a single row of m elements

17. Transpose of a row matrix is

a. zero matrix

b. row matrix

c. column matrix

d. diagonal matrix

Answer: C

Explanation:

Column matrix is a matrix consisting of a single column of m elements

18. Matrices obtained by changing rows and columns is called

a. symmetric

b. transpose

c. rectangular matrix

d. None of Above

Answer: B

Explanation:

Transpose of a Matrix. A matrix which is formed by turning all the rows of a given matrix into columns and vice-versa.

19. A matrix having m rows and n columns with $m = n$ is said to be a

- A. scalar matrix
- B. identity matrix
- C. Square matrix
- D. rectangular matrix

Answer: C

Explanation:

Square matrix is a **square matrix** is a matrix with the same number of rows and columns. An n -by- n matrix is known as a **square matrix** of order n .

20. Which of the following property does not hold for matrix multiplication?

- Associative
- Commutative
- Distributive
- None of the mentioned

Answer: c

Explanation:

In matrix multiplication, $AB \neq BA$

21. Solve the equations by using Cramer's Rule

$2x - y + z = 4$

$x + 3y + 2z = 12$

$3x + 2y + 3z = 16$

a. infinite solutions

b. finite solutions

c. either a or b

d. none

ANSWER a

SOLUTION:

Considering the equations: $2x - y + z = 4$

$x + 3y + 2z = 12$ $3x + 2y + 3z = 16$

$$x = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} 4 & -1 & 1 \\ 12 & 3 & 2 \\ 16 & 2 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \end{vmatrix}} = \frac{4(9 - 4) + 1(36 - 32) + 1(24 - 48)}{2(9 - 4) + 1(3 - 6) + 1(2 - 9)}$$

By using Cramer's Rule, the solution of the equations are given below:

$$\frac{4 \times 5 + 1 \times 4 + (-24)}{2 \times 5 - 3 - 7}$$

Since $\Delta = 0$; $\Delta_x = 0$, $\Delta_y = 0$ and $\Delta_z = 0$,

There the equations are dependent and will have infinite solutions.

22. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction)

a. $A^n = nA + (n-1)I$

b. $A^n = 2^{n-1}A + (n-1)I$

c. $A^n = nA - (n-1)I$

d. $A^n = 2^{n-1}A - (n-1)I$

ANSWER c

SOLUTION:

$$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \quad \therefore A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

$$nA = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix}, (n-1)I = \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix} \quad nA - (n-1)I = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$$

23. In a skew symmetric matrix, the diagonal elements are all

Different from each other

Zero

One

None of these

ANSWER b

SOLUTION:

In a skew symmetric matrix, the diagonal elements are all Zero

24. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{pmatrix}$ then AB is equal to

a. I_3

b. $2I_3$

c. $4I_3$

d. $18I_3$

ANSWER d

SOLUTION:

We have $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{pmatrix}$

$$\therefore AB = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{pmatrix}$$

$$AB = \begin{pmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix} = 18 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$AB = 18I_3$

25. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

a. $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$

b. $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$

c. $\begin{bmatrix} 42 \\ 44 \end{bmatrix}$

d. $\begin{bmatrix} 41 \\ 44 \end{bmatrix}$

ANSWER A

SOLUTION:

$$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} \\ = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$$

26. Assuming that the sums and products given below are defined, which of the following is not true for matrices

a. $A+B=B+A$

b. $AB=AC$ does not imply $B=C$

c. $AB=O$ implies $A=O$ or $B=O$

d. $(AB)'=B'A'$

ANSWER C

SOLUTION:

Assume that A, B are each non-singular - i.e. they are invertible.

Thus, $A^{-1}AB = -1AB = B$, and, $ABB^{-1} = A$.

But AB is a zero matrix, so $A=B=0$

27. What is the size of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 12 & 3 & 4 & 1 \\ 13 & 14 & 1 & 2 \end{bmatrix}$?

a. 2×3

b. 3×2

c. 3×4

d. 4×3

ANSWER C

SOLUTION:

A has 3 rows and 4 columns and hence it is a 3×4 matrix.

28. If I is a unit matrix of order 10, then the determinant of I is equal to

- a. 10
c. 1/10
b. 1
d. 9

ANSWER: B

SOLUTION:

Determinants of unit matrix of any order = 1.

29. Which is true about matrix multiplication

- a. It is commutative
c. Both (a) and (b)
b. It is associative
d. None of these

Answer: B

Solution:

Matrix multiplication distributive and associative not commutative

30. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ then determinant of $A^2 - 2A$ is

- a. 5
c. -5
b. 25
d. -25

Answer: B

Solution:

$$A \neq 0 \setminus A^2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} \text{ and}$$

$$A^2 - 2A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, (A^2 - 2A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 25$$

31. If two matrices A and B are of order $p \times q$ and $r \times s$ respectively, can be subtracted only, if

- a. $p=q$
c. $p=r, q=s$
b. $p=q, r=s$
d. None of these

Answer: C

Solution :

For subtraction of two matrix, they should be of the same order i.e.

$$abc \left(1 + \sum \frac{1}{a} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$q = s$$

32. Choose the correct answer

- a. Every identity matrix is a scalar matrix
 b. Every scalar matrix is an identity matrix
 c. Every diagonal matrix is an identity matrix
 d. A square matrix whose each element is 1 is an identity matrix

Answer: A

Solution:

We know that every identity matrix is a scalar matrix.

33. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then $A^2 =$

- a. Unit matrix
 b. Null matrix
 c. A
 d. - A

Answer: A

Solution:

$$A^2 = A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & 1 \end{bmatrix} = I$$

34. $AB=0$, if and only if

- a. $A \neq 0, B=0$
 b. $A=0, B \neq 0$
 c. $A=0$ or $B=0$
 d. None of these

Answer: D

Solution:

Since $AB=0$, even if $A \neq 0$ and $B \neq 0$

35. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $A^n =$

a. $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$

c. $\begin{bmatrix} n & 1 \\ 0 & n \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$

Answer: A

Solution:

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \text{ and}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^n = A^{n-1} \cdot A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

36. If $\begin{bmatrix} m & n \\ m & n \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = [25]$ and $m < n$, then (m, n)

a. (2, 3)

b. (3, 4)

c. (4, 3)

d. None of these

Answer: B

Solution:

It is obvious that $(m, n) = (3, 4)$.

37. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix}$, then

a. $A' = A$

b. $A' = -A$

c. $A' = 2A$

d. None of these

Answer: B

Solution:

$$A' \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -5 \\ -2 & 5 & 0 \end{bmatrix} = -A$$

38. If $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^2 - 6A =$

a. $3I$ c. $-5I$ b. $5I$

d. None of these

Answer: C**Solution:**

$$\begin{aligned} A^2 - 6A &= \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} - 6 \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 19 & 6 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 24 & 6 \\ 18 & 12 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} \\ &= -5I \end{aligned}$$

39. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $AA' =$

a. 14

b. $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

d. None of these

Answer: C**Solution:**

$$A' = [1 \ 2 \ 3], \text{ therefore } AA' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [1 \ 2 \ 3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

40. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 =$

- a. 5A
- c. 16A

- b. 10A
- d. 32A

Answer: C

Solution:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 2^5 & 0 & 0 \\ 0 & 2^5 & 0 \\ 0 & 0 & 2^5 \end{bmatrix}$$

$$2^4 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$



CHAPTER 3

LINEAR INEQUALITIES

INEQUALITIES	<p>Inequalities are statements where two quantities are unequal but a relationship exists between them. These type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.</p>
LINEAR INEQUALITIES IN ONE VARIABLE AND THE SOLUTION SPACE	<p>Any linear function that involves an inequality sign is a linear inequality. It may be of one variable, or, of more than one variable. Simple example of linear inequalities are those of one variable only; viz., $x > 0$, $x \leq 0$ etc.</p> <div style="text-align: center; border: 1px solid #003366; padding: 5px; margin: 10px 0;"> </div>
SUMMARY OF GRAPHICAL METHOD	<p>It involves:</p> <ol style="list-style-type: none"> i. Formulating the linear programming problem, i.e. expressing the objective function and constraints in the standardised format. ii. Plotting the capacity constraints on the graph paper. For this purpose normally two terminal points are required. This is done by presuming simultaneously that one of the constraints is zero. When constraints concerns only one factor, then line will have only one origin point and it will run parallel to the other axis.

- iii. Identifying feasible region and coordinates of corner points. Mostly it is done by breading the graph, but a point can be identified by solving simultaneous equation relating to two lines which intersect to form a point on graph.
- iv. Testing the corner point which gives maximum profit. For this purpose the coordinates relating to the corner point should put in objectives function and the optimal point should be ascertained.
- v. For decision – making purpose, sometimes, it is required to know whether optimal point leaves some resources unutilized. For this purpose value of coordinates at the optimal point should be put with constraint to find out which constraints are not fully utilized.
- vi. Linear inequalities in two variables may be solved easily by extending our knowledge of straight lines.



1. The Linear relationship between two variables in an inequality

a. $x + by \leq c$

b. $ax + by \leq c$

c. $axy + by \leq c$

d. $ax + bxy \leq c$

Answer: a

Explanation:

The linear relationship between two variables in an inequality is given by $ax + by \leq c$

Any linear function that involves an inequality sign is a linear inequality It may be of one variable, or, of more than one variable.

Ex : $3x + y < 6$, $x - y < 2$, etc

2. On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$, we get the following situation

$(0, 18), (12, 0), (4, 2)$ & $(7, 6)$
 $(5, 0), (0, 10), (4, 2)$ & $(7, 6)$

$(3, 0), (0, 3), (4, 2)$, & $(7, 6)$
 $(0, 18), (12, 0), (4, 2), (0, 0)$ and $(7, 6)$

Answer: a

Explanation:

We draw the graph of $6x + y \geq 18$, $x + 4y \geq 12$, and $2x + y \geq 10$ in-the same plane. The solution set of system is that portion of the graphs of the given inequality which is represented by the intersection of the above three equations.

3. Solve $x + 2 < 4$

a. $x < 2$

b. $x > 2$

c. $x \neq 2$

d. $x < 4$

Answer: a

Explanation:

We need to subtract 2 from both sides of the inequality.

$$x + 2 < 4$$

$$x < 4 - 2$$

$$x < 2$$

4. Solve the inequality $3 - 2x \geq 15$

a. $x \leq 6$

b. $x \leq -6$

c. $x > -6$

d. $x > 6$

Answer: b

Explanation:

we need to subtract 3 from both sides; then divide both sides by -2 (remembering to change the direction of the inequality).

$$= 3 - 2x \geq 15$$

$$= -2x \geq 15 - 3$$

$$= -2x \geq 12$$

$$= x \leq \frac{12}{-2}$$

$$= x \leq -6$$

5. Solve $-1 < 2x + 3 < 6$

a. $-2 < x < 3/2$

b. $2 < x < 3/2$

c. $2 < x < 3/2$

d. $-3 < x < 3/2$

Answer: a

Explanation:

$$= -1 < 2x + 3 < 6$$

Subtract 3 from all 3 sides

$$= -1 - 3 < 2x + 3 - 3 < 6 - 3$$

$$= -4 < 2x < 3$$

Divide all sides by 2

$$= -2 < x < 1.5$$

6. Solve $\frac{x}{2} > 8$

a. $x < 8$

b. $x > 16$

c. $x = 8$

d. $x = 4$

ANSWER : b

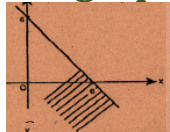
Explanation:

$$= \frac{x}{2} > 8$$

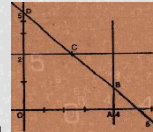
$$= x > 8 \times 2$$

$$= x > 16$$

7. The graph to express the inequality $x + y \leq 6$ is:



a.



b.

c. Either a or b

d. None of these

ANSWER: a

Explanation:

$x + y = 6$ is graphically represent by



8. On the average, experienced person does 5 units of work while a fresh one 3 units work daily but the employer have to maintain the output of at least

30 units of work per day. The situation can be expressed as

- a. $5x + 3y = 30$
- b. $5x + 3y = 30$
- c. $5x + 3y = 30$
- d. None of these

ANSWER: b

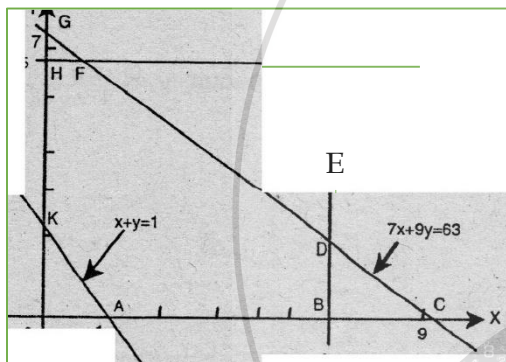
Explanation:

Let Experience Person X unit work per day

Fresh one = Y unit work per day

So situation is $5x + 3y = 30$

9. Common region of the inequalities is:



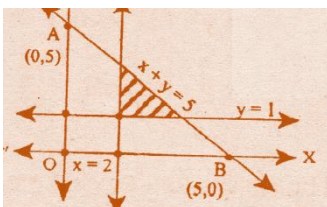
- a. BCDB and DEFD
- b. Unbounded
- c. HFGH
- d. ABDFHKA

ANSWER: d

Explanation:

Common Region of the inequalities is ABDFHKA.

10. The shaded region represents:



- a. $X + y \leq 5, X \geq 1.2, y \geq 1$
- b. $X + y \geq 1.5, X \geq 1.2, y \geq 1$

c. $X + y \leq 5, X \geq 1, y \geq 1$

d. None of these

ANSWER : b**Explanation:**

Region represented by the line $x+y=5$ touch the coordinate axes at $(5, 0)$ and $(0, 5)$ since the shaded region lies below the line $x+y=5$. Hence it is represented by the inequation $x+y \leq 5$

11. A company produces two products A and B, each of which requires processing in two machines. The first machine can be used at most for 60 hours, the second machine can be used at most for 40 hours. The product A requires 2 hours on machine one and one hour on machine two. The product B requires one hour on machine one and two hours on machine two. Above situation is using linear inequalities?

a. True

b. False

c. Partial

d. None

ANSWER : a**Explanation:**

Let the company produce, x number of product A and y number of product B. As each of product A requires 2 hours in machine one and one hour in machine two, x number of product A requires $2x$ hours in machine one and x hours in machine two. Similarly, y number of product B requires y hours in machine one and $2y$ hours in machine two. But machine one can be used for 60 hours and machine two for 40 hours. Hence $2x + y$ cannot exceed 60 and $x + 2y$ cannot exceed 40. In other words,

$$2x + y = 60 \quad \text{and} \quad x + 2y = 40.$$

Thus, the conditions can be expressed using linear inequalities.

12. The inequalities $5x_1 + 4x_2 \geq 9, x_1 + x_2 \geq 3, x_1 \geq 0$ and $x_2 \geq 0$ is correct?

a. True

b. False

c. Not sure

d. None

ANSWER: a

Explanation:

We draw the straight lines $5x_1 + 4x_2 = 9$ and $x_1 + x_2 = 3$.

Table for $5x_1 + 4x_2 = 9$

x_1	0	$9/5$
x_2	$9/4$	0

Table for $x_1 + x_2 = 3$

x_1	0	3
x_2	3	0

Now, if we take the point (4, 4), we find

$$5x_1 + 4x_2 = 9$$

$$\text{i.e., } 5 \cdot 4 + 4 \cdot 4 = 9$$

$$\text{or, } 36 = 9 \text{ (True)}$$

$$x_1 + x_2 = 3$$

$$\text{i.e., } 4 + 4 = 3$$

$$8 = 3 \text{ (True)}$$

Hence (4, 4) is in the region which satisfies the inequalities

13. Solve the inequality $-2(x+3) < 10$

a. $x > -8$

b. $x > 16$

c. $x > 8$

d. $x > -16$

ANSWER : a**Explanation:**

$$-2x - 6 < 10 - 2x - 6 < 10$$

$$-2x - 6 + 6 < 10 + 6 - 2x - 6 + 6 < 10 + 6$$

$$-2x < 16 - 2x < 16$$

$$-2x - 2 > 16 - 2 - 2x - 2 > 16 - 2$$

$$x > -8$$

14. Is (1, 2) a solution to the inequality

a. $7 < 1$

b. $7 > 1$

c. $7 > -1$

d. none

ANSWER : b**Explanation:**

$$2x+3y > 12x+3y > 1$$

$$2 \cdot 1 + 3 \cdot 2 > ? 12 \cdot 1 + 3 \cdot 2 > ? 1$$

$$2 + 5 > ? 12 + 5 > ? 1$$

$$7 > 1$$

15. Solve the absolute value inequality $2|3x+9| < 36$

a. $-9 < x > 3$

b. $-9 < x < 3$

c. $9 < x > 3$

d. $9 < x < 3$

ANSWER : b**Explanation:**

$$2|3x+9| < 36 \quad 2|3x+9| < 36 \quad 2$$

$$|3x+9| < 18 \quad |3x+9| < 18$$

$$-18 < 3x+9 < 18 \quad -18 < 3x+9 < 18$$

$$-18-9 < 3x+9-9 < 18-9 \quad -18-9 < 3x+9-9 < 18-9$$

$$-27 < 3x < 9 \quad -27 < 3x < 9$$

$$-27 \div 3 < 3x \div 3 < 9 \div 3 \quad -27 \div 3 < 3x \div 3 < 9 \div 3$$

$$-9 < x < 3.$$

16. Solve $x + 2 < 4$

a. $x < 1$

b. $x > 1$

c. $x > -2$

d. $x < 2$

ANSWER : d**Explanation:**

We need to subtract 2 from both sides of the inequality.

$$x+2 < 4$$

$$x < 4 - 2$$

$$x < 2$$

17. Solve $\frac{x}{2} > 4$

a. $x < 4$

c. $x > -4$

b. $x > 8$

d. $x < 2$

ANSWER: b

Explanation:

We need to multiply both sides of the inequality by 2.

$$\frac{x}{2} > 4$$

$$x > 4 \times 2$$

$$x > 8$$

18. Solve the inequality $\frac{3}{2}(1 - x) > \frac{1}{4} - x$

a. $x < \frac{5}{2}$

c. $x < \frac{10}{2}$

b. $x < 5$

d. $x < \frac{5}{6}$

ANSWER: a

Explanation:

$$\frac{3}{2}(1 - x) > \frac{1}{4} - x$$

$$6 - 6x > 1 - 4x$$

$$-6x + 4x > 1 - 6$$

$$-2x > -5$$

$$x < \frac{5}{2}$$

19. The solution of the inequality $8x + 6 < 12x + 14$ is:

a. $(-2, 2)$

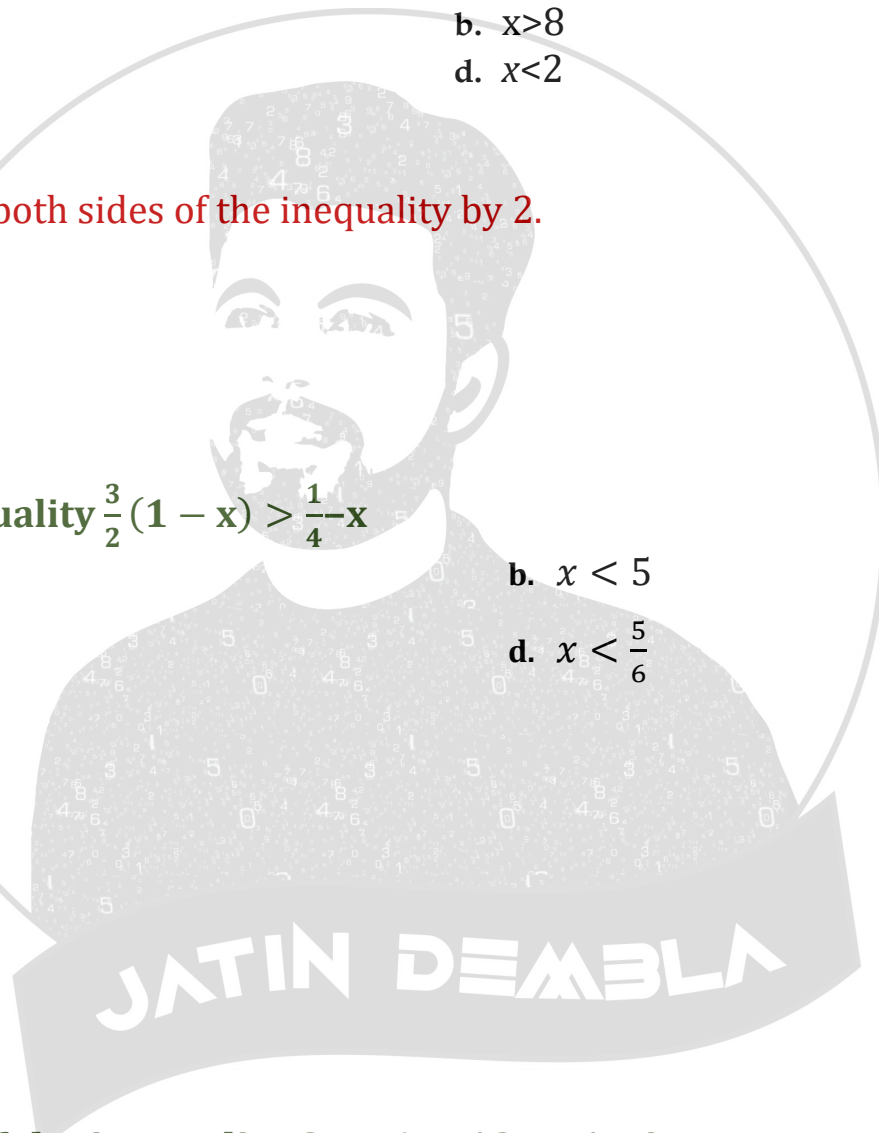
c. $(2,)$

b. $(0, -2)$

d. $(-2,)$

ANSWER: d

Explanation:



$$2x < -5 + 1 \quad \text{or} \quad 2x > 5 + 1$$

$$2x < -4 \quad \text{or} \quad 2x > 6$$

$$x < -2 \quad \text{or} \quad x > 3$$

23. Find all pair of consecutive even positive integers, both of the which are larger than 5 such that their sum is less than 23.

a. (7,8),(7,3)and(2,3)

b. (6,8),(8,10)and(10,12)

c. (5,7),(7,9)and(2,6)

d. (2,3),(4,5)and(3,1)

Answer: b

Explanation:

Let x and $x+2$ be two consecutive even positive integers.

Since both the integer are larger then 5. $x > 5$ $x+2 > 5$ -----(1)

Also sum of two is less than 23

$$x + x + 2 < 23$$

$$\Rightarrow 2x + 2 < 23$$

Adding -2 to both sides

$$2x < 23 - 2$$

$$2x < 21$$

Diving by 2 on both sides,

$$\frac{2x}{2} < 23 - 2$$

$$x < \frac{21}{2}$$

$$x < 10.5$$

Step 2:

Since x is an even positive integer greater than 5 and less then 10.5 x can take value 6,8,10.

Thus the required pair of numbers is (6,8),(8,10) and (10,12)

Hence B is the correct answer.

24. The longest side of a triangle is three times the shortest side and third side is 2cm shorter than the longest side. If the perimeter of the triangle is at least 61cm find the minimum length of the shortest side.

a. 9cm

b. 3cm

c. 5cm

d. 5cm

Answer: A**Explanation:**Let the length of the shortest side be x cmLength of the largest side is $3x$ cmLength of the third side is $3x-2$ cm

Since the perimeter of the triangle is at least 61 cm, we get,

$$x+3x+3x-2 \geq 61$$

$$\Rightarrow 7x-2 \geq 61$$

Adding 2 on both sides,

$$\Rightarrow 7x \geq 61+2$$

$$7x \geq 63$$

Dividing both sides by positive number 7.

$$\frac{7x}{7} \geq \frac{63}{7}$$

$$x \geq 9$$

Step 2:

The minimum length of the shortest side is 9 cm.

Hence A is the correct answer.

25. Solve the inequality : $2 \leq 3x-4 \leq 5$

a. [2,8]

b. [4,5]

c. [3,4]

d. [2,3]

Answer: D**Explanation:**The given inequality is $2 \leq 3x-4 \leq 5$ Adding +4 throughout the inequality $2+4 \leq 3x-4+4 \leq 5+4$

$$\Rightarrow 6 \leq 3x \leq 9$$

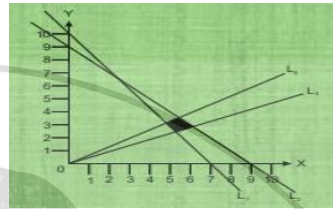
Dividing by positive number 3 through out the inequality $\Rightarrow 2 \leq x \leq 3$

$$\Rightarrow 2 \leq x \leq 3$$

Step 2:

Thus all real number , which are greater than or equal to 2, and less than or equal to 3,

are solutions to the given inequality .
 The solution set is [2,3]
 Hence D is the correct answer.



26 Graphs of Inequations are drawn below:

L1: $5x+3y=30$

L2: $x+y=9$

L3: $y= x/3$

L4: $y=x/2$

The common region region (shaded part) shown in the diagram refers to the inequalities

(a) $5x+3y \leq 30$

(b) $5x+3y \geq 30$

$x+y \leq 9$

$x+y \leq 9$

$y \leq 1/2x$

$y \geq x/3$

$y \leq x/2$

$y \leq x/2$

$x \geq 0, y \geq 0$

$x \geq 0, y \geq 0$

(c) $5x+3y \geq 9$

(d) None of these

$x+y \geq 9$

$y \leq x/3$

$y \geq x/2$

$x \geq 0, y \geq 0$

Answer: d

Explanation:

$5x + 3y > 30$

$x + y < 9$

$y \geq 9$

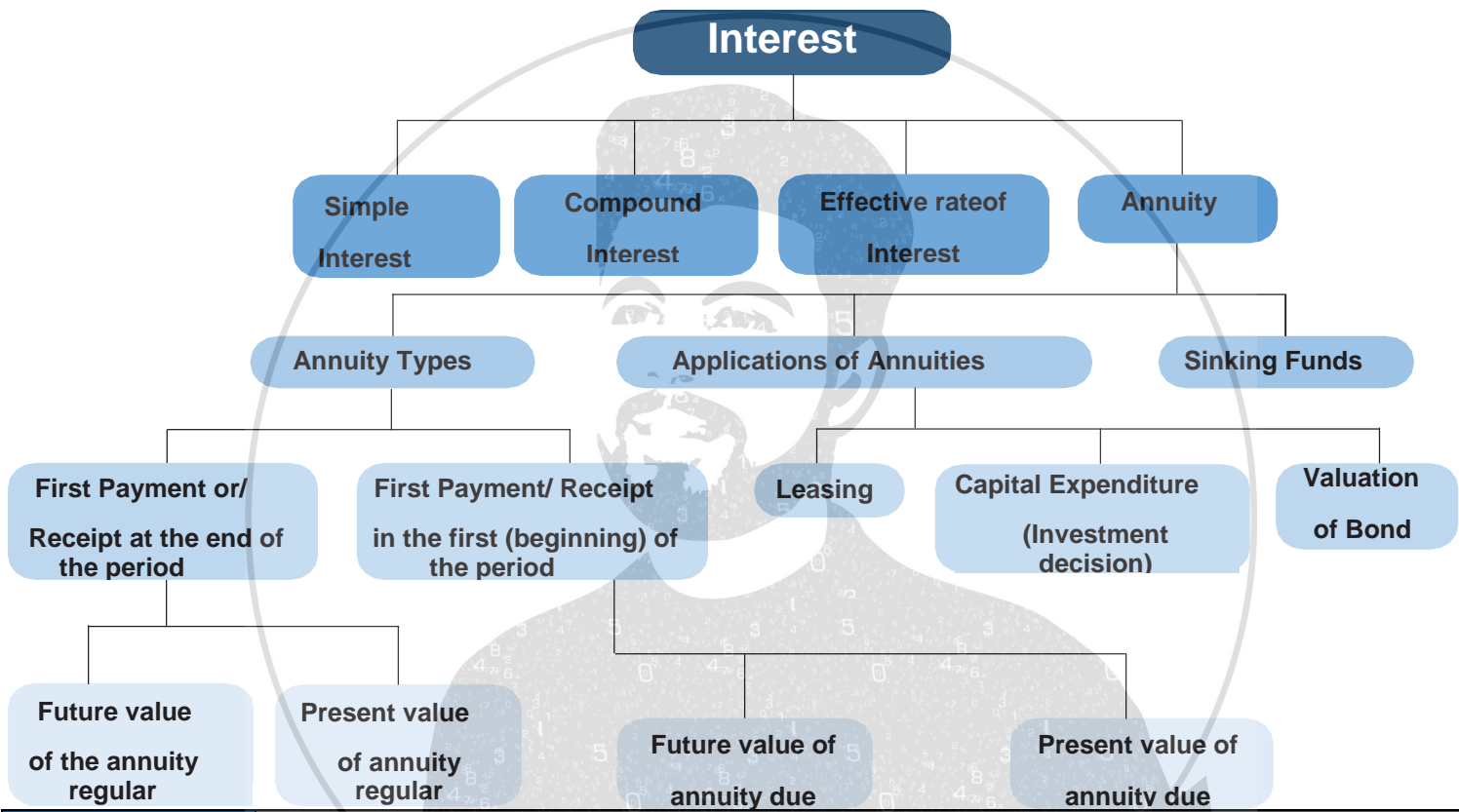
$y \leq x/2$

$x \geq 0; y \geq 0$

JATIN DEMBLA

CHAPTER 4

TIME VALUE OF MONEY

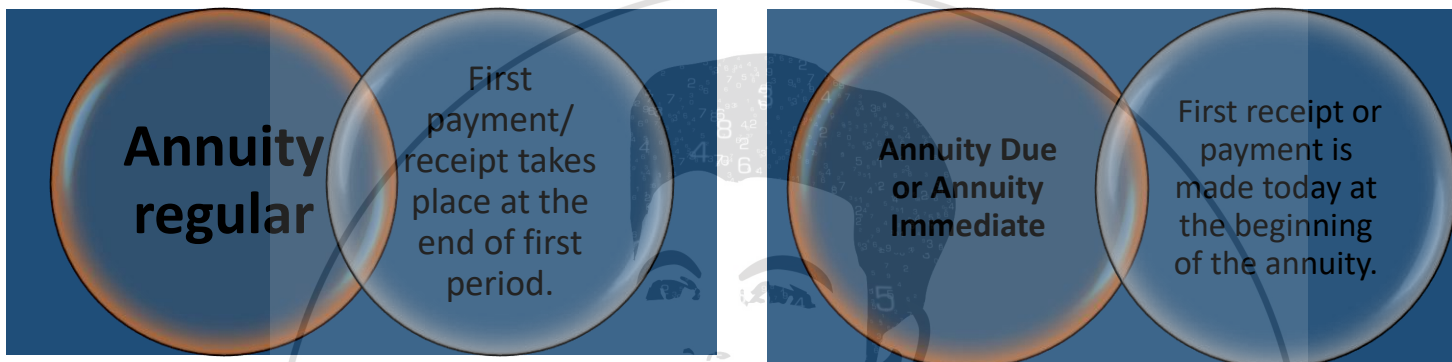


TIME VALUE OF MONEY	Time value of money means that the value of a unity of money is different in different time periods. The sum of money received in future is less valuable than it is today. In other words the present worth of money received after some time will be less than a money received today.
INTEREST	Interest is the price paid by a borrower for the use of a lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending).
SIMPLE INTEREST	The interest computed on the principal for the entire period of borrowing.

	$I = Pit$ $A = P + I$ $I = A - P$ <p>Here, A = Accumulated amount (final value of an investment) P = Principal (initial value of an investment) i = Annual interest rate in decimal. I = Amount of Interest t = Time in years</p>
<p>COMPOUND INTEREST</p>	<p>The interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.</p> <p>Formula for compound interest:</p> $A_n = P (1 + i)^n$ <p>where, i = Annual rate of interest n = Number of conversion periods per year</p> $\text{Interest} = A_n - P = P (1 + i)^n - P$ <p>n is total conversions i.e. t x no. of conversions per year</p>
<p>EFFECTIVE RATE OF INTEREST</p>	<p>The effective interest rate can be computed directly by following formula:</p> $E = (1 + i)^n - 1$ <p>Where E is the effective interest rate i = actual interest rate in decimal n = number of conversion period</p>
<p>FUTURE VALUE</p>	<p>Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore</p> $F = C.F. (1 + i)^n$

ANNUITY	Annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.
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TYPES OF ANNUITY



FUTURE VALUE OF AN ANNUITY DUE/ANNUITY IMMEDIATE	<p>Future value of an Annuity due/Annuity immediate = Future value of annuity regular $\times (1+i)$ where i is the interest rate in decimal.</p> <p>The present value P of the amount A_n due at the end of n period at the rate of i per interest period may be obtained by solving for P the below given equation</p> $A_n = P(1 + i)^n$	
PRESENT VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE	<p>Present value of annuity due/ immediate for n years is the same as an annuity regular for $(n-1)$ years plus an initial receipt or payment in beginning of the period. Calculating the present value of annuity due involves two steps.</p> <p>Step 1: Compute the present value of annuity as if it were a annuity regular for one period short.</p> <p>Step 2: Add initial cash payment/receipt to the step 1 value.</p>	
SINKING FUND	<p>It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Size of the sinking fund deposit is computed from $A = P.A(n,i)$ where A is the amount to be saved the periodic payment, in the payment period.</p>	
ANNUITY	Leasing	Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset

$$I = P \times i \times t$$

$$50000 \times \frac{5.5}{100} \times 2$$

$$= 5500$$

3. Sachin deposited ` 1, 00,000 in his bank for 2 years at simple interest rate of 6%. How much interest would he earn? How much would be the final value of deposit?

- a. 11200
c. 124000
- b. 112000
d. 12400

ANSWER: b

EXPLANATION:

i Required interest amount is given by

$$I = P \times it$$

$$100000 \times \frac{6}{100} \times 2$$

$$= 12,000$$

ii. Final value of deposit is given by

$$= A = P + I$$

$$= (1, 00,000 + 12,000)$$

$$= 1, 12,000$$

4. Rohika invested ` 70,000 in a bank at the rate of 6.5% p.a. simple interest rate. He received ` 85,925 after the end of term. Find out the period for which sum was invested by Rahul.

- a. 3.5 years
c. 0.35 years
- b. 35 years
d. 36 years

ANSWER: b

EXPLANATION:

We know $A = P(1+it)$

$$i.e. 85925 = 70000(1 + \frac{6.5}{100} \times t)$$

$$\frac{85925}{70000} = \frac{100 + 6.5t}{100}$$

$$\frac{85925 \times 100}{70000} - 100 = 6.5t$$

$$22.75 = 6.5t$$

$$t = 3.5$$

$$= \text{time} = 3.5 \text{ years}$$

$$\text{i. e. } 28,600 = P \times \frac{2.5}{100} \times 3 \frac{3}{12}$$

$$28600 = \frac{2.5}{100} P \times \frac{13}{4}$$

$$28600 = \frac{32.5}{400} P$$

$$P = \frac{28600 \times 400}{32.5}$$

$$= 352000$$

`3, 52,000 will produce ` 28,600 interest in 3 years and 3 months at 2.5% p.a. simple interest.

8. In what time Vansh will do ` 85,000 amount to ` 1, 57,675 at 4.5 % p.a.?

- a. 9 years
b. 91 years
c. 19 years
d. 1year

ANSWER: c

EXPLANATION:

We know

$$A = P (1 + it)$$

$$157675 = 85000 \left(1 + \frac{4.5}{100} \times t \right)$$

$$\frac{157675}{85000} = \frac{100 + 4.5t}{100}$$

$$4.5t = \left(\frac{157675}{85000} \times 100 \right) - 100$$

$$t \frac{4.5}{4.5} = 19$$

In 19 years ` 85,000 will amount to ` 1, 57,675 at 4.5% p.a. simple interest rate.

9. A sum of money doubles itself in 10 years. The number of years it would treble itself is:

- a. 25 years
b. 20 years
c. 15 years
d. 18 years

ANSWER: b

EXPLANATION:

Let the sum of money invested be P.

Then, Amount = 2P

$$A = P(1 + it)$$

$$2p = p \left(1 + r \times \frac{10}{100} \right)$$

$$2 = \frac{100 + 10r}{100}$$

$$10r = 100$$

$$R = 10\% \text{ p.a.}$$

Now, year be 20 years

10. A company establishes a sinking fund to provide for the payment of 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if interest is 5% per annum

a. 6142

b. 6049

c. 6052

d. 6159

ANSWER: b

EXPLANATION:

Let the annual deposit be a

$$\text{F.Y.} = [(1 + i)^n - 1]$$

$$2,00,000 = [(1 + 0.05)^{20} - 1]$$

$$10,000 = a(1.6533)$$

$$a = \frac{10000}{1.6533}$$

$$a = 6049$$

11. A machine worth 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to 2,00,000:

a. 5 years 6 months

b. 5 years 7 months

c. 5 years 5 months

d. None

ANSWER: a

EXPLANATION:

$$\text{Amount} = 2,00,000$$

$$\text{In case of depreciation } A = P(1 - i)^n$$

$$2,00,000 = 4,90,740(1 - 0.15)^n$$

$$0.4075 = (0.85)^n$$

$$.(0.85)^{.55} = (0.85)$$

$n = 5.5$ or 5 years 6 months (approx)

12. A sum amount to 1,331 at a principal of 1,000 at 10% compounded annually. Find the time.

- a. 3.31 years
 c. 3 years
 b. 4 years
 d. 2 years

ANSWER: c

EXPLANATION:

$P = 1,000$

$A = 1,331$

$i = 0.10$

Time = n years

$A = P(1 + i)^n$

$1331 = 1000(1 + 0.10)^n$

$1.331 = (1.10)^n$

$(1.10)^3 = (1.10)^n$

$\therefore n = 3$

Therefore, Rs. 1,000 amounts to 1,331 at 10% p.a. C. I. in 3 years

13. If a sum triples in 15 years at simple rate of interest, the rate of interest per annum will be

- a. 13.0%
 c. 1.33%
 b. 13.3%
 d. 13.66%

ANSWER: b

EXPLANATION:

Let Principal $P = P$

Amount $A = 3P$

$T = 15$ years

$S.I. = A - P$

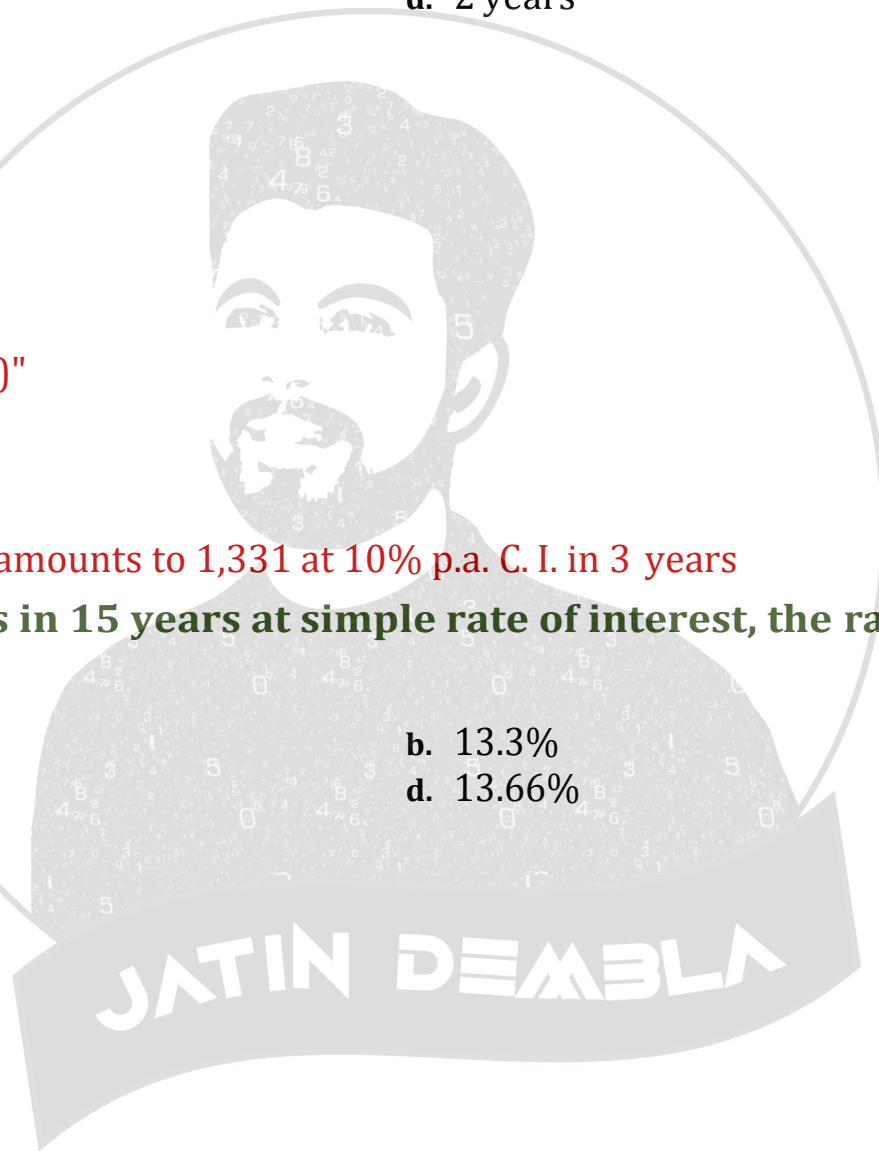
$= 3P - P$

$= 2P$

$$R = \frac{S.I. \times 100}{P \times T}$$

$$R = \frac{40}{3}$$

$$= 13.33\%$$



$i = \text{rate of interest} = 0.12$

$n = \text{time period} = 2$

$F = `3,000(1+0.12)^2$

$= `3,000 \times 1.2544$

$= `3,763.20$

21. Ascertain the compound value and compound interest of an amount of ` 75,000 at 8 percent compounded semiannually for 5 years.

a. 30615

b. 36051

c. 36501

d. 36015

ANSWER: d

EXPLANATION:

Computation of Compound Value and Compound Interest

Semiannual Rate of Interest $(i) = 8/2 = 4\%$

$n = 5 \times 2 = 10$, $P = `75,000$

Compound Value $= P(1+i)^n$

$= 75,000(1+4\%)^{10}$

$= 75,000 \times 1.4802$

$= `1,11,015$

Compound Interest $= `1,11,015 - `75,000 = `36,015.$

22. A doctor is planning to buy an X-Ray machine for his hospital. He has two options. He can either purchase it by making a cash payment of ` 5 lakhs or ` 6, 15,000 are to be paid in six equal annual installments. Which option do you suggest to the doctor assuming the rate of return is 12 percent? Present value of annuity of Re. 1 at 12 percent rate of discount for six years is 4.111

a. 421378

b. 412378

c. 487321

d. 421387

ANSWER: a

EXPLANATION:

Option I:

Cash Down Payment Cash down payment = `5,00,000

Option II:

Annual Installment Basis

$$\text{Annual installment} = 615000 \times \frac{1}{6} = 102500$$

Present Value of 1 to 6 instalments @12%

$$= 1,02,500 \times 4.111$$

$$= 4,21,378$$

23. Calculate if ` 10,000 is invested at interest rate of 12% per annum, what is the amount after 3 years if the compounding of interest is done?

- a. 14049.28
b. 14185.19
c. 14857.61
d. 14094.28

ANSWER: b

EXPLANATION:

$$10,000 \left[1 + \frac{12}{100 \times 2} \right]^{3 \times 2}$$

$$10,000 (1 + 0.06)^6$$

$$= 10,000 \times 1.418519$$

$$= \text{` } 14,185.19$$

24. Present Value” is the current value of a “Future Amount”. The statement is correct or not?

- a. Correct
b. Incorrect
c. Not sure
d. None

ANSWER: a

EXPLANATION:

Present Value” is the current value of a “Future Amount”. It can also be defined as the amount to be invested today (Present Value) at a given rate over specified period to equal the “Future Amount”

24. Simple Interest may be defined as Interest that is calculated as a simple percentage of the restructured amount, is true or false?

- a. True
b. False
c. Partial
d. None

ANSWER: b

EXPLANATION:

Simple Interest may be defined as Interest that is calculated as a simple percentage of the *original principal amount*.

25. Time value of money indicates that

- a. A unit of money obtained today is worth more than a unit of money obtained in future
- b. A unit of money obtained today is worth less than a unit of money obtained in future
- c. There is no difference in the value of money obtained today and tomorrow
- d. None of the above

ANSWER: a

EXPLANATION:

A unit of money obtained today is worth more than a unit of money obtained in future.

26. Time value of money supports the comparison of cash flows recorded at different time period by

- a. Discounting all cash flows to a common point of time
- b. Compounding all cash flows to a common point of time
- c. Using either a or b
- d. None of the above

ANSWER: c

EXPLANATION:

Time value of money supports the comparison of cash flows recorded at different time period by Discounting and compounding all cash flows to a common point of time

27. Accounting; financial management → liquidity decisions

- a. True
- b. False
- c. Partial
- d. None

ANSWER: b

EXPLANATION:

False

It should be → the controller's responsibilities are primarily - in nature, while the treasurer's responsibilities are primarily related to this.

28. Richa borrowed a sum of Rs. 4800 from Ankita as a loan. She promised Ankita that she will pay it back in two equal installments. If the rate of Interest be 5% per annum compounded annually, find the amount of each installment.

- a. 14049.28
- b. 2581.46
- c. 24857.61
- d. 14094.28

ANSWER: b

EXPLANATION:

Given that principal value = 4800

Rate = 5%

Two equal installments annually = 2 years

Applying the formula, $P = X / (1+r/100)^n + \dots + X / (1+r/100)^n$

so, we have here two equal installments.

$$P = X / (1+r/100)^2 + X / (1+r/100)$$

$$4800 = X / (1+5/100)^2 + X / (1+5/100)$$

on simplifying

we have $x = \text{Rs. } 2581.46$

so, the amount of each installment is Rs 2581.46

29. A builder borrows Rs. 2550 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years in two equal yearly installments. How much will each installment be?

- a. Rs. 1352
- b. Rs. 1377
- c. Rs. 1275
- d. Rs. 1283

Answer A

Explanation

Amount = Rs 2550

Rate = 4% per annum

Time = 2 years

Applying the formula

$$P = X / (1+r/100)^n + \dots + X / (1+r/100)^n$$

Here we have two equal installments, so

$$P = \frac{X}{\left[1 + \frac{r}{100}\right]^2} + \frac{X}{\left[1 + \frac{r}{100}\right]}$$

$$= 2550 = \frac{X}{\left[\frac{4}{100}\right]^2} + \frac{X}{\left[1 + \frac{4}{100}\right]}$$

$$= \text{Rs. } 1352$$

30. A man buys a scooter on making a cash down payment of Rs. 16224 and promises to pay two more yearly installments of equivalent amount in next two years. If the rate of interest is 4% per annum, compounded yearly, the cash value of the scooter, is

- a. Rs. 40000
- b. Rs. 46824

c. Rs. 46000

d. Rs. 50000

Answer B**Explanation**

Concept used in this question is: you need to calculate principal for every year unlike simple interest where principal used to be same for every year.

Let principal (present worth) for first year be P_1 and that for two years be P_2 .

$$\therefore 16224 = P_1 \left[1 + \frac{4}{100} \right]$$

$$P_1 = \frac{16224 \times 25}{26} = \text{Rs. } 15600$$

$$\text{Again, } 6224 = P_2 \left[1 + \frac{4}{100} \right]^2$$

$$P_2 = \frac{16224 \times 625}{676} = \text{Rs. } 15000$$

The total payment will be (cash down payment + installments paid)

Cash value of the scooter

$$= \text{Rs. } (16224 + 15600 + 15000) = \text{Rs. } 46824.$$

31. The population of Chandigarh is increases at a rate of 1% for first year, it decreases at the rate of 4% for the second year and for third year it again increases at the rate of 5%. Then what will be the population after 3 years if present population of Chandigarh is 50000.

a. Rs. 51006

b. Rs. 50904

c. Rs. 50836

d. Rs. 51125

Answers: B**Explanations**

Since the rate growth of population is increasing first and then decreasing for the second year and again it increases for third year, then the population after T years will be

$$50,000 \times \left(1 + \frac{1}{100} \right)^1 \times \left(1 + \frac{4}{100} \right)^1 \times \left(1 + \frac{5}{100} \right)^1 = 50904$$

32. A person bought a new machine. The value of the machine is Rs. 10000. If rate of depreciation is 5 % per annum, then what will be the value of the machine after 2 years?

a. Rs. 9025

b. Rs. 9044

c. Rs. 9110

d. Rs. 9080

Answer: A**Explanation**

Here P = Rs 10000

Rate of depreciation = 5%

T = 2 years

Therefore, the value after 2 years will be = $P(1 - R/100)^t$

$$= 10,000 \left(1 - \frac{5}{100}\right)^2 = \text{Rs } 9025.$$

33. A sum of Rs. 6600 was taken as a loan. This is to be repaid in two equal annual instalments. If the rate of interest be 20% compounded annually then the value of each instalment is

a. Rs. 4320

b. Rs. 4400

c. Rs. 2220

d. Rs. 4420

Answer: A**Explanation**

Present worth of Rs. x due T years hence is given by

$$\text{Present worth (PW)} = \frac{x}{\left(1 + \frac{R}{100}\right)^T}$$

$$\frac{x}{\left(1 + \frac{20}{100}\right)^1} + \frac{x}{\left(1 + \frac{20}{100}\right)^2} = 6600$$

$$\frac{x}{\left(\frac{6}{5}\right)} + \frac{x}{\left(\frac{6}{5}\right)^2} = 6600$$

$$\frac{5x}{6} + \frac{25x}{36} = 6600$$

$$\frac{55x}{36} = 6600.$$

$$x = \frac{6600 \times 36}{55} = 4320$$

34. Simple interest on a sum at 5% per annum for 2 years is Rs. 60. The compound interest on the same sum for the same period is

a. Rs. 62.4

b. RS. 61.5

c. Rs. 62

d. Rs. 60.5

Answer: B

Apply log both sides, we get

$$\log(1/60) = n * \log(9/10)$$

$$-1.7781 = n * -0.0457$$

$$38.9 = n$$

Value of type writer becomes 200 after 38.9 years.

37. An annuity with an extended life is classified as

- a. extended life
- b. perpetuity
- c. deferred perpetuity
- d. due perpetuity

Answer: B

Explanation:

A **perpetuity** is a type of annuity that receives an infinite amount of periodic payments. An annuity is a financial instrument that pays consistent periodic payments. As with any annuity, the **perpetuity** value formula sums the present value of future cash flows.

38. Periodic rate if it is multiplied with per year number of compounding periods is called

- a. extrinsic rate of return
- b. intrinsic rate of return
- c. annual rate of return
- d. nominal annual rate

Answer: D

Explanation:

An **interest rate** is called **nominal** if the frequency of compounding (e.g. a month) is not identical to the basic time unit in which the **nominal rate** is quoted (normally a year).

39. A deposit of Rs. 100 is placed into a college fund at the beginning of every month for 10 years. The fund earns 9% annual interest, compounded monthly, and paid at the end of the month. How much is in the account right after the last deposit?

- a. 193751.43
- b. 11244.43
- c. 11231.67
- d. 61658.67

Answer: A

Explanation

The value of the initial deposit is Rs. 100, so $a_1=100$. A total of 120 monthly deposits are made in the 10 years, so $n = 120$. To find r , divide the annual interest rate by 12 to find

the monthly interest rate and add 1 to represent the new monthly deposit.

$$r = 1 + \frac{0.09}{12} = 1.0075$$

Substitute $a_1=100$, $r= 1.0075$

, and $n = 120$ into the formula for the sum of the first n terms of a geometric series, and simplify to find the value of the annuity.

$$S_{120} = \frac{100(1 - 1.0075^{120})}{1 - 1.0075}$$

$$= 19351.73$$

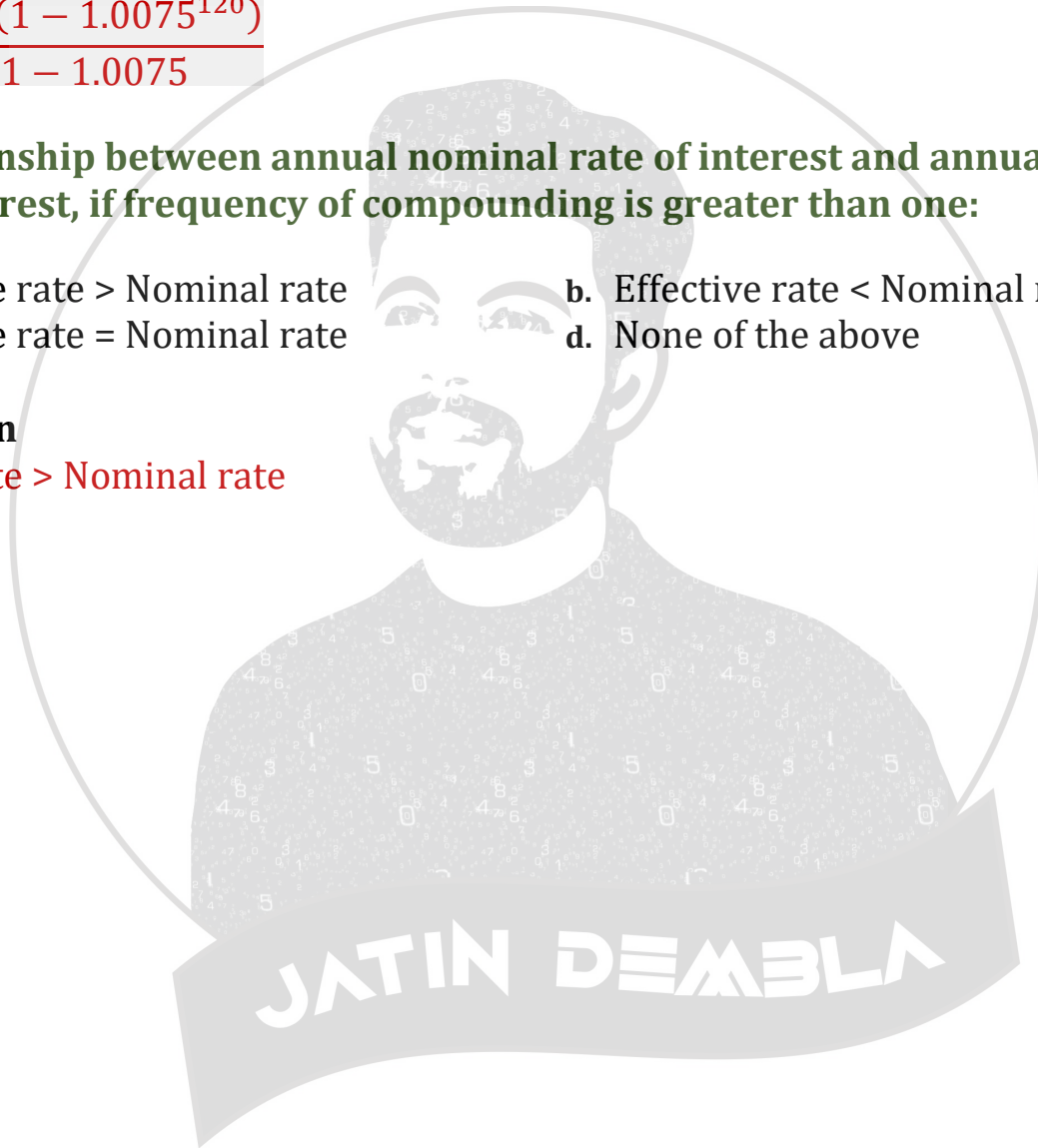
40. Relationship between annual nominal rate of interest and annual effective rate of interest, if frequency of compounding is greater than one:

- a. Effective rate > Nominal rate
- b. Effective rate < Nominal rate
- c. Effective rate = Nominal rate
- d. None of the above

Answer: a

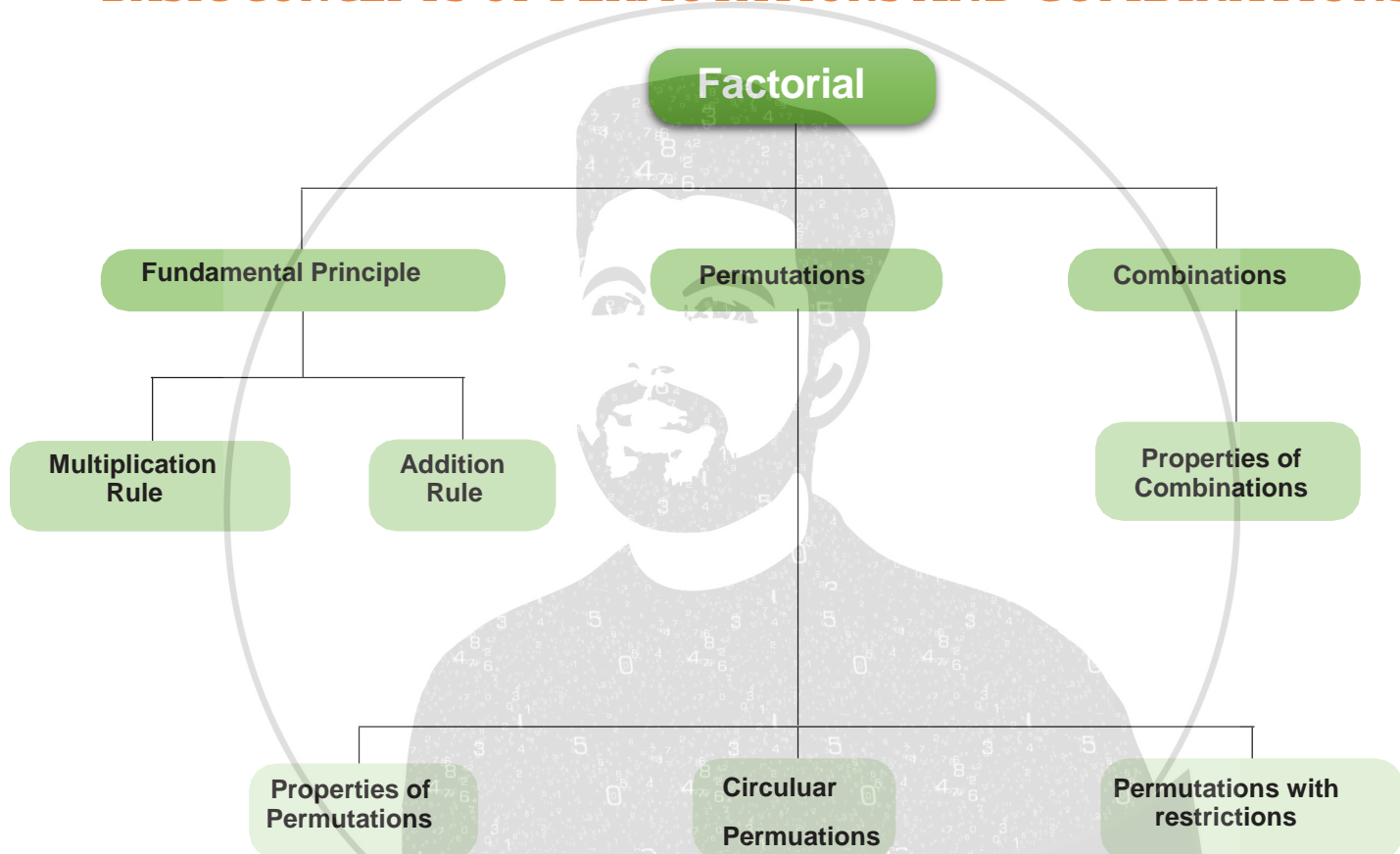
Explanation

Effective rate > Nominal rate



CHAPTER 5

BASIC CONCEPTS OF PERMUTATIONS AND COMBINATIONS



Fundamental principles of counting	Multiplication Rule	If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways then total number of ways of doing both things simultaneously = $m \times n$.
	Addition Rule	If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in $(m + n)$ ways.
Factorial	The factorial n, written as $n!$ or n , represents the product of all integers from 1 to n both inclusive. To make the notation meaningful, when $n = 0$, we define $0!$ or $0 = 1$.	

	<p>Thus, $n! = n (n - 1) (n - 2) \dots$ 3.2.1</p>
<p>Permutations</p>	<p>The ways of arranging or selecting smaller or equal number of persons or objects from a group of persons or collection of objects with due regard being paid to the order of arrangement or selection, are called permutations.</p> <p>The number of permutations of n things chosen r at a time is given by</p> ${}^n P_r = n (n - 1) (n - 2) \dots (n - r + 1)$ <p>Where the product has exactly r factors.</p>
<p>Circular Permutations</p>	<p>(a) n ordinary permutations equal one circular permutation.</p> <p>Hence there are ${}^n P_n / n$ ways in which all the n things can be arranged in a circle. This equals $(n-1)!$.</p> <p>(b) The number of necklaces formed with n beads of different colours</p> <ul style="list-style-type: none"> • Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1} P_r$. • Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement
<p>Combinations</p>	<p>The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations.</p> ${}^n C_r = n! / r! (n - r)!$ ${}^n C_r = {}^n C_{n-r}$ ${}^n C_0 = n! / \{0! (n-0)!\} = n! / n! = 1.$ ${}^n C_n = n! / \{n! (n-n)!\} = n! / n! \cdot 0! = 1.$ <p>${}^n C_r$ has a meaning only when r and n are integers $0 \leq r \leq n$ and ${}^n C_{n-r}$ has a meaning only when $0 \leq n - r \leq n$.</p> <ul style="list-style-type: none"> • ${}^{n+1} C_r = {}^n C_r + {}^n C_{r-1}$

The candidate can select 8 Questions by selecting at least three from each part in the following ways:

3 questions from part A and 5 questions from part B $= {}^7C_3 \times {}^5C_5 = 35$ ways

4 questions from part A and part B each

$= {}^7C_4 \times {}^5C_4 = 175$ ways.

5 questions from part A and 3 questions from part B $= {}^7C_5 \times {}^5C_3 = 210$ ways.

Hence, the total number of ways in which the candidate can select the question will be $= 35 + 175 + 210 = 420$ ways

2. Code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. How many such code words are there?

a. 6,15,800

b. 46,800

c. 7,19,500

d. 4,10,800

ANSWER: b

EXPLANATION:

The number of ways of filling the first two places with English alphabets $= 26 \times 25 = 650$

The number of ways of filling the last two places with distinct numbers $= 9 \times 8 = 72$

The number of code words that can be formed are $= 650 \times 72$

$= 46800$

3. A boy has 3 library tickets and 8 books of his interest in the library of these 8, he does not want to borrow Mathematics part-II unless Mathematics part-I is also borrowed? In how many ways can he choose the three books to be borrowed?

a. 41

b. 51

c. 61

d. 71

ANSWER: a**EXPLANATION:**

There are two cases possible

CASE 1: When Mathematics Part - II is borrowed (i.e. it means Mathematics Part-I has also been borrowed)

Number of ways = ${}^6C_1 = 6$ ways

CASE 2: When Mathematics part-II is not borrowed (i.e. 3 books are to be selected out of 7)

Number of ways = ${}^7C_3 = 35$ ways

Therefore, total number ways

 $35 + 6 = 41$ ways**4. Find 5! , 4! And 6!**

a. 720

b. 120

c. 380

d. 620

ANSWER: a**EXPLANATION:** $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$; $4! = 4 \times 3 \times 2 \times 1 = 24$; $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ **5. Find 9! / 6! ; 10! / 7!**

a. 630,504

b. 504,720

c. 920,630

d. 121,720

ANSWER: b**EXPLANATION:**

$$\frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = \frac{9 \times 8 \times 7}{7!} = \frac{504}{7!}; \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7}{7!}$$

$$10 \times 9 \times 8 = 720$$

6. Find x if $1/9! + 1/10! = x/11!$

a. 121

b. 112

c. 211

d. 111

ANSWER: a

EXPLANATION:

$$1/9! (1 + 1/10) = x/11 \times 10 \times 9! \text{ or, } 11/10 = x/11 \times 10 \text{ i.e., } x = 121$$

7. Find n if n + 1 = 30

a. $n=30$

b. $n=-6$

c. $n=31$

d. $n=29$

ANSWER: d

EXPLANATION:

$$n = 30 - 1$$

$$n = 29$$

8. Evaluate each of 5P_3 , ${}^{10}P_2$, ${}^{11}P_5$.

a. 540

b. 55440

c. 5440

d. 5540

ANSWER: b

EXPLANATION:

$${}^5P_3 = 5 \times 4 \times (5-3+1) = 5 \times 4 \times 3 = 60,$$

$${}^{10}P_2 = 10 \times \dots \times (10-2+1) = 10 \times 9 = 90,$$

$${}^{11}P_5 = 11! / (11-5)! = 11 \times 10 \times 9 \times 8 \times 7 \times 6! / 6! = 11 \times 10 \times 9 \times 8 \times 7 = 55440$$

9. How many three letters words can be formed using the letters of the word SQUARE?

a. 110

b. 12

c. 120

d. 210

ANSWER: c

EXPLANATION:

Since the word 'SQUARE' consists of 6 different letters, the number of permutations of choosing 3 letters out of six equals ${}^6P_3 = 6 \times 5 \times 4 = 120$

10. In how many different ways can five persons stand in a line for a group photograph?

a. 110 ways

b. 120ways

c. 130ways

d. 20ways

ANSWER: b**EXPLANATION:**

Here we know that the order is important. Hence, this is the number of permutations of five things taken all at a time. Therefore, this equals

$${}^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways.}$$

11. How many three letters words can be formed using the letters of the word HEXAGON?

a. 110

b. 12

c. 120

d. 210

ANSWER: d**EXPLANATION:**

Since the word 'HEXAGON' contains 7 different letters, the number of permutations is ${}^7P_3 = 7 \times 6 \times 5 = 210$.

12. First, second and third prizes are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many different ways can the prizes be awarded?

a. 1110 ways

b. 1320ways

c. 1830ways

d. 1716ways

ANSWER: d**EXPLANATION:**

Here, order of selection is important and repetitions are not meaningful as no exhibit can receive more than one prize. Hence, the answer is the number of permutations of 13 things taken three at a time. Therefore, we find ${}^{13}P_3 = 13! / 10! = 13 \times 12 \times 11 = 1,716$ ways

13. In how many different ways can 3 students be associated with 4 chartered accountants, assuming that each chartered accountant can take at most one student?

a. 10

b. 12

c. 20

d. 24

ANSWER: d**EXPLANATION:**

This equals the number of permutations of choosing 3 persons out of 4. Hence, the answer is ${}^4P_3 = 4 \times 3 \times 2 = 24$.

14. Compute the sum of 4 digit numbers which can be formed with the four digits 1, 3, 5, 7, if each digit is used only once in each arrangement.

a. 1,06,656

b. 1,46,800

c. 7,19,500

d. 4,10,800

ANSWER: a**EXPLANATION:**

The number of arrangements of 4 different digits taken 4 at a time is given by ${}^4P_4 = 4! = 24$. All the four digits will occur equal number of times at each of the positions, namely ones, tens, hundreds, thousands.

Thus, each digit will occur $24 / 4 = 6$ times in each of the positions. The sum of digits in one's position will be $6 \times (1 + 3 + 5 + 7) = 96$. Similar is the case in ten's, hundred's and thousand's places. Therefore, the sum will be $96 + 96 \times 10 + 96 \times 100 + 96 \times 1000 = 1,06,656$.

15. In how many different ways can a club with 10 members select a President, Secretary and Treasurer, if no member can hold two offices and each member is eligible for any office?

a. 720

b. 780

c. 960

d. 630

ANSWER: a**EXPLANATION:**

The answer is the number of permutations of 10 persons chosen three at a time. Therefore, it is ${}^{10}P_3 = 10 \times 9 \times 8 = 720$

16. When Jiana arrives in New York, she has eight shops to see, but he has time only to visit six of them. In how many different ways can he arrange her schedule in New York?

- a. 20,160
c. 26105
- b. 2016
d. 21560

ANSWER: a

EXPLANATION:

She can arrange his schedule in ${}^8P_6 = 8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20,160$ ways

17. When Dr. Ramanujan arrives in his dispensary, he finds 12 patients waiting to see him. If he can see only one patient at a time, find the number of ways, he can schedule his patients if they all want their turn.

- a. 479,001,600 ways
c. 79,333,600 ways
- b. 79,833,000 ways
d. 78,833,600 ways

ANSWER: d

EXPLANATION:

There are $12 - 3 = 9$ patients. They can be seen ${}^{12}P_9 = 79,833,600$ ways.

18. How many arrangements can be made out of the letters of the word 'DRAUGHT', the vowels never be separated?

- a. 1440
c. 740
- b. 720
d. 750

ANSWER: a

EXPLANATION:

The word 'DRAUGHT' consists of 7 letters of which 5 are consonants and two are vowels. In the arrangement we are to take all the 7 letters but the restriction is that the two vowels should not be separated.

We can view the two vowels as one letter. The two vowels A and U in this one letter can be arranged in $2! = 2$ ways. (i) AU or (ii) UA. Further, we can arrange the six letters: 5 consonants and one letter compound letter consisting of two vowels. The total number of ways of arranging them is ${}^6P_6 = 6! = 720$ ways.

Hence, by the fundamental principle, the total number of arrangements of the letters of the word DRAUGHT, the vowels never being separated = $2 \times 720 = 1440$ ways.

19. A code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. How many such code words are there?

- a. 6,15,800
c. 7,19,500
- b. 46,800
d. 4,10,800

ANSWER: b**EXPLANATION:**

The number of ways of filling the first two places with English alphabets = $26 \times 25 = 650$

The number of ways of filling the last two places with distinct numbers = $9 \times 8 = 72$

The number of code words that can be formed are = 650×72

= 46,800

20. A boy has 3 library tickets and 8 books of his interest in the library of these 8, he does not want to borrow Mathematics part-II unless Mathematics part-I is also borrowed? In how many ways can he choose the three books to be borrowed?

a. 61

b. 51

c. 41

d. 31

ANSWER: c**EXPLANATION:**

When Mathematics part-II is not borrowed (i.e. 3 books are to be selected out of 7)

Number of ways = ${}^7C_3 = 35$ ways

Therefore, total number of ways

$35 + 6 = 41$ ways.

21. An examination paper with 10 questions consists of 6 questions in mathematics and 4 questions in statistic part. At least one question from each part is to be attempted in how many ways can this be done?

a. 1024

b. 945

c. 1000

d. 1022

ANSWER: b**EXPLANATION:**

Total question = 10

No. of Mathematics questions = 6 No. of Statistics questions = 4

No. of ways at least one question of Mathematics

= $(2^6 - 1) = (64 - 1) = 63$

No. of ways at least one question of statistics

= $(2^4 - 1) = (16 - 1) = 15$

Total no. of ways = $63 \times 15 = 945$

c. 220

d. 230

ANSWER: b**EXPLANATION:**

The number of ways in which 6 articulated clerks can be selected out of 10 candidates
 $= {}^{10}C_6 = 210$ ways.

25. Six persons A, B, C, D, E and F are to be seated at a circular table. In how many ways can this be done, if A must always have either B or C on his right and B must always have either C or D on his right?

a. 3

b. 6

c. 12

d. 18

ANSWER: d**EXPLANATION:**

Using the given restrictions, we must have AB or AC and BC or BD.

Therefore, we have the following alternatives

ABC, D, E, F which gives $(4-1)!$ or 3! ways.

ABD, C, E, F which gives $(4-1)!$ or 3! ways.

AC, B, E, F which gives $(4-1)!$ or 3! ways.

Hence, the total number of ways are

$$= 3! + 3! + 3!$$

$$= 6 + 6 + 6 = 18 \text{ ways}$$

26. Fundamental principles of counting is:

a. $m \times n, m - n$ b. $m \times n, m + n$ c. $m + n, m \div n$ d. $m \div n, m - n$ **ANSWER: b****EXPLANATION:**

Fundamental principles of counting

a. Multiplication Rule : $m \times n$

b. Addition Rule: $m+n$

If ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$, then the value of n is 27.

a. 3

b. 4

c. 2

d. 5

ANSWER: A

$$= 989 - \left\{ \begin{array}{l} 20, 30, 40, \dots, 100 = 9 \\ 101, 102, \dots, 200 = 19 \\ 201, \dots, 300 = 19 \\ \dots \\ \dots \\ 901, \dots, 990 = 18. \end{array} \right\}$$

$= 989 - (9 + 18 + 19 \times 8) = 810$. Aliter: Between 10 and 1000, the numbers are of 2 digits and 3 digits. Since repetition is allowed, so each digit can be filled in 9 ways. Therefore number of 2 digit numbers $= 9 \times 9 = 81$ and number of 3 digit numbers $9 \times 9 \times 9 = 729$. Hence total ways $= 81 + 729 = 810$

39. The number of ways in which the letters of the word TRIANGLE can be arranged such that two vowels do not occur together is

- a. 1200
- b. 2400
- c. 14400
- d. 14400

ANSWER: C

EXPLANATION:

$\cdot T \cdot R \cdot N \cdot G \cdot L$ Three vowels can be arrange at 6 places in ${}^6P_3 = 120$ ways. Hence the required number of arrangements $= 120 \times 5! = 14400$

40. There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball does not go to box of its own colour is

- a. 8
- b. 7
- c. 9
- d. None of these

ANSWER: C

EXPLANATION:

Since number of derangements in such a problems is given by

$$n! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \dots \dots \dots (-1)^n \frac{1}{n!} \right\}$$

$$\therefore \text{Number of derangements are} = 4! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right\} = 12 - 4 + 1 = 9$$

41. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then $r =$

a. 31

c. 51

b. 41

d. None of these

ANSWER: B**EXPLANATION:**

$$\frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!}$$

$$\frac{30800}{1} = 56 \times 55 \times (51-r) = 30800$$

$$r = 41$$

42. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player just one card, is

a. $\frac{52!}{(17!)^3}$

c. 52!

b. $\frac{52!}{(17!)^2}$

d. none

ANSWER: A**EXPLANATION:**

For the first set number of ways $52C_{17}$. Now out of 35 cards left 17 cards can be put for second in $35C_{17}$ ways similarly for 3rd in $18C_{17}$. One card for the last set can be put in only one way. Therefore the required number of ways for the proper

$$\text{distribution} = \frac{52!}{35!17!} \times \frac{35!}{18!17!} \times \frac{18!}{17!1!} \times 1! = \frac{52!}{(17!)^3}$$

43. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then the number of ways in which they can be seated is

a. $\frac{m!(m+1)!}{(m-n+1)!}$

c. $\frac{(m-1)!(m+1)!}{(m-n+1)!}$

b. $\frac{m!(m-1)!}{(m-n+1)!}$

d. none

ANSWER: A**EXPLANATION:**

First arrange m men, in a row in $m!$ ways. Since $n < m$ and no two women can sit together, in any one of the $m!$ arrangement, there are places in which n women can

$$\text{be arranged in } m + 1P_n = \frac{m!(m+1)!}{\{(m+1)-n\}!} = \frac{m!(m+1)!}{(m-n+1)!}$$

44. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

a. 369

b. 300

c. 271

d. 302

ANSWER: B**EXPLANATION:**

To find the number of times 3 occurs in listing the integer from 1 to 999. (Since 3 does not occur in 1000). Any number between 1 to 999 is a 3 digit number xyz where the digit x, y, z are any digits from 0 to 9. Now, we first count the numbers in which 3 occurs once only. Since 3 can occur at one place in $3C_1$ ways, there are $3C_1 \cdot (9 \times 9) + 3 \times 1 = 300$

45. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is

a. $\frac{10!}{6}$ b. $3!7!$ c. $10P_3 \cdot 7!$

d. None of these

ANSWER: A**EXPLANATION:**

For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in 1. $3C_2 = 3$ ways. The remaining 7 persons can speak in $7!$ ways. Hence, the number of ways in which all the 10 person can speak is $10C_3 \cdot 7! = \frac{10!}{3!} = \frac{10!}{6}$

46. How many words can be made out from the letters of the word INDEPENDENCE, in which vowels always come together

a. 16800

b. 16630

c. 1663200

d. None of these

ANSWER: A**EXPLANATION:**

Required number of ways are $\frac{8!}{2!3!} \times \frac{5!}{4!} = 16800$. {Since IEEEEENDPNDNC = 8 letters}.

47. The exponent of 3 in 100! is

a. 33

b. 44

c. 48

d. 52

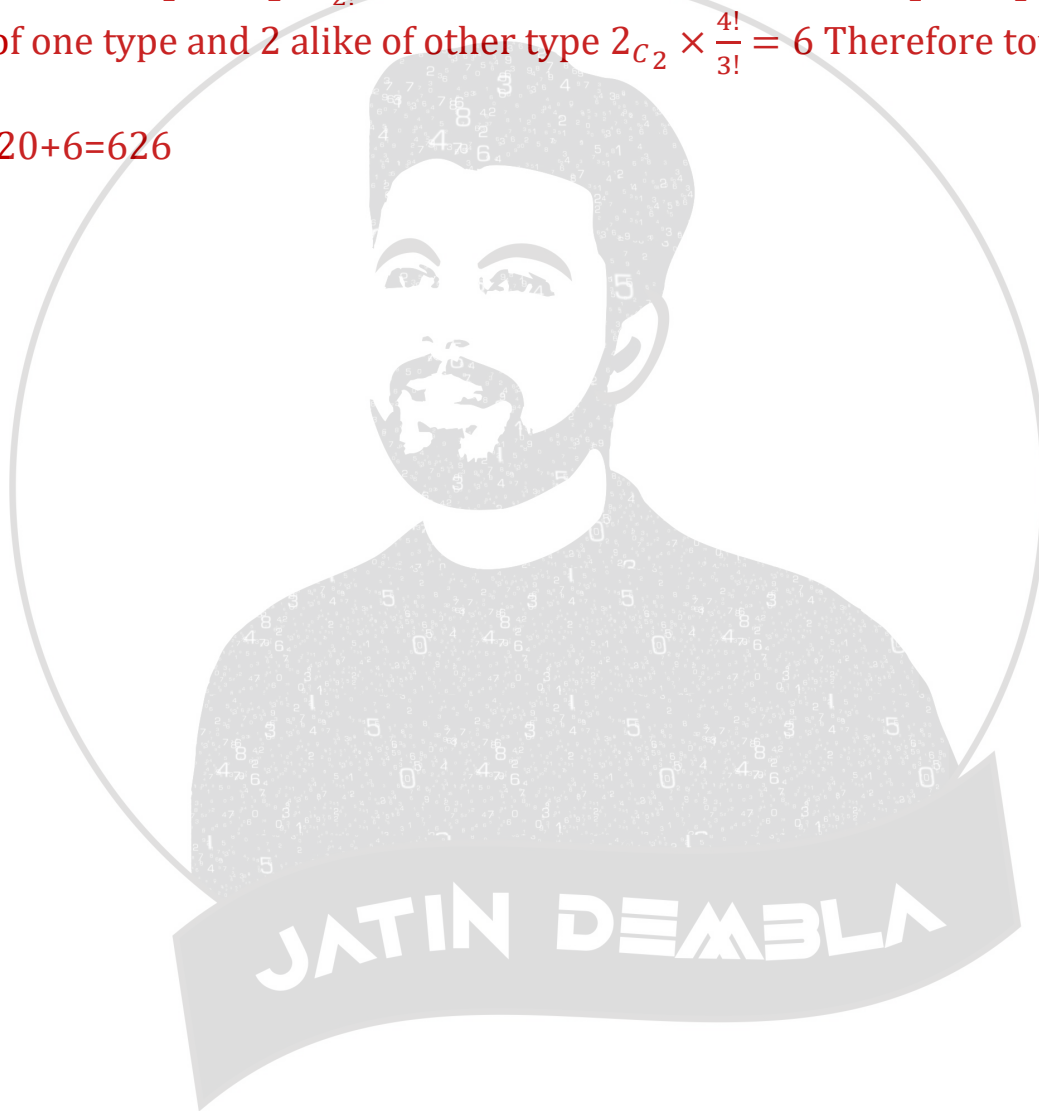
ANSWER: C

c. 620

d. 626

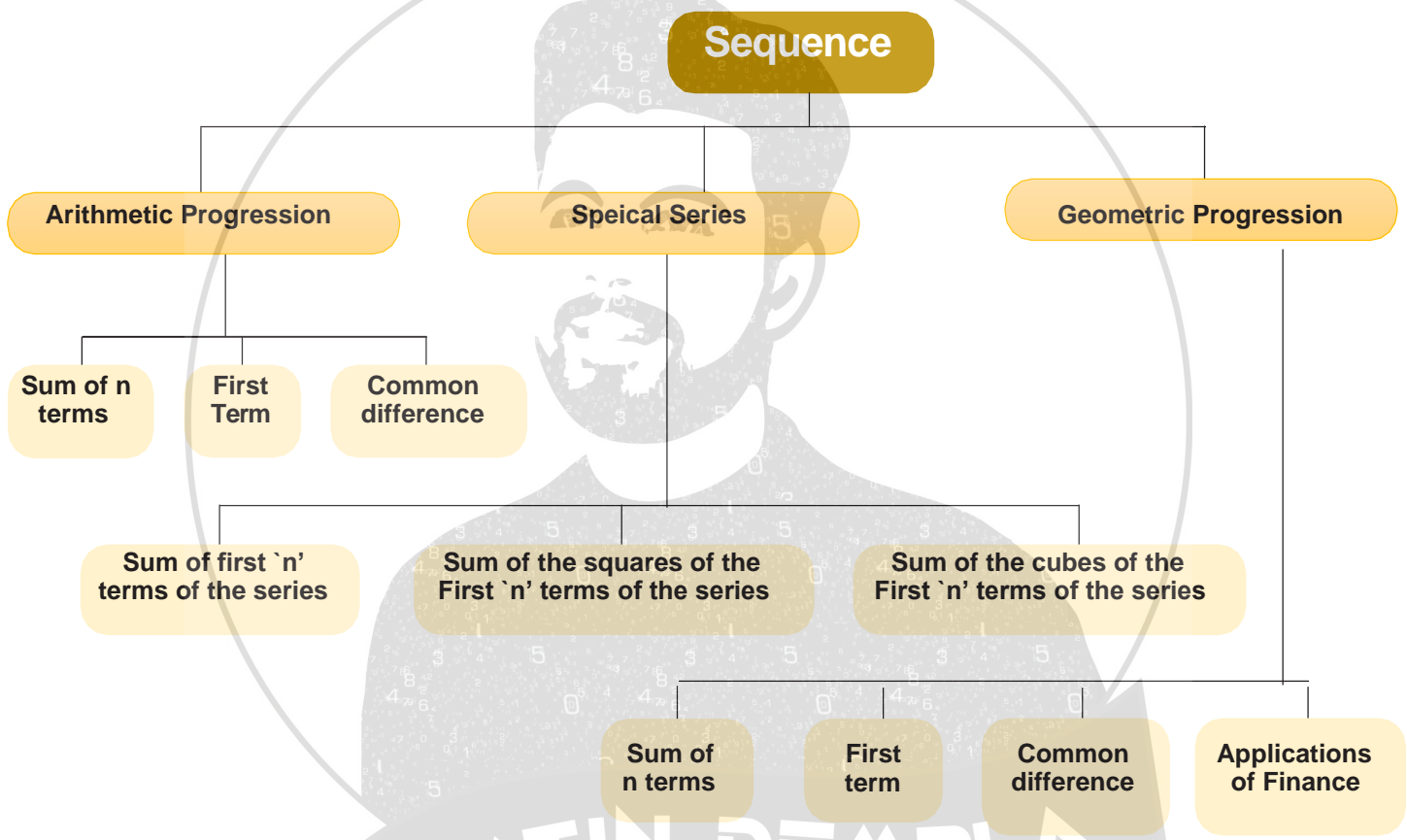
ANSWER: D**EXPLANATION:**

In MORADABAD, we have 6 different types of letters $3A^S$, $2D^S$ and rest four different. We have to form words of 4 letters. (i) All different $6P_4 = 6 \times 5 \times 4 \times 3 = 360$. (ii) Two different two alike $2C_1 \times 5C_2 \times \frac{4!}{2!} = 240$ (iii) 3 alike 1 different $1C_1 \times 5C_1 \times \frac{4!}{3!} = 20$ (iv) 2 alike of one type and 2 alike of other type $2C_2 \times \frac{4!}{2!} = 6$ Therefore total number of words
 $= 360 + 240 + 20 + 6 = 626$



CHAPTER 6

SEQUENCE AND SERIES-ARITHMETIC AND GEOMETRIC PROGRESSIONS



Sequence	<p>An ordered collection of numbers $a_1, a_2, a_3, a_4, \dots, a_n, \dots$ is a sequence if according to some definite rule or law, there is a definite value of a_n, called the term or element of the sequence, corresponding to any value of the natural number n</p> <p>An expression of the form $a_1 + a_2 + a_3 + \dots + a_n + \dots$ which is the sum of the elements of the sequence $\{ a_n \}$ is called a <i>series</i>. If the series contains a</p>
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finite number of elements, it is called a *finite series*, otherwise called *an infinite series*.

Arithmetic Progression

A sequence $a_1, a_2, a_3, \dots, a_n$ is called an Arithmetic Progression (A.P.) when $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$. That means A. P. is a sequence in which each term is obtained by adding a constant d to the preceding term. This constant ' d ' is called the *common difference* of the A.P. If 3 numbers a, b, c are in A.P., we say $b - a = c - b$ or $a + c = 2b$; b is called the arithmetic mean between a and c .

n^{th} term (t_n) = $a + (n - 1)d$

Where a = First Term

$D = \text{Common difference} = t_n - t_{n-1}$

Sum of 1st n natural or counting numbers

Sum of n terms of AP	$s = \frac{n}{2} [2a + (n - 1)d]$
Sum of the first n terms	Sum of 1st n natural or counting numbers $S = n(n + 1) / 2$
Sum of 1st n odd number	$S = n^2$
Sum of the Squares of the first, n natural numbers	$\frac{n(n + 1) (2n + 1)}{6}$

Geometric Progression (G.P)

If in a sequence of terms each term is constant multiple of the proceeding term, then the sequence is called a Geometric Progression (G.P). The constant multiplier is called the *common ratio*

$$\frac{\text{Any term}}{\text{Preceding term}} = \frac{t_n}{t_{n-1}}$$

$$= \frac{ar^{n-1}}{ar^{n-2}} = r$$

Sum of first n terms of a G P	$S_n = a (1 - r^n) / (1 - r)$ when $r < 1$
---------------------------------	--

$$t_5 = a + 4d = 14$$

$$t_{12} = a + 11d = 35$$

On solving the above two equations,

$$7d = 21 \text{ i.e., } d = 3$$

$$\text{and } a = 14 - (4 \times 3) = 14 - 12 = 2$$

Hence, the required A.P. is 2, 5, 8, 11, 14,

3. Divide 69 into three parts which are in A.P. and are such that the product of the first two parts is 483.

a. 21, 23, 25.

b. 21, 22, 23.

c. 22, 23, 25.

d. 21, 22, 25.

ANSWER: a

EXPLANATION:

Given that the three parts are in A.P., let the three parts which are in A.P. be $a - d$, a , $a + d$

$$\text{Thus } a - d + a + a + d = 69$$

$$\text{Or } 3a = 69$$

$$\text{or } a = 23$$

So the three parts are $23 - d$, 23 , $23 + d$

Since the product of first two parts is 483, therefore, we have $23 (23 - d) = 483$

$$\text{Or } 23 - d = 483 / 23 = 21$$

$$\text{or } d = 23 - 21 = 2$$

Hence, the three parts which are in A.P. are $23 - 2 = 21$, 23 , $23 + 2 = 25$

Hence the three parts are 21, 23, 25.

4. Find the arithmetic mean between 4 and 10.

a. 5

b. 7

c. 10

d. 3

ANSWER: b

EXPLANATION:

We know that the A.M. of a & b is $= (a + b) / 2$ Hence, The A. M between 4 & 10 = $(4 + 10) / 2 = 7$

5. Find the G.P. series where 4th term is 8 and 8th term is 128/625

a. 125, 50, 20, 9,

b. 125, 50, 20, 10,

c. 125, 5, 20, 8,

d. 125, 50, 20, 8,

ANSWER: d**EXPLANATION:**

$$t_4 = ar^3 = 8$$

$$t_8 = ar^7 = 128/625$$

Dividing the two terms

$$t_8 / t_4 = ar^7 / ar^3 = 128 / 625 * 1 / 8$$

$$= r^4 = 16 / 625$$

$$= r^4 = (2 / 5)^4$$

$$= r = 2 / 5$$

$$\text{Now } ar^4 = a (2 / 5)^4 = 8$$

$$\text{Solving, } a = 125$$

Thus $a = 125, r = 2/5$, the G.P Series is 125, 50, 20, 8,

6. Insert three Geometric Means between 1/9 and 9

a. $1/9, 1/3, 1, 3, 9.$

b. $1/8, 1/5, 1, 3, 9$

c. $11/9, 1/3, 1, 3, 9$

d. $121/9, 1/3, 1, 3$

ANSWER: a**EXPLANATION:**G.P. Series $1/9, --, --, --, --, 9$

$$\text{Here } t_1 = a = 1/9$$

$$t_5 = a.r^4 = 9$$

$$\text{Now, } t_5 = 1/9.r^4 = 9$$

$$= r^4 = 81$$

$$= r^4 = 3^4$$

$$= r = 3$$

$$t_2 = ar = 1/9 * 3 = 1/3$$

$$t_3 = ar^2 = 1/9 * 3^2 = 1$$

$$t_4 = ar^3 = 1/9 * 3^3 = 3$$

Thus the series $1/9, 1/3, 1, 3, 9.$

7. Find the sum of 1st 8 terms of G.P series $1+2+4+8+.....$

155

255

185

-822

ANSWER: b**EXPLANATION:**

$$\text{Here } a = 1, r = 2, n = 8$$

$$S_n = a.(r^n - 1) / (r - 1) \text{ when } r > 1$$

$$S_8 = 1.(2^8 - 1) / (2 - 1)$$

$$= 1(256 - 1) = 255$$

$$\text{Thus } S_8 = 255$$

8. Find the sum of the series -2, 6, -18.....7 terms?

1554

-1094

1094

-8223

ANSWER: b

EXPLANATION:

Here $a = -2, r = -3, n = 7$

$$S_n = a.(1 - r^n) / (1 - r) \text{ when } r < 1$$

$$S_7 = (-2) [1 - (-3)^7] / [1 - (-3)]$$

$$= (-2)(1 + 2187) / 4$$

$$= (-2)(2188) / 4$$

$$S_7 = -1094$$

9. In a G.P. the product of the 1st three terms 27/8. The middle term is

a. 27/8

b. 3/2

c. 2/9

d. 8/27

ANSWER: b

EXPLANATION:

Let the three terms of GP are $a/r, a, ar$

Now Product of terms

$$a/r * a * ar = 27/8$$

$$a^3 = 27/8$$

$$a^3 = (3/2)^3$$

$$a = 3/2$$

Thus the middle term, $a = 3/2$

10. If you save 1 paisa today, 2 paisa the next day and 4 paisa the succeeding day and so on, then your total savings in two weeks will be.

a. Rs. 168.32

b. Rs.163.98

c. Rs.163.83

d. None

ANSWER: c**EXPLANATION:**

Here the pattern of savings shows the G.P series 0.01, 0.02, 0.04,.....

Here $a = 0.01$, $r = 2$, $n = 14$

$$S_n = a.(r^n - 1) / (r - 1) \text{ when } r > 1 \quad S_{14} = 0.01(2^{14} - 1) / (2 - 1)$$

$$= 0.01(16384 - 1) / 1$$

$$= 0.01 * 16383$$

$$S_{14} = 163.83$$

Thus the total savings in 14 days would be Rs 163.83.

11. The sum of the Infinite G.P Series $1 - 1/3 + 1/9 - 1/27 + \dots$

a. 0.75

b. 75

c. 0.57

d. 57

ANSWER: a**EXPLANATION :**Here $a = 1$, $r = (-1/3)$

$$S_\infty = a / (1 - r) = 1 / [1 - (-1/3)]$$

$$= 1 / [1 + 1/3]$$

$$= 1 / [4/3]$$

$$= 3/4$$

$$= 0.75$$

12. Find the 10th term of the A. P. : 2, 4, 6,.....

a. 20

b. 40

c. 2

d. 0.20

ANSWER: a**EXPLANATION :**Here the first term (a) = 2 and common difference $d = 4 - 2 = 2$ Using the formula $t_n = a + (n - 1) d$, we have

$$t_{10} = 2 + (10 - 1) 2 = 2 + 18 = 20$$

Hence, the 10th term of the given A. P. is 20

13. The 10th term of an A. P. is - 15 and 31st term is -57, find the 15th term.

a. -20

b. 20

c. -25

d. 25

ANSWER: c**EXPLANATION :**

Let a be the first term and d be the common difference of the A. P. Then from the formula:

$t_n = a + (n - 1) d$, we have

$$t_{10} = a + (10 - 1) d = a + 9d \quad t_{31} = a + (31 - 1) d = a + 30d$$

We have,

$$a + 9d = -15 \quad \dots(1)$$

$$a + 30d = -57 \dots(2)$$

Solve equations (1) and (2) to get the values of a and d . Subtracting (1) from (2), we have

$$21d = -57 + 15 = -42$$

$$\text{Again from (1), } a = -15 - 9d = -15 - 9(-2) = -15 + 18 = 3$$

$$\text{Now } t_{15} = a + (15 - 1)d$$

$$= 3 + 14(-2) = -25$$

14. Which term of the A. P.: 5, 11, 17 ... is 119?

a. $n = 20$ b. $n = 2$ c. $n = 30$ d. $n = 19$ **ANSWER: a****EXPLANATION :**

Here $a = 5$, $d = 11 - 5 = 6$

$t_n = 119$ We know that

$$t_n = a + (n - 1) d$$

$$\Rightarrow 119 = 5 + (n - 1) \times 6$$

$$(n - 1) = \frac{119 - 5}{6} = 19$$

$n = 20$, Therefore, 119 is the 20th term of the given A. P.

15. Is 600 a term of the A. P.: 2, 9, 16, ... ?

yes

no

not sure

none

ANSWER: b**EXPLANATION:**

Here, $a = 2$, and $d = 9 - 2 = 7$.

Let 600 be the n^{th} term of the A. P. We have $t_n = 2 + (n - 1) 7$

According to the question

$$2 + (n - 1) 7 = 600$$

$$(n - 1) 7 = 598$$

$$\text{Or } n = \frac{598}{7} + 1 \quad n = 86\frac{3}{7}$$

Since n is a fraction, it cannot be a term of the given A. P. Hence, 600 is not a term of the given A.P.

16. The common difference of an A. P. is 3 and the 15th term is 37. Find the first term.

a. -5

b. 5

c. 42

d. -42

ANSWER: a

EXPLANATION :

Here, $d = 3$, $t_{15} = 37$, and $n = 15$ Let the first term be a . We have

$$t_n = a + (n - 1) d$$

$$37 = a + (15 - 1) 3$$

$$\text{or, } 37 = a + 42$$

$$a = -5$$

Thus, first term of the given A. P. is - 5

17. Geometric mean G between two numbers a and b is

a. ab

b. ab^2

c. a^2b

d. \sqrt{ab}

ANSWER: d

EXPLANATION:

If a single geometric mean 'G' is inserted between two given numbers 'a' and 'b', then G is known as the geometric mean between 'a' and 'b'.

$$\text{G.M.} = G = G^2 = \sqrt{ab}$$

18. If A and G are arithmetic and geometric mean respectively between two positive numbers a and b, then A (AM) < G (GM) is correct?

a. yes

b. no

c. not sure

d. none

ANSWER: b**EXPLANATION:**

We have

$$\text{A.M.} = A = \frac{a+b}{2} \text{ and G.M.} = G = \sqrt{ab}$$

$$A - G = \frac{a+b}{2} - \sqrt{ab}$$

$$= \frac{a + b - 2\sqrt{ab}}{2}$$

$$\frac{(\sqrt{a} - \sqrt{b})^2}{2} > 0$$

$$\therefore A - G > 0$$

$$\Rightarrow A > G$$

19. Find the sum of the AP : 11, 17, 23, 29, ... of first 10 terms.

a. 380

b. 568

c. 960

d. 593

ANSWER: a**EXPLANATION:**

$$\Rightarrow \text{nth term for the given AP} = 5 + 6n$$

$$\Rightarrow \text{First term} = 5 + 6 = 11$$

$$\Rightarrow \text{Tenth term} = 5 + 60 = 65$$

$$\Rightarrow \text{Sum of 10 terms of the AP} = 0.5n(\text{first term} + \text{last term}) = 0.5 \times 10(11 + 65)$$

$$\Rightarrow \text{Sum of 10 terms of the AP} = 5 \times 76 = 380$$

20. Find the G. M. between $\frac{3}{2}$ and $\frac{27}{2}$

a. 9/2

b. 2/9

c. 6/3

d. 3/6

ANSWER: a**EXPLANATION:**We know that if a is the G. M. between a and b , then

$$G = \sqrt{ab}$$

$$\text{G. M. between } \frac{3}{2} \text{ and } \frac{27}{2} = \sqrt{\frac{3}{2} \times \frac{27}{2}}$$

$$a = -5$$

Thus, first term of the given A. P. is - 5.

26. If a,b,c are in G.P., then

a. $a(b^2 + a^2) = c(b^2 + c^2)$

c. $b(b^2 + a^2) = c(b^2 + c^2)$

b. $a(b^2 + a^2) = c(a^2 + b^2)$

d. None

ANSWER: B

EXPLANATION:

If a, b, c are in G.P. Then $b^2 = ac$

$$b^2 = (a - c) = ac \quad (a-c)$$

$$b^2 a - b^2 c = a^2 c - ac^2$$

$$a(b^2 + c^2) = c(a^2 + b^2)$$

Trick: Put a=1, b=2, c=4 and check the alternates.

27. The sum to infinity of the progression 9-3+1-13+..... is

a. 9

c. 27/4

b. 9/2

d. 15/2

ANSWER: B

EXPLANATION:

Infinite series $9-3+1-\frac{1}{3} \dots \infty$ is a G.P. with

$$a=9, r=\frac{-1}{3} \quad \therefore S_{\infty} = \frac{a}{1-r} = \frac{9}{1+\left(\frac{1}{3}\right)} = \frac{9 \times 3}{4} = \frac{27}{4}$$

28 . The product (32) (32) 1/6(32)1/36 to ∞ is

a. 16

c. 64

b. 32

d. 0

ANSWER: C

EXPLANATION:

$$(32)(32)1/6(32)1/36 \dots \infty = (32)^{1+\frac{1}{6}+\frac{1}{36}+\dots \infty} = (32)^{\frac{1}{1-(1/6)}}$$

$$(32)^{\frac{1}{5/6}} = (32)^{6/5} = 2^6 = 64$$

29. Obtain the sum of all positive integers up to 1000, which are divisible by 5 and not divisible by 2.

a. 10050

b. 5050

c. 5000

d. 50000

ANSWER: D**EXPLANATION:**

The positive integers, which are divisible by 5, are 5, 10, 15, ..., 1000

Out of these 10, 20, 30, ..., 1000 are divisible by 2

Thus, we have to find the sum of the positive integers 5, 15, 25, ..., 995

If n is the number of terms in the sequence then

$$995 = 5 + 10(n - 1) \Rightarrow 1000 = 10n$$

Therefore, $n = 100$.

$$\text{Thus the sum of the series} = (n/2)(a + l) = (100/2)(5 + 995) = 50000.$$

30. If s is the sum of an infinite G.P., the first term a then the common ratio r given by

a. $\frac{a-s}{s}$

b. $\frac{s-a}{s}$

c. $\frac{s}{1-s}$

d. none

ANSWER: B**EXPLANATION:**

$$\frac{a}{1-r}$$

$$s - sr = a$$

$$-sr = a - s$$

$$r = \frac{s-a}{s}$$

31. If in an infinite G.P. first term is equal to the twice of the sum of the remaining terms, then its common ratio is

a. 1

c. 1/3

b. 2

d. -1/3

ANSWER: C**EXPLANATION:**

$$\text{Given, } a = 2 \left(\frac{ar}{1-r} \right)$$

$$1-r = 2r$$

$$r = \frac{1}{3}$$

32. If n geometric means between a and b be G_1, G_2, \dots, G_n and a geometric mean be G , then the true relation is

a. $G_1, G_2, \dots, G_n = G$

c. $G_1, G_2, \dots, G_n = G^n$

b. $G_1, G_2, \dots, G_n = G^{\frac{1}{n}}$

d. none

ANSWER: C**EXPLANATION:**Here $G = (ab)^{\frac{1}{2}}$ and $G_1 = ar^1, G_2 = ar^2, \dots, G_n = ar^n$. Therefore $G_1 \cdot G_2 \cdot G_3 \dots G_n = a^n r^{1+2+\dots+n} = a^n r^{n(n+1)/2}$ but

$$ar^{n+1} = b$$
$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Therefore, the required product is $a^n \left(\frac{b}{a}\right)^{\frac{1}{n+1} \cdot n(n+1)/2}$

$$= (ab)^{n/2}$$

$$= \{(ab)^{1/2}\}^n$$

$$= G^n$$

Note: It is a well-known fact.

33. 7th term of the sequence $\sqrt{2}, \sqrt{10}, 5\sqrt{2} \dots$ is

a. $125\sqrt{10}$

c. 125

b. $25\sqrt{2}$

d. $125\sqrt{2}$

ANSWER: D**EXPLANATION:**Given sequence is $\sqrt{2}, \sqrt{10}, \sqrt{50} \dots$ common ratio $r = \sqrt{5}$, first term $a = \sqrt{2}$, then 7th term

$$t_7 = \sqrt{2} (\sqrt{5})^{7-1} = \sqrt{2} (\sqrt{5})^6 = \sqrt{2} (5)^3$$

$$125\sqrt{2}$$

34. If the first term of a G.P. be 5 and common ratio be -5 , then which term is 3125?

a. 6th

c. 7th

b. 5th

d. 8th

ANSWER: B**EXPLANATION:**Given that first term $a=5$ and common ratio $r=-5$. Suppose that n^{th} term is 3125,

$$\text{then } ar^{n-1} = 3125$$

$$5(-5)^{n-1} = 5^4.$$

$$\text{Hence} = 5^{\text{th}}$$

35. The sums of n terms of three A.P.'s whose first term is 1 and common differences are 1, 2, 3 are S_1, S_2, S_3 respectively. The true relation is

a. $S_1 + S_2 = S_3$

b. $S_1 + S_3 = 2S_2$

c. $S_1 + S_2 = 2S_3$

d. None

ANSWER: B

EXPLANATION:

$$\text{We have } a_1 = a_2 = a_3 = 1$$

$$d_1 = 1, d_2 = 2, d_3 = 3$$

$$\text{Therefore, } S_1 = \frac{n}{2}(n+1) \dots\dots (i)$$

$$S_2 = \frac{n}{2}(2n) \dots\dots (ii)$$

$$S_3 = \frac{n}{2}(3n+1)$$

.... (iii) Adding (i) and (iii),

$$S_1 + S_3 = \frac{n}{2}[(n+1) + (3n-1)]$$

$$= 2 \left[\frac{n}{2}(2n) \right] = 2S_2$$

$$\text{Hence correct relation } S_1 + S_3 = 2S_2$$

36. What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

a. 897

b. 164,850

c. 164,749

d. 149,700

ANSWER: B

EXPLANATION:

The smallest 3 digit number that will leave a remainder of 2 when divided by 3 is 101.

The next number that will leave a remainder of 2 when divided by 3 is 104, 107,

The largest 3 digit number that will leave a remainder of 2 when divided by 3 is 998.

So, it is an AP with the first term being 101 and the last term being 998 and common difference being 3.

$$\text{Sum of an AP} = \frac{\text{First term} + \text{Last Term}}{2} \times \text{Number of Term}$$

We know that in an A. P., the nth term $a_n = a_1 + (n - 1)d$

In this case, therefore, $998 = 101 + (n - 1) \cdot 3$

i.e., $897 = (n - 1) \cdot 3$

Therefore, $n - 1 = 299$

Or $n = 300$.

Sum of the AP will therefore, be $\frac{101+998}{2} \times 300 = 164,850$

37. Obtain the sum of all positive integers up to 1000, which are divisible by 5 and not divisible by 2.

- | | |
|----------|----------|
| a. 10050 | b. 5050 |
| c. 5000 | d. 50000 |

ANSWER: D

EXPLANATION:

The positive integers, which are divisible by 5, are 5, 10, 15, ..., 1000

Out of these 10,20,30,... 1000 are divisible by 2

Thus, we have to find the sum of the positive integers 5, 15, 25, ..., 995

If n is the number of terms in it the sequence then

$$995 = 5 + 10(n - 1) \Rightarrow 1000 = 10n$$

Therefore, $n = 100$.

Thus the sum of the series = $(n/2)(a + l) = (100/2) (5 + 995) = 50000$.

38. What is the sum of the following series? -64, -66, -68,, -100

- | | |
|----------|----------|
| a. -1458 | b. -1558 |
| c. -1568 | d. -1664 |

ANSWER: B

EXPLANATION:

The sequence is -64, -66, -68,, -100.

The given set of numbers are in an arithmetic progression.

Key Data: First term is -64. The common difference is -2. The last term is -100.

Sum of the first n terms of an AP = $\frac{n}{2}[2a_1 + (n-1)d]$

To compute the sum, we know the first term $a_1 = -64$ and the common difference $d = -2$. We do not know the number of terms n . Let us first compute the number of terms and then find the sum of the terms.

Step to compute number of terms of the sequence

$$a_n = a_1 + (n - 1)d$$

$$-100 = -64 + (n - 1)(-2)$$

Therefore, $n = 19$.

$$\text{Sum } S_n = \frac{19}{2} [2(-64) + (19-1)(-2)]$$

$$S_n = \frac{19}{2} [-128 - 36]$$

$$S_n = 19 \times (-82) = -1558$$

39. The sum of third and ninth term of an A.P is 8. Find the sum of the first 11 terms of the progression.

a. 44

b. 22

c. 19

d. None of these

ANSWER: A

EXPLANATION:

The third term $t_3 = a + 3d$

The ninth term $t_9 = a + 8d$

$$t_3 + t_9 = 2a + 10d = 8$$

Sum of first 11 terms of an AP is given by

$$S_{11} = \frac{11}{2} [2a + 10d]$$

$$S_{11} = \frac{11}{2} [8] = 44$$

40. The sum of the three numbers in A.P is 21 and the product of the first and third number of the sequence is 45. What are the three numbers?

a. 9, 7 and 5

b. 3, 7, and 11

c. Both A & B

d. None of these

ANSWER: C**EXPLANATION:**Let the numbers are be $a - d, a, a + d$ Then $a - d + a + a + d = 21$

$$3a = 21$$

$$a = 7$$

and $(a - d)(a + d) = 45$

$$a^2 - d^2 = 45$$

$$d^2 = 4$$

$$d = \pm 2$$

Hence, the numbers are 5, 7 and 9 when $d = 2$ and 9, 7 and 5 when $d = -2$. In both the cases numbers are the same.**41. If the first term of G.P. is 7, its n^{th} term is 448 and sum of first n terms is 889, then find the fifth term of G.P.**

a. 112

c. 62

b. 110

d. 39

ANSWER: A**EXPLANATION:**Given $a = 7$ the first term $t_n = ar^{n-1} = 7(r)^{n-1} = 448$

$$7r^n = 448r$$

Also $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{7(r^n - 1)}{r - 1}$

$$889 = \frac{448r - 7}{r - 1}$$

$$R = 2.$$

Hence $T_5 = ar^4 = 7(2)^4 = 112$

42. If the third and fourth terms of an arithmetic sequence are increased by 3 and 8 respectively, then the first four terms form a geometric sequence. Find**(i) the sum of the first four terms of A.P.**

a. 54

c. 23

b. 27

d. 79

ANSWER: A**EXPLANATION:**

Sol. $a, (a + d), (a + 2d), (a + 3d)$ in A.P.

$a, a + d, (a + 2d + 3), (a + 3d + 8)$ are in G.P.

hence $a + d = ar$

$$\text{also } r = \frac{a+d}{a} = \frac{a+2d+3}{a+d} = \frac{a+3d+8}{a+2d+3}$$

$$\therefore \frac{d+3}{d} = \frac{d+5}{d+3}$$

$$\Rightarrow d^2 + 6d + 9 = d^2 + 5d \Rightarrow d = -9$$

$$\therefore \frac{a-9}{a} = \frac{a-15}{a-9}$$

$$\Rightarrow a^2 - 18a + 81 = a^2 - 15a \Rightarrow 3a = 81 \Rightarrow a = 27$$

hence A.P. is 27, 18, 9, 0,

Sum of the first four terms of AP = 54

43. Three positive numbers form a G.P. If the second term is increased by 8, the resulting sequence is an A.P. In turn, if we increase the last term of this A.P. by 64, we get a G.P. Find the three numbers.

a. 4, 12, 36

c. 5, 15, 20

ANSWER: A

b. 4, 8, 16

d. none

44. The sum of the first five terms of a geometric series is 189, the sum of the first six terms is 381, and the sum of the first seven terms is 765. What is the common ratio in this series.

EXPLANATION:

a. 3

c. 6

ANSWER: B

b. 2

d. 56

Sol. Let the numbers be a, ar, ar^2 where $r > 0$

Hence $a, (ar + 8), ar^2$ in A.P. - (1)

Also $a, (ar + 8), ar^2 + 64$ in G.P. - (2)

$$\Rightarrow (ar + 8)^2 = a(ar^2 + 64) \quad a = 4/4-r \quad (3)$$

$$\text{Also(1)} \Rightarrow 2(ar + 8) = (a + ar^2) \Rightarrow (1-r)^2 = 16/a \quad (4)$$

From (3) and (4) $r = 3$ or -5 (rejected).

Hence $a = 4$ numbers are : 4, 12, 36

EXPLANATION:

$$S_5 = 189; S_6 = 381; S_7 = 765; t_6 = S_6 - S_5 = 381 - 189 = 192$$

$$t_7 = S_7 - S_6 = 765 - 381 = 384$$

$$\text{now common ratio} = \frac{t_7}{t_6} = \frac{384}{192} = 2$$

45. Find the 3rd n th term for the AP : 11, 17, 23, 29, ...

a. 23

b. 17

c. 11

d. 6

ANSWER: A

EXPLANATION:

Here, $a = 11, d = 17 - 11 = 23 - 17 = 29 - 23 = 6$

We know that nth term of an AP is $a + (n - 1) d$

$$\Rightarrow \text{nth term for the given AP} = 11 + (n - 1) 6$$

$$\Rightarrow \text{nth term for the given AP} = 5 + 6n$$

We can verify the answer by putting values of 'n'.

$$\Rightarrow n = 1 \rightarrow \text{First term} = 5 + 6 = 11$$

$$\Rightarrow n = 2 \rightarrow \text{Second term} = 5 + 12 = 17$$

$$\Rightarrow n = 3 \rightarrow \text{Third term} = 5 + 18 = 23$$

46. The sum of three numbers in a GP is 26 and their product is 216. and the numbers.

a. 2, 6 and 18.

b. 3, 7, and 11

c. Both

d. None of these

ANSWER: C

EXPLANATION:

Let the numbers be $a/r, a, ar$.

$$\Rightarrow (a/r) + a + ar = 26$$

$$\Rightarrow a(1 + r + r^2) / r = 26$$

Also, it is given that product = 216

$$\Rightarrow (a / r) \times (a) \times (a r) = 216$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

$$\Rightarrow 6(1 + r + r^2) / r = 26$$

$$\Rightarrow (1 + r + r^2) / r = 26 / 6 = 13 / 3$$

$$\Rightarrow 3 + 3r + 3r^2 = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow (r - 3)(r - (1/3)) = 0$$

$$\Rightarrow r = 3 \text{ or } r = 1/3$$

Thus, the required numbers are 2, 6 and 18.

47. A sequence in which the ratio of two consecutive terms is always constant (1, 0) is called

- | | |
|-------|-------|
| a. AP | b. GP |
| c. HP | d. NP |

ANSWER: b

EXPLANATION :

A sequence in which the ratio of two consecutive terms is always constant (1, 0) is called a Geometric Progression (G. P.)

48. For the elements 4 and 6, verify

- | | |
|------------------------|-------------------|
| a. $A \geq G \geq H$. | b. $A < G \geq H$ |
| c. $A > G \geq H$ | d. None |

ANSWER: A

EXPLANATION:

$$A = \text{Arithmetic Mean} = (4 + 6) / 2 = 5$$

$$G = \text{Geometric Mean} = \sqrt{4 \times 6} = 4.8989$$

$$H = \text{Harmonic Mean} = (2 \times 4 \times 6) / (4 + 6) = 48 / 10 = 4.8$$

Therefore, $A \geq G \geq H$

49. Which term of the G. P.: 5, -10, 20, -40, ... is 320?

- | | |
|---------------------|--------------------|
| a. 7 th | b. 8 th |
| c. 10 th | d. 1 st |

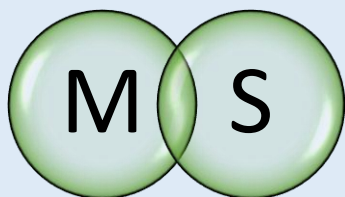
ANSWER: A

CHAPTER 7

SETS, RELATIONS AND FUNCTIONS



set	A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described. Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.	
	Singleton Set	A set containing one element is called Singleton
	Equal Set	Two sets A & B are said to be equal, written as $A = B$ if every element of A is in B and every element of B is in A.
VENN DIAGRAMS	A Venn diagram is a diagram that shows all possible logical relation between a fine collections of different sets. These diagram depict elements as point in the plane, and sets as regions inside closed curves.	



Equivalent Set	Two finite sets A & B are said to be equivalent if $n(A) = n(B)$.
Power Set	The collection of all possible subsets of a given set A is called the power set of A, to be denoted by $P(A)$. <ol style="list-style-type: none"> 1. A set containing n elements has 2^n subsets. 2. A set containing n elements has $2^n - 1$ proper subsets
PRODUCT SETS	Ordered Pair Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b) .
	Cartesian Product of sets If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B, is called the Cartesian product of A and B, to be denoted by $A \times B$. Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$ If
Relation and Function	Any subset of the product set $X \times Y$ is said to define a relation from X to Y and any relation from X to Y in which no two different ordered pairs have the same first element is called a function . Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by $f(x)$ of B, is called a function or mapping from A to B and we write $f : A \rightarrow B$.
Domain & Range of a function	Let $f : A \rightarrow B$, then A is called the domain of f, while B is called the co-domain of f. The set $f(A) = \{f(x) : x \in A\}$ is called the range of f.

VARIOUS TYPES OF FUNCTION

IDENTITY FUNCTION

• Let A be a non-empty set. Then, the function I defined by $I : A \rightarrow A : I(x) = x$ for all $x \in A$ is called an identity function on A

EQUAL FUNCTION

• Two functions f and g are said to be equal, written as $f = g$ if they have the same domain and they satisfy the condition $f(x) = g(x)$, for all x .

INVERSE FUNCTION

• Let f be a one-one onto function from A to B . Let y be an arbitrary element of B . Then f being onto, there exists an element x in A such that $f(x) = y$. A function is invertible if and only if f is one-one onto.

ONE -ONE FUNCTION

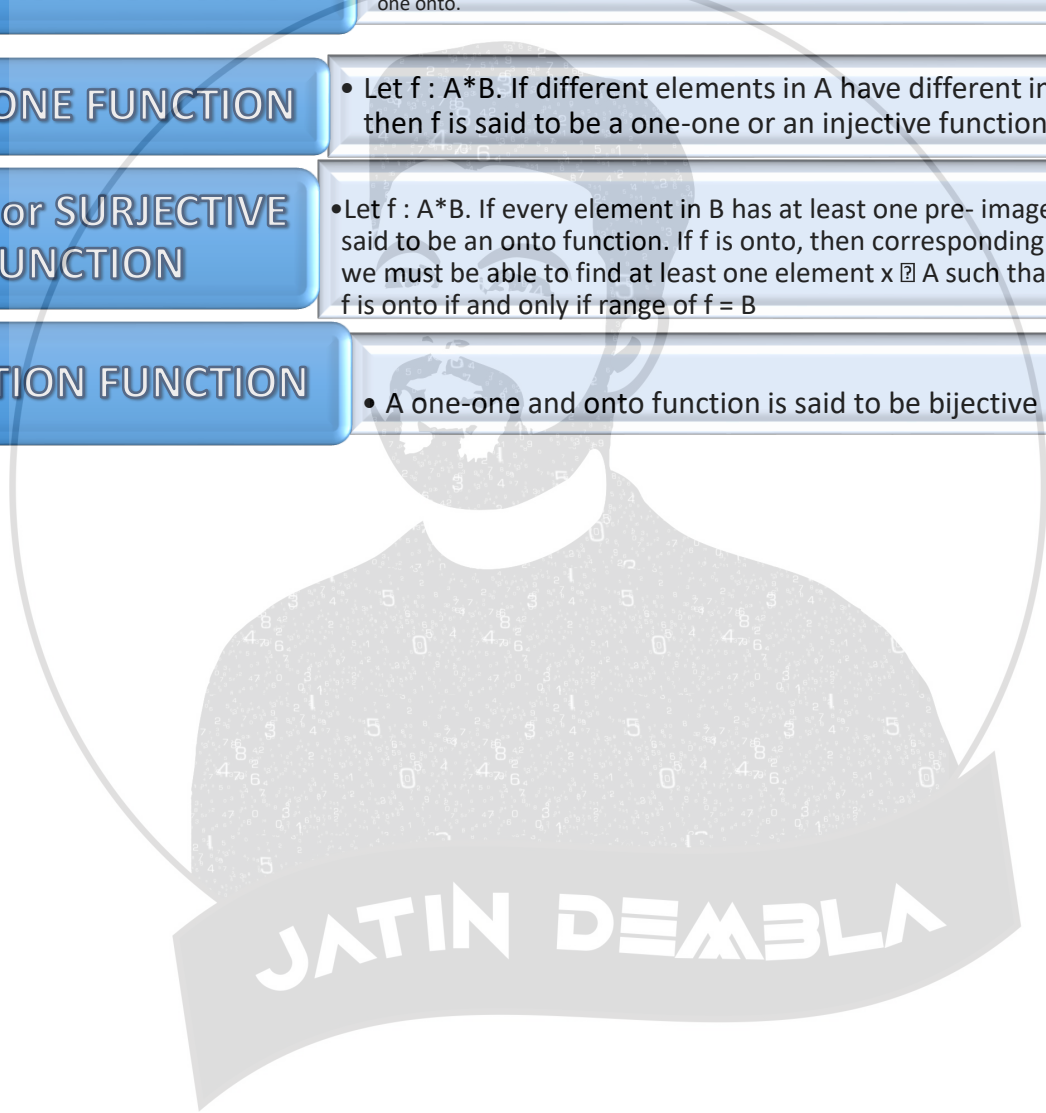
• Let $f : A \rightarrow B$. If different elements in A have different images in B , then f is said to be a one-one or an injective function or mapping

ONTO or SURJECTIVE FUNCTION

• Let $f : A \rightarrow B$. If every element in B has at least one pre- image in A , then f is said to be an onto function. If f is onto, then corresponding to each $y \in B$, we must be able to find at least one element $x \in A$ such that $y = f(x)$. Clearly, f is onto if and only if $\text{range of } f = B$

BIJECTION FUNCTION

• A one-one and onto function is said to be bijective



Different types of relations

Let $S = \{a, b, c, \dots\}$ be any set then the relation R is a subset of the product set $S \times S$

- i) If R contains all ordered pairs of the form (a, a) in $S \times S$, then R is called reflexive. In are *flexive* relation 'a' is related to itself.

For example, 'Is equal to' is a reflexive relation for $a = a$ is true.

- ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$ then R is called symmetric

For Example $a = b \Rightarrow b = a$. Hence the relation 'is equal to' is a symmetric relation.

- iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for every $a, b, c \in S$ then R is called *transitive*.

For Example $a = b, b = c \Rightarrow a = c$. Hence the relation 'is equal to' is a transitive relation.

A relation which is reflexive, symmetric and transitive is called an *equivalence relation* or simply an *equivalence*. 'is equal to' is an equivalence relation.

Similarly, the relation "is parallel to" on the set S of all straight lines in a plane is an equivalence relation.

Domain & Range of a relation

If R is a relation from A to B , then the set of all first co-ordinates of elements of R is called the domain of R , while the set of all second co-ordinates of elements of R is called the range of R .



1. Which of the following statements is used to create an empty set?

(a) { }

(b) Set()

(c) []

(d) ()

ANSWER: b**EXPLANATION:**

{ } creates a dictionary not a set. Only set () creates an empty set.

2. What is the output of the following piece of code when executed in the python shell?

```
a= {1,2,3} a.intersection update({2,3,4,5})
```

a. {2,3}

b. Error, duplicate item present in list

c. Error, no method called intersection update for set data type

d. {1,4,5}

ANSWER: a**EXPLANATION:**

The method intersection update returns a set which is an intersection of both the sets.

3. Which of the following lines of code will result in an error?

a. s={abs}

b. s={4, 'abc', (1,2)}

c. s={2, 2.2, 3, 'xyz'}

d. s={san}

ANSWER: d**EXPLANATION:**

The line: s={san} will result in an error because 'san' is not defined. The line s={abs} does not result in an error because abs is a built-in function. The other sets shown do not result in an error because all the items are hashable.

4. What is the output of the code shown below?

```
s=set([1, 2, 3])
```

```
s.union([4, 5])
```

```
s|([4, 5])
```

a. {1, 2, 3, 4, 5} {1, 2, 3, 4, 5}

b. Error {1, 2, 3, 4, 5}

c. {1, 2, 3, 4, 5} Error

d. Error Error

ANSWER: c**EXPLANATION:**

The first function in the code shown above returns the set $\{1, 2, 3, 4, 5\}$. This is because the method of the function union allows any alterable. However the second function results in an error because f unsupported data type, that is list and set.

5. What is the output of the line of code shown below, if $s1 = \{1, 2, 3\}$? $s1.is\ subset(s1)$

- a. True
- b. Error
- c. No output
- d. False

ANSWER: a

EXPLANATION:

Every set is a subset of itself and hence the output of this line of code is true.

6. A _____ is an ordered collection of objects.

- a. Relation
- b. Function
- c. Set
- d. Proposition

Answer: c

Explanation:

By the definition of set.

7. The set O of odd positive integers less than 10 can be expressed by _____

- a. $\{1, 2, 3\}$
- b. $\{1, 3, 5, 7, 9\}$
- c. $\{1, 2, 5, 9\}$
- d. $\{1, 5, 7, 9, 11\}$

Answer: b

Explanation:

Odd numbers less than 10 is $\{1, 3, 5, 7, 9\}$.

8. Power set of empty set has exactly _____ subset.

- a. 1
- b. 2
- c. 0
- d. 3

Answer: a

Explanation:

Power set of null set has exactly one subset which is empty set.

9. What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b\}$?

- a. 2
c. 6
- b. 4
d. 8

Answer: b

Explanation:

The set A has got 2 elements so $n(P(A))=4$.

23. Two sets A and B contains a and b elements respectively .If power set of A contains 16 more elements than that of B, value of 'b' and 'a' are respectively

- a. 5, 4
c. 2, 3
- b. 6, 7
d. None of the mentioned

Answer: a

Explanation:

$32-16=16$, hence $a=5$, $b=4$.

24. Let A be {1, 2, 3, 4}, U be set of all natural numbers, then $U-A'$ (complement of A) is given by set.

- a. {1, 2, 3, 4, 5, 6,}
c. {1, 2, 3, 4}
- b. {5, 6, 7, 8, 9,}
d. All of the mentioned

Answer: c

Explanation:

$U - A' \equiv A$.

25. Which sets are not empty?

- a. {x: x is a even prime greater than 3}
c. {x: x is an even number and x+3 is even}
- b. {x : x is a multiple of 2 and is odd}
d. { x: x is a prime number less than 5 and is odd}

Answer: d

Explanation:

Because the set is {3}

26. If A, B and C are any three sets, then $A-(B \cap C)$ is equal to

- a. $(A - B) \cup (A - C)$
b. $(A - B) \cap (A - C)$

c. $(A - B) \cup C$

d. *NONE***Answer: A****Explanation:**From De Morgan's Law, $A - (B \cap C) = (A - B) \cup (A - C)$ **27. Which of the following is the empty set**

a. $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$

b. $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

c. $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$

d. $\{x : x \text{ is a real number and } x^2 = x + 2\}$

Answer: D**Explanation:**Since $x^2 - 1 = 0$, given $x^2 = -1$

$x = \pm i$

 \therefore No value of x is possible**28. If a set A has n elements, then the total number of subsets of A is**

a. n

b. n^2

c. 2^n

d. 2n

Answer: c**Explanation:**Number of subsets of A = $nC_0 + nC_1 + \dots + nC_n = 2^n$ **29. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to**

a. A

b. B

c. A^c

d. B^c

Answer: A**Explanation:** $A \cap B \subseteq A$. Hence $A \cup (A \cap B) = A$ **30. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are**

a. 2^{99}

b. 99^2

c. 100

d. 18

Answer: B**Explanation:**

$$n((A \times B) \cap (B \times A))$$

$$= n((A \cap B) \times (B \cap A)) = n(A \cap B) \cdot n(B \cap A)$$

$$= n(A \cap B) \cdot n(A \cap B) = (99)(99) = 99^2$$

31. If $A = \{x : x \text{ is a multiple of } 4\}$ and $B = \{x : x \text{ is a multiple of } 6\}$ then $A \cap B$ consists of all multiples of ?

a. 16

b. 12

c. 8

d. 4

Answer: B**Explanation:**

$$A = \{4, 8, 12, 16, 20, 24, \dots\}$$

$$B = \{6, 12, 18, 24, 30, \dots\} \quad A \cap B = \{12, 24, \dots\}$$

$$= \{x : x \text{ is a multiple of } 12\}.$$

32. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$, $C = \{3, 4, 6\}$, then $(A \cup B) \cap C$ is

a. $\{3, 4, 6\}$ b. $\{1, 2, 3\}$ c. $\{1, 4, 3\}$

d. None of these

Answer: A**Explanation:**

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \quad (A \cup B) \cap C = \{3, 4, 6\}$$

33. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C) =$

a. 288

b. 1

c. 2

d. 17

Answer: C

Explanation:

$$n(A)=4, n(B)=3 \Rightarrow n(A) \times n(B) \times n(C) = n(A \times B \times C) = 4 \times 3 \times n(C) = 24$$

$$n(C) = \frac{24}{12} = 2$$

34. If $A = \{2, 3, 5\}$, $B = \{2, 5, 6\}$, then $(A - B) \times (A \cap B)$ is

- a. $\{(3, 2), (3, 3), (3, 5)\}$ b. $\{(3, 2), (3, 5), (3, 6)\}$
 c. $\{(3, 2), (3, 5)\}$ d. None of these

Answer: C

Explanation:

$$A - B = \{3\}, A \cap B = \{2, 5\}$$

$$(A - B) \times (A \cap B) = \{(3, 2), (3, 5)\}$$

35. The set of intelligent students in a class is [AMU 1998]

- a. A null set b. A singleton set
 c. A finite set d. Not a well defined collection

Answer: D

Explanation:

Since, intelligence is not defined for students in a class i.e., Not a well defined collection.

36. If A and B be any two sets, then $(A \cap B)'$ is equal to

- a. $A' \cap B'$ b. $A' \cup B'$
 c. $A \cap B$ d. $A \cup B$

Answer: D

Explanation:

From De' Morgan's law, $(A \cap B)' = A' \cup B'$

43. If A and B are two sets then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to

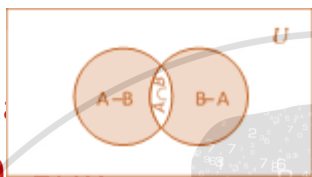
- a. $A \cup B$
- b. $A \cap B$
- c. A
- d. B'

Answer: A

Explanation:

From Venn-Euler's diagram

$\therefore (A - B) \cup (B - A) \cup (A \cap B) = A \cup B$



44. The shaded region in the given figure is

- a. $A \cap (B \cup C)$
- b. $A \cup (B \cap C)$
- c. $A \cap (B - C)$
- d. $A - (B \cup C)$

Answer: D

Explanation:

From Venn-Euler's diagram, $A - (B \cup C)$



45. If A and B are two sets, then $A \cup B = A \cap B$

- a. $A * B$
- b. $B + A$
- c. $A = B$
- d. None of these

Answer: C

Explanation:

Let $x \in A \Rightarrow x \in A \cup B, [\because A \subseteq A \cup B]$

$= x \in A \cap B, [\because A \cup B = A \cap B]$

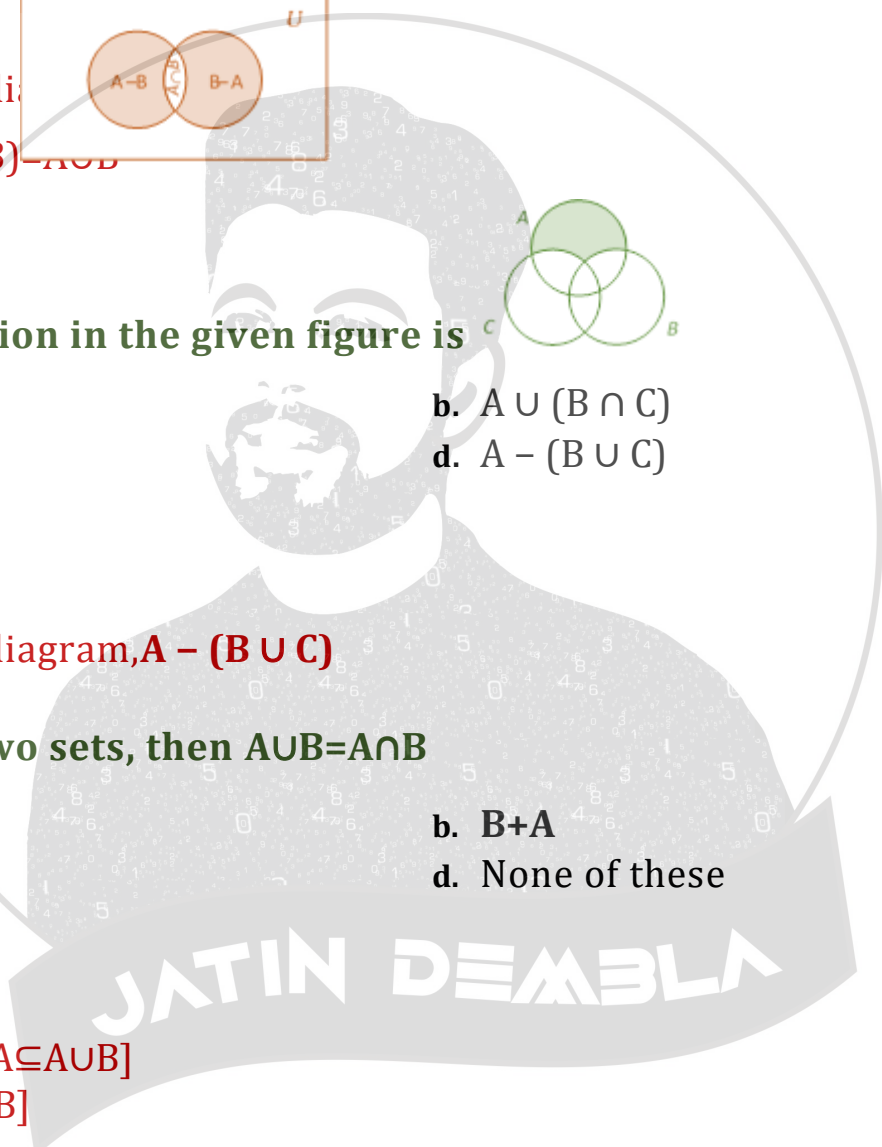
$= x \in A$ and $x \in B$

$\forall x \in B, A \subseteq B$

Similarly $x \in B$

$= x \in A \setminus B \subseteq A$ Now $A \subseteq B, B \subseteq A$

$= A = B$



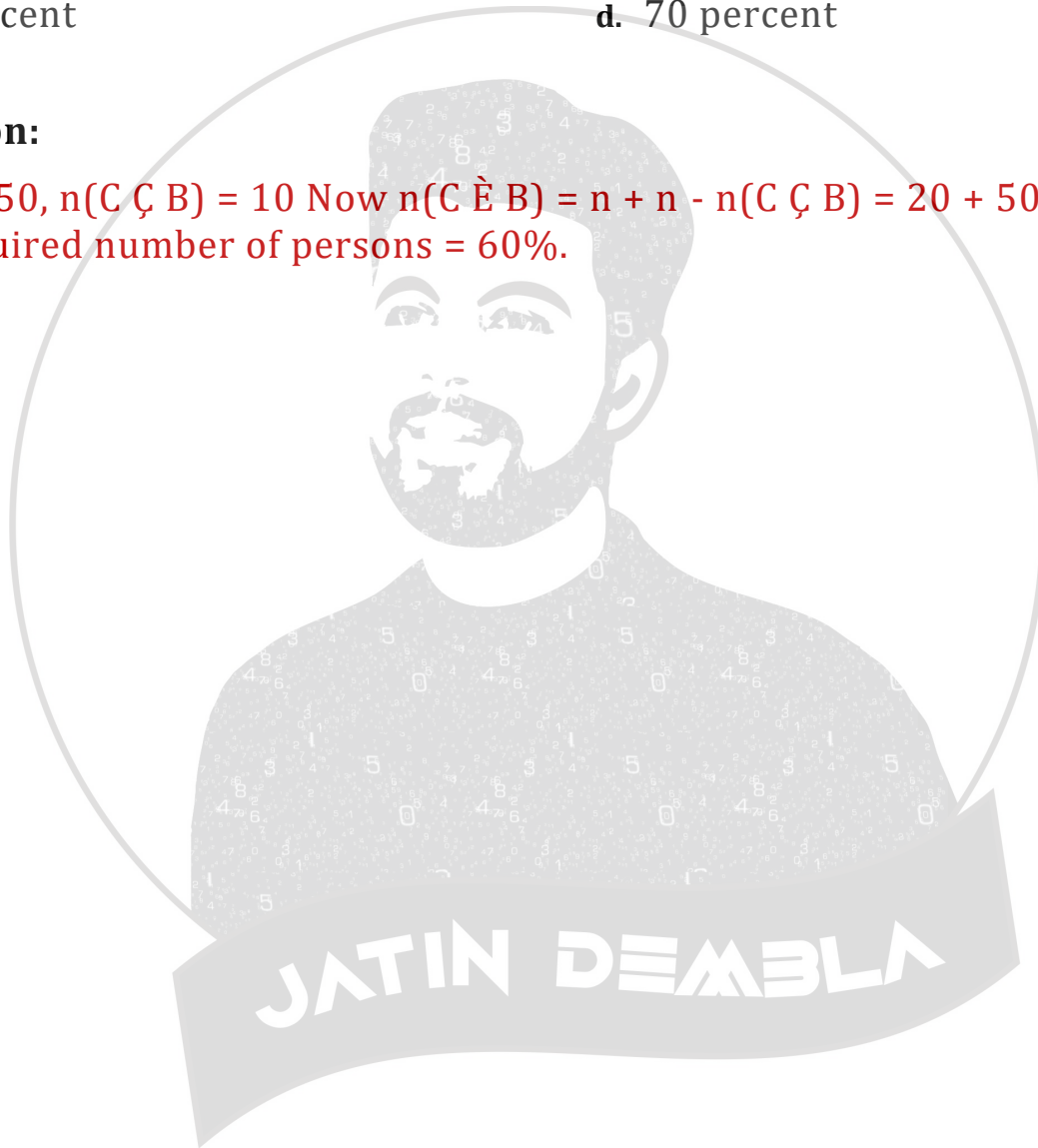
50. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

- a. 80 percent
- b. 40 percent
- c. 60 percent
- d. 70 percent

Answer: C

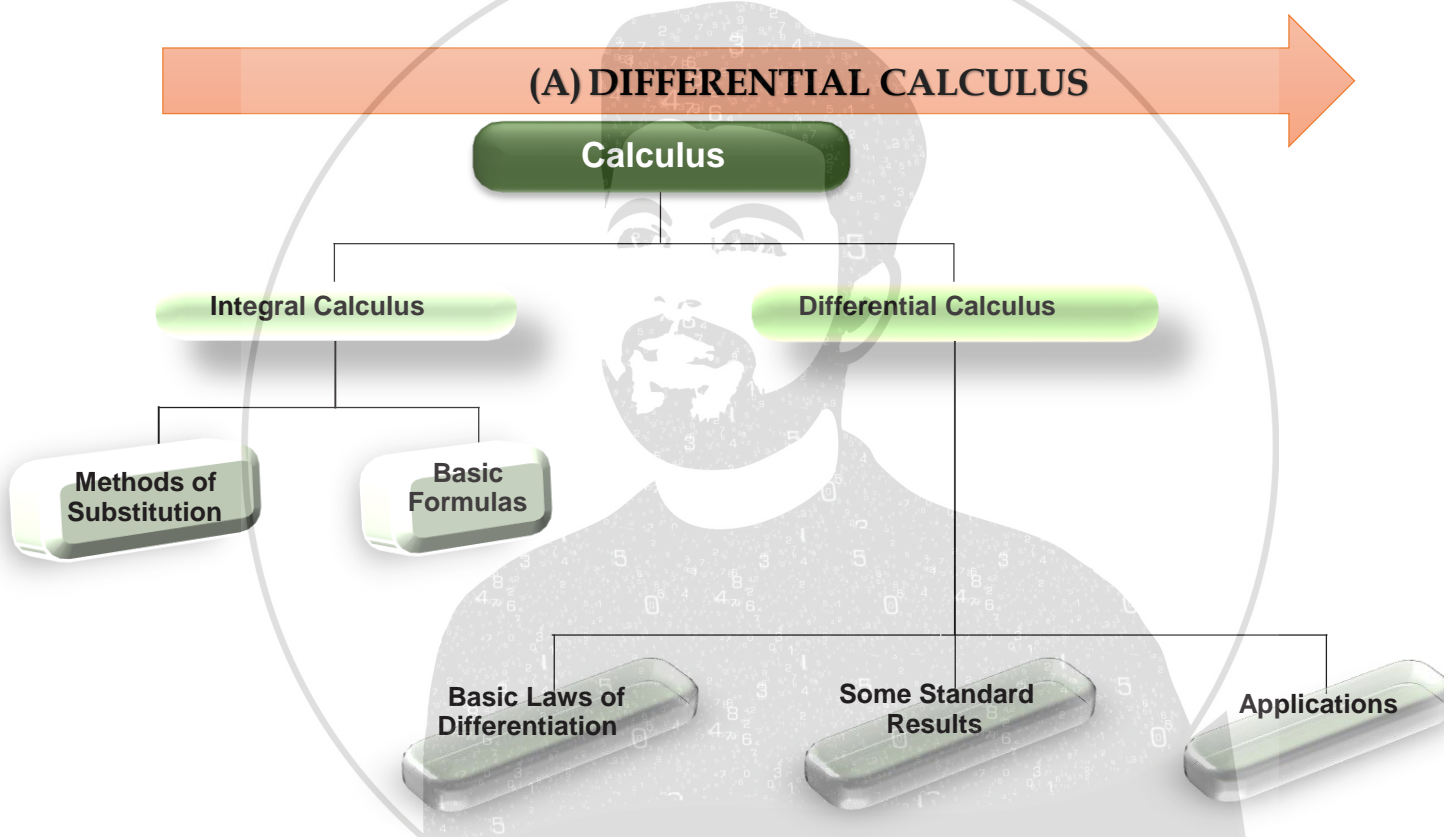
Explanation:

$n = 20, n = 50, n(C \cap B) = 10$ Now $n(C \cup B) = n + n - n(C \cap B) = 20 + 50 - 10 = 60$.
Hence, required number of persons = 60%.



CHAPTER 8

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS

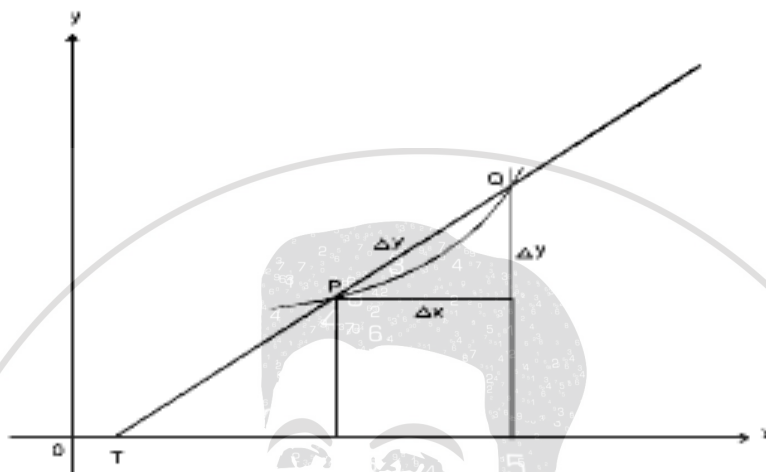


INTRODUCTION	Differentiation is one of the most important fundamental operations in calculus. Its theory primarily depends on the idea of limit and continuity of function.
DERIVATIVE OR DIFFERENTIAL COEFFICIENT	Let $y = f(x)$ be a function. If h be the small increment in x and the corresponding increment in y or $f(x)$ be $y = f(x+h) - f(x)$

<p>STANDARD FORMULAS</p>	$\frac{d}{dx}(a) = 0$ $\frac{d}{dx}(x) = 1$ $\frac{d}{dx}(au) = a \frac{du}{dx}$ $\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$ $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v} \frac{du}{dx} - \frac{u}{v^2} \frac{dv}{dx}$ $\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$ $\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}} \frac{du}{dx}$ $\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2} \frac{du}{dx}$ $\frac{d}{dx}\left(\frac{1}{u^n}\right) = -\frac{n}{u^{n+1}} \frac{du}{dx}$ $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)] \frac{du}{dx}$ $\frac{d}{dx}[\ln u] = \frac{d}{dx}[\log_e u] = \frac{1}{u} \frac{du}{dx}$ $\frac{d}{dx}[\log_a u] = \log_a e \frac{1}{u} \frac{du}{dx}$ $\frac{d}{dx} e^u = e^u \frac{du}{dx}$ $\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$ $\frac{d}{dx}(u^v) = vu^{v-1} \frac{du}{dx} + \ln u \cdot u^v \frac{dv}{dx}$ $\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$ $\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$ $\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$ $\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$ $\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$ $\frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx}$
<p>IMPLICIT FUNCTIONS</p>	<p>A function in the form $f(x, y) = 0$. For example $x^2y^2 + 3xy + y = 0$ where y cannot be directly defined as a function of x is called an implicit function of x.</p>
<p>PARAMETRIC EQUATION</p>	<p>When both the variables x and y are expressed in terms of a parameter (a third variable), the involved equations are called parametric equations. For the parametric equations $x = f(t)$ and $y = h(t)$ the differential coefficient $\frac{dy}{dx}$</p>
<p>LOGARITHMIC DIFFERENTIATION</p>	<p>The process of finding out derivative by taking logarithm in the first instance is called logarithmic differentiation.</p>



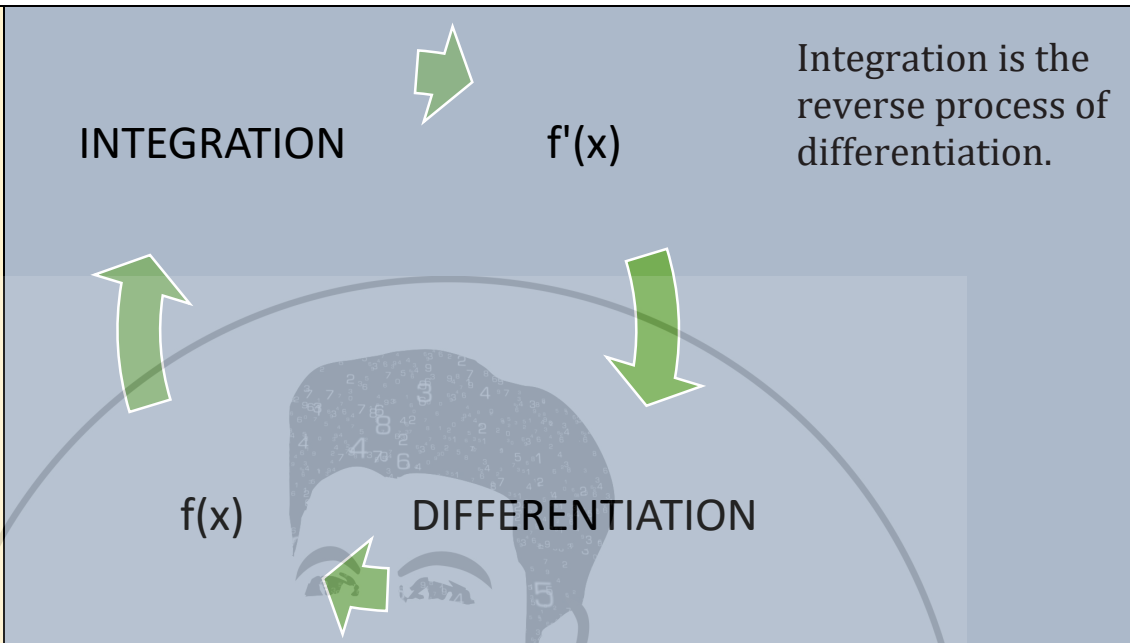
GEOMETRIC INTERPRETATION OF THE DERIVATIVE



COST FUNCTION	Total cost consists of two parts (i) Variable Cost (ii) Fixed Cost.	
	Average cost (AC or \bar{C})	$\frac{\text{Total Cost } C(X)}{\text{Output } \bar{X}}$
	Average variable cost (AVC)	$\frac{\text{Variable Cost } V(X)}{\text{Output } \bar{X}}$
	Average Fixed Cost (AFC)	$\frac{\text{Fixed Cost } F(X)}{\text{Output } \bar{X}}$
MARGINAL COST	If $C(x)$ the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units	
REVENUE FUNCTION	Revenue, $R(x)$, gives the total money obtained (Total turnover) by selling x units of a product. If x units are sold at 'P' per unit, then $R(x) = P \cdot X$	
PROFIT FUNCTION	Profit $P(x)$, the difference of between total revenue $R(x)$ and total Cost $C(x)$.	



INTEGRATION



Integration is the reverse process of differentiation.

Calculus: Integral Formulas

Common Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Trig Integrals

$$\int \sin x = -\cos x + C$$

$$\int \cos x = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Double Angle Formulas

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2\sin^2 \theta$$

$$= 2\cos^2 \theta - 1$$

Power Reducing Formulas (helpful for trig integrals)

$$\sin^2 X = \frac{1 - \cos 2X}{2}$$

$$\cos^2 X = \frac{1 + \cos 2X}{2}$$

Washer and Shell Formulas

$$V = \pi \int_a^b (R^2 - r^2) dx$$

$$V = 2\pi \int_a^b h(x) \cdot r(x) dx = 2\pi \int_a^b [f(x) - g(x)](x - c) dx$$

Definition of Definite Integral

If **f** is defined on the closed interval **[a,b]** and the limit of Riemann sums over partitions Δ exists, then **f** is integrable on **[a,b]** and

$$\lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx$$

The limit is called the **definite integral** of **f** from **a** to **b**. The number **a** is the **lower limit** of integration and the number **b** is the **upper limit** of integration.

<p>DEFINITE INTEGRATION</p>	<p>Definite Integral</p> <p>Let f be continuous function defined on the closed interval $[a, b]$ and F be an anti derivative of f. Then $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$.</p>
<p>IMPORTANT PROPERTIES</p>	<p>Properties of Definite Integrals</p> <p>P_0: $\int_a^b f(x) dx = \int_a^b f(t) dt$</p> <p>$P_1$: $\int_a^b f(x) dx = -\int_b^a f(x) dx$. In particular, $\int_a^a f(x) dx = 0$</p> <p>P_2: $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$</p> <p>$P_3$: $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$</p> <p>$P_4$: $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ (Note that P_4 is a particular case of P_3)</p> <p>P_5: $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$</p> <p>$P_6$: $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$, if $f(2a-x) = f(x)$ and 0 if $f(2a-x) = -f(x)$</p> <p>P_7: (i) $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if f is an even function, i.e., if $f(-x) = f(x)$. (ii) $\int_{-a}^a f(x) dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.</p>



1. Find an expression for y given $\frac{dy}{dx} = 7x^5$

- a. 6
- b. 2
- c. 3
- d. 5

ANSWER: A
EXPLANATION:

$$7x^5 = \int 7x^5 dx$$

$$= \frac{7}{6} x^6 + c$$

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$,
 $n = 5, n + 1 = 5 + 1 = 6$

2. Find an expression for y given $\frac{dy}{dx} = x^{-\frac{3}{4}}$

- a. 2/3
- b. 1/4
- c. 5/4
- d. NONE

ANSWER: B
EXPLANATION:

$$x^{-\frac{3}{4}} = \int x^{-\frac{3}{4}} dx$$

$$= \frac{1}{\frac{1}{4}} x^{\frac{1}{4}} + c$$

$$= 4x^{\frac{1}{4}} + c$$

Use $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

$$n = -\frac{3}{4}, n+1 = -\frac{3}{4} + 1 = \frac{-3+4}{4} = \frac{1}{4}$$

divide 1 by $\frac{1}{4}$

3. $-12x^{-4} = \int -12x^{-4} - dx$ solve it:

a. 6

c. 3

b. 2

d. 4

ANSWER: D

EXPLANATION:

$$-12x^{-4} = \int -12x^{-4} dx$$

$$= -12 \int x^{-4} dx$$

$$= \left(\frac{-12x^{-3}}{-3} \right) + c$$

$$= 4x^{-3} + c$$

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$

$$n = -4, n+1 = -4 + 1 = -3$$

Simplifying fraction, $\frac{-12}{-3} = 4$

4. Given $f'(x) = \frac{1}{2}x^{\frac{1}{3}} - \frac{1}{4}x^{\frac{1}{5}} + \pi$, find $f(x)$

a. 6

c. 3

b. 2

d. NONE

ANSWER: D

EXPLANATION:

$$\frac{1}{2}x^{\frac{1}{3}} - \frac{1}{4}x^{\frac{1}{5}} + \pi$$

$$= \int \frac{1}{2}x^{\frac{1}{3}} dx - \int \frac{1}{4}x^{\frac{1}{5}} dx + \int \pi dx$$

$$= \frac{1}{2} \int x^{\frac{1}{3}} dx - \frac{1}{4} \int x^{\frac{1}{5}} dx + \pi \int dx$$

$$= \frac{\frac{1}{2}x^{\frac{4}{3}}}{\frac{4}{3}} - \frac{\frac{1}{4}x^{\frac{6}{5}}}{\frac{6}{5}} + \frac{\pi x}{1} + c$$

$$= \frac{3x^{\frac{4}{3}}}{8} - \frac{5x^{\frac{6}{5}}}{24} + \pi x + c$$

Use $\int f(x) dx + g(x) dx = \int f(x) dx + \int g(x) dx$

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$

Divide $\frac{1}{2}$ by $\frac{4}{3}$ and $\frac{1}{4}$ by $\frac{6}{5}$

5. Given $f'(x) = \int \left(\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x^5} \right) dx$

a. -6

c. -3

b. 2

d. None

ANSWER: C

EXPLANATION:

$$\int \left(\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right) dx$$

$$= \int \left(\frac{2}{x} + 3x^{-2} + x^{-3} \right) dx$$

Write as negative exponent

$$= \int \frac{2}{x} dx + \int 3x^{-2} dx + \int x^{-3} dx$$

Use $\int f(x) dx + g(x) dx = \int f(x) dx + \int g(x) dx$

$$= 2 \ln x + \frac{3x^{-1}}{-1} + \frac{x^{-2}}{-2} + c$$

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$

$$= 2 \ln x - \frac{3}{x} - \frac{1}{2x^2} + c$$

Simplify $\frac{3}{-1}$

6. Integrate $\int \frac{3}{x^2} dx$

a. $6\sqrt{x+c}$

b. $\sqrt{x+c}$

c. $8\sqrt{x+c}$

d. $9\sqrt{x+c}$

ANSWER: A

EXPLANATION:

$$\int \frac{3}{x^2} dx = \int 3x^{-2} dx$$

$$= \frac{3x^{-2+1}}{-2+1} + c$$

$$= \frac{3x^{-1}}{-1} + c$$

$$= -3x^{-1} + c$$

$$= -\frac{3}{x} + c$$

7. Find y as a function of x if

$$\frac{d^2y}{dx^2} = 2x \quad \text{when } x = 2, y = 7$$

a. $y = \frac{x^3}{3} + c$

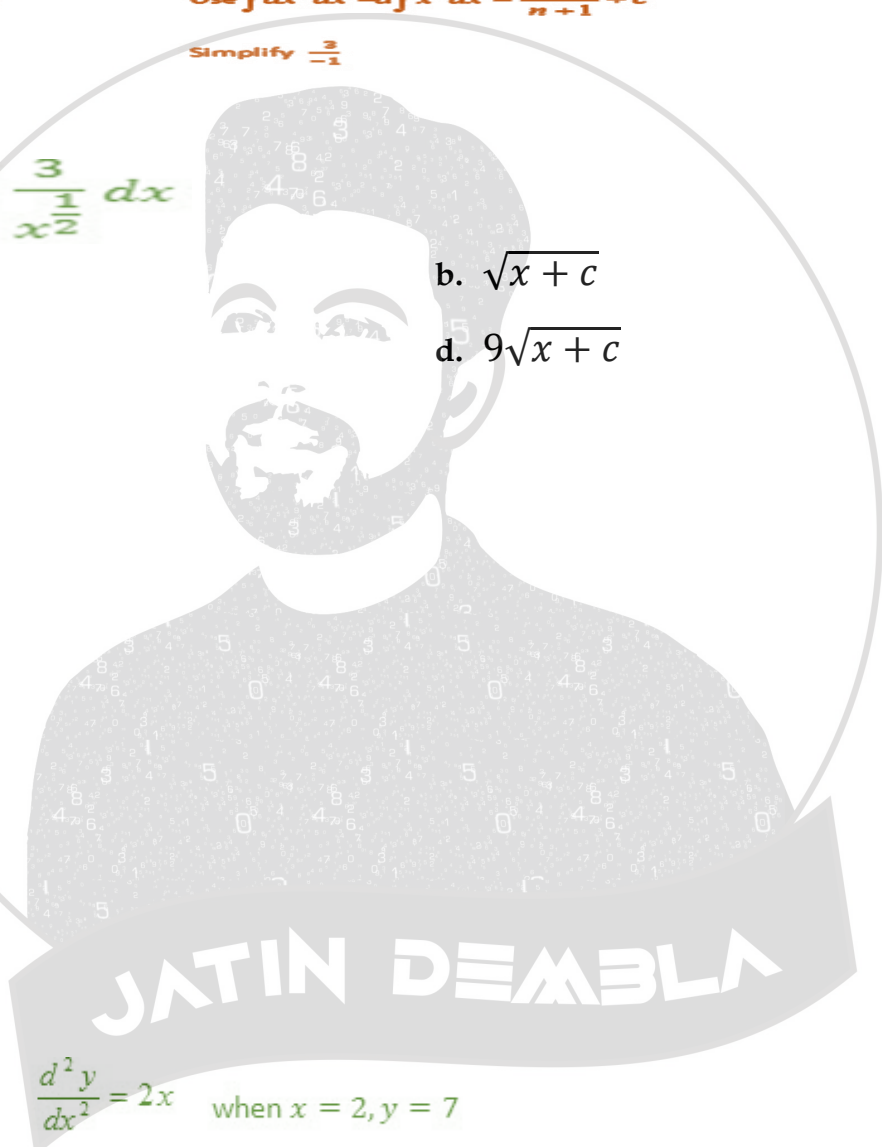
b. $y = \frac{x^2}{3} + c$

c. $y = \frac{x}{3} + c$

d. None

ANSWER: A

EXPLANATION:



$$\int 2x dx = 2 \int x dx$$

$$= \left(\frac{2x^{1+1}}{1+1} \right) + c$$

$$\frac{dy}{dx} = x^2 + c$$

Finding $y = \int \frac{dy}{dx} = \int x^2 dx$

$$y = \frac{x^3}{3} + c$$

At (2, 7),

$$7 = \frac{2^3}{3} + c$$

$$c = \frac{21}{8}$$

Thus, the function is $y = \frac{x^3}{3} + c$.

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$

Multiplication of fractions/simplify

Use $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

Substituting $x = 2$ and $y = 7$ to find c

8 Integrate $\int \left(w + \frac{1}{w} \right) \left(w - \frac{1}{w} \right) dw$

a. $\frac{w^3}{3} + \frac{1}{w}$

b. $\frac{w^3}{3} + \frac{1}{w} + c$

c. $\frac{w}{3} + \frac{1}{w} + c$

d. None

ANSWER: B

EXPLANATION:

$$\int \left(w + \frac{1}{w} \right) \left(w - \frac{1}{w} \right) dw$$

$$= \int \left(w^2 - \frac{1}{w^2} \right) dw$$

$$= \int w^2 dw - \int \frac{1}{w^2} dw$$

$$= \int w^2 dw - \int w^{-2} dw$$

$$= \frac{w^3}{3} + \frac{1}{w} + c$$

Express the product as a difference of two squares

Use $\int f(x) dx + g(x) dx = \int f(x) dx + \int g(x) dx$

Express in negative exponential form

Use $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$. Simplify

9 If $\frac{d^2y}{dx^2} = 10 - 3x$, find $\frac{dy}{dx} + c$

a. $10x - \frac{3}{2}x^2 + c$

b. $10x - \frac{3}{2} + c$

c. $10 - \frac{3}{2}x^2 + c$

d. none

ANSWER: A

EXPLANATION:

$$\frac{dy}{dx} = \int (10 - 3x) dx = \int 10 dx - \int 3x dx$$

Use $\int f(x) dx + g(x) dx = \int f(x) dx + \int g(x) dx$

$$= 10x - \left(\frac{3x^{1+1}}{1+1} \right) + c$$

Use $\int ax^n dx = a \int x^n dx = \frac{ax^{n+1}}{n+1} + c$

$$= 10x - \left(\frac{3x^2}{2} \right) + c$$

Simplify

$$= 10x - \frac{3}{2}x^2 + c$$

10. Calculate $\int x^7 dx$

a. $\frac{1}{8}x^7 + c$

b. $\frac{1}{7}x^7 + c$

c. $\frac{1}{8}x^8 + c$

d. None

ANSWER: C

EXPLANATION

$$\int x^7 dx = \frac{1}{7+1} x^{7+1} + c$$

$$= \frac{1}{8} x^8 + c$$

use $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$ and substitute $n = 7$

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12. If $\int f(x) dx = xe^{-\log|x|} + f(x)$, then $f(x)$ is

a. 1

b. 0

c. ce^x

d. $\log x$

Answer: C

$$\int f(x)dx = xe^{\log|\frac{1}{x}|} + f(x) \Rightarrow \int f(x)dx = \frac{x}{|x|} + f(x)$$

On differentiating both sides , we get

$$f(x) = 0 + f'(x)$$

We know

$$\frac{d}{dx}(e^x) = e^x, \therefore f(x) = ce^x.$$

Explanation:

13. If $f(t) = \int_{-t}^t \frac{dx}{1+x^2}$, then $f'(1)$ is

a. 0

b. 2/3

c. -1

d. -1

Answer: D

$$\text{Given } f(t) = \int_{-t}^t \frac{dx}{1+x^2} = [\tan^{-1}x]_{-t}^t = 2\tan^{-1}t$$

$$\text{Differentiating with respect to } t, f'(t) = \frac{2}{1+t^2}$$

$$\therefore f'(1) = \frac{2}{2} = 1.$$

Explanation:

14. The existence of first order partial derivatives implies continuity

True

False

Not Sure

Invalid Question

Answer: b

Explanation:

The mere existence cannot be declared as a condition for continuity because the second order derivatives should also be continuous.

15. $y = (x^2(1 + x^3))$

a. $-(2x + 5x^4)\sin(x^2 + x^5)$

b. $(2x + 5x^4)\sin(x^2 + x^5)$

c. $(2x + 5x^4)(x^2 + x^5)$

d. none

ANSWER: D

EXPLANATION:

$$\frac{dy}{dx} = -\sin(x^2 + x^5) \frac{d}{dx}(x^2 + x^5) \quad \text{using the Chain Rule}$$

$$\frac{dy}{dx} = -(\sin(x^2 + x^5))(2x + 5x^4) \quad \text{using basic derivatives}$$

$$\frac{dy}{dx} = -(2x + 5x^4)\sin(x^2 + x^5) \quad \text{reordering factors}$$

16. If $f(x) = x^k$ and $f'(1) = 10$, then the value of k is

- a. 10
 b. -10
 c. 1/10
 d. None

ANSWER: A

EXPLANATION:

$$f(x) = x^k$$

$$f(x) = \frac{d}{dx}(x^k)$$

$$10 = k \times 1$$

$$K = 10$$

17. The points of discontinuity of the function, $f(x) = \frac{x^2+2x+5}{x^2-3x+2}$ are

- a. $X=0, x=1$
 b. $X=1, x=2$
 c. $X=0, x=2$
 d. None

ANSWER: B

EXPLANATION:

$$f(x) = \frac{x^2 + 2x + 5}{x^2 - 3x + 2}$$

$$\text{Denominator} = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$X = 1, x = 2$$

18. The gradient of a function is parallel to the velocity vector of the level curve

- True
 False
 Not Sure
 Invalid Question

Answer: b

Explanation:

The gradient is perpendicular and not parallel to the velocity vector of the level curve.

$$19. y = (8 + x^3)(x^3 - 8)$$

a. $6x^5$

b. x^5

c. $6x$

d. None

ANSWER: A**EXPLANATION:**

This problem is solvable as a product but if you realize that you are looking at a difference of two squares, it becomes very simple.

$$y = (8 + x^3)(x^3 - 8) = x^6 - 64$$

$$\frac{dy}{dx} = 6x^5$$

$$20. \text{ If } (x, y, z, t) = xy + zt + x^2 yzt; x = k^3; y = k^2; z = k; t = \sqrt{k}$$

Find $\frac{df}{dt}$ at $k = 1$

a. 34

b. 16

c. 32

d. 61

Answer: b**Explanation:**

Using Chain rule we have

$$\begin{aligned} \frac{df}{dt} &= f_x \cdot \frac{dx}{dk} + f_y \cdot \frac{dy}{dk} + f_z \cdot \frac{dz}{dk} + f_t \cdot \frac{dt}{dk} \\ &= (y + 2xyzt) \cdot (3k^2) + (x + x^2zt) \cdot (2k) + (t + x^2yt) \cdot (1) + (z + x^2yz) \cdot \left(\frac{1}{2\sqrt{k}}\right) \end{aligned}$$

Put $k=1$; we have $x=y=z=t=1$

$$9 + 4 + 2 + 1 = 16.$$

$$21. \text{ If } (x, y) = x^2 + y^3; X = t^2 + t^3; y = t^3 + t^9 \text{ Find } \frac{df}{dt} \text{ at } t=1.$$

a. 0

b. 1

c. -1

d. 164

Answer: d**Explanation:**

Using chain rule we have

$$\frac{df}{dt} = f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt}$$

$$= (2x) \cdot (2t + 3t^2) + (3y^2) \cdot (3t^2 + 9t^8)$$

Put $t = 1$; we have $x = 2$; $y = 2$

$$= 4 \cdot (5) + 12 \cdot (12) = 164.$$

22. $f(x, y) = x^2 + xyz + z$ Find f_x at $(1, 1, 1)$

a. 0

c. 3

b. 1

d. -1

Answer: c

Explanation:

$$f_x = 2x + yz$$

Put $(x, y, z) = (1, 1, 1)$

$$f_x = 2 + 1 = 3.$$

23. Necessary condition of euler's theorem is

z should be homogeneous and of order n

z should not be homogeneous but of order n

z should be implicit

z should be the function of x and y only

Answer: a

Explanation:

Answer `a` is correct as statement of euler's theorem is "if z is an homogeneous function of x and y of order `n` then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$ "

Answer `b` is incorrect as z should be homogeneous.

Answer `c` is incorrect as z should not be implicit.

Answer `d` is incorrect as z should be the homogeneous function of x and y not non-homogeneous functions.

24. If $f(x, y) = x^{x+y}/y$, $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = ?$

a. 0

c. 2

b. 1

d. 3

Answer: a

Given function $f(x,y) = \frac{x+y}{y}$ Can be written as $f(x,y) = \frac{[1+\frac{y}{x}]}{\frac{y}{x}} = x^0 f(\frac{y}{x})$,

Hence by euler's theorem,

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$$

Explanation:

25. Find the approximate value of $[0.98^2 + 2.01^2 + 1.94^2]^{(1/2)}$

- a. 1.96
- b. 2.96
- c. 0.04
- d. -0.04

Answer: b

Explanation:

Let $f(x,y,z) = (x^2 + y^2 + z^2)^{(1/2)}$ (1)

Hence, $x = 1, y = 2, z = 2$ so that, $dx = -0.02, dy = 0.01, dz = -0.06$

From (1),

$$\frac{\partial f}{\partial x} = \frac{x}{f}$$

$$\frac{\partial f}{\partial y} = \frac{y}{f}$$

$$\frac{\partial f}{\partial z} = \frac{z}{f}$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = \frac{(xdx+ydy+zdz)}{f} = \frac{-0.02+0.02-0.12}{3} = -0.04$$

Hence,

$$[0.98^2 + 2.01^2 + 1.94^2]^{\frac{1}{2}} = f(1,2,2) + df = 3 - 0.04 = 2.96$$

26. $f(x,y) = \frac{x^3+y^3}{x^{99}+y^{98}x+y^{99}}$ find the value of f_y at $(x,y) = (0,1)$

- a. 101
- b. -96
- c. 210
- d. 0

Answer: b

Explanation:

Using Euler theorem

$$xf_x + yf_y = n f(x,y)$$

Substituting $x = 0$; $n = -96$ and $y = 1$ we have
 $f_y = -96$. $f(0, 1) = -96 \cdot (1/1)$
 $= -96$.

27. $f(x, y) = x^3 + xy^2 + 901$ satisfies the Eulers theorem

- a. True
- b. False
- c. Not Sure
- d. Invalid Question

Answer: b

Explanation:

The function is not homogenous and hence does not satisfy the condition posed by eulers theorem.

28. For a homogenous function if critical points exist the value at critical points is

- 1 equal to its degree
- 0
- 1

Answer: c

Explanation:

Using Euler theorem we have

$$xf_x + yf_y = n f(x, y)$$

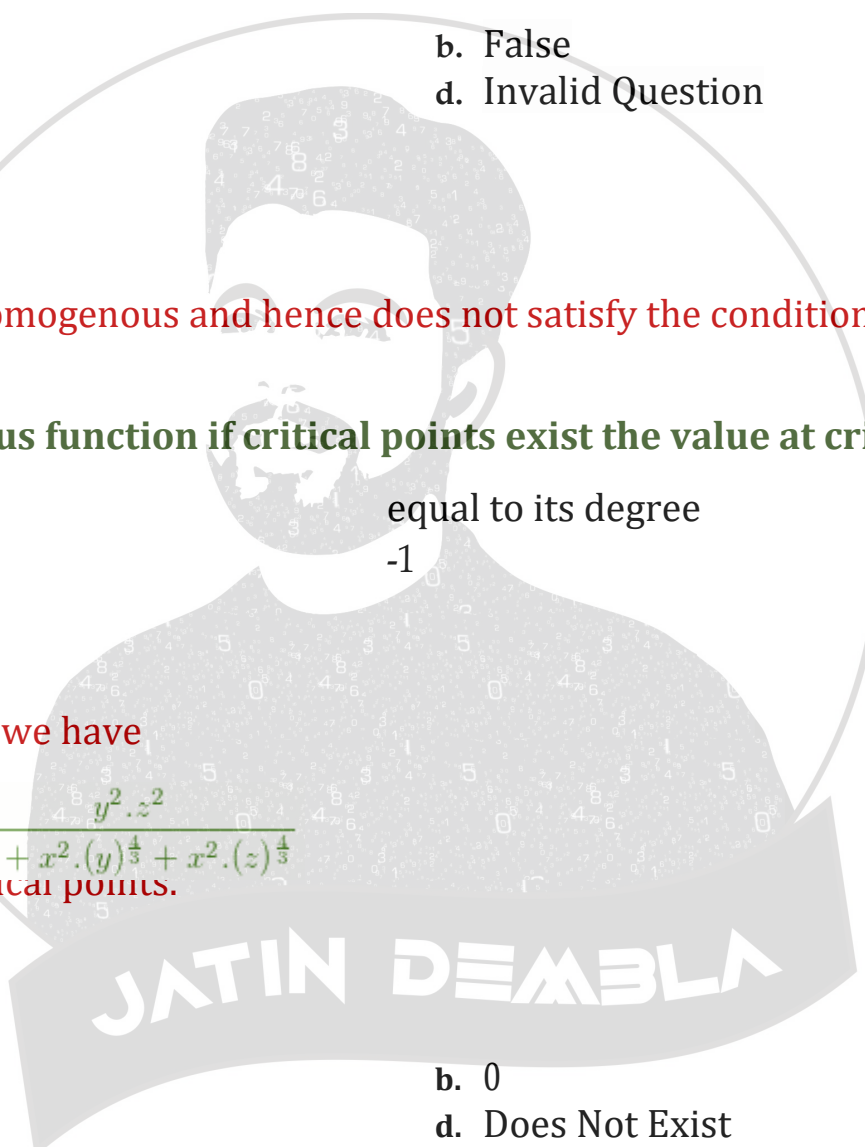
At criti $lt_{(x,y,z) \rightarrow (0,0,0)} \frac{y^2 \cdot z^2}{x^3 + x^2 \cdot (y)^{\frac{4}{3}} + x^2 \cdot (z)^{\frac{4}{3}}}$
 $f(a, b) = 0(a, 0) \rightarrow$ critical points.

29. Find

- a. 1
- b. 0
- c. ∞
- d. Does Not Exist

Answer: d

Explanation:



Put $x = t : y = a_1 * t^{3/4} : z = a_2 * t^{3/4}$

$$= \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{(a_1)^2 \cdot t^{3/2} \cdot (a_2)^2 \cdot t^{3/2}}{t^3 + t^2 \cdot t \cdot (a_1)^{4/3} + t^2 \cdot t \cdot (a_2)^{4/3}}$$

$$= \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{t^3}{t^3} \times \frac{(a_1)^2 \cdot (a_2)^2}{1 + (a_1)^{4/3} + (a_2)^{4/3}}$$

$$= \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{(a_1)^2 \cdot (a_2)^2}{1 + (a_1)^{4/3} + (a_2)^{4/3}}$$

30. $\lim_{n \rightarrow \infty} \left[\frac{n}{1+n^2} + \frac{n}{4+n^2} + \frac{n}{9+n^2} + \dots + \frac{1}{2n} \right]$ is equal to

a. $\frac{\pi}{2}$

b. $\frac{\pi}{4}$

c. 1

d. None of these

Answer: d

Explanation:

We have, $\lim_{n \rightarrow \infty} \left[\frac{n}{1+n^2} + \frac{n}{4+n^2} + \dots + \frac{1}{2n} \right]$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{r^2 + n^2} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 \left(1 + \frac{r^2}{n^2} \right)}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n \left(1 + \frac{r^2}{n^2} \right)} = \int_0^1 \frac{dx}{1+x^2}$$

{ Applying formula, $\lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \left\{ f\left(\frac{r}{n}\right) \right\} \cdot \frac{1}{n} = \int_0^1 f(x) dx$ }

$$= [\tan^{-1} x]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4}$$

31. For homogenous function with no saddle points we must have the minimum value as

90

1

equal to degree

0

Answer: d

Explanation:

Substituting $f_x = f_y = 0$ At critical points in euler theorem we have $nf(a, b) = 0 \Rightarrow f(a, b) = 0(a, b) \rightarrow$ critical points.

32. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)},$

- a. $\frac{9}{100}$
- c. $\frac{1}{99}$

- b. $\frac{-1}{2}$
- d. $\frac{1}{101}$

Answer: B

Explanation:

$$\lim_{n \rightarrow \infty} \frac{1^{99} + 2^{99} + \dots + n^{99}}{n^{100}} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{r^{99}}{n^{100}} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^{99} = \int_0^1 x^{99} dx = \left[\frac{x^{100}}{100} \right]_0^1 = \frac{1}{100}.$$

33. Let $f(x) = \int_1^x \sqrt{2 - t^2} dt$. Then real roots of the equation $x^2 - f'(x) = 0$ are

- a. ± 1
- c. $\pm \frac{1}{2}$

- b. $\pm \frac{1}{\sqrt{2}}$
- d. 0 and 1

Answer: A

Explanation:

$$f'(x) = \sqrt{2 - x^2} \Rightarrow x^2 - \sqrt{2 - x^2} = 0 \quad \text{or}$$

$$x^4 + x^2 - 2 = 0 \text{ or } (x^2 + 2)(x^2 - 1) = 0$$

$$\therefore x^2 - 1 = 0, \therefore x = \pm 1.$$

34. The value of the integral $\sum_{k=1}^n \int_0^1 f(k - 1 + x) dx$ is

- A. $\int_0^1 f(x) dx$
- B. $\int_0^2 f(x) dx.$

C. $n \int_0^n f(x) dx$

D. $n \int_0^1 f(x) dx$

A.

Answer: C

Explanation:

Let $I = \int_0^1 f(k-1+x) dx$

$I = \int_{k-1}^k f(t) dt$, where $t = k-1+x \Rightarrow I = \int_{k-1}^k f(x) dx$

$\therefore \sum_{k=1}^n \int_{k-1}^k f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx + \dots + \int_{n-1}^n f(x) dx$

$= \int_0^n f(x) dx.$

35. The derivative of $F(x) = \int_{x^2}^{x^3} \frac{1}{\log t} dt, (x > 0)$ is

a. $\frac{1}{3 \log x} - \frac{1}{2 \log x}$

b. $\frac{1}{3 \log x}$

c. $\frac{3x^2}{3 \log x}$

d. $(\log x)^{-1} \cdot x(x-1)$

Answer: D

Explanation:

We know that

$\frac{d}{dx} \left(\int_a^b f(t) dt \right) = \frac{db}{dx} f(b) - \frac{da}{dx} f(a)$ a and b

are functions of x. $\therefore F(x) = \int_{x^2}^{x^3} \frac{1}{\log t} dt$

$F'(x) = \frac{d}{dx} (x^3) \frac{1}{\log x^3} - \frac{d}{dx} (x^2) \frac{1}{\log x^2}$

$= \frac{3x^2}{3 \log x} - \frac{2x}{2 \log x} = x(x-1)(\log x)^{-1}.$

36. The greatest value of the

function $F(x) = \int_1^x |t| dt$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is given by ?

a. $\frac{3}{8}$

b. $-\frac{1}{2}$

c. $-\frac{3}{8}$

d. $\frac{2}{5}$

Answer: C

Explanation:

$F'(x) = |x| > 0 \forall x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ Hence the
 function is increasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and therefore $F(x)$ has
 maxima at the right end point of $\left[-\frac{1}{2}, \frac{1}{2}\right]$.
 $\therefore \text{Max } F(x) = F\left(\frac{1}{2}\right) = \int_1^{1/2} |t| dt = -\frac{3}{8}$.

37. For homogenous function the linear combination of rates of independent change along x and y axes is

Integral multiple of function value no relation to function value
 real multiple of function value depends if the function is a polynomial

Answer: c

Explanation:

Eulers theorem is nothing but the linear combination asked here, The degree of the

38. $\int_0^{b-c} f''(x+a) dx =$

homogenous function can be a real number. Hence, the value is integral multiple of real number.

- a. $f'(a) - f'(b)$
- b. $f'(b-c+a) - f'(a)$
- c. $f'(b+c-a) + f'(a)$
- d. None of these

Answer: B

Explanation:

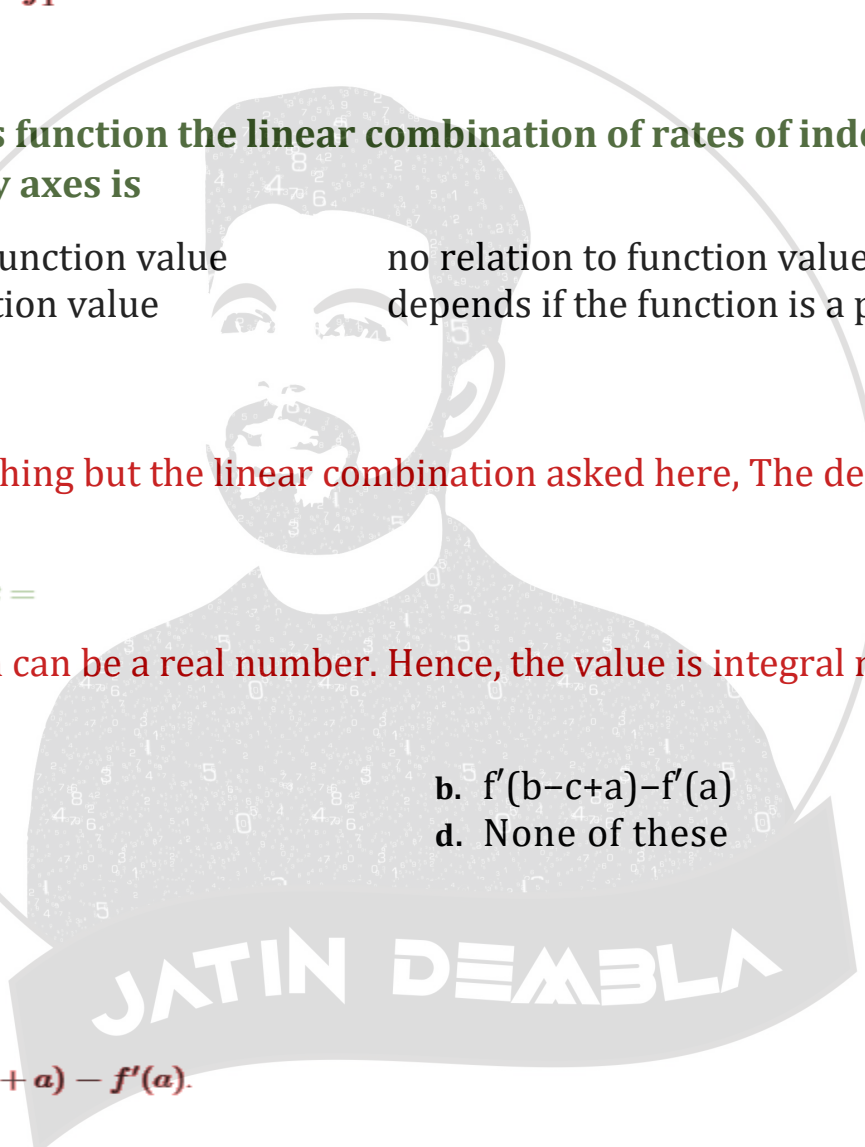
$$= \int_0^{b-c} f''(x+a) dx = [f'(x+a)]_0^{b-c} = f'(b-c+a) - f'(a)$$

39 $\int_0^{\infty} \frac{x^3 dx}{(x^2 + 4)^2} =$

- a. 0
- b. ∞
- c. 1/2
- d. None of these

Answer: B

Explanation:



$$\begin{aligned} &= \frac{1}{2} \int_0^\infty \frac{x^3 dx}{(x^2 + 4)^2} = \frac{1}{2} \int_0^\infty \frac{x^2 \cdot x dx}{(x^2 + 4)^2} dx \\ &= \frac{1}{2} \int_0^\infty \frac{t}{(t + 4)^2} dt, \quad [\text{Putting } x^2 = t] \\ &= \frac{1}{2} \int_0^\infty \left[\frac{1}{t + 4} - \frac{4}{(t + 4)^2} \right] dt = \frac{1}{2} \left[\log(t + 4) + \frac{4}{t + 4} \right]_0^\infty \\ &= \frac{1}{2} [\log \infty + 0 - (\log 4 + 1)] = \infty. \end{aligned}$$

40. The points of intersection of $F_1(x) = \int_2^x (2t - 5) dt$ and $F_2(x) = \int_0^x 2t dt$, are

- a. $(\frac{6}{5}, \frac{36}{25})$
- b. $(\frac{2}{3}, \frac{4}{5})$
- c. $(\frac{1}{3}, \frac{3}{6})$
- d. $(\frac{5}{4}, \frac{5}{7})$

Answer: A

Explanation:

Let $F_1(x) = y_1 = \int_2^x (2t - 5) dt$ and $F_2(x) = y_2 = \int_0^x 2t dt$. Now point of intersection means those point at which $y_1 = y_2 = y \Rightarrow y_1 = x^2 - 5x + 6$ and $y_2 = x^2$. On solving, we get $x^2 = x^2 - 5x + 6 \Rightarrow x = \frac{6}{5}$ and $y = x^2 = \frac{36}{25}$. Thus point of intersection is $(\frac{6}{5}, \frac{36}{25})$.

41. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is

- a. e+1
- b. e-1
- c. 1-e
- d. e

Answer: B

Explanation:

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}} = \int_0^1 e^x dx = [e^x]_0^1 = e - 1.$$

42. $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}} =$

- a. $\frac{1}{p+1}$
- c. $\frac{1}{p} - \frac{1}{p-1}$

- b. $\frac{1}{1-p}$
- d. None

Answer: A

Explanation:

$$\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \left[\frac{r^p}{n^{p+1}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n (r/n)^p = \int_0^1 x^p dx = \left[\frac{x^{p+1}}{p+1} \right]_0^1 = \frac{1}{p+1}$$

43. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \dots + \frac{1}{\sqrt{n^2+(n-1)n}} \right]$ is equal to .?

- a. $2+2\sqrt{2}$
- c. $2\sqrt{2}$
- b. $2\sqrt{2}-2$
- d. 2

Answer: B

Explanation:

$$y = \lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{\sqrt{n^2+n}} + \dots + \frac{1}{\sqrt{n^2+(n-1)n}} \right]$$

$$y = \lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{n\sqrt{1+\frac{1}{n}}} + \dots + \frac{1}{n\sqrt{1+\frac{(n-1)}{n}}} \right]$$

$$\Rightarrow y = \frac{1}{n} \lim_{n \rightarrow \infty} \left[1 + \frac{1}{\sqrt{1+\frac{1}{n}}} + \dots + \frac{1}{\sqrt{1+\frac{(n-1)}{n}}} \right]$$

$$y = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{\sqrt{1+\frac{(k-1)}{n}}}, \text{ Put } \frac{k-1}{n} = x \text{ and } \frac{1}{n} = dx$$

$$\Rightarrow y = \lim_{n \rightarrow \infty} \int_0^{\frac{n-1}{n}} \frac{dx}{\sqrt{1+x}}$$

$$= \lim_{n \rightarrow \infty} 2[\sqrt{1+x}]_0^{\frac{n-1}{n}}$$

$$y = 2 \lim_{n \rightarrow \infty} \left[\sqrt{\frac{2n-1}{n}} - 1 \right] = 2 \lim_{n \rightarrow \infty} \sqrt{2 - \frac{1}{n}} - 2$$

$$\Rightarrow y = 2 \lim_{n \rightarrow \infty} \sqrt{2 - \frac{1}{n}} - 2 = 2\sqrt{2} - 2$$

44. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right] =$

a. 0

c. $\log_e 3$

b. $\log_e 4$

d. $\log_e 2$

Answer: D

Explanation:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right] = \\ & \lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} \right] \\ & = \frac{1}{n} \lim_{n \rightarrow \infty} \left[1 + \frac{1}{1 + \frac{1}{n}} + \frac{1}{1 + \frac{2}{n}} + \dots + \frac{1}{1 + \frac{n}{n}} \right] \\ & = \frac{1}{n} \lim_{n \rightarrow \infty} \sum_{r=0}^n \left[\frac{1}{1 + \frac{r}{n}} \right] = \int_0^1 \frac{1}{1+x} dx \\ & = [\log_e(1+x)]_0^1 = \log_e 2 - \log_e 1 = \log_e 2. \end{aligned}$$

45. The solution of the equation $\frac{x^2 d^2y}{dx^2} = \ln x$, when $x=1, y=0$ and $\frac{dy}{dx} = -1$

a. $\frac{1}{2}(\ln x)^2 + \ln x$

b. $\frac{1}{2}(\ln x)^2 - \ln x$

c. $-\frac{1}{2}(\ln x)^2 + \ln x$

d. $-\frac{1}{2}(\ln x)^2 - \ln x$

Answer: D

Explanation:

$$\begin{aligned} & \frac{d^2y}{dx^2} = \frac{\log x}{x^2} \Rightarrow \frac{dy}{dx} = \frac{-(\log x + 1)}{x} + c \\ & \text{At } \frac{dy}{dx} = -1, x=1, y=0, \wedge c=0 \\ & y = - \int \frac{\log x + 1}{x} dx = -\frac{1}{2}(\log x)^2 - \log x. \end{aligned}$$

46. The rate of increase of bacteria in a certain culture is proportional to the number present. If it double in 5 hours then in 25 hours, its number would be

a. 8 times the original

b. 16 times the original

c. 32 times the original

d. 64 times the original

Answer: C

Explanation:

Let P_0 be the initial population and let the population after t years be P . Then $\frac{dP}{dt} = kP \Rightarrow \frac{dP}{P} = k dt$

On integrating, we have $\log P = kt + c$ At $t = 0$,
 $P = P_0 \therefore \log P_0 = 0 + c, \therefore \log P = kt + \log P_0$
 $\log \frac{P}{P_0} = kt$ When $t = 5$ hrs, $P = 2P_0 \therefore$

$\log \frac{2P_0}{P_0} = 5k \Rightarrow K = \frac{\log 2}{5}; \therefore \log \frac{P}{P_0} = \frac{\log 2}{5} t$ When

$t = 25$ hours, we have

$\log \frac{P}{P_0} = \frac{\log 2}{5} \times 25 = 5 \log 2 = \log 32; \therefore P = 32P_0.$

47. The degree of the equation

$3 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{3/2}$ is differential

- a. 1
- b. 2
- c. 3
- d. 6

Answer: B

Explanation:

$3 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{3/2}$ On squaring, we
 get $9 \left(\frac{d^2 y}{dx^2} \right)^2 = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^3$ Obviously the
 highest derivative $\frac{d^2 y}{dx^2}$ contains a degree 2.

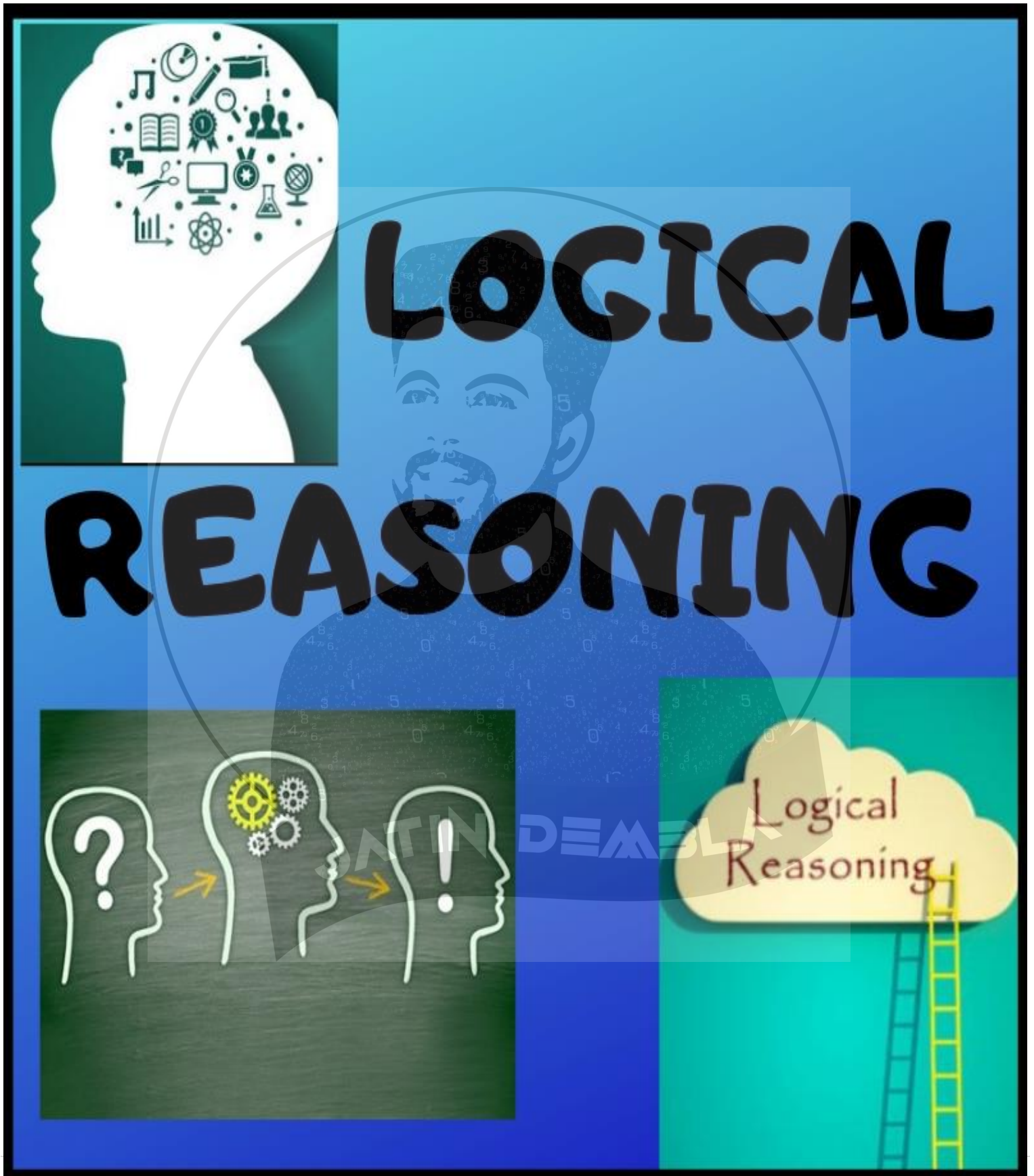
48. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of

- a. Order 1
- b. Order 2
- c. Degree 3
- d. Degree 4

Answer: A

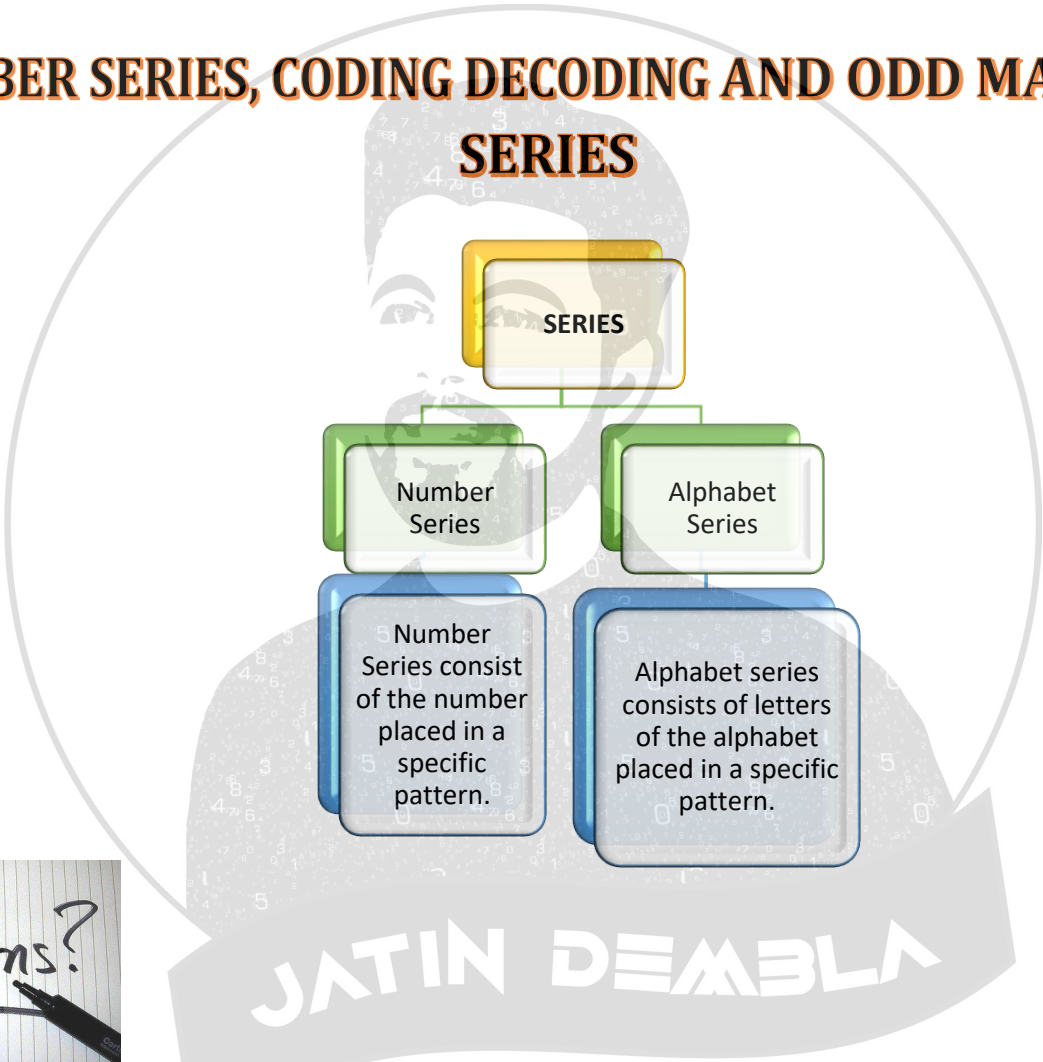
Explanation:

Given curve is $y^2 = 2c(x + \sqrt{c})$.
 Differentiate w.r.t. x , $2y \frac{dy}{dx} = 2c \Rightarrow c = y \frac{dy}{dx}$ Hence
 differential equation is $y^2 = 2y \frac{dy}{dx} \left(x + \sqrt{y \frac{dy}{dx}} \right)$
 $\frac{y}{2dy/dx} - x = \sqrt{y \frac{dy}{dx}}$ Squaring and multiplying by
 $\left(\frac{dy}{dx} \right)^2$ $y \left(\frac{dy}{dx} \right)^3 - x^2 \left(\frac{dy}{dx} \right)^2 + xy \left(\frac{dy}{dx} \right) - \frac{y^2}{4} = 0$
 Hence order is 1 and degree is 3.



CHAPTER 9

NUMBER SERIES, CODING DECODING AND ODD MAN OUT SERIES



<p>CODING AND DECODING</p>	<p>Before transmitting, the data is encoded and at receiver side encoded at a is decoded in order to obtain original data by determining common key in encoded data. Type 1: Letter Coding Type 2: Number coding</p>
<p>ODD MANOUT</p>	<p>Classification means 'to assort the items' of a given group on the</p>

basis of a certain common quality they possess and then spot the stranger or 'odd one out'.

1. Find the missing term of the series 2, 7, 16, __, 46, 67, 92

a. 29

b. 30

c. 19

d. 39

ANSWER: a

EXPLANATION:

Here the terms of the series are +5, +9, +13, +17, +21, +25...

Thus, $2 + 5 = 7$; and $7 + 9 = 16$...

So missing term = $16 + 13 = 29$

2. Find the wrong terms of the series 9, 29, 65, 126, 217, 344

a. 30

b. 29

c. 28

d. 27

30

a. 29

b. 28

c. 27

ANSWER: b

EXPLANATION:

$2^3+1, 3^3+1, 4^3+1, \dots$ Here 29 is wrong term of series

3. Find the missing term of the series 1, 9, 25, 49, 81, 121,

a. 129

b. 149

c. 169

d. 139

ANSWER: c

EXPLANATION:

The given terms of the series are consists square of consecutive odd number $1^2, 3^2, 5^2, 7^2, \dots$ So missing value = $13^2 = 169$

4. Find the next term of the series BKS, DJT, FIU, HHV?

- a. JGW
- c. JVG

- b. a. JGV
- d. d. BBA

ANSWER: a

EXPLANATION:

This type of question usually consist of a series of small letters which follow a certain pattern. However some letters are missing from the series. These missing letters are then given in a proper sequence as one of the alternatives.

5. 3, 5, 11, 14, 17, 21 Find the odd man out

- a. 21
- c. 14
- b. 17
- d. 3

ANSWER: C

EXPLANATION:

Each of the numbers except 14 is an odd number.

The number '14' is the only EVEN number.

6. 8, 27, 64, 100, 125, 216, 343 Find the odd man out ?

- a. 27
- c. 125
- b. 100
- d. 343

Answer: B

Explanation:

EXCEPT 100 ALL ARE CUBE OF 2,3,4,5,6 and 7

7. 6, 9, 15, 21, 24, 28, 30

Answer: Option D

Explanation:

The numbers are $1^3, 2^3, 3^3, 4^3$ etc. So, 124 is wrong; it must have been 5^3 i.e., 125
11. 8, 13, 21, 32, 47, 63, 83. Find out the wrong number in the given sequence of numbers.

- A. 47
- B. 63
- C. 32
- D. 83

Answer: Option A

Explanation:

Go on adding 5, 8, 11, 14, 17, 20.

So, the number 47 is wrong and must be replaced by 46

12. Insert the missing number.

16, 33, 65, 131, 261, (....)

- a. 523
- b. 521
- c. 613
- d. 721

Answer: Option A

Explanation:

Each number is twice the preceding one with 1 added or subtracted alternatively.

So, the next number is $(2 \times 261 + 1) = 523$

13. Insert the missing number

2, 4, 12, 48, 240, (....)

- a. 960
- b. 1440

c. 1080

d. 1920

Answer: Option B**Explanation:**

Go on multiplying the given numbers by 2, 3, 4, 5, 6.

So, the correct next number is 1440

14. Insert the missing number 8, 7, 11, 12, 14, 17, 17, 22, (....)

a. 27

b. 20

c. 22

d. 24

Answer: Option B**Explanation:**

There are two series (8, 11, 14, 17, 20) and (7, 12, 17, 22) increasing by 3 and 5 respectively.

15. Find out the wrong number in the series.**7, 8, 18, 57, 228, 1165, 6996**

a. 8

b. 18

c. 57

d. 228

Answer: Option D**Explanation:**

Let the given numbers be A, B, C, D, E, F, G.

Then, $A, A \times 1 + 1, B \times 2 + 2, C \times 3 + 3, D \times 4 + 4, E \times 5 + 5, F \times 6 + 6$ are the required numbers.

Clearly, 228 is wrong

16. Find out the wrong number in the series 1, 2, 6, 24, 96, 720

- a. 720
- b. 96
- c. 24
- d. 6

Answer: Option B

Explanation:

Go on multiplying with 1, 2, 3, 4, 5, 6 to get next number.

So, 96 is wrong

17. Find out the wrong number in the series 196, 169, 144, 121, 100, 80, 64

- a. 169
- b. 144
- c. 121
- d. 80

Answer: Option D

Explanation:

Numbers must be $(14)^2, (13)^2, (12)^2, (11)^2, (10)^2, (9)^2, (8)^2$.

So, 80 is wrong

18. Find out the wrong number in the series 445, 221, 109, 46, 25, 11, 4

- a. 221
- b. 109
- c. 46
- d. 25

Answer: Option C

Explanation:

Go on subtracting 3 and dividing the result by 2 to obtain the next number.

Clearly, 46 is wrong.

19. Find out the wrong number in the series 190, 166, 145, 128, 112, 100, 91

- a. 100
- b. 166
- c. 145
- d. 128

Answer: Option D

Explanation:

Go on subtracting 24, 21, 18, 15, 12, 9 from the numbers to get the next number.

$$190 - 24 = 166$$

$$166 - 21 = 145$$

$$145 - 18 = 127 \text{ [Here, 128 is placed instead of 127]}$$

$$127 - 15 = 112$$

$$112 - 12 = 100 \dots \text{ and so on.}$$

Therefore, 128 is wrong

20 . In a certain code DELHI is written as CDKGH. How is SUSPECT written in code?

- a. RTRODBS.
- c. RTIODBS

- b. QTRODBS
- d. RTROIBS.

Answer: A

Explanation:

Clearly, we can see that each letter of the word DELHI is moved one step backward to obtain the code.

D	E	L	H	I
-1↓	-1↓	-1↓	-1↓	-1↓
C	D	K	G	H

Similarly, SUSPECT will be coded as RTRODBS.

21. In a certain code COURAGE is written as UOCREGA. How will JOURNAL be written in the code.

- a. UOJRLAN.
- c. UPJRLAN

- b. UOMRLAN.
- d. ULOJRLAN

Answer:

Explanation: A

Clearly, when COURAGE is coded, some letters are interchange with respect to their positions, i.e., odd position are interchanged.

1	2	3	4	5	6	7
C	O	U	R	A	G	E

↔ ↔

Position of 1 changes to 3 and 3 to 1. Position of 5 changes to 7 and 7 to 5.



can be coded as UOJRLAN

22. Find out the wrong number in the series.

19, 26, 33, 46, 59, 74, 91

- a. 26
- b. 33
- c. 46
- d. 59

Answer: Option B

Explanation:

Go on adding 7, 9, 11, 13, 15, 17 respectively to obtain the next number.

So, 33 is wrong. It must be 35

23. Find out the wrong number in the series 1, 3, 10, 21, 64, 129, 356, 777

- a. 10
- b. 21
- c. 64
- d. 356

Answer: Option D

Explanation:

$A \times 2 + 1$, $B \times 3 + 1$, $C \times 2 + 1$, $D \times 3 + 1$ and so on.

So, 356 is wrong

24. Find out the wrong number in the series 6, 12, 48, 100, 384, 768, 3072

- a. 768
- b. 384
- c. 100
- d. 48

Answer: Option C

Explanation:

Each even term of the series is obtained by multiplying the previous term by 2.

$$2^{\text{nd}} \text{ term} = (1^{\text{st}} \text{ term}) \times 2 = 6 \times 2 = 12$$

$$4^{\text{th}} \text{ term} = (3^{\text{rd}} \text{ term}) \times 2 = 48 \times 2 = 96.$$

$$6^{\text{th}} \text{ term} = (5^{\text{th}} \text{ term}) \times 2 = 384 \times 2 = 768.$$

\therefore 4th term should be 96 instead of 100

25. Insert the missing number. 7, 26, 63, 124, 215, 342, (....)

a. 391

b. 421

c. 481

d. 511

Answer: D

Explanation:

Numbers are $(2^3 - 1)$, $(3^3 - 1)$, $(4^3 - 1)$, $(5^3 - 1)$, $(6^3 - 1)$, $(7^3 - 1)$ etc.

So, the next number is $(8^3 - 1) = (512 - 1) = 511.$

26. Find the odd man out? 396, 462, 572, 427, 671, 264

a. 671

b. 462

c. 427

d. 264

Answer: C

Explanation:

Here the given series is 396, 462, 572, 427, 671, 264.

In all the terms, the middle digit is the sum of first and third digit except 427.

So the Odd number in the given series is 427.

27. Insert the missing number. 2, 4, 12, 48, 240, (....)

a. 960

b. 1440

c. 1080

d. 1920

Answer: B

Explanation:

Go on multiplying the given numbers by 2, 3, 4, 5, 6.

So, the correct next number is 1440.

28. Find the odd man out. 41, 43, 47, 53, 61, 71, 73, 81

a. 41

b. 61

c. 71

d. 81

Answer: D

Explanation:

Each of the numbers except 81 is a prime number

29. Find out the wrong number in the given sequence of numbers. 582, 605, 588, 611, 634, 617, 600

a. 634

b. 611

c. 605

d. 600

Answer: A

Explanation:

Alternatively 23 is added and 17 is subtracted from the terms. So, 634 is wrong.

30. Find out the wrong number in the given sequence of numbers. 1, 2, 6, 15, 31, 56, 91

a. 31

b. 91

c. 101

d. 15

Answer: B

Explanation:

$1, 1 + 1^2 = 2, 2 + 2^2 = 6, 6 + 3^2 = 15, 15 + 4^2 = 31, 31 + 5^2 = 56, 56 + 6^2 = 92$
Last number of given series must be 92 not 91

31. find odd number: 324, 244, 136, 352, 514

a. 324

b. 244

c. 136

d. 352

Answer: B

Explanation:

Sum of the digits in each other number is 10.

32. find odd number: 43, 53, 63, 73, 83

a. 43

b. 53

c. 63

d. 73

Answer: c

Explanation:

Each of the numbers except 63, is a prime number.

33. find odd number: 10 , 26 , 24 , 21 , 18

- a. 10
- b. 26
- c. 24
- d. 21

Answer: D

Explanation:

Each of the numbers except 21, is an even number.

34. Find odd number: 51 , 144 , 64 , 121 , 256

- a. 51
- b. 144
- c. 64
- d. 121

Answer: A

Explanation:

Each of the number except 51, is a perfect square.

35. find odd number: 15 , 21 , 24 , 28 , 30

- a. 15
- b. 21
- c. 24
- d. 28

Answer: D

Explanation:

Each of the numbers except 28, is divisible by 3.

36. Find odd number: 2384 , 1592 , 3756 , 4298 , 3629

- a. 2384
- b. 1592
- c. 3756
- d. 3629

Answer: D

Explanation:

In all other numbers, the last digit is two times the first, All are EVEN but 3629 is ODD.

37. Choose odd number: 7359 , 1593 , 9175 , 3781 , 9317

- a. 7359
- b. 1593
- c. 9175
- d. 3781

Answer: D

Explanation:

All other numbers consist of odd digits only. Sum of all digits is a prime in D.

38. find odd number: 8314, 2709, 1315, 2518, 3249

a. 8314

b. 2709

c. 1315

d. 2518

Answer: A

Explanation:

In all number except 8314, the sum of first three digits is equal to the unit's digit. Hence, the answer is (a).

39. Find odd number: 48, 12, 36, 24, and 59

a. 48

b. 12

c. 36

d. 59

Answer: D

Explanation:

In all numbers except 59, the unit's digit is twice the ten's digit. Hence, the answer is (d), and all are multiples of 12 too except 59

40. Find odd number: 2345, 3456, 5467, and 5678

a. 2345

b. 3456

c. 5467

d. 567

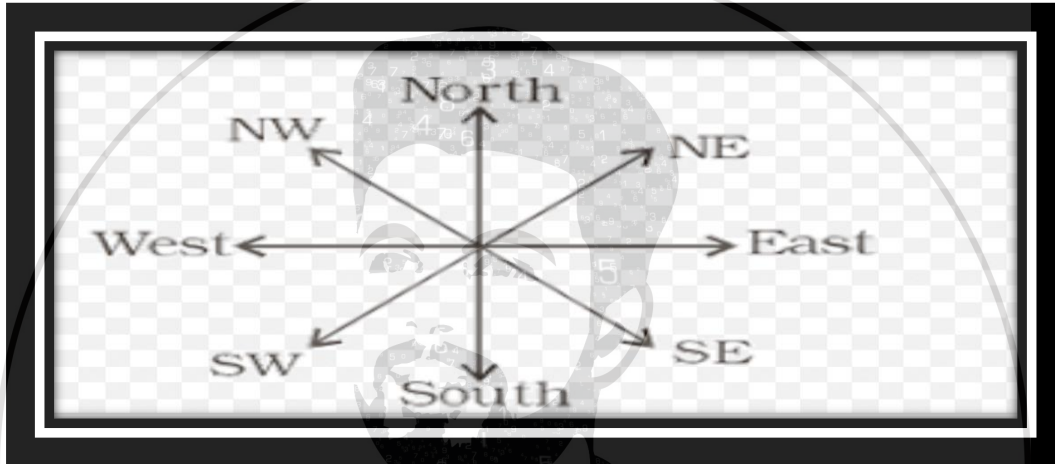
Answer: C

Explanation:

All other numbers contain four consecutive digits in order.

CHAPTER 10

DIRECTION SENSE TEST



Always Remember



LEFT & LEFT	Down
RIGHT & LEFT	UP
LEFT & RIGHT	UP
RIGHT & RIGHT	Down
UP & LEFT	Left
UP & RIGHT	Right
DOWN & LEFT	Right
DOWN & RIGHT	Left

1. One morning Udai and Vishal were talking to each other face to face at a crossing. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?



- a. East.
- b. West
- c. North
- d. South

Answer: Option C

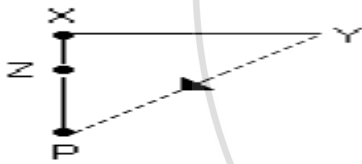
Explanation:

2. Y is in the East of X which is in the North of Z. If P is in the South of Z, then in which direction of Y, is P?

- A. North
- B. South
- C. South-East
- D. None of these

Answer: Option D

Explanation:



P is in South-West of Y

3. If South-East becomes North, North-East becomes West and so on. What will West become?

- A. North-East
- B. North-West
- C. South-East
- D. South-West

Answer: Option C

Explanation:



It is clear from the diagrams that new name of West will become South-East

4. A man walks 5 km toward south and then turns to the right. After walking 3 km he turns to the left and walks 5 km. Now in which direction is he from the starting place?

a. North-East

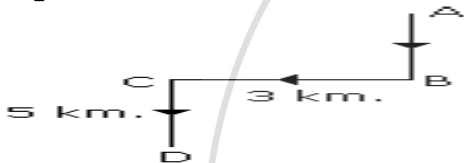
b. North-West

c. South-East

d. South-West

Answer: Option D

Explanation:



Hence required direction is South-West.

5. Rahul put his timepiece on the table in such a way that at 6 P.M. hour hand points to North. In which direction the minute hand will point at 9.15 P.M.?

a. South-East

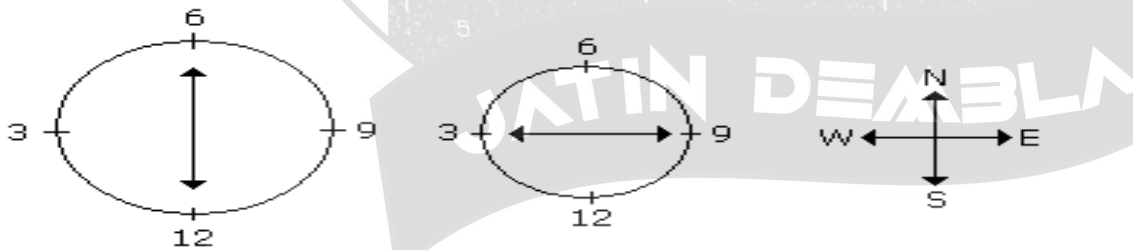
b. South

c. North

d. West

Answer: Option D

Explanation:



At 9.15 P.M., the minute hand will point towards west

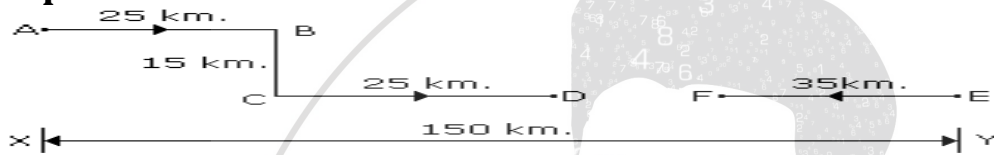
3. Two cars start from the opposite places of a main road, 150 km apart. First car runs for 25 km and takes a right turn and then runs 15 km. It then turns left and then runs for another 25 km and then takes the direction back to reach the main road. In the meantime, due to minor break down the other car has run only 35

km along the main road. What would be the distance between two cars at this point?

- a. 65 km
- b. 75 km
- c. 80 km
- d. 85 km

Answer: Option A

Explanation:



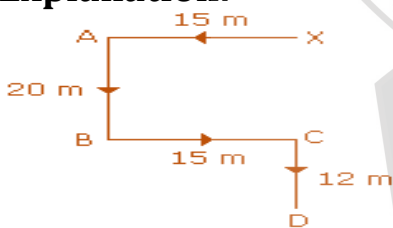
$$\begin{aligned}
 \text{Required distance} &= DF \\
 &= 150 - (25 + 25 + 35) \\
 &= 150 - 85 \\
 &= 65 \text{ km.}
 \end{aligned}$$

4. Starting from the point X, Jayant walked 15 m towards west. He turned left and walked 20 m. He then turned left and walked 15 m. After this he turned to his right and walked 12 m. How far and in which directions is now Jayant from X?

- a. 32 m, South
- b. 47 m, East
- c. 42 m, North
- d. 27 m, South

Answer: Option A

Explanation:



$$\begin{aligned}
 \text{Required distance} &= 20 + 12 \\
 &= 32 \text{ m in south direction}
 \end{aligned}$$

5. One evening before sunset Rekha and Hema were talking to each other face to face. If Hema's shadow was exactly to the right of Hema, which direction was Rekha facing?

- A. North
- B. South

C. East

D. Data is inadequate

Answer: Option B

Explanation:



In the evening sun sets in West. Hence then any shadow falls in the East. Since Hema's shadow was to the right of Hema. Hence Rekha was facing towards South.

6. A boy rode his bicycle Northward, then turned left and rode 1 km and again turned left and rode 2 km. He found himself 1 km west of his starting point. How far did he ride northward initially?

a. 1 km

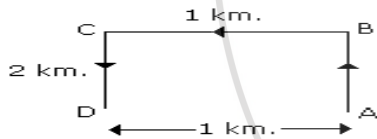
b. 2 km.

c. 3 km

d. 5 km

Answer: Option B

Explanation:



The boy rode 2 km. Northward.

7. K is 40 m South-West of L. If M is 40 m South-East of L, then M is in which direction of K?

a. East

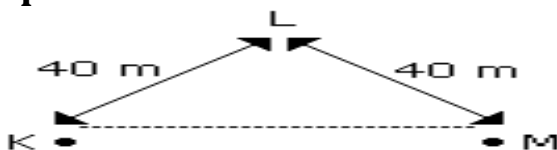
b. West

c. North-East

d. Sout

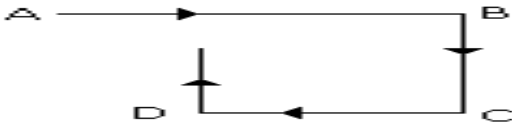
Answer: Option A

Explanation:



Hence M is in the East of K.

Explanation:



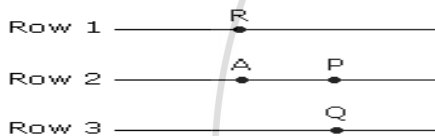
Hence finally Sujata will face towards North.

11. Some boys are sitting in three rows all facing north such that A is in the middle row. P is just to the right of A but in the same row. Q is just behind of P while R is in the North of A. In which direction of R is Q?

- a. North
- b. South- East
- c. North-East
- d. South-West

Answer: Option b

Explanation:



Q is in South-East of R

12. One morning after sunrise, Vimal started to walk. During this walking he met Stephen who was coming from opposite direction. Vimal watch that the shadow of Stephen to the right of him (Vimal). To Which direction Vimal was facing?

- a. East
- b. West
- c. South
- d. Data inadequate

Answer: Option C

Explanation:

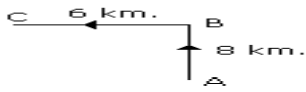
Sun rises in the east. So the shadow of a man will always falls towards the west. Since the shadow of Stephen is to the right of Vimal. Hence Vimal is facing towards South.

13. Golu started from his house towards North. After covering a distance of 8 km. he turned towards left and covered a distance of 6 km. What is the shortest distance now from his house?

- 10 km
- 14 km
- 14 km
- 2 km

Answer: Option A

Explanation:



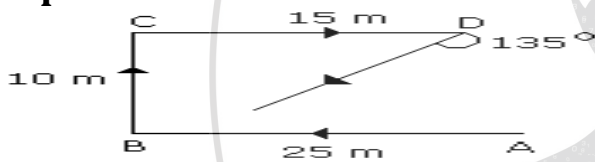
$$\begin{aligned}
 \text{Required distance} &= AC \\
 &= \sqrt{8^2 + 6^2} \\
 &= \sqrt{64 + 36} \\
 &= \sqrt{100} \\
 &= 10 \text{ km.}
 \end{aligned}$$

14. P started from his house towards west. After walking a distance of 25 m. He turned to the right and walked 10 m. He then again turned to the right and walked 15 m. After this he is to turn right at 135° and to cover 30 m. In which direction should he go?

- a. West
- b. South
- c. South-West
- d. South-East

Answer: Option C

Explanation:



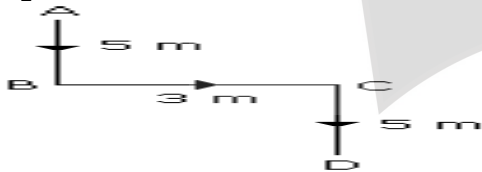
Hence he should go in the South-West direction.

15. X-Men started to walk straight towards south. After walking 5 m he turned to the left and walked 3 m. After this he turned to the right and walked 5 m Now to which direction X is facing?

- a. North-East
- b. South
- c. North
- d. South-West

Answer: Option B

Explanation:



Hence X-Men will face in the end towards South.

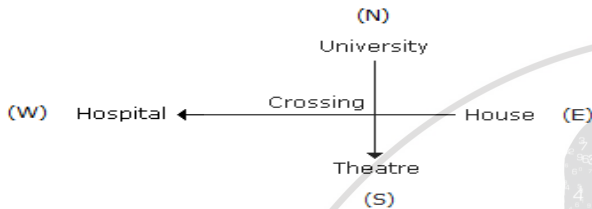
16. Hemant in order to go to university started from his house in the east and came to a crossing. The road to the left ends in a theatre, straight ahead is the

hospital. In which direction is the university?

- a. North
- b. South
- c. East
- d. West

Answer: Option A

Explanation:



Therefore university is in North

17. If a boy starting from Nilesh, met to Ankur and then to Kumar and after this he to Dev and then to Pintu and whole the time he walked in a straight line, then how much total distance did he cover?

- a. 215 m
- b. 155 m
- c. 245 m
- d. 185 m

Answer: Option A

Explanation:

Required distance = 25 m + 40 m + 60 m + 90 m

Required distance = 215 m

18. Each of the following questions is based on the following information:

1. Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U.
2. Q gets a North facing flat and is not next to S.
3. S and U get diagonally opposite flats.
4. R next to U, gets a south facing flat and T gets North facing flat.

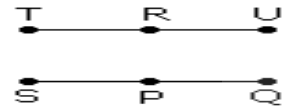
1. If the flats of P and T are interchanged then whose flat will be next to that of U?

- a. P
- b. Q
- c. R
- d. T

Answer: Option C



Interchanging flats P and T



Explanation:

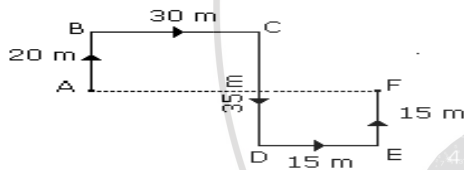
Hence flat R will be next to U.

19. Which of the following combination get south facing flats?

- a. QTS
- b. UPT
- c. URP
- d. Data is inadequate

Answer: Option C

Explanation:



$$\begin{aligned} \text{Required distance} &= AF \\ &= 30 + 15 \\ &= 45 \text{ m.} \end{aligned}$$

From the above diagram, F is in East direction from A.

Hence the required answer is '45 m East'.

Hence URP flat combination get south facing flats.

20. Rasik walked 20 m towards north. Then he turned right and walks 30 m. Then he turns right and walks 35 m. Then he turns left and walks 15 m. Finally he turns left and walks 15 m. In which direction and how many meters is he from the starting position?

- a. 15 m West
- b. 30 m East
- c. 30 m West
- d. 45 m East

Answer: Option D

Explanation:

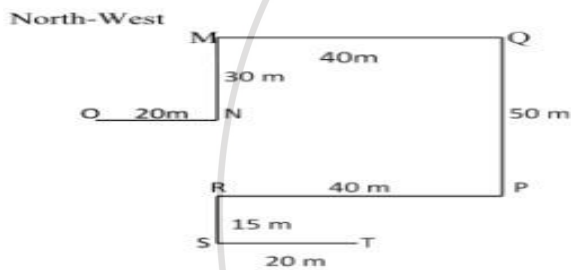
21. Eight persons M through T are standing in such a way that O is 20 m apart from N towards West, N is 30 m South with respect to M. M is 40 m towards West with respect to Q. P is 50 m towards South with respect to Q. R is 15 m apart from S towards North. T is 20 m towards East with respect to S. R is 40 m towards West with respect to P .In which direction is Q standing with respect to R?

- a. North-West
- c. North-East

- b. North
- d. Cannot be determined

Answer: Option c

Explanation:

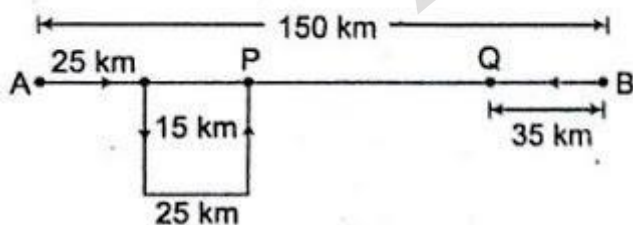


22. Two buses start from the opposite points of a main road, 150km apart. The first bus runs for 25 km and takes a right turn and then runs for 15km. It then turns left and runs for another 25km and takes the direction back to reach the main road. In the meantime, due to the minor break down the other bus has run only 35km along the main road. What would be the distance between the two buses at this point?

- a. 65km
- c. 75km

- b. 80km
- d. 85km

Answer: Option a



Explanation:

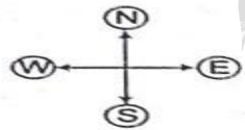
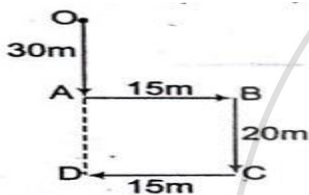
Required distance = $PQ = 150 - (25 + 25 + 35) = 65\text{km}$

23. Mohan walked 30m towards South, took a left turn and walked 15m. He, then took a right turn and walked 20m. He again took a right turn and walked 15m. How far is he from the starting point?

- a. 95m
- b. 50m
- c. 70m
- d. Cannot be determined

Answer: Option b

Explanation:



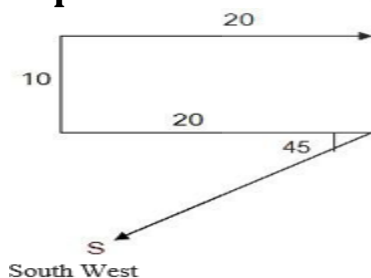
Required distance = $OD = OA + AD = OA + BC = 30 + 20 = 50\text{m}$

24. Starting from the point X, Jayant walked 15 m towards west. He turned left and walked 20 m. He then turned left and walked 15 m. After this he turned to his right and walked 12 m. How far and in which directions is now Jayant from X?

- a. 32 m, South
- b. 47 m, East
- c. 42 m, North
- d. 27 m, South

Answer: Option A

Explanation:

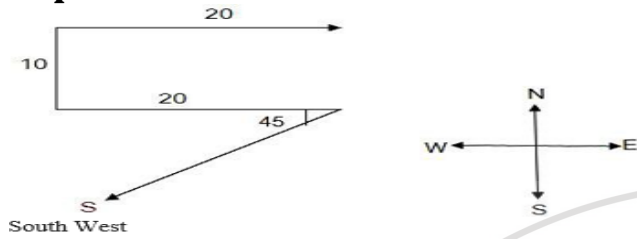


25. Lakshman went 15 kms to the West of his house, then turned left and walked 20 kms. He then turned East and walked 25 kms and finally turning left covered 20 kms. How far was he from his house?

- a. 5 kms
- b. 10 kms
- c. 40 kms
- d. 80 kms

Answer: Option C

Explanation:

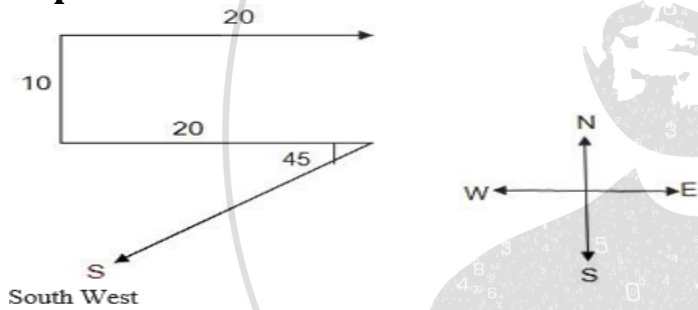


26. A starts from a point and walks 5kms north, then turns left and walks 3kms. Then again turns left and walks 5kms. Point out the direction in which he is going.

- a. West
- b. South
- c. North
- d. East

Answer: Option b

Explanation:

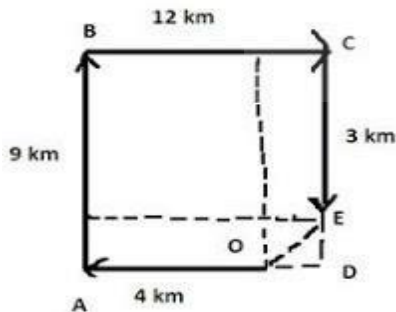


27. A person walks 4 km towards west, then turns to his right to travel 9 km. He turns towards east and travels 12 km. Finally, he travels 3 km towards south. How far is he from the initial position (in km)?

- a. 15
- b. 23
- c. 18
- d. 10

Answer: Option d

Explanation:



JATIN DEMBLA

From the figure, the distance OE is to be calculated. In triangle ODE, $OE = \sqrt{(OD^2) + (DE^2)}$

$$= \sqrt{(BC - AO)^2 + (AB - CE)^2} \quad OE = \sqrt{(8^2 + 6^2)} = 10\text{km.}$$

28. One evening before sunset two friends Sumit and Mohit were talking to each other face to face. If Mohit's shadow was exactly to his right side, which direction was Sumit facing?

- a. North
b. south
c. West
d. Data inadequate

Answer: Option b

Explanation:

In the evening, sun is in the west and so the shadows fall towards east. Now, since Mohit's shadow fell towards right, therefore, Mohit is facing North. So, Sumit standing face to face with Mohit, was facing South.

29. A girl leaves from her home. She first walks 30 meters in North-west direction and then 30 meters in South-west direction. Next, she walks 30 meters in South-east direction. Finally, she turns towards her house. In which direction is she moving?

- a. North-East
b. North-West
c. South-East
d. South-East

Answer: Option a

Explanation:



The movements of the girl are as shown in Fig. (A to B, B to C, C to D, D to A).

Clearly, she is finally moving in the direction DA i.e. north east.

30. A man goes towards East 5km, then he takes a turn to South-West and goes 5km. He again takes a turn towards North-West and goes 5km With respect to the point from where he started, where is he now?

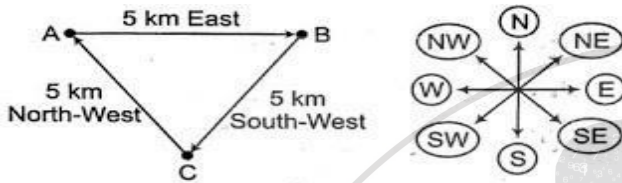
- a. At the starting point
- c. In the East

- b. In the West
- d. In the North East

Answer: Option a

Explanation:

According to the question, the direction diagram is as follows



It is clear from the diagram that both starting and finishing point are same i.e., the man is at starting point 'A'.

31. Nikhil walked 30m towards East took a left turn and walked 20m. He again took a left turn and walked 30m. How far and in which direction is he from his starting point?

- a. 20m, North
- b. 80m, North
- c. 20m, South
- d. 80m, South

Answer: Option a

Explanation:

According to the question, the direction diagram is as follows



Required distance = $AD = BC = 20m$

So, Nikhil is 20m North from his starting point

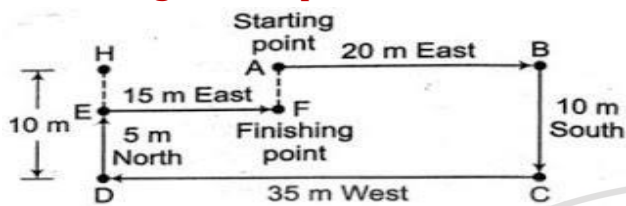
32. Rakesh is standing at a point. He walks 20m towards the East and further 10m towards the South, then he walks 35m towards the West and further 5 m towards the North, then he walks 15 m towards the East. What is the straight distance (in m) between his starting point and the point where he reached last?

- a. 0
- b. 5
- c. 10
- d. CANNOT BE DETERMINED

Answer: Option b

Explanation:

According to the question. The direction diagram is as follows



From diagram, AB
 $= 20\text{m}$ $BC = HD$
 $= 10\text{m}$

$ED =$
 5m CD
 $= 35\text{m}$
 $HE =$
 AF

Required distance, $AF = HF = HD - ED$

$= 10 - 5 = 5\text{m}$

33. Anoop starts walking towards South. After walking 15m he turns towards North. After walking 20m, he turns towards East and walks 10m. He, then turns towards South and walks 5m. How far is he from his original position in which direction?

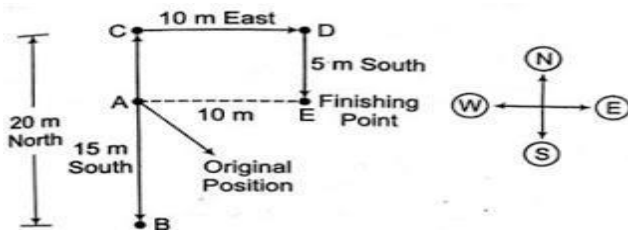
- a. 10m, North
 b. 10m, South
 c. 10m, West
 d. 10m, East

Answer: Option

Explanation:

According to the question, the direction diagram is as follows

A = Original position, E = Finishing point



$BC = 20, AB = 15m, AC = ED = 5m, CD = AE = 10m$

Clearly, at finishing point E, Anoop is 10 m East from original position A.

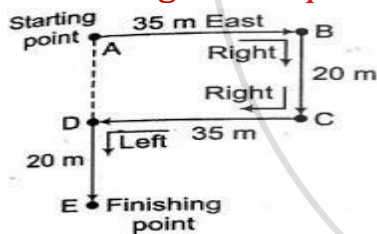
34. From a point, Rajneesh started walking East and walked 35m. He, then turned on his right and walked 35m. He, then turned on his right and walked 20m and he again turned to right and walked 35m. Finally, he turned his left and walked 20m and reached his destination. Now, how far is he from the starting point?

- a. 50m
- b. 55m
- c. 20m
- d. 40m

Answer: Option d

Explanation:

According to the question, the direction diagram is as follows



$AB = CD$
 $= 35m$
 $BC = AD$
 $20m$
 $DE = 20m$

Required distance, $AE = AD + DE$

$= 20 + 20 = 40m$

35. A rat runs 20m towards East and turns to right, then runs 10m and turns to right, runs 9m and again turns to left, runs 5m and then turns to left, runs 12m and finally turns to left and runs 6m. Now, which direction is the rat facing?

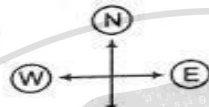
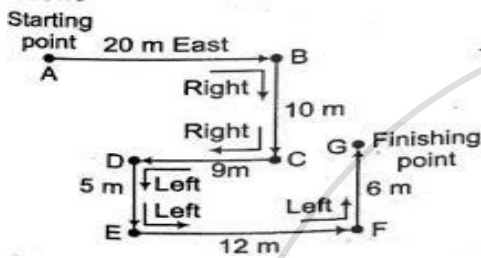
- a. East
- c. West

- b. North
- d. South

Answer: Option b

Explanation:

According to the question, the direction diagram is as follows



Clearly, the rat is facing North at finishing point.

36. Starting from a point S, Mahesh walked 25m towards South. He turned to his left and walked 50m. He, then again turned to his left and walked 25m. He again turned to his left and walked 60m and reached a point T. How far Mahesh is from point S and in which direction?

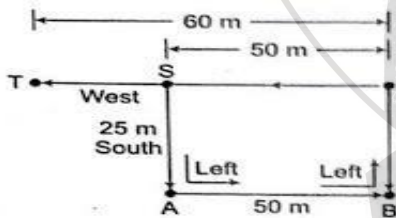
- a. 10m, West
- c. 10m, East

- b. 25m, North
- d. 25m, West

Answer: a

Explanation:

According to the question, the direction diagram is as follows



S = Starting point, T =
Finishing point AS = BC =
25m

AB = SC =
50m CT =
60m

Required distance, $ST = CT - SC = 60 - 50 = 10\text{m}$ clearly, at point T, Mahesh is 10 m West from S.

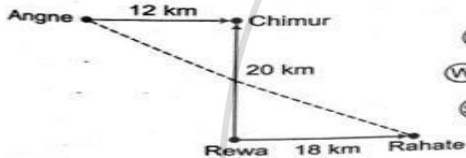
37. Village Chimur is 20 km to the North of village Rewa. Village Rahate is 18 km to the East of village Rewa. Village Angne is 12 km to the West of Chimur. If Sanjay starts from village Rahate and goes to village Angne, in which direction is he from his starting point?

- a. North
- b. North-West
- c. South
- d. South-East

Answer: Option b

Explanation:

According to the question, the direction diagram will be as follows



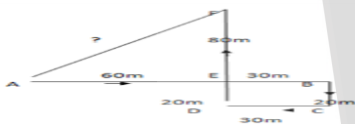
Clearly, Sanjay will go North-West starting from Rahate to reach Angne.

38. A boy is looking for his mother. He went 90 metres in the east before turning to his right. He went 20 metres before turning to his right again to look for his mother at his uncle's place 30 metres from this point. His mother was not there. From here he went 100 metres to his north before meeting his mother in a street. How far did the son meet his mother from the starting point?

- a. 110m
- b. 100m
- c. 90m
- d. 240m

Answer: b

Explanation:

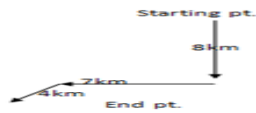


39. Kashmira facing towards south moved straight 8 km and from there turned to her right 90° and travelled 7 km. Then she took a 45° turn to her left and travelled 4 km. Where would she be now with respect to the starting point?

- a. South
- b. South-west
- c. North-east
- d. South-east

Answer: b

Explanation:

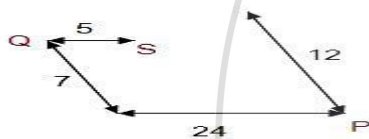


40. Pinky walks 12m towards southeast and stops at point P and then she walks 24m towards west and again she walks 7m towards northwest direction and stops at point Q. Finally she walks 5m towards east and stops at point S. She is facing which direction from starting point?

- a. Northeast
- b. Northwest
- c. East
- d. Southwest

Answer: d

Explanation:



South West

41. A man walks 40m towards north and he turns his left and walked 40m. He then turns his left and walked 15m. He finally turns his right and walked 20m. What is the distance he is from starting point and in which direction?

- a. 55m , Northwest
- b. 36m, Northeast
- c. 65m, Southeast
- d. 65m, Northwest

Answer: d

Explanation:



$$40 + 20 = 60$$

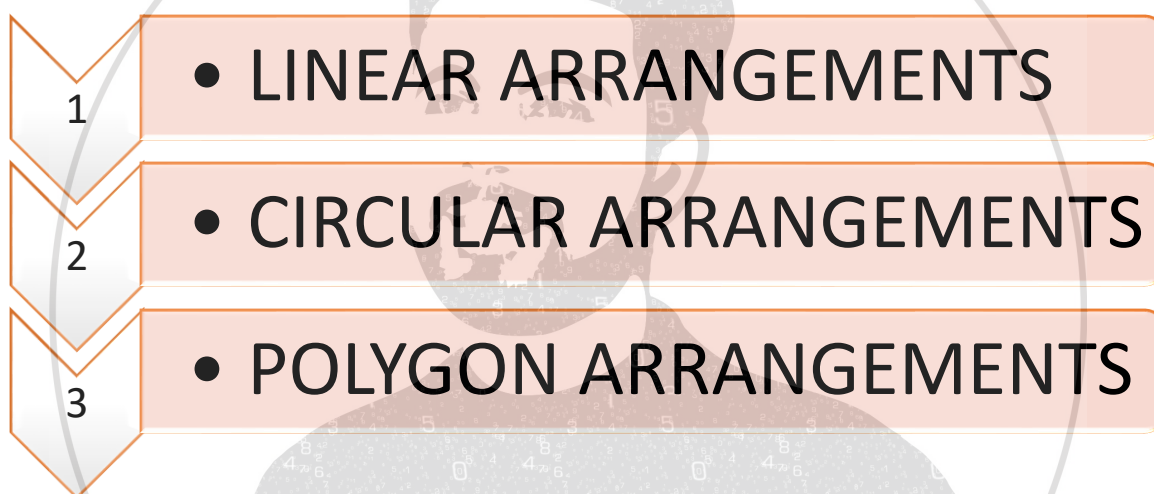
$$40 - 15 = 25$$

$$= \sqrt{60^2 + 25^2} = \sqrt{4225} = 65\text{m, North West}$$

CHAPTER 11

SEATING ARRANGEMENTS

VARIOUS PATTERN OF SITTING ARRANGEMENTS



LINEAR ARRANGEMENTS

we arrange objects or persons in a line or row. The arrangement is done only on one 'axis' and hence, the position of persons or objects assumes importance in terms of order like positions. In this type of arrangement, we take directions according to our left and right.

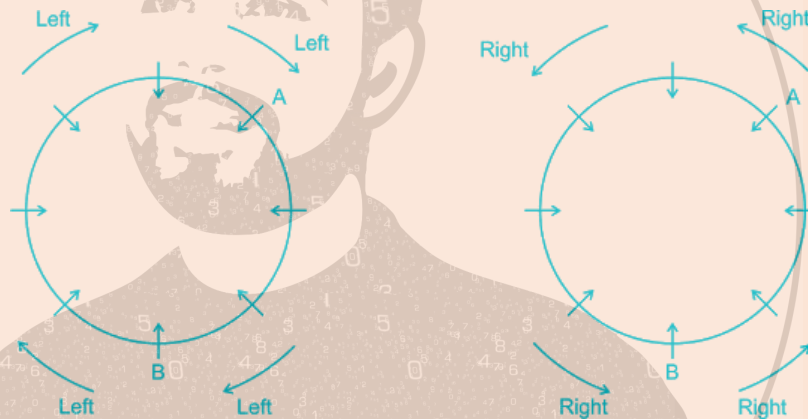
Steps to Solve the Linear Arrangements:

- (a) Identify the number of objects and their names.
- (b) Use pictorial method to represent the people or objects and their positions.
- (c) Arrange the information with relevant facts and their positions and try to find out the solution.
- (d) Answer the questions based on the arrangement having made.

One Row Sequence	When direction of face is not clear.
Two Row Sequence	When direction of face is clear at every level to each and every person.

CIRCULAR ARRANGEMENT

some persons are sitting around a circle and they are facing the center



QUESTIONS:

1. Four Children's are sitting in arrow. A is occupying seat next to B but not next to C. If C is not sitting next to D? Who is occupying seat next to adjacent to D.?

- a. B
- b. B and A
- c. Impossible to tell
- d. A

ANSWER: (d)

EXPLANATION:

The arrangements as per given information is possible only if C is sitting next to B and D is sitting next to A.

Therefore, two possible arrangements are C, B, A, D, or D, A, B, C Clearly, only A is sitting adjacent to D

2. P, Q, R, S, T, U, V and W are sitting in a row facing North.

- a. P is fourth to the right of T
- b. W is fourth to the left of S
- c. R and U, which are not at the ends, are neighbours of Q and T respectively
- d. W is next to the left of P and P is the neighbour of Q, who are sitting at the extreme ends

ANSWER: a

EXPLANATION:

There are three persons between P and TXXXXP.

In the information (iv), it is given that W is next to the left of P and Q is the neighbour of P. Using the information with (i), we get TXXWPQ.

3. A, P, R, X, S and Z are sitting in a row. S and Z are in the centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P ?

- a. A
- b. X
- c. S
- d. Z

Answer: Option B

Explanation:

The seating arrangement is as follows:

• • • • • •
P X S Z R A

Therefore, right of P is X

4. A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting?

- a. Between B and D
- b. Between B and C
- c. Between E and D
- d. Between C and E

Answer: Option B

Explanation:



Therefore, A is sitting in between B and C

5. P, Q, R, S, T, U, V and W are sitting round the circle and are facing the centre:

1. P is second to the right of T who is the neighbour of R and V.
2. S is not the neighbour of P.
3. V is the neighbour of U.
4. Q is not between S and W. W is not between U and S

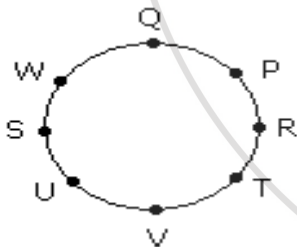
According to this answer bellowed Questions:

5.1. Which two of the following are not neighbours?

- | | |
|-------|-------|
| a. RV | b. UV |
| c. RP | d. QW |

Answer: Option A

Explanation:

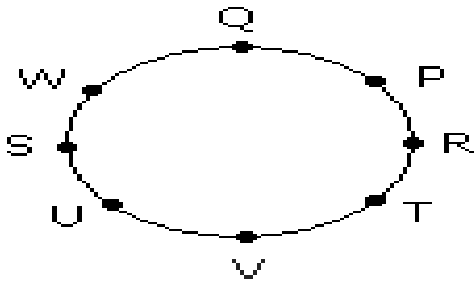


5.2. Which one is immediate right to the V ?

- | | |
|------|------|
| a. P | b. U |
| c. R | d. T |

Answer: Option D

Explanation:

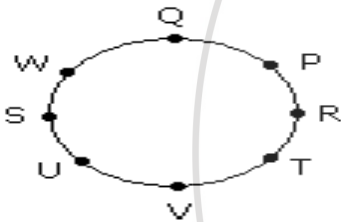


5.3. Which of the following is correct?

- a. P is to the immediate right of Q
- b. R is between U and V
- c. Q is to the immediate left of W
- d. U is between W and S

Answer: Option C

Explanation:

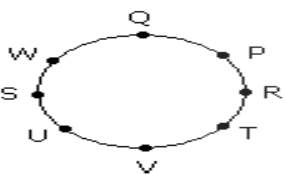


5.4. What is the position of S?

- a. Between U and V
- b. Second to the right of P
- c. To the immediate right of W
- d. Data inadequate.

Answer: Option C

Explanation:



6 . Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. According to this answer bellowed Questions:

6.1. Who is sitting immediate right to Reeta?

- a. Bindu
- b. Rani
- c. Mary
- d. Seema

Answer: Option C

Explanation:



Mary is sitting immediate right to Reeta.

6.2. Who is in the middle of the photograph?

- a. Bindu
- b. Rani
- c. Reeta
- d. Seema

Answer: Option B

Explanation:



Rani is in the middle of the photograph.

6.3. Who is second from the right?

- a. Mary
- b. Rani
- c. Reeta
- d. Bindu

Answer: Option C

Explanation:



Reeta is sitting second from the right.

6.4. Who is second from the left in photograph?

- a. Reeta
- b. Mary
- c. Bindu
- d. Seema

Answer: Option D

Explanation:



Seema is sitting second from the left in photograph.

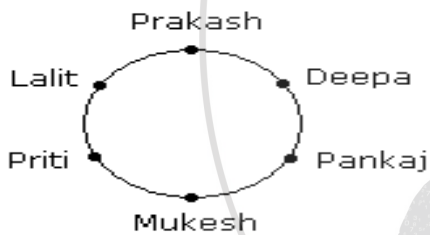
7. Six friends are sitting in a circle and are facing the centre of the circle. Deepa is between Prakash and Pankaj. Priti is between Mukesh and Lalit. Prakash and Mukesh are opposite to each other.

7.1. Who is sitting right to Prakash ?

- a. Mukesh
- b. Deepa
- c. Pankaj
- d. Lalit

Answer: Option D

Explanation:



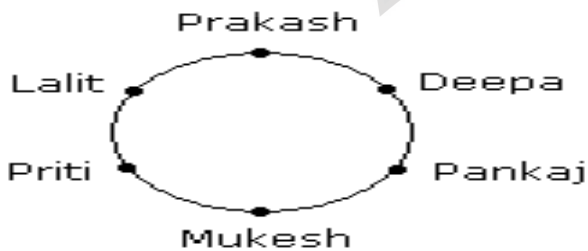
Hence, Lalit is sitting right to Prakash.

7.2. .Who is just right to Pankaj?

- a. Deepa
- b. Lalit
- c. Prakash
- d. Priti

Answer: Option A

Explanation:



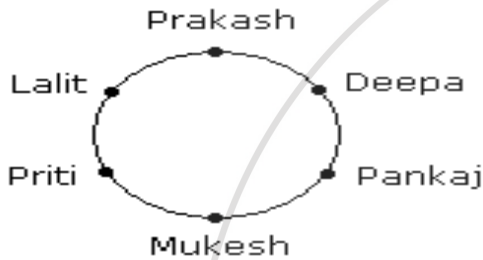
Hence, Deepa is sitting just right to Pankaj.

8. Who are the neighbours of Mukesh?

- a. Prakash and Deepa
- b. Deepa and Priti
- c. Priti and Pankaj
- d. Lalit and Priti

Answer: Option C

Explanation:



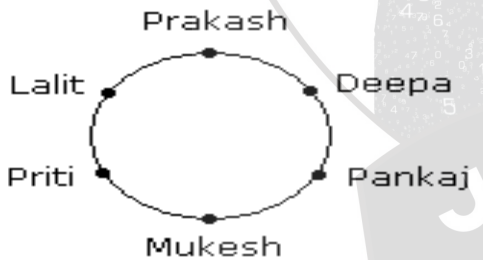
Hence, Priti and Pankaj are the neighbours of Mukesh.

9. Who is sitting opposite to Priti ?

- a. Prakash
- b. Deepa
- c. Pankaj
- d. Lalit

Answer: Option B

Explanation:



Hence, Deepa is sitting opposite to Priti.

10. In an Exhibition seven cars of different companies - Cadillac, Ambassador, Fiat, Maruti, Mercedes, Bedford and Fargo are standing facing to east in the following order :

1. Cadillac is next to right of Fargo.
2. Fargo is fourth to the right of Fiat.

3. Maruti car is between Ambassador and Bedford.

4. Fiat which is third to the left of Ambassador, is at one end.

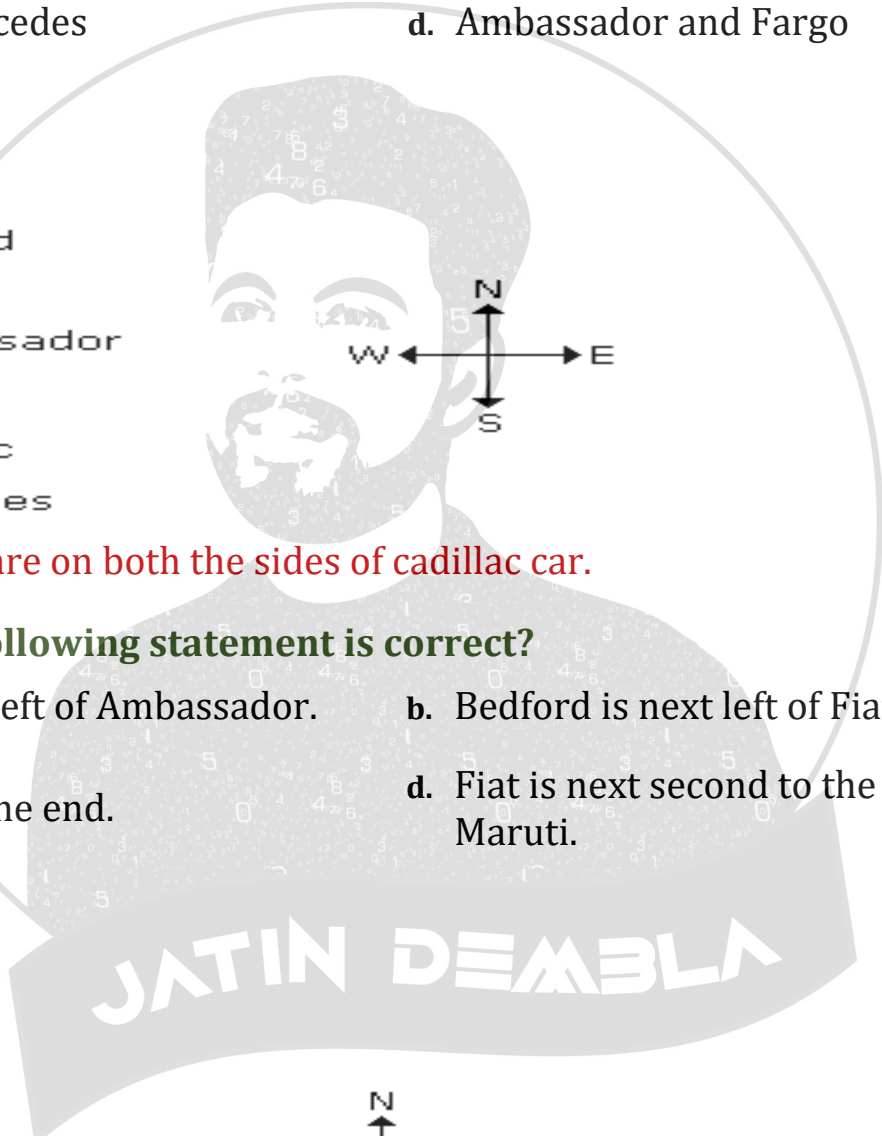
10.1. Which of the cars are on both the sides of cadillac car?

- a. Ambassador and Maruti
- b. Maruti and Fiat
- c. Fargo and Mercedes
- d. Ambassador and Fargo

Answer: Option C

Explanation:

- → Fiat
- → Bedford
- → Maruti
- → Ambassador
- → Fargo
- → Cadillac
- → Mercedes



Fargo and Mercedes are on both the sides of cadillac car.

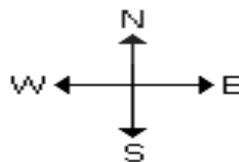
10.2. Which of the following statement is correct?

- a. Maruti is next left of Ambassador.
- b. Bedford is next left of Fiat.
- c. Bedford is at one end.
- d. Fiat is next second to the right of Maruti.

Answer: Option A

Explanation:

- → Fiat
- → Bedford
- → Maruti
- → Ambassador
- → Fargo
- → Cadillac
- → Mercedes



Therefore, Maruti is next left of Ambassador.

11. Which one of the following statements is correct?

- a. Fargo car is in between Ambassador and Fiat.
- b. Cadillac is next left to Mercedes car.
- c. Fargo is next right of Cadillac.
- d. Maruti is fourth right of Mercedes.

Answer: Option B

Explanation:



Therefore, Cadillac is next left to Mercedes car.

12. Which of the following groups of cars is to the right of Ambassador?

- a. Cadillac, Fargo and Maruti
- b. Mercedes, Cadillac and Fargo
- c. Maruti, Bedford and Fiat
- d. Bedford, Cadillac and Fargo

Answer: Option B

Explanation:



Mercedes, Cadillac and Fargo cars are to the right of Ambassador.

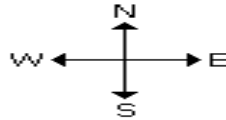
13. Which one of the following is the correct position of Mercedes ?

- a. Next to the left of Cadillac
- b. Next to the left of Bedford
- c. Between Bedford and Fargo
- d. Fourth to the right of Maruti.

Answer: Option D

Explanation:

- Fiat
- Bedford
- Maruti
- Ambassador
- Fargo
- Cadillac
- Mercedes



The correct position of Mercedes is fourth to the right of Maruti.

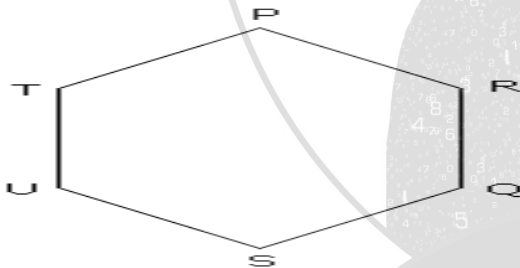
14. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S.

14.1. Which one is sitting opposite to P ?

- a. R
- b. Q
- c. T
- d. S

Answer: Option D

Explanation:



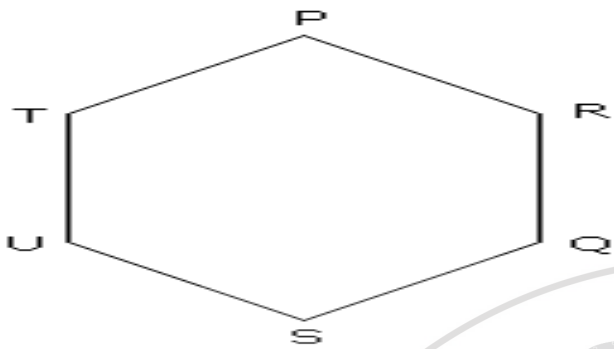
S is sitting opposite to P.

14.2. Who is the fourth person to the left of Q ?

- a. P
- b. U
- c. R
- d. Data inadequate

Answer: Option A

Explanation:



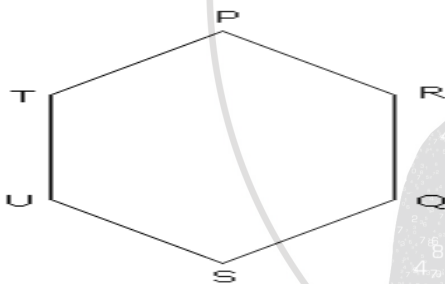
P is the fourth person to the left of Q.

14.3. Which of the following are the neighbours of P ?

- a. U and P
- b. T and R
- c. U and R
- d. Data inadequate

Answer: Option B

Explanation:



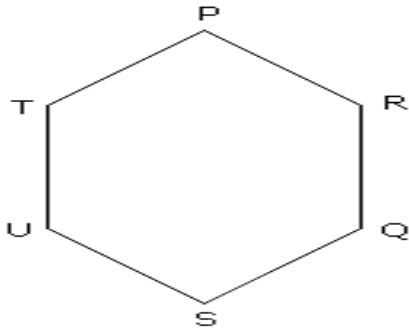
T and R are the neighbours of P.

14.4. Which one is sitting opposite to T ?

- a. R
- b. Q
- c. Cannot be determined
- d. S

Answer: Option B

Explanation:



Q is sitting opposite to T.

15. Each of these questions are based on the information given below:

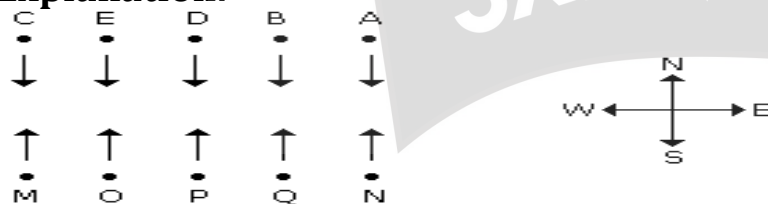
1. A, B, C, D and E are five men sitting in a line facing to south - while M, N, O, P and Q are five ladies sitting in a second line parallel to the first line and are facing to North.
2. B who is just next to the left of D, is opposite to Q.
3. C and N are diagonally opposite to each other.
4. E is opposite to O who is just next right of M.
5. P who is just to the left of Q, is opposite to D.
6. M is at one end of the line

15.1. Who is sitting third to the right of O?

- | | |
|------|--------------------|
| a. Q | b. N |
| c. M | d. Data inadequate |

Answer: Option B

Explanation:



15.2. If B shifts to the place of E, E shifts to the place of Q, and Q shifts to the place of B, then who will be the second to the left of the person opposite to O?

- | | |
|------|------|
| a. Q | b. P |
|------|------|

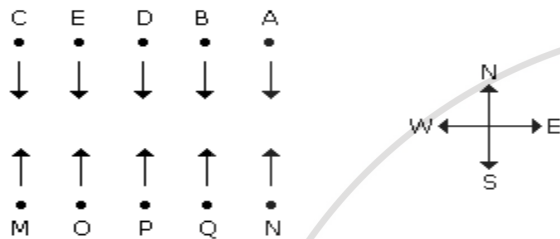
c. E

d. D

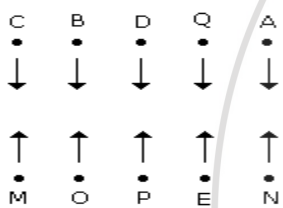
Answer: Option A

Explanation:

Initial arrangement :



New arrangement after shifting :



B is opposite to O and second person left to B is Q.

15.3. Which of the following pair is diagonally opposite to each other?

a. EQ

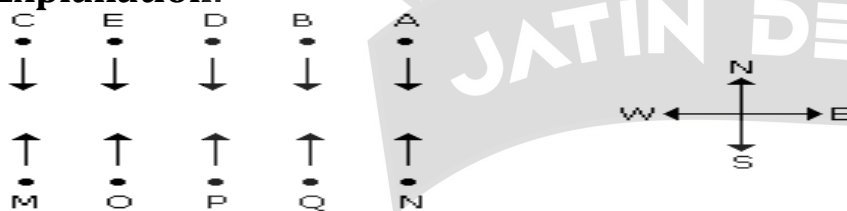
b. BO

c. AN

d. AM

Answer: Option D

Explanation:



15.4. If O and P, A and E and B and Q interchange their positions, then who will be the second person to the right of the person who is opposite to the person second of the right of P ?

- a. D
- b. A
- c. E
- d. O

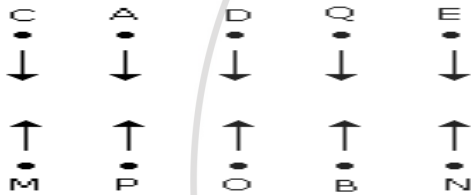
Answer: Option B

Explanation:

Old arrangement :



New arrangement :

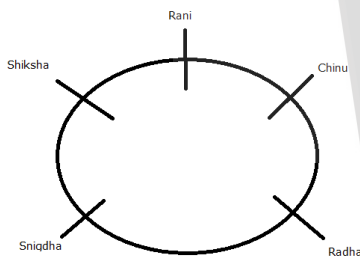


16. Who sits to the left of Shiksha?

- a. Rani
- b. Radha
- c. Chinu
- d. Snigdha

Answer – A
Explanation

After observation, we can conclude that the sitting arrangement is like this –



So, Rani sits to the left of Shiksha.

17. If Radha and Snigdha change their places then who will be second to the left of Rani?

- a. Radha
- c. Shiksha

- b. Snigdha
- d. None of the above

Answer – B
Explanation

Second to the left of Rani will be Snigdha. Hence, option B is correct.

18. How many girls are there in between Shiksha and Chinu if we count anti clockwise?

- a. 1
- c. 3
- b. 2
- d. None of the above

Answer – B
Explanation –

Only two girls are there in between Shiksha and Chinu if we count anti clockwise?

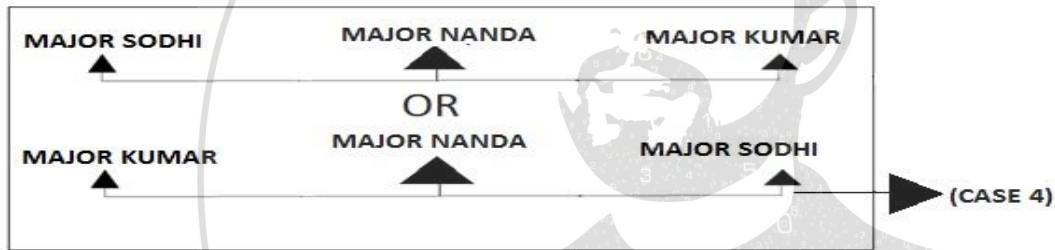
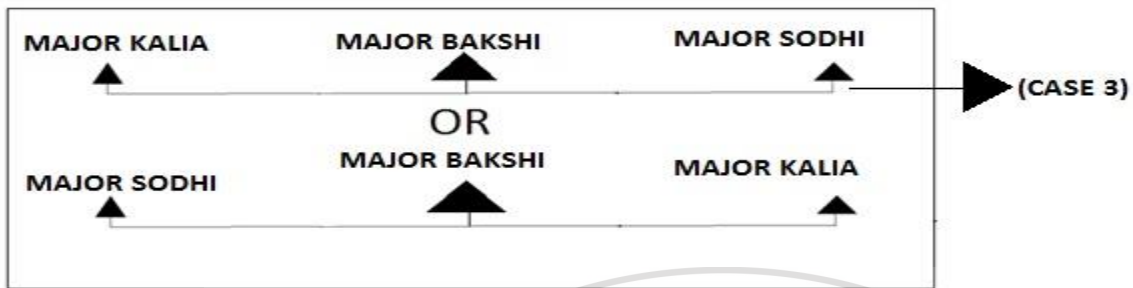
19. What is the position of Major Batra?

- a. Major Batra is sitting between Major Kumar and Major Kalia.
- c. Major Batra is sitting to the immediate right of Major Kumar
- b. Major Batra is sitting to the left of Major Kalia.
- d. All the above are true.

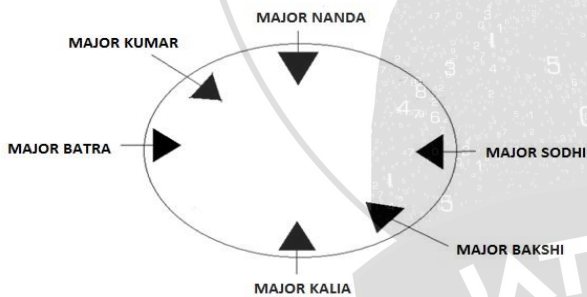
Answer – Option D
Explanation –



JATIN DEMBLA



By applying (CASE 1), (CASE 2), (CASE 3) and (CASE 4), we get



All the options (A),(B) and (C) satisfy our condition. Therefore option (D) is correct.

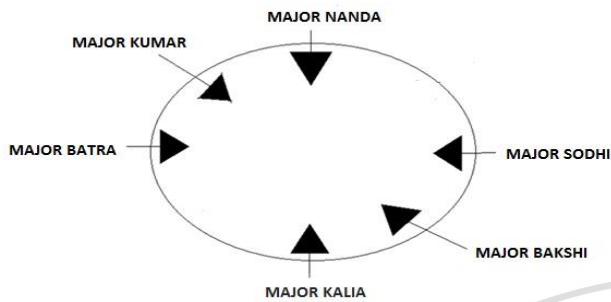
20. Who is sitting to the immediate left of Major Kumar?

- a. Major Bakshi
- b. Major Batra
- c. Major Nanda
- d. Major Sodhi

Answer – Option C

Explanation –

According to the diagram –



By observing the diagram, we can clearly say that Major Nanda is sitting to the immediate left of Major Kumar.

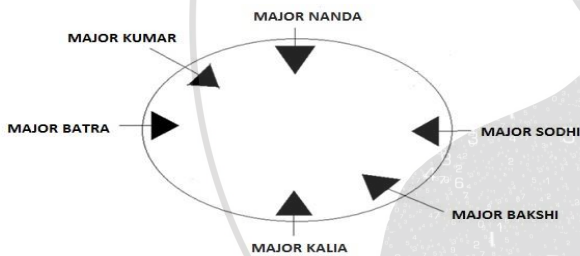
21. Who is sitting to the immediate right of Major Kalia?

- a. Major Nanda
- b. Major Kumar
- c. Major Kalia
- d. Major Bakshi

Answer – Option D

Explanation –

According to the diagram –



By observing the diagram, one can easily conclude that major Bakshi is sitting to the immediate right of Major Kalia.

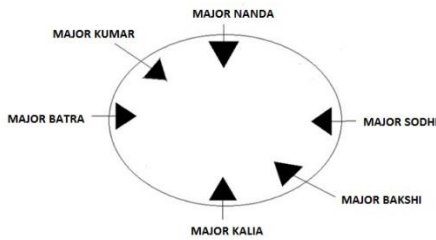
22. Which of the following statement is true?

- a. Major Sodhi is sitting second to the left of Major Bakshi.
- b. Major Kalia is sitting between Major Nanda and Major Kumar
- c. Major Batra is sitting to the left of Major Kalia
- d. Major Nanda is sitting to the left of Major Kalia.

Answer – Option C

Explanation –

According to the diagram –



By observing the diagram, we can conclude that options (A), (B) and (D) do not satisfy the condition. But option (C) does.

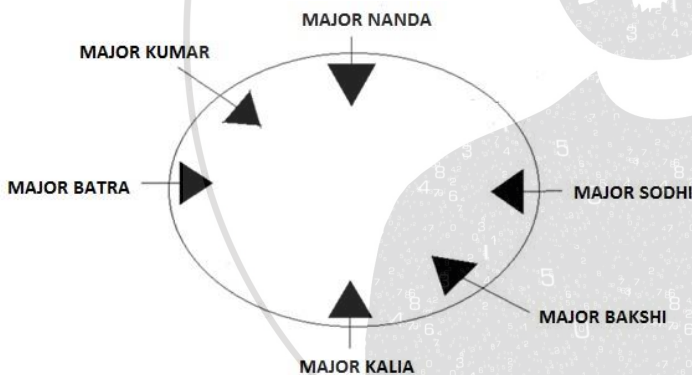
23. How many Majors are sitting between Major Sodhi and Major Kumar, if counted in clockwise direction?

- a. Six
- b. Two
- c. Three
- d. Five

Answer – Option C

Explanation –

According to the diagram –



Major Bakshi, Major Kalia and Major Batra are the three majors sitting between Major Sodhi and Major Kumar.

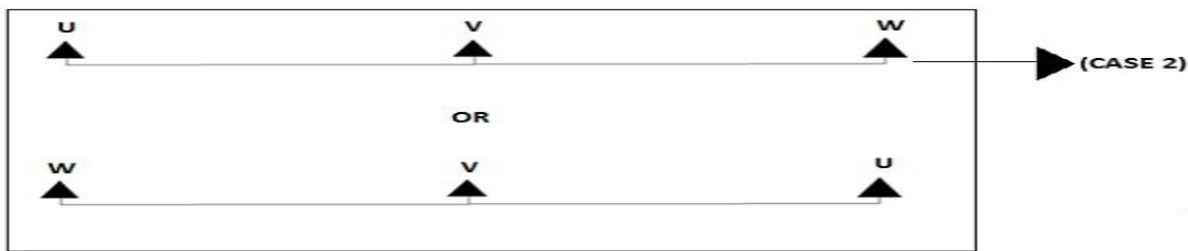
24. What is Major Batra position with respect to Major Sodhi?

- a. Second to the left
- b. Immediate Right.
- c. Fourth to the Right
- d. Third to the left

Answer – Option D

Explanation –

According to the diagram –



By applying (CASE 1) and (CASE 2), we get



From the diagram, it is clear that W is sitting immediate right of V.

25.2. Who is sitting to the immediate left of X?

- a. W
- b. U
- c. V
- d. None of these

Answer – Option A

Explanation –

According to the diagram –



By observing the diagram, we can conclude that W is sitting immediate left of X.

25.3. What is the number of People sitting between X and U?

- a. Three
- b. Five
- c. One
- d. Two

Answer – Option D

Explanation –

According to the diagram –



By observing the diagram, we can conclude that V and W are the two people sitting between U and X.

25.4. Which statement is true?

- a. X is sitting to the left of U
- b. U and X are sitting at extreme ends.

c. U is sitting between V and X

d. Three people are sitting between U and X

Answer – Option B

Explanation –

According to the diagram –



U and X are the two people sitting at extreme ends whereas V is sitting to the right of U and W is sitting to the left of X.

25.5. Which of the following pairs is the first person sitting to the immediate right of second person?

VW

WV

UV

None of these

Answer – Option B

Explanation –

According to the diagram –



Here in option (B), the second person is V and the first person is W. So according to the condition, that is the first person sitting to the immediate right of the second person, only satisfies in option (B).

25.6. How many persons are there to the right of U?

a. Two

b. Four

c. One

d. Three

Answer – Option D

Explanation –

According to the diagram –



By observing the diagram, we can conclude that three person V, W and X are sitting to the right of U.

26. Study the following information carefully and answer the questions given below.

Certain number of people was sitting in a circle facing towards the Centre. Some of the person's arrangements are known. A was sitting fourth to the left of B. J was sitting seventh to the right of A. Number of person sitting between A and B was same as the number of persons sitting between A and F. J was the neighbor of D who sits at the seventh position from F (either left or right of F). Number of person sitting between F and M was same as the number of persons sitting between M and D. K was the neighbor of J. M is not the immediate neighbor of A.

26.1. What is the position of M with respect to A?

- | | |
|----------------------|---------------------|
| Third to the left | Immediate right |
| Seventh to the right | Second to the right |

Answer: D

26.2 How many persons were sitting in a circle?

- | | |
|-------|-------|
| a. 07 | b. 08 |
| c. 16 | d. 19 |

Answer: D

26.3. How many known persons were sitting between A and J when counted from left of A?

- | | |
|----------|---------|
| a. Three | b. Four |
| c. Five | d. Two |

Answer: D

26.4. Who sits second to the right of B?

- | | |
|------|------|
| a. K | b. F |
| c. A | d. J |

Answer: A

Explanation:

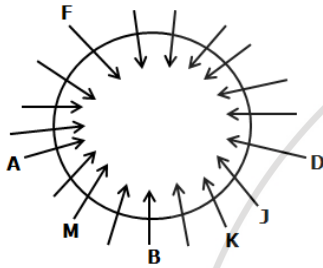
26.5. If C sits exactly between A and K when counted from right of A, then what is the

position of C with respect to D?

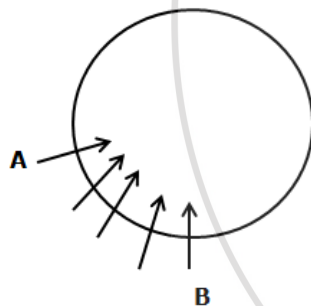
- a. Fifth to the left
- b. Fourteenth to the right
- c. Fifth to the left
- d. Either (a) or (b)

Answer: D

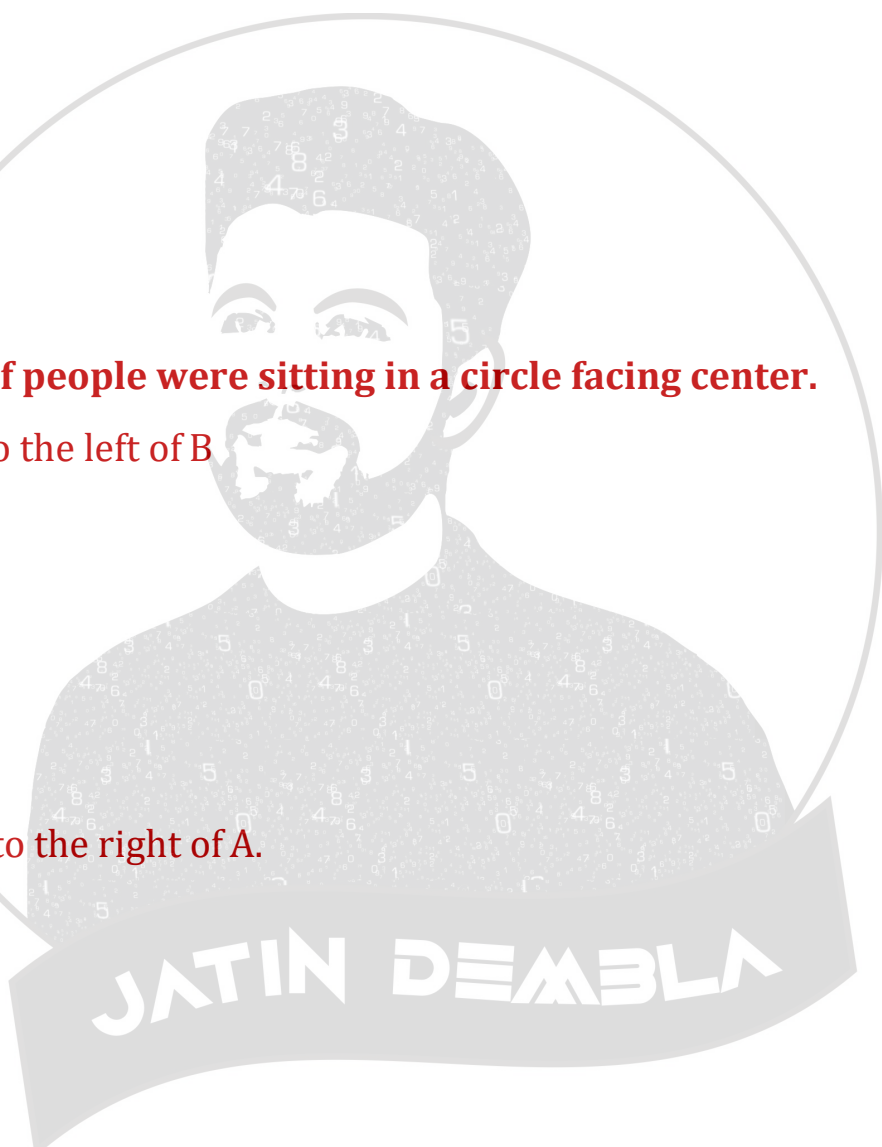
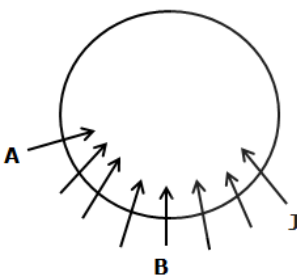
Explanation of Question 26 is:



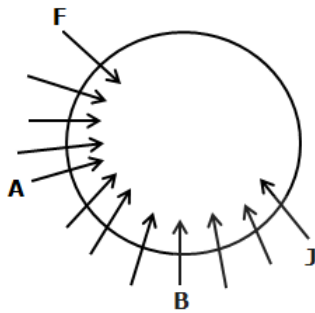
**A certain number of people were sitting in a circle facing center.
A was sitting fourth to the left of B**



J was sitting seventh to the right of A.

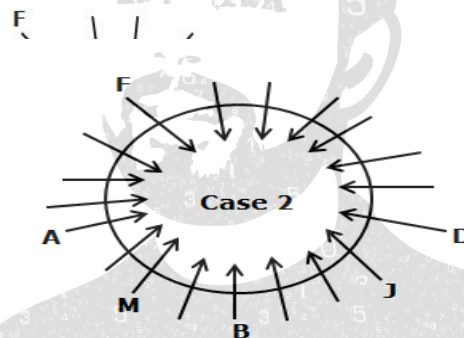
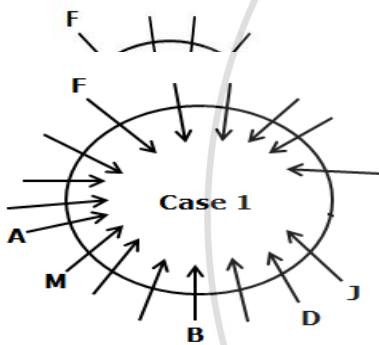


Number of person sitting between A and B was same as the number of persons sitting



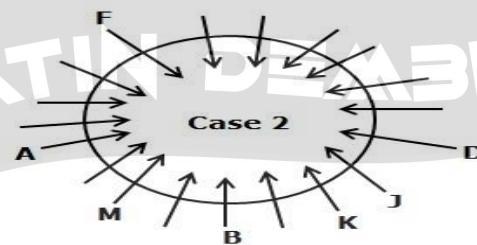
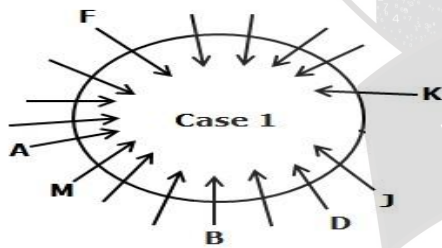
between A and F.

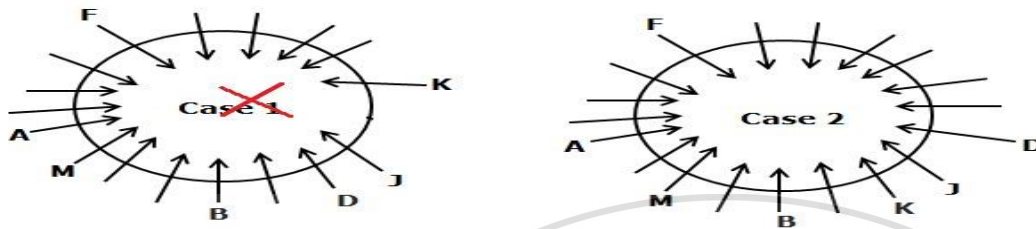
J was the neighbor of D who sits at the seventh position from F (either left or right of F).



Number of person sitting between F and M was same as the number of persons sitting between M and D.

K was the neighbor of J.





M is not the immediate neighbor of A. From this statement case 1 is eliminated because M and A are immediate neighbors.

26. Direction(27.1 to 27.5) Twelve persons A, B, C, D, E, F, P, Q, R, S, T and U are sitting in two parallel rows with equidistance from each other. In Row-1, A, B, C, D, E and F are sitting and all of them are facing south and in Row-2, P, Q, R, S, T and U are sitting and all of them are facing north but not necessary in the same order.

E sits second to the left of the one who faces P and either one of them sits at the extreme ends of the rows. Two persons are sitting between P and Q. F faces one of the immediate neighbour of Q. U faces the person the one who sits to the immediate right of A. Two persons are sitting between U and S. As many persons sitting to the right of T is same as the number of persons sitting to the right of C and neither of them sits at the extreme ends of the rows. R is not an immediate neighbour of S. C does not face Q. B sits one of the places to the left of E.

27.1. Who sits diagonally opposite to S?

- a. B
- b. A
- c. D
- d. F

Answer: c)

27.2. How many persons are sitting between T and the one who faces D?

- a. None
- b. One
- c. Two
- d. Three

Answer: d

27.3. Four of the following five are alike in a certain way and hence form a group. Which one of the following that does not belong to the group?

- a. Q
- b. P
- c. D
- d. S

Answer: a)

27.4. Which of the following statements is true?

- a. Only two persons are sitting to the right of A
- b. U faces E
- c. Q sits exactly between T and R
- d. C sits at one of the extreme ends of the row

Answer: b)

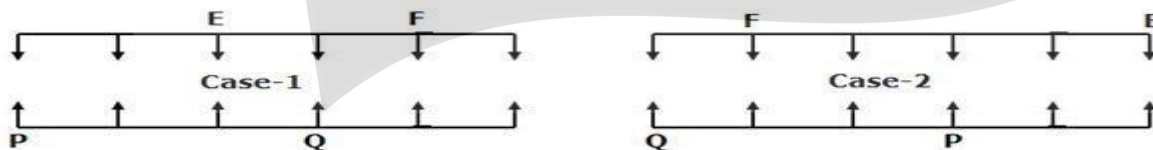
27.5. If R is related to A and F is related to U in a certain way. Then, Q is related to which of the following?

- a. C
- b. R
- c. E
- d. B

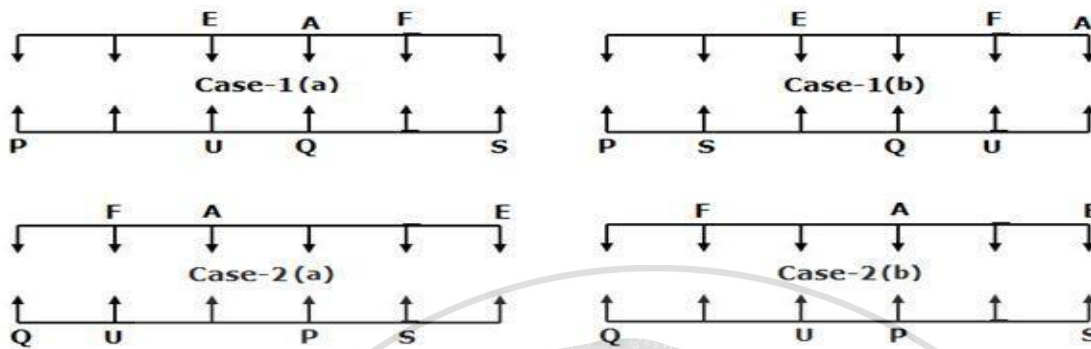
Answer: d)

Explanation:

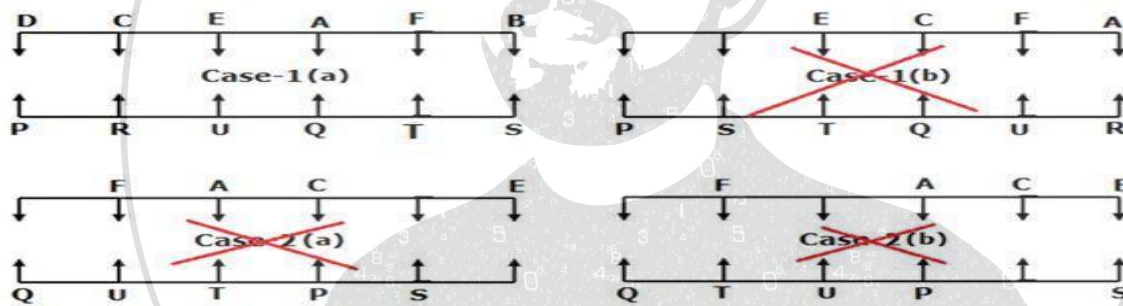
- E sits second to the left of the one who faces P and either one of them sits at the extreme ends of the rows. Two persons are sitting between P and Q. F faces one of the immediate neighbors of Q.



- U faces the person the one who sits to the immediate right of A. Two persons are sitting between U and S.



- As many persons sitting to the right of T is same as the number of persons sitting to the right of C and neither of them sits at the extreme ends of the rows. R is not an immediate neighbor of S. C does not face Q.
 - So, Case-1(b), Case-2(a) and Case-2(b) will be dropped.
- B sits one of the places to the



28. Direction (28.1 to 28.5): Read the following information carefully and answer the questions given below. Eight persons P, Q, R, S, T, U, V and W are sitting in a square table such that four of them are sitting at the corners and remaining are sitting at the middle of the each side. The persons who are sitting at the corners are facing towards centre of the table and the persons who are sitting at the middle of the sides are facing away from the centre of the table. R sits third to the left of T, who does not sit at one of the middle side of the table. Only one person sits between R and P (Either from right or left). Q sits second to the left of U and not an immediate neighbour of R. W sits opposite to S, who is not an immediate neighbour of P. More than one persons sit between W and R (Either from left or right)

28.1. Who among the following persons sits third to the right of the one who sits to the immediate left of Q?

- a. S
- b. R
- c. Q
- d. P

Answer: b)

28.2. How many persons are sitting between P and T, when counted from left of T?

- a. Two
- b. One
- c. Four
- d. Three

Answer: c)

28.3. Four of the following five are alike in a certain way and hence form a group. Which one of the following that does not belong to the group?

- a. V
- b. P
- c. U
- d. R

Answer: a)

28.4. If R is related to Q and U is related to P in a certain way. Then, V is related to which of the following?

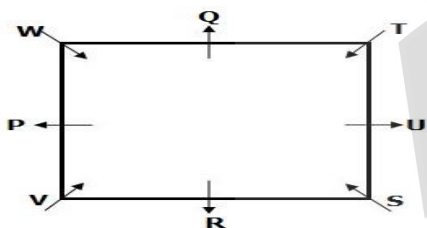
- a. W
- b. S
- c. R
- d. T

Answer: d)

28.5. Which of the following statements is true?

- a. U sits second to the right of R
- b. V sits at one of the corners
- c. P sits opposite to T
- d. W faces outside from the center

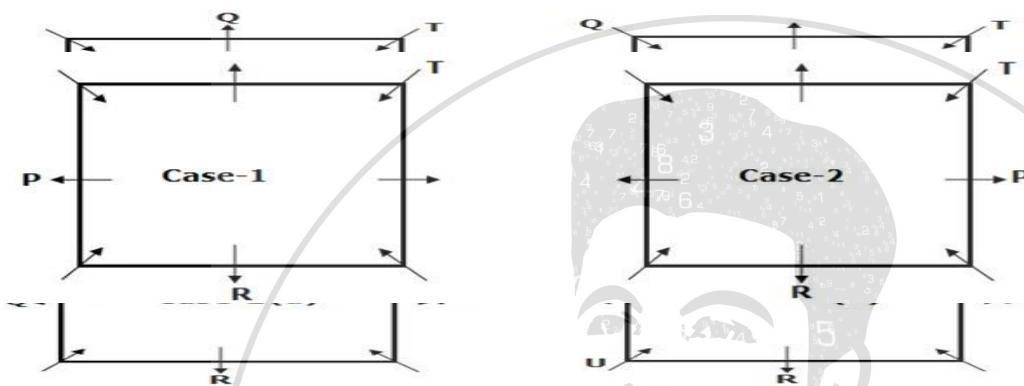
Answer: b)



Explanation:

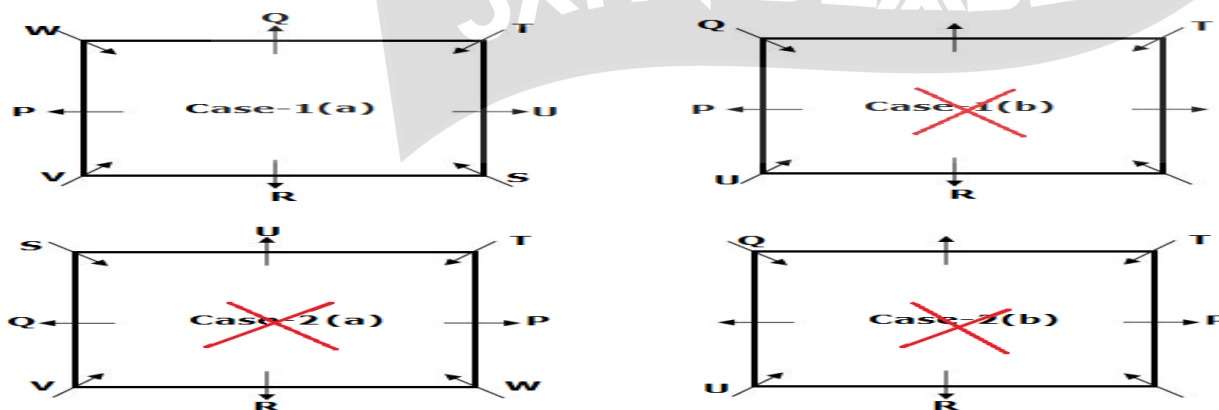
R sits third to the left of T, who does not sit at one of the middle of the sides. Only one person sits between R and P (Either from right or left).

Q sits second to the left of U and not an immediate neighbour of R.



W sits opposite to S, who is not an immediate neighbour of P.

More than one persons sit between W and R (Either from left or right). So, Case-1(b), Case-2(a) and Case-2(b) will be dropped.



29. Direction (21-25): Read the following information carefully and answer the questions given below.

A certain number of people sitting in the linear row facing north. Only three people sit between A and R. Only four people sit between K and W. Only five people sit between R and K. T sits third to the right of W. Only six people sit between R and Y. Not more than three people sit between K and Y. More than four people are between T and Y. Q sits third to the right of Y. None of them sits between Q and W. J sits eighth to the left of K. Not more than three persons sit between A and J.

29.1 How many people are sitting in the linear row?

- a. Nineteen
- b. Twenty
- c. Twenty One
- d. Twenty Two

Answer: a)

29.2 How many people sits between A and J?

- a. Seven
- b. Three
- c. Ten
- d. One

Answer: d)

29.3 If three people sits between W and H, then which of the following statement is definitely true?

- a. Three people sit between T and H
- b. W sits fourth to the right of H.
- c. More than six people sit between Q and H.
- d. More than five people sit between Y and H.

Answer: d)

29.4 How many people sits between Y and W?

- a. Sixteen
- b. Three
- c. Ten
- d. Eight

Answer: b)

29.5 How many people sit to the left of K?

- a. Ten
- b. Eight
- c. Sixteen
- d. Thirteen

Answer: a)

Explanation:

i). Only three people sit between A and R. ii). Only six people sit between R and Y.

Case(i): A _____ R _____ Y

Case(ii): Y _____ A _____ R

Case(iii): Y _____
 _____ R _____ A Case(iv): R _____
 _____ A _____ Y

iii). Only five people sit between R and K. iv). Only four people sit between K and W. v). T sits third to the right of W.

vi). Not more than three people sit between K and Y

Case (i): A _____ R _____ K Y _____ W _____ T Case

(ii)a: Y K _____ A _____ W R _____ T

Case (ii)b: W _____ T Y K _____ A _____ R Case

(iii)a: Y K _____ W R _____ T _____ A

Case (iii)b: W _____ T Y K _____ R

_____ A Case (iv)a: R _____ A _____ K Y

_____ W _____ T Case (iv)b: R W _____ A _____ K Y

does not follow condition (v) vii). More than four people are between T and Y.

viii). Q sits third to the right of Y.

ix). None of them sits between Q and W.

x). Not more than three persons sit between A and J. xi). J sits eighth to the left of K.

Case (i): A J _____ R _____ K Y _____ Q W _____ T

Case (ii)a: Y K A _____ W R _____ T does not follow condition (viii)

Case (ii)b: W _____ T Y K _____ A _____ R does not follow condition (vii)
Case (iii)a: Y K Q _____ W R _____ T _____ A does not follow condition (ix)

Case (iii)b: W _____ T Y K _____ R _____ A does not follow condition (vii)
Case (iv)a: J _____ R _____ A _____ K Y _____ Q W _____ T does not follow condition (x)

30. Direction (30.1 to 30.5): Study following information carefully and answer the questions given below.

Seven friends- S, T, U, V, W, X and Y are sitting in a straight line. Some of them facing north and some of them are facing south.

Y faces north. Only two persons sit to the left of V. S sits second to the left of W. Only one person sits between S and

U. X sits third to the left of U. The immediate neighbours of S face the opposite directions. T is not an immediate neighbour of W. The immediate neighbours of U face same the directions. Only two persons sit between V and W. S faces the same direction as U. W sits to the immediate left of Y.

30.1. How many persons sit between Y and T?

- a. One
- b. Two
- c. Three
- d. None of these

Answer: d)

30.2. Four of the following five are alike in certain way and thus form a group as per the given arrangement. Which of the following does not belong to that group?

- a. X
- b. T
- c. W
- d. Y

Answer: a)

30.3. What is the position of V with respect to T?

- a. Second to the left
- b. Fourth to the right
- c. Immediate left
- d. Second to the right

Answer: d)

30.4. How many persons face south directions?

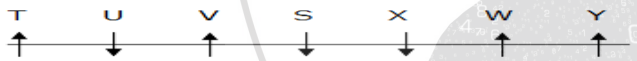
- a. Two
- b. Three
- c. Four
- d. Five

Answer: b)

30.5. Which of the following statement is correct?

- a. W sits at the extreme ends of the line
- b. T faces south and sits to the immediate right of U
- c. Only three person sits between Y and the one who sits second to the right of T
- d. W faces north and sits second to the right of the one who sits immediate left of T

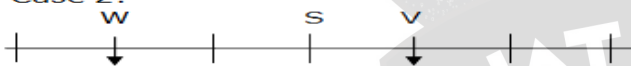
Answer: c)



Case 1:



Case 2:

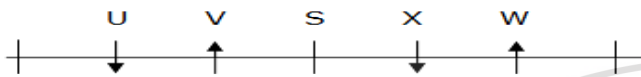


Explanation:

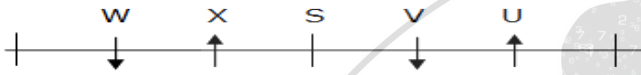
- Only two persons sit to the left of V.
- Only two persons sit between V and W.
- S sits second to the left of W.
- Now we have 2 Cases.
- The immediate neighbors of S face the opposite directions.

- Only one person sits between S and U.
- X sits third to the left of U

Case 1:

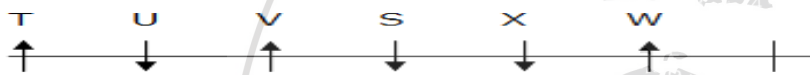


Case 2:

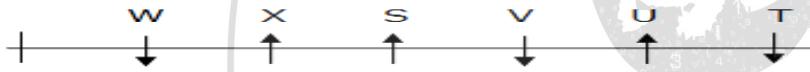


- T is not an immediate neighbour of W.
- The immediate neighbor of U faces same the directions.

Case 1:

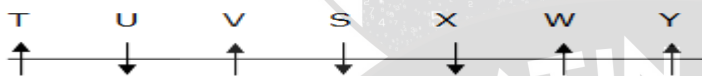


Case 2:



- S faces the same direction as U
- W sits to the immediate left of Y.
- Y faces north.
- From the above condition Case 2 was dropped.
 - So the final arrangement is..

Case 1:



31. Direction (31.1 to 31.5): Read the following information carefully and answer the questions given below.

Eight persons are sitting in a circular table and all of them are facing away from the center of the table. V sits third to the right of M. Only one person sits between V and N (Either from right or left from V). K sits second to the left of G, who is not an immediate neighbour of M. As many persons are sitting between N and S is same as the number of persons sitting between R and S. T sits to the immediate

right of R. T does not sit opposite to K.

31.1. Who among the following sits second to the right of S?

- | | |
|------|------|
| a. R | b. T |
| c. N | d. G |

Answer: a)

31.2. How many persons are sitting between G and V, when counted from left of G?

- | | |
|----------|---------|
| a. Three | b. Four |
| c. One | d. Two |

Answer: d)

31.3. Who among the following sits opposite to T?

- | | |
|------|------|
| a. V | b. S |
| c. K | d. G |

Answer: d)

31.4. Four of the following five are alike in a certain way and hence form a group. Which one of the following that does not belong to the group?

- | | |
|-------|-------|
| a. TG | b. KM |
| c. MN | d. VS |

Answer: c)

31.5. If all the persons in the final arrangement are made to sit in the alphabetical order as in the English alphabetical series from G in clockwise direction, then how many of them remains their original position (Excluding G)?

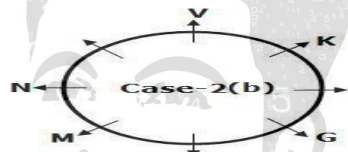
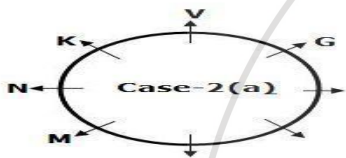
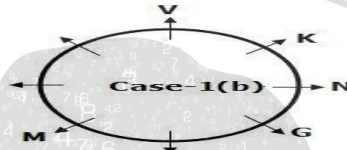
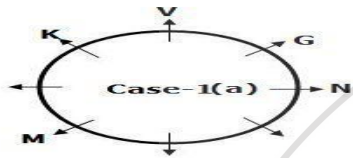
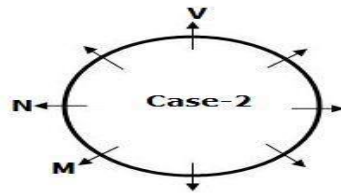
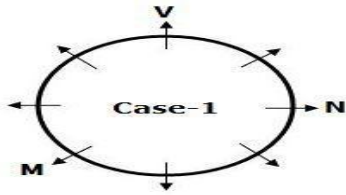
- | | |
|---------|----------|
| a. None | b. One |
| c. Two | d. Three |

Answer: b)

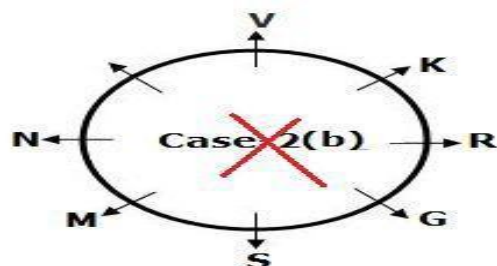
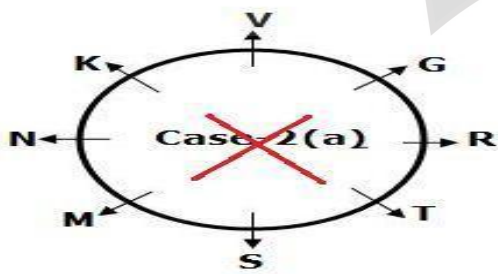
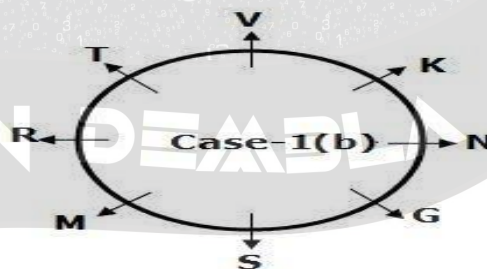
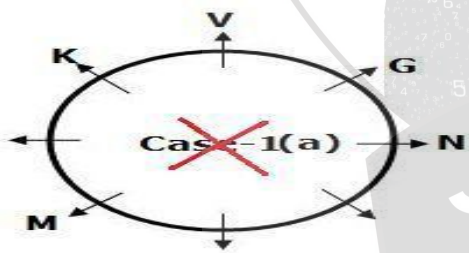
Explanation:

- V sits third to the right of M. Only one person sits between V and N (Either from right or left from V).

- K sits second to the left of G, who is not an immediate neighbour of M.



- As many persons are sitting between N and S is same as the number of persons sitting between R and S. T sits to the immediate right of R.
- So, Case-1(a) and Case-2(b) will be dropped.
- T does not sit opposite to K.
- So, Case-2(a) will be dropped



32. Direction (32.1 to 32.5): study the given information carefully and the answer the following question below.

Ten persons are sitting in a parallel row. In Row 1 A, B, C, D and E are sitting in Row 1 facing north. In Row 2 P, Q, R, S and T are sitting in Row 2 facing south. The person in row 1 exactly faces the person in row 2.

R doesn't sit opposite to C. A sits second from the extreme end. Only one person sits between one who faces A and Q. B is not an immediate neighbour of A and doesn't sit opposite to Q. E sits second to the left of B. T doesn't face E and never sits at extreme ends. S is not an immediate neighbour of T. C doesn't sit opposite to Q.

32.1. Four of the five among the following are similar in the arrangement to form a group, which one of the following doesn't belong to the group?

- a. CA
- b. ED
- c. SQ
- d. BD

Answer: d)

32.2. What is the position of A with respect to B?

- a. Third to the left
- b. Third to the right
- c. Second to the left
- d. Immediate left

Answer: a)

32.3. Which of the following statements is not true?

- a. A sits to the immediate left of E
- b. The one who sits opposite to D sits second to the left of T
- c. C and B doesn't sit at the extreme ends
- d. Q is the immediate neighbour of R and S

Answer: c)

32.4. Who sits second to the right of R?

- a. The one who sits opposite to A
- b. S
- c. P
- d. None of these

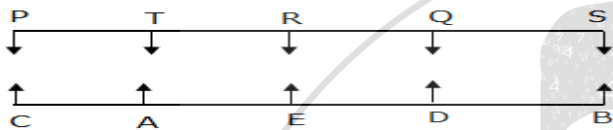
Answer: d)

32.5. How many person sits between D and B?

- a. One
- b. Two
- c. Three
- d. None

Answer: d)

Explanation:



- A sits second from the extreme end.
- Only one person sits between one who faces A and Q.



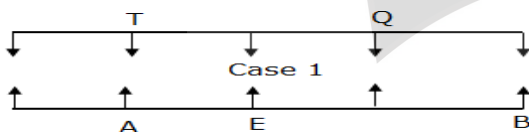
- B is not an immediate neighbour of A and doesn't sit opposite to Q.



- E sits second to the left of B.



- T doesn't faces E and never sits at extreme ends.



of Cook and Mark, when counted from left of Mark?

- a. One
- b. Two
- c. Three
- d. No one

Answer: b)

33.3. Which of the following statement is correct?

- a. Cook sits to the immediate left of Trump
- b. Only three persons sit between Satya and Sundar
- c. Sundar and Mark is an immediate neighbours
- d. Only one person sits between Nitish and the one who sits to the immediate left to Sundar

Answer: c)

33.4. What is the position of the one who sits second to the left of Sundar with respect to Trump?

- a. Immediate right
- b. Fourth to the right
- c. Second to the right
- d. Second to the right

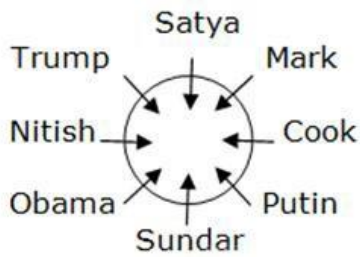
Answer: a)

33.5. Which of the following statement is correct?

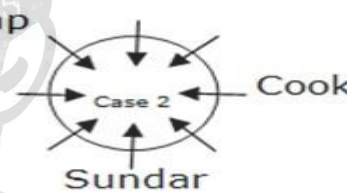
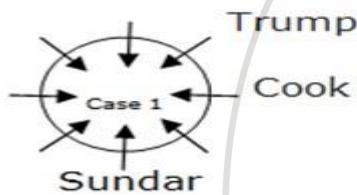
- a. Nitish sits to the immediate left of Obama
- b. Cook is an immediate neighbour of Satya
- c. Only two persons sit between Mark and Nitish when counted from left of Nitish
- d. Both a and c

Answer: d)

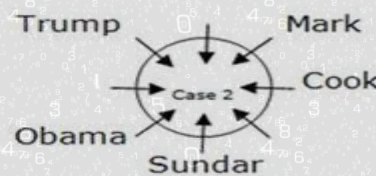
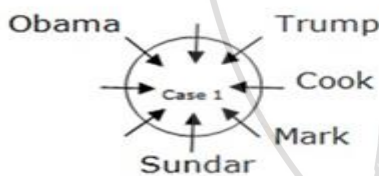
Explanation:



- Only two persons sit between Sundar and Trump.
- Sundar sits second to the left of Cook.
- Now we have 2 cases

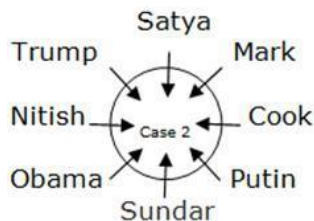


- Obama is not an immediate neighbour of Trump and Cook.
- Only three persons sit between Mark and Obama



- Putin is not an immediate neighbour of Mark and Trump.
- Satya sits not opposite of Cook.
- Putin and Obama are not immediate neighbours.
- Nitish is not an immediate neighbour of Putin. Trump is not an immediate neighbour of Putin.
- From the above condition Case 1 was dropped.

- So the final arrangement is..



34. Direction (34.1 to 34.5): Read the following information carefully and answer the questions given below.

Eight people I, J, K, L, M, N, O and P are sitting in a rectangular table. Only three persons are sitting in the longer side of the rectangle. The people sits at longer side of table faces outside the table, while the people sits at smaller side of the table faces inside the table. All the information is not necessary to be in the same order.

N sits second to the left of M. Only two persons sit between M and P, who is not an immediate neighbour of N. J sits immediate right of L. P sits opposite to I. Neither K nor L is an immediate neighbour of P. J sits second to the right of O and both are facing same direction. J does not sit opposite to O.

34.1. Name the person who sits opposite to J?

- | | |
|------|------|
| a. M | b. O |
| c. N | d. I |

Answer: c)

34.2. Who sits second to the left of P?

- | | |
|------|------|
| a. L | b. K |
| c. J | d. O |

Answer: b)

34.3. If all the people are made to sit in alphabetical order in clockwise direction from I, then how many of them remains in their original position?

- | | |
|--------|--------|
| a. One | b. Two |
|--------|--------|

c. Three

d. More than Three

Answer: a)

34.4. Name the person who sits second to the right of L?

a. L

b. K

c. J

d. None of these

Answer: d)

34.5. Who sits fourth to the left of J?

a. L

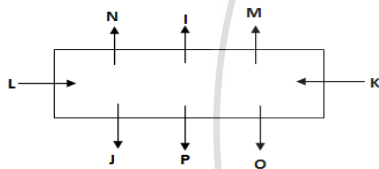
b. M

c. J

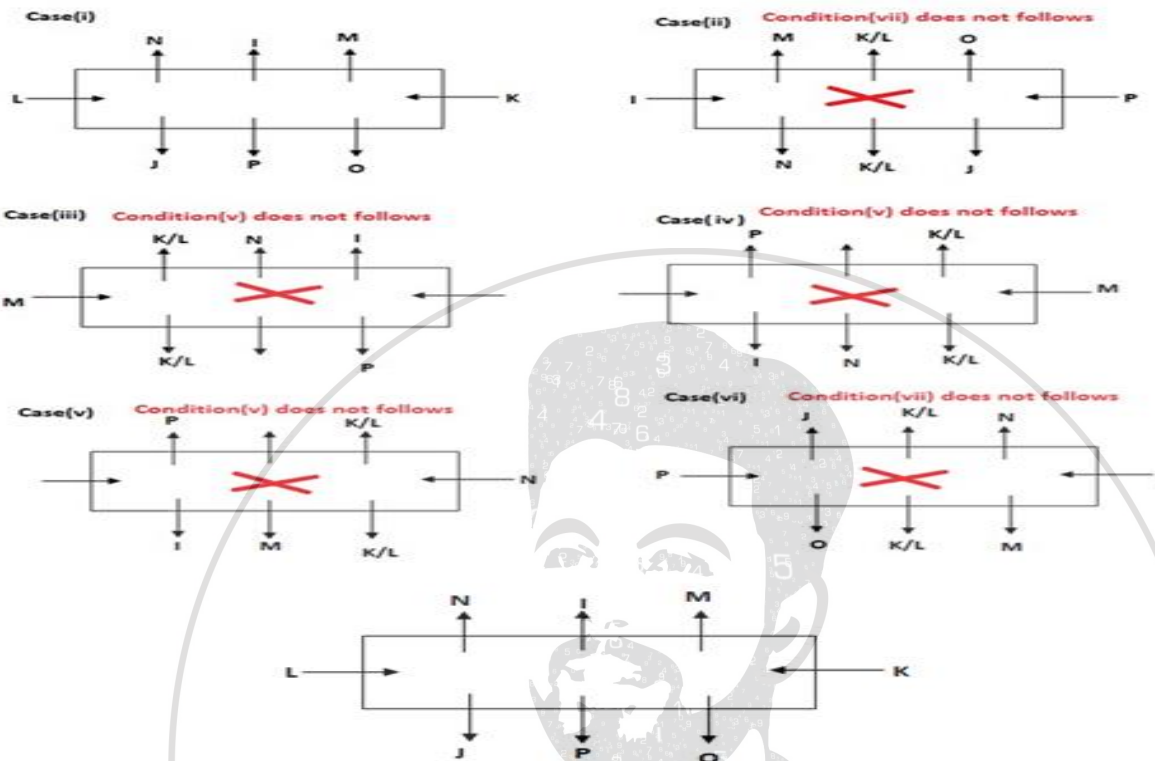
d. O

Answer: b)

Explanation:



- i). N sits second to the left of M.
- ii). Only two persons sit between M and P, who is not an immediate neighbour of N.
- iii). P sits opposite to I.
- iv). Neither K nor L is an immediate neighbour of P.
- v). J sits second to the right of O and both are facing same direction.
- vi). J does not sit opposite to O.
- vii). J sits immediate right of L.



35. Direction (35.1 to 35.5): Study the following information carefully to answer the given questions.

Six Students- Sita, Smita, Sunita, Sarita, Sujitha and Sneha lives on a building which has Six Floors with top floor numbered as 6. They got different Ranks from 1 to 6 in a School exam. They are also having different Lucky numbers from 1 to 6. These Six Students are also sitting in a row which has six seats and all are facing north. All students are having unique floor number, Lucky number, and Rank (i.e., No two numbers will be same for a particular student). Students who sit at extreme ends of the row live neither on the top floor nor on the bottom floor. Sujitha lives on an even numbered floor. Sarita Floor number and Sita Lucky number are same. Two students live between Sita and Smita. Smita sits third to the left of Sujitha. One who lives on top floor sits third to the left of

Sneha. A student whose Lucky number is 3 sits third to the right of the student whose lucky number is 5. Sita's Rank is 5. Two students live between Sujitha and Sarita. Smita Lucky number is same as Sita Floor number. Sunita Rank is 6 and she lives on an even numbered floor. Sneha's Rank is same as Sita's Lucky number. Sneha sits second to the right of Smita. Sneha's Lucky number is same as Sarita's Rank. Sujitha Rank is same as Sarita Lucky number.

35.1. Which of the following Pair is sitting at extreme ends?

- a. . Sita and Sneha
- b. Sunita and Sarita
- c. Sujitha and Smita
- d. Sunita and Sarita

Answer: d

35.2. Who among the following is living on Bottom Floor?

- a. Sarita
- b. Smita
- c. Sneha
- d. Sita

Answer: a

35.3. What is the Lucky number of Sita?

- a. One
- b. Two
- c. Three
- d. Four

Answer: a

35.4. Who among the following got Rank 2?

- a. Sita
- b. Sneha
- c. Smita
- d. Sujitha

Answer: c

35.5. Which of the following statement is false?

Sita lives on top floor

Sujitha Lucky number is 2

Sneha’s Rank is 1

Sarita Rank is 3

Answer: d

Explanation-

Floor No	Rank	Lucky No	Person
6	5	1	Sita
5	1	4	Sneha
4	3	2	Sujitha
3	2	6	Smita
2	6	5	Sunita
1	4	3	Sarita



36. Direction (36.1 to 36.5): Eight persons – A, B, C, D, E, F, G, and H are sitting in two rows having Five seats in each row. In each row, one seat is vacant. Some of them are facing north and some are facing south.

Two persons are sitting between D and B. C sits opposite to D. G sits opposite to E. H sits opposite to the person who is sitting second to the left of F. F is not adjacent to E. Vacant seats are not opposite to each other. A, C and G face the same direction (i.e., All face either North or South). D, B, and E face the same direction (i.e., All face either North or South). C sits second to the right of E. H faces north. C doesn’t sit at the extreme end. E sits second to the right of C. E sits to the adjacent left of H.

36.1. How many persons are sitting between A and H?

a. One

b. Two

c. Three

d. Four

Answer - 2.Two

36.2. Who among the following pair is sitting opposite to vacant seats?

a. A and D

b. B and A

c. C and F

d. A and F

Answer - 5.A and F

36.3. Who among the following is facing South?

a. A

b. B

c. F

d. G

Answer - 3.F

36.4. Which of the following pair is sitting in the same row?

a. A and D

b. C and F

c. E and B

d. B and F

Answer - 5.B and F

36.5. Which of the following statement is false based on above arrangement?

a. A faces North

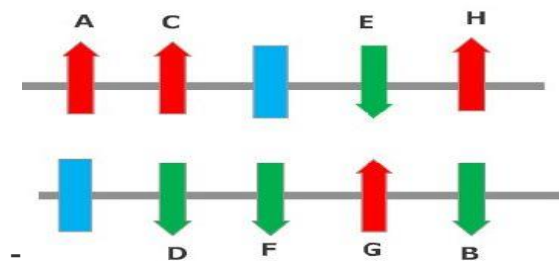
b. B sits at one of the extreme ends

c. D sits third to the right of B

d. Both the vacant seats are at extreme ends

Answer - 4.Both the vacant seats are at extreme ends

Explanation:



37. Study the following information carefully to answer the given questions.

Eight members P, Q, R, S, T, U, V and W of a family are sitting around a rectangular table with all of them facing outwards. Each one of them like different type of music instruments viz. XYLOPHONE, Balafon, Guitar, Piano, VIOLIN, TRUMPET, Accodion and Flute. Three married couples are there in the family.

W is the only sister-in-law of P whereas Q likes TRUMPET and daughter-in-law of RP who is the father of U and uncle of V, sits to the left of the person who likes XYLOPHONE. U is an immediate neighbor of her aunty W who does not sit next to S. R does not like Flute or Accodion. The two youngest members sit next to each other. The one who likes the Balafon sits between V and the one who likes VIOLIN. V is third to the left of S. The one who likes TRUMPET sits between the persons who like Accodion and Flute Respectively. S's husband and son sit next to her. Piano is not liked by V's father. V does not like Guitar or Accodion. S is the mother of P and T, and sits second to the left of T.

37.1. Which of the following statements is true regarding the family?

- | | |
|--------------------------|--------------------------------|
| a. P is the brother of W | b. R is the father-in-law of P |
| c. Q is the aunty of V | d. U and V are married couple |

Answer - c. Q is the aunty of V

37.2. Who among the following sits between Q and the one who likes Balafon?

- | | |
|------|------|
| a. P | b. T |
| c. S | d. V |

Answer - d. V

37.3. What is the position of the person who likes Piano with respect to the one who likes TRUMPET?

- a. Third to the right
- b. Second to the left
- c. Immediate left
- d. Third to the left

Answer - a. Third to the right

37.4. Who among the following likes Guitar?

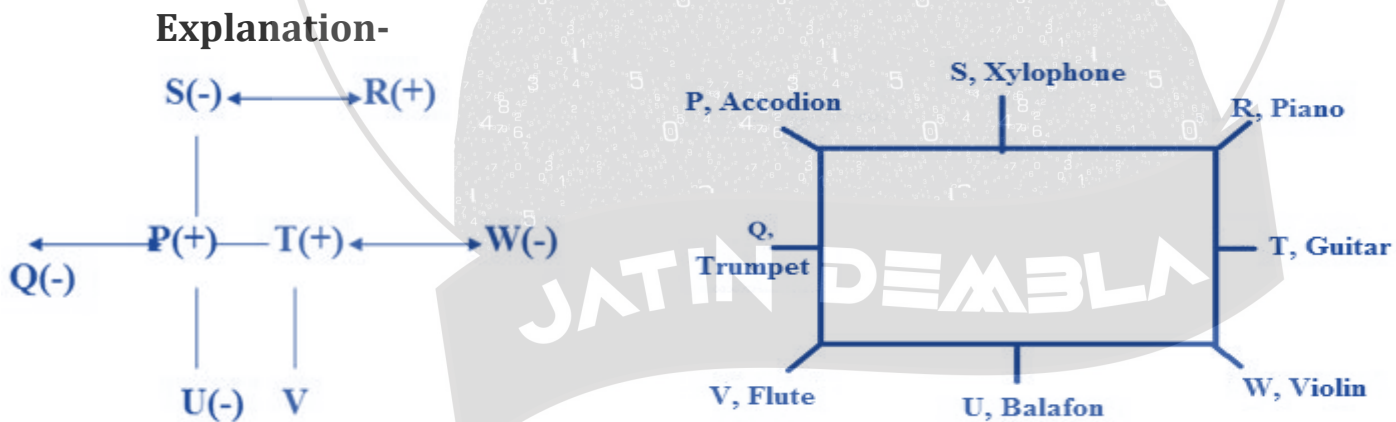
- a. W
- b. U
- c. V
- d. T

Answer - d. T

37.5. Which of the following options represent a pair?

- a. Y, X
- b. W, T
- c. W, R
- d. S, U

Answer - b. W, T



38. Study the following information carefully to answer the given questions.

Eight players - P, Q, R, S, T, U, V and W sit around a square table in such a way that four of them sit on the four sides while the rest at corners. They play different instruments namely Xylophone, Balafon, Guitar, Piano, Violin, Trumpet, Accodion

and Flute. Some of them are facing the centre while some are facing outside.(i.e away from the centre)

Note: Same directions means that if one person facing the centre then the other person also faces the centre and vice versa. Opposite direction means if one person is facing the centre then the other person faces outside and vice versa.

- Q faces the centre of the table and does not sit on any corner. V sits on one of the corner between the Flute player and Trumpet player. W sits second to the right of Balafon player who faces the centre.
- The Violin player sits third to the left of Q. S sits opposite to W. P sits on the corner exactly opposite to T. The Balafon player sits third to the right of Accodion player. The Xylophone player does not facing the centre.
- The Trumpet sits opposite to Q, also faces in opposite direction of Q and sits between Accodion player and Violin player. T who is the Violin player sits immediate right to the Piano player.
- The Piano player faces the same direction of the U. The immediate neighbours of Q are facing opposite directions. The Accodion player sits exactly opposite to Guitar player.
- The one who is on the immediate left of U is facing the same direction as W. R sits third to the left of W.

38.1. Who among the following is a Trumpet player?

- | | |
|------|------------------------|
| a. P | b. U |
| c. T | d. Can't be determined |

Answer – B. U

38.2. R is related to which of the following Instruments?

- | | |
|-------------|------------------------|
| a. Guitar | b. Xylophone |
| c. Accodion | d. Can't be determined |

Answer – A. Guitar

38.3. Who among the following sits exactly between R and the Xylophone Player?

- a. The person who plays Flute
- b. The person who plays Accodion
- c. The person who plays Balafon
- d. Trumpet

Answer – C. The person who plays Balafon

38.4. How many persons sit facing the centre?

- a. None
- b. One
- c. Two
- d. Four

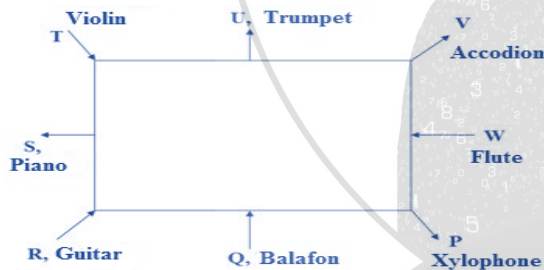
Answer – D. Four

38.5. Which of the following pairs are the immediate neighbors of the Flute player?

- a. P, V
- b. P, R
- c. V, R
- d. Q, R

Answer – A. P, V

Explanation-



39. Study the following information carefully to answer the given questions. There are 16 persons – B,C,D,E,F,G,H,I,P,Q,R,S,T,U,V and W standing in a square plot. Inside a Square plot, a square shaped garden is developed. The persons who are standing inside the garden facing outside. The persons who are standing outside the garden facing inside the centre and likes colours namely viz., – Red, Blue, Black, Brown, Yellow, Green, Violate and Pink. So all the persons standing in the inner square faces the persons standing in the outer Square and likes fruits namely viz., – Apple, Orange, Mango, Grapes, Papaya, Pomegranate, Guava and Banana.

G faces the centre and W faces G. D sits second to the right of G. There are four persons sits between G and E. D is not an immediate neighbour of E. There are three persons standing between I and E. There are two persons standing between I and B. B stands exactly between the E and F. F stands to the immediate left of G. There are two persons standing between W and U. U faces H. T faces outside. There are two persons standing between T and Q. T faces C. Q stands to the immediate left of W. R, the one who faces B stands exactly between the persons P and V. P faces E. The one who sits in the corner of the square likes Red. The one who likes Red sits between the persons who like Black and Blue. The one who likes Blue sits second to the right of the person who likes Green. Three persons sit between one who likes Black and one who likes Green. Two persons sit between one who likes Black and one who likes Yellow. Two persons sit between one who likes Yellow and one who likes Pink. G and F do not like Violate and Yellow respectively. The one who likes Red faces P. The immediate neighbours of P are the one who likes apple and the one who likes Grapes. The one who likes Apple faces the one who likes Black. Three persons sit between the one who likes apple and the one who likes Guava. The immediate neighbours of the person who likes Orange are the one who likes apple and the one who likes Pomegranate. The one who likes Papaya sits exactly behind to the one who likes Orange. The one who likes Banana sits exactly behind to the one who likes Mango. The one who likes Banana faces E.

39.1. In the given arrangement, if three people come and stand to the immediate left of E, how many people will sit between F (From the left of F) and C?

- | | |
|---------|-------------------|
| a. Two | b. Three |
| c. Five | d. More than four |

Answer – C. Five

39.2. Who amongst the following likes Green?

- | | |
|--------------------------------------|------|
| a. C | b. B |
| c. Other than those given as options | d. D |

Answer – D. D

39.3. How many people stand between V and U?

- e. Two
- f. Three
- g. Four
- h. More than four

Answer – B. Three

39.4. Four of the following five are alike in a certain way based upon their arrangement and so form a group. Which of the following does not belong to the group?

- a. FV
- b. UH
- c. EP
- d. GW

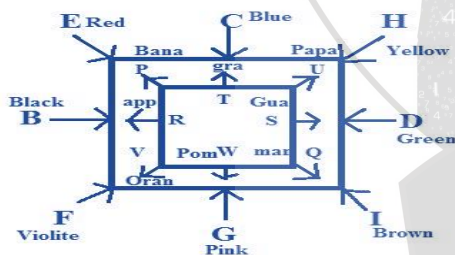
Answer – D. GW

39.5. Who amongst the following likes Papaya?

- a. P
- b. U
- c. Q
- d. E

Answer – B. U

Explanation-



40. Study the following information carefully to answer the given questions.

There are 16 persons – B,C,D,E,F,G,H,I,P,Q,R,S,T,U,V and W standing in a Circular plot. Inside a circular plot, a circularly shaped garden is developed. The persons who are standing inside the garden facing outside. The persons who are standing outside the garden facing inside the centre and lives in a different number of floors. So all the persons standing in the inner circle faces the persons standing in the outer circle and hold a different number of chocolates.

G faces outside and S faces G. D sits immediate right of G. There are four persons sits between G and E. H is not an immediate neighbour of E. There are two persons standing between D and H. H faces R. There are three persons standing between R and U. U stands exactly between the B and F. B faces D. There are two persons standing between P and C. Neither S nor R is an immediate neighbour of P. I stands to the immediate left of H. I faces T. The one who faces F stands exactly between the persons Q and W. W faces P. H stands second to the left of G. B lives on the second floor and sits exactly opposite to the person who lives on the floor which is the square number of the floor of B. F lives on the third floor and stands exactly opposite to the person who lives on the floor which is the square number of the floor of F. P lives on 6th floor and S lives immediately above P. U lives immediately below B. R lives immediately above T. The one who faces P holds chocolates two less than the number of the floor occupied by P. The one who faces U holds chocolates six more than the number of the floor occupied by U. Number of chocolates hold by E is the difference between the number of chocolates hold by D and W. Number of chocolates hold by G is the sum of the number of chocolates hold by D and E also equals to number of chocolates hold by V and H. Number of chocolates hold by I is the square of the number of chocolates hold by H.

40.1. In the given arrangement, how many people will sit between B and T?

- | | |
|---------|-------------------|
| a. Five | b. Three |
| c. Four | d. More than four |

Answer - B. Three

40.2. Who amongst the following lives on the seventh floor?

- | | |
|--------------------------------------|------|
| a. S | b. Q |
| c. Other than those given as options | d. U |

Answer - A. S

40.3. If persons counted from the right of G, then how many people stand between G and E as per the given arrangement?

- | | |
|---------|----------|
| e. Five | f. Three |
|---------|----------|

facing North. Therefore in the given seating arrangement, each member seated in a row either faces another member of the other row or seated behind each other. (All the information given above does not necessarily represent the order of seating in the final arrangement.) Each person stays in ten different floors numbered 1 to 12. (From Ground floor to Top floor)

There is only one floor between the person from Mumbai and the person from Pune. S is not from Bengaluru. D is neither from Pune nor from Hyderabad. P sits immediate right of the person from Surat. R sits one of the extreme ends of the line and from Surat. C sits third to the right of the person from Chennai. P does not face A and faces south direction. The person from Mumbai sits exactly between the persons from Kochi and Pune. The person from Hyderabad faces the person from Kochi. The person from Surat stays on the odd numbered floor. T faces North Direction and sits immediate left of Q. Only one person sits between the persons from Bengaluru and Kolkatta. The person from Kolkatta sits to the immediate right of Q, who seated exactly in the middle of the row. P faces one of the immediate neighbors of the person from Chennai. D faces one of the immediate neighbors of the person from Bengaluru. The person from Kochi stays on the top floor. Only One person sits between the person from Surat and Q. C sits to the immediate right of the person who faces S. The person from Hyderabad stays on the 4th floor. Only two people sit between C and E. S is neither from Mumbai nor from Ahmedabad. The person from Pune sits second to the right of the one who faces North Direction. One of the immediate neighbors of the person from Pune behind the person from Bengaluru. A faces the opposite direction to the person from Jaipur. The persons from Bengaluru, Jaipur and Kolkatta stay on the consecutive floors. The floor number of the person from Chennai is the double of the floor number of the person from Surat. The floor number of the B is the square of the floor number of P. Neither E nor A stays on floor numbered 6.

41.1. Who amongst the following faces the person from Hyderabad?

- | | |
|---------------------------|--------------------------|
| a. The person from Mumbai | b. D |
| c. The person from Pune | d. The person from Surat |

Answer - B. D

41.2. T stays on which of the following floors?

- a. 1
- b. 2
- c. 4
- d. 6

Answer - A. 1

41.3. Which of the following is true regarding C?

- a. C faces south direction
- b. None of the given options is true
- c. C is from Bangladesh
- d. The person from India faces C

Answer. C faces south direction.

41.4. R is related to Kolkatta in the same way as C is related to Pune based on the given arrangement, To who amongst the following is T related to the following same pattern?

- a. Mumbai
- b. Sri Nagar
- c. Bengaluru
- d. Hyderabad

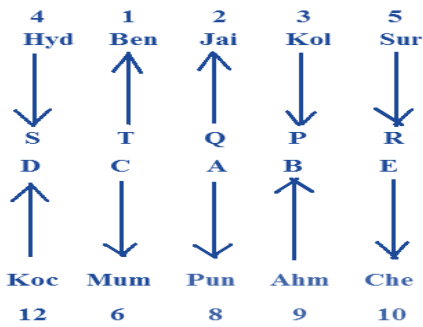
Answer - D. Hyderabad

41.5. Who amongst the following sit at extreme ends of the row?

- a. The person stays on 8th floor and R
- b. The persons from Ahmedabad and A
- c. D and the person stays on 10th floor
- d. The persons from Hyderabad and Bengaluru

Answer - C. D and the person stays on 10th floor

Explanation-



42. Study the following information carefully to answer the given questions.

Ten friends are sitting in two parallel rows of six seats each. One seat is vacant in each row. M, N, O, P and Q are sitting in row-1 facing South. D, E, F, G and H are facing North. Each likes a different Chocolate i.e. 5star, Dairy Milk, Munch, Kitkat, Perk, Snickers, Bourneville, Gems, Eclairs and Galaxy. Each person has different number of their favourite chocolates – 2, 3, 4, 6, 7, 8, 9, 11, 15 and 16.

The difference between the chocolates hold by N and O is 3. G sits third to the right of F and likes Kitkat. Only two people sit between E and the vacant seat. E does not like Perk or Munch Chocolate. Q is not an immediate neighbour of O. N likes Galaxy. The persons who sit at the extreme end of the line have chocolates in consecutive order. Neither E nor H has 8 chocolate. One of the neighbors of vacant seat in both rows have chocolates in odd number. The one who likes Munch Chocolate faces the one who likes Gems. The one who likes Munch sits opposite to the one who sits third right of the person who sits opposite to G. O is not an immediate neighbour of P. H, who likes neither Perk nor Snickers, does not face the vacant seat. Neither G nor F sits at any of the extreme ends of the row. P faces F. Vacant seats are not opposite to each other. Two seats are there between O and N, who sits third right of the one who likes Bourneville. The one who likes Eclairs Chocolate faces the one who likes Kitkat. The persons who like the 5star and Gems are adjacent to each other. Vacant seat of row – 1 is not an immediate neighbour of P. E sits at one of the extreme ends of the row. F does not like 5star and Gems. Vacant seat of row-1 does not face G who doesn't sit at any of the extreme ends of the row. The person who likes 5star has 3 chocolates. The total number of chocolates hold by Q is the half of the total number of chocolates hold by H. The total numbers of chocolates hold by M, F and G is the Square of the total number of chocolates hold by P, Q and M respectively. Neither P nor G has 4 chocolate.

42.1. In the given arrangement, if two people come and sit to the immediate left of E, how many people will sit between D and E?

- a. Two
- b. Three
- c. Four
- d. More than four

Answer - B. Three

42.2. Who amongst the following sits third to the right of F?

- a. The one who likes Kitkat
- b. E
- c. Other than those given as options
- d. D

Answer - A. The one who likes Kitkat

42.3. Which of the following faces the vacant seat of Row - 1?

- a. The one who likes Kitkat
- b. E
- c. Other than those given as options
- d. The one who has 15 chocolate

Answer - D. The one who has 15 chocolate

42.4. Four of the following five are alike in a certain way based upon their seating arrangement and so form a group. Which of the following does not belong to the group?

- a. QE
- b. ND
- c. HO
- d. FP

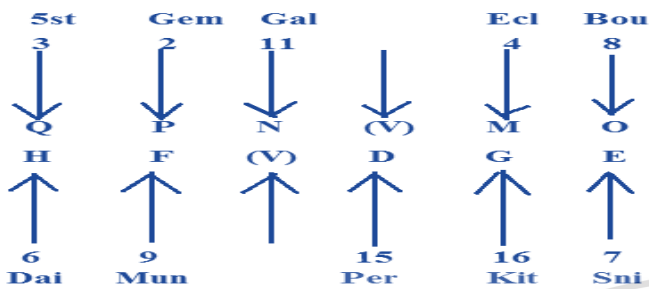
Answer - D. FP

42.5. Who among the following has 11 chocolate?

- a. Q
- b. N
- c. D
- d. E

Answer - B. N

Explanation-



43. Study the following information carefully to answer the given questions.

Eight friends C, D, E, F, L, M, N and O are seated in a straight line, but not necessarily in the same order. Some of them are facing north while some face South. Only three people sit to the right of M. E sits second to the left of M. F sits third to the right of O. O is not an immediate neighbour of M. O does not sit at any of the extreme ends of the line. Both the immediate neighbours of O face south. D sits second to the right of N. As many people sit between M and D as between M and L. Immediate neighbours of F face opposite directions (i.e., If one person faces north then the other person faces south and vice-versa). C faces south. L and F face direction opposite to C. (i.e If C faces north then both L and F face south and vice-versa)

43.1. Which of the following is true, based on the given arrangement?

- a. D faces North
- b. Only three people face South
- c. L sits at one of the extreme ends of the line
- d. O and E face the same directions

Answer - D. O and E face the same directions

43.2. How many people sit to the left of O?

- a. Two
- b. Three
- c. One
- d. More than four

Answer - C. One

43.3. Who amongst the following faces South?

- a. E
- b. M

c. F

d. L

Answer: B. M

43.4. Who amongst the following sits second to the left of L?

a. O

b. F

c. D

d. No one as less than two people sit to the left of L

Answer -B. F

43.5. Who amongst the following represent the persons sitting at extreme ends of the line?

a. D, N

b. C, D

c. L, N

d. D, L

Answer - B. C, D

Explanation-



44. Study the following information carefully to answer the given questions. Eight people B, C, D, E, F, G, H and I are sitting in a straight line with equal distances between each other, but not necessarily in the same order. Some of them are facing north and some of them are facing south.

- E sits immediate right of the person who sits at one of the extreme ends of the line. Only three people sit between E and G. B sits exactly between D and H.
- C sits third to the right of H. F is an immediate neighbour of G and faces south. G sits second to the right of C. D sits third to the left of G. B and E face the same direction as C(i.e if C faces north then B and E also face North and Vice-Versa).
- Immediate neighbours of G face opposite directions(i.e. if one neighbour faces North then the other neighbour faces south and Vice-Versa)

- Person who sit at the extreme ends of the line face opposite directions(i.e. if one neighbour faces North then the other neighbour faces south and Vice-Versa)
- D and H face the same direction as I(i.e if I faces north then D and H also face North and Vice-Versa).

44.1. In the given arrangement, how many people will sit between D and G?

- | | |
|---------|-------------------|
| a. Two | b. Three |
| c. Four | d. More than four |

Answer - A. Two

44.2. Who amongst the following sits third to the right of B?

- | | |
|--------------------------------------|------|
| a. E | b. I |
| c. Other than those given as options | d. F |

Answer - B. I

44.3. How many people face South as per the given arrangement?

- | | |
|---------|-------------------|
| e. Two | f. Three |
| g. Four | h. More than four |

Answer - C. Four

44.4. Four of the following five are alike in a certain way based upon their seating arrangement and so form a group. Which of the following does not belong to the group?

- | | |
|-------|-------|
| a. IH | b. EG |
| c. DF | d. EB |

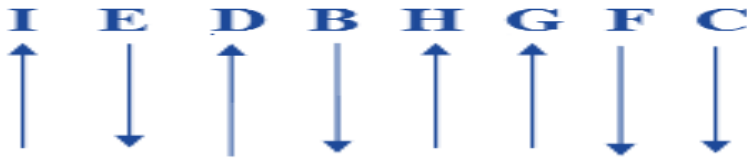
Answer - D. EB

44.5. Who amongst the following sits at the extreme right end of the row?

- | | |
|------|------|
| a. G | b. C |
| c. I | d. H |

Answer - B. C

Explanation-



45. Study the following information carefully to answer the given questions.

Ten persons from ten different countries viz. Switzerland, Spain, Italy, USA, UK, Australia, New Zealand, Brazil, Canada and Singapore are sitting in two parallel rows containing five people each, in such a way that there is an equal distance between adjacent persons. In row 1- A, B, C, D and E are seated and some of them are facing South and some of them are facing North. In row 2 - P, Q, R, S and T are seated and some of them are facing South and some of them are facing North. Therefore in the given seating arrangement, each member seated in a row either faces another member of the other row or seated behind each other. (All the information given above does not necessarily represent the order of seating in the final arrangement.). Each person stays in ten different floors numbered 1 to 12. (From Ground Floor to Top floor)

There is only one floor between the person from Switzerland and the person from UK. S is not from Italy. D is neither from UK nor from Australia. P sits immediate right of the person from Canada. R sits one of the extreme ends of the line and from Canada. C sits third to the right of the person from Spain. P does not face A and faces south direction. The person from Switzerland sits exactly between the persons from Singapore and UK. The person from Australia faces the person from Singapore. The person from Canada stays on the odd numbered floor. T faces North Direction and sits immediate left of Q. Only one person sit between the persons from Italy and USA. The person from USA sits to the immediate right of Q, who seated exactly in the middle of the row. P faces one of the immediate neighbors of the person from Spain. D faces one of the immediate neighbors of the person from Italy. The person from Singapore stays on the top floor. Only One person sits between the person from Canada and Q. C sits to the immediate right of the person who faces S. The person from Australia stays on the 4th floor. Only two people sit between C and E. S is neither from Switzerland nor from Brazil. The person from UK sits second to the right of the one who faces North Direction. One

of the immediate neighbors of the person from UK behind the person from Italy. A faces the opposite direction to the person from New Zealand. The persons from Italy, New Zealand and USA stay on the consecutive floors. The floor number of the person from Spain is the double of the floor number of the person from Canada. The floor number of the B is the square of the floor number of P. Neither E nor A stays on floor numbered 6.

45.1. Who amongst the following faces the person from Australia?

- a. The person from Switzerland
- b. D
- c. The person from UK
- d. The person from Canada

Answer - B. D

45.2. T stays on which of the following floors?

- a. 1
- b. 2
- c. 4
- d. 6

Answer - a. 1

45.3. Which of the following is true regarding C?

- a. C faces south direction
- b. None of the given options is true
- c. C is from Bangladesh
- d. The person from India faces C

Answer - A. C faces south direction.

45.4. R is related to USA in the same way as C is related to UK based on the given arrangement, To who amongst the following is T related to the following same pattern?

- a. Switzerland
- b. Sri Nagar
- c. Italy
- d. Australia

Answer - D. Australia

45.5. Who amongst the following sit at extreme ends of the row?

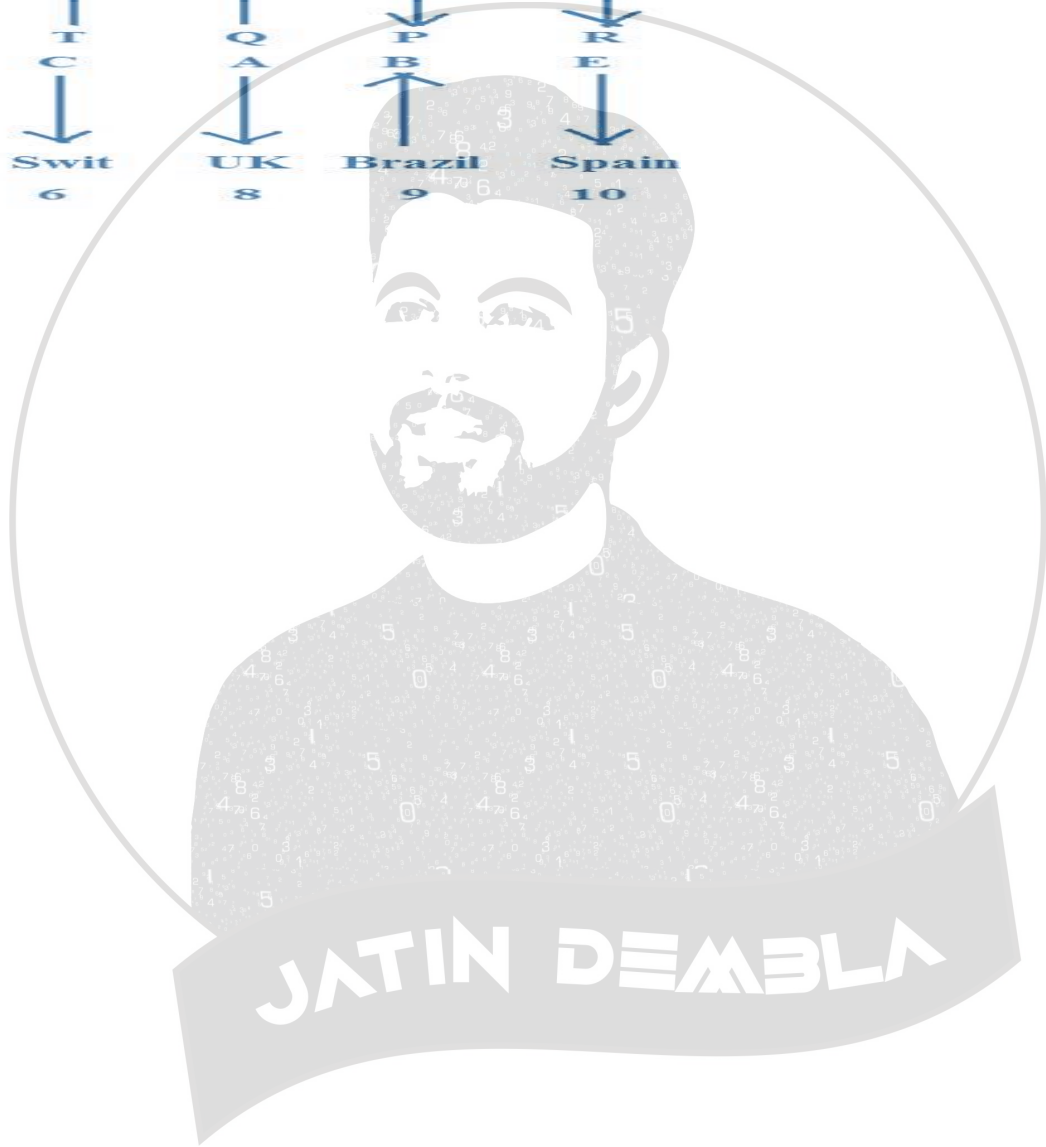
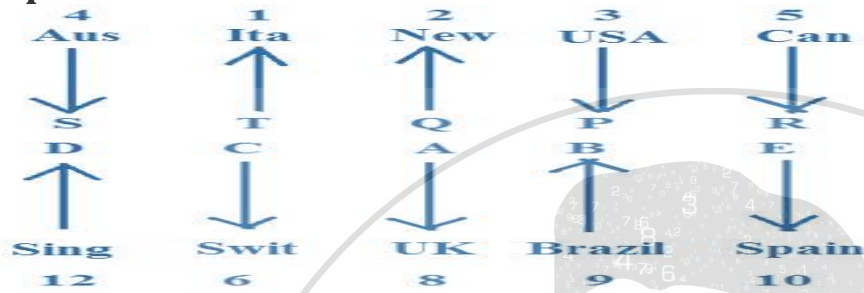
- a. The person stays on 8th floor and R
- b. The persons from Brazil and A

c. D and the person stays on 10th floor

d. The persons from Australia and Italy

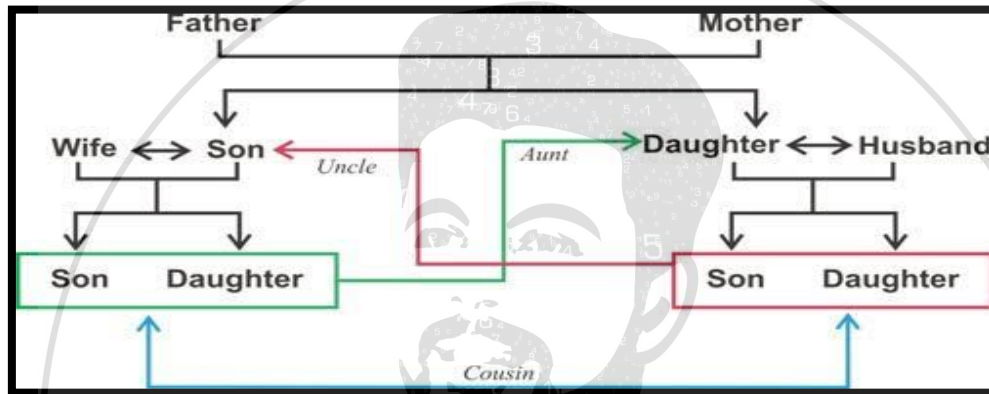
Answer - C. D and the person stays on 10th floor

Explanation-



CHAPTER 12

BLOOD RELATIONS



A person who is related to another by birth rather than by marriage.

S.no	Relationship	Commonly Used Terms
1	Father's son (or) mother's son	Brother
2	Father's daughter (or) mother's daughter	Sister
3	Mother's brother	Uncle (Paternal Uncle)
4	Father's brother	Uncle (Maternal Uncle)
5	Mother's sister	Aunt (Paternal Aunt)
6	Father's sister	Aunt (Maternal Aunt)
7	Son's wife	Daughter-in-law
8	Daughter's husband	Son-in-law
9	Sister's husband	Brother-in-law
10	Husband's brother (or) wife's brother	Brother-in-law
11	Brother's wife	Sister-in-law
12	Husband's sister (or) wife's sister	Sister-in-law
13	Husband's father (or) wife's father	Father-in-law
14	Husband's mother (or) wife's mother	Mother-in-law
15	Brother's son (or) sister's son	Nephew
16	Brother's daughter (or) Sister's daughter	Niece
17	Uncle's daughter (or) Aunt's daughter	Cousin
18	Uncle's Grand son (or) Aunt's Grand son	Nephew
19	Father's father (or) mother's father	Grandfather
20	Father's mother (or) mother's mother	Grandmother
21	Father of Grandfather or Father of grandmother	Great Grandmother
22	Father of Grandfather (or) father of grandmother	Great Grandfather

3. If A is the brother of B; B is the sister of C; and C is the father of D, how D is related to A?

- a. Brother
- b. Sister
- c. Nephew
- d. Cannot be determined

Answer: Option D

Explanation:

If D is Male, the answer is Nephew.

If D is Female, the answer is Niece.

As the sex of D is not known, hence, the relation between D and A cannot be determined.

Note: Niece - A daughter of one's brother or sister, or of one's brother-in-law or sister-in-law. Nephew - A son of one's brother or sister, or of one's brother-in-law or sister-in-law

4. If A + B means A is the brother of B; A - B means A is the sister of B and A x B means A is the father of B. Which of the following means that C is the son of M?

- a. $M - N \times C + F$
- b. $F - C + N \times M$
- c. $N + M - F \times C$
- d. $M \times N - C + F$

Answer: Option D

Explanation:

$M \times N \rightarrow M$ is the father of N

$N - C \rightarrow N$ is the sister of C

and $C + F \rightarrow C$ is the brother of F.

Hence, M is the father of C or C is the son of M

5. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl?

- a. Brother
- b. Nephew
- c. Uncle
- d. Son-in-law

Answer: Option A

Explanation:

The father of the boy's uncle → the grandfather of the boy and daughter of the grandfather → sister of father

6. Pointing to a photograph Lata says, "He is the son of the only son of my grandfather." How is the man in the photograph related to Lata?

- a. Brother
- b. Uncle
- c. Cousin
- d. Data is inadequate

Answer: Option A

Explanation:

The man in the photograph is the son of the only son of Lata's grandfather i.e., the man is the son of Lata's father. Hence, the man is the brother of Lata

7. If $A + B$ means A is the brother of B; $A \times B$ means A is the son of B; and $A \% B$ means B is the daughter of A then which of the following means M is the maternal uncle of N?

- a. $M + O \times N$
- b. $M \% O \times N + P$
- c. $M + O \% N$
- d. None of these

Answer: Option D

Explanation:

Because the sex of O is not known

8. If D is the brother of B, how B is related to C? To answer this question which of the statements is/are necessary?

The son of D is the grandson of C.

B is the sister of D.

- a. Only 1
- b. Only 2
- c. Either 1 or 2
- d. 1 and 2 both are required

Answer: Option D

Explanation:

Given: D is the brother of B.

From statement 1, we can detect that D is son of C (son of D is the grandson of C).

From statement 2, we can detect that B is 'Female' (sister of D).

Therefore, B is daughter of C.

9. If $A + B$ means A is the father of B; $A - B$ means A is the brother B; $A \% B$ means A is the wife of B and $A \times B$ means A is the mother of B, which of the following shows that M is the maternal grandmother of T?

a. $M \times N \% S + T$

b. $M \times N - S \% T$

c. $M \times S - N \% T$

d. $M \times N \times S \% T$

Answer: Option A

Explanation:

$M \times N \rightarrow$ M is the mother of N

$N \% S \rightarrow$ N is the wife of S

and $S + T \rightarrow$ S is the father of T.

Hence, M is the maternal grandmother of T

10. Pointing to a photograph. Bajpai said, "He is the son of the only daughter of the father of my brother." How Bajpai is related to the man in the photograph?

a. Nephew

b. Brother

c. Father

d. Maternal Uncle

Answer: Option D

Explanation:

The man in the photo is the son of the sister of Bajpai. Hence, Bajpai is the maternal uncle of the man in the photograph

11. Deepak said to Nitin, "That boy playing with the football is the younger of the two brothers of the daughter of my father's wife." How is the boy playing football related to Deepak?

a. Son

b. Brother

c. Cousin

d. Brother-in-law

Answer: Option B

Explanation:

Father's wife \rightarrow mother. Hence, the daughter of the mother means sister and sister's younger brother means brother. Therefore, the boy is the brother of Deepak.

12. Pointing a photograph X said to his friend Y, "She is the only daughter of the father of my mother." How X is related to the person of photograph?

- a. Daughter
- b. Son
- c. Nephew
- d. Cannot be decided

Answer: Option B

Explanation:

'The only daughter of the father of X's mother' means mother of X.

Hence X is the son of the lady in the photograph.

Note: Still have doubt like "How X is a male?"

13. Veena who is the sister-in-law of Ashok, is the daughter-in-law of Kalyani. Dheeraj is the father of Sudeep who is the only brother of Ashok. How Kalyani is related to Ashok?

- a. Mother-in-law
- b. Aunt
- c. Wife
- d. None of these

Answer: Option D

Explanation:

Ashok is the only brother of Sudeep and Veena is the sister-in-law of Ashok. Hence Veena is the wife of Sudeep. Kalyani is the mother-in-law of Veena. Kalyani is the mother of Ashok.

14. If $A + B$ means A is the sister of B; $A \times B$ means A is the wife of B, $A \% B$ means A is the father of B and $A - B$ means A is the brother of B. Which of the following means T is the daughter of P?

- a. $P \times Q \% R + S - T$
- b. $P \times Q \% R - T + S$
- c. $P \times Q \% R + T - S$
- d. $P \times Q \% R + S + T$

Answer: Option B

Explanation:

$P \times Q \rightarrow P$ is the wife of Q

$Q \% R \rightarrow Q$ is the father of R

$R - T \rightarrow R$ is the brother of T

$T + S \rightarrow T$ is the sister of S .

Therefore, T is the daughter of P .

15. Pointing to a woman, Abhijit said, "Her granddaughter is the only daughter of my brother." How is the woman related to Abhijit?

- a. Sister
- b. Grandmother
- c. Mother-in-law
- d. Mother

Answer: Option D

Explanation:

Daughter of Abhijit's brother \rightarrow niece of Abhijit. Thus the granddaughter of the woman is Abhijit's niece.

Hence, the woman is the mother of Abhijit.

16. Amit said - "This girl is the wife of the grandson of my mother". How is Amit related to the girl?

- a. Brother
- b. Grandfather
- c. Husband
- d. Father-in-law

Answer: Option D

Explanation:

The girl is the wife of grandson of Amit's mother i.e., the girl is the wife of son of Amit. Hence, Amit is the father-in-law of the girl.

17. A and B are children of D. Who is the father of A? To answer this question which of the statements (1) and (2) is necessary?

C is the brother of A and the son of E.

F is the mother B.

- a. Only (1)
- b. Only (2)
- c. Either (1) or (2)
- d. (1) and (2) both

Answer: Option B

Explanation:

A and B are children of D.

From (1), C is the brother B and son of E.

Since, the sex of D and E are not known. Hence (1) is not sufficient to answer the question.

From (2). F is the mother of B. Hence, F is also the mother of A. Hence D is the father of A.

Thus, (2) is sufficient to answer the question.

18. Pointing towards a man, a woman said, "His mother is the only daughter of my mother." How is the woman related to the man?

- a. Mother
- b. Grandmother
- c. Sister
- d. Daughter

Answer: Option A

Explanation:

Only daughter of my mother → myself.

Hence, the woman is the mother of the man.

19. If P \$ Q means P is the brother of Q; P # Q means P is the mother of Q; P * Q means P is the daughter of Q in A # B \$ C * D, who is the father?

- a. D
- b. B
- c. C
- d. Data is inadequate

Answer: Option A

Explanation:

A is the mother of B, B is the brother of C and C is the daughter of D. Hence, D is the father.

A (Parents) D



B - is - Brother - of - C

20. Introducing Sonia, Aamir says, "She is the wife of only nephew of only brother of my mother." How Sonia is related to Aamir?

- a. Wife
- b. Sister
- c. Sister-in-law
- d. Data is inadequate

Answer: Option A

Explanation:

Brother of mother means maternal uncle. Hence only nephew of Aamir's maternal uncle means Aamir himself. Therefore Sonia is the wife of Aamir

21. If A + B means A is the brother of B; A % B means A is the father of B and A x B means A is the sister of B. Which of the following means M is the uncle of P?

- a. M % N x P
- b. N x P % M
- c. M + S % R % P
- d. M + K % T x P

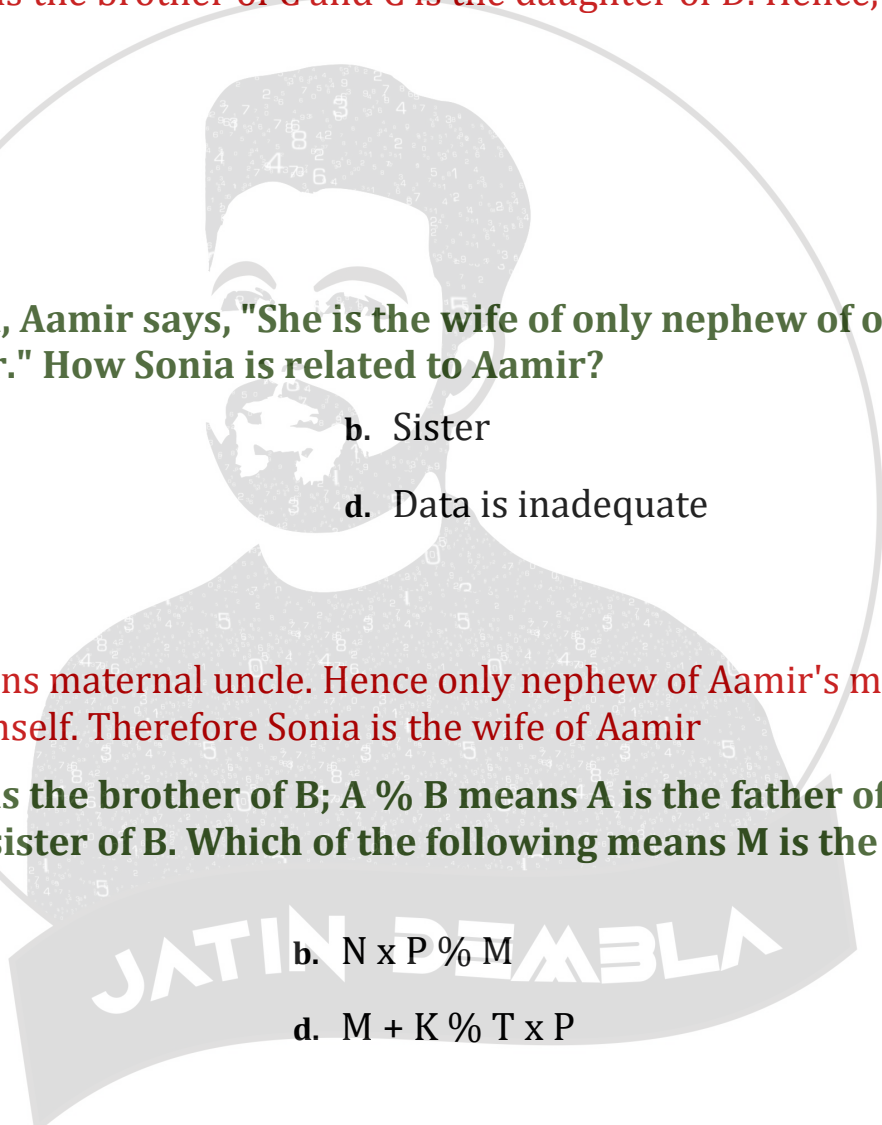
Answer: Option D

Explanation:

M + K → M is the brother of K

K % T → K is the father of T

T x P → T is the sister of P



Therefore, K is the father of P and M is the uncle of P.

22. Pointing to Varman, Madhav said, "I am the only son of one of the sons of his father." How is Varman related to Madhav?

- a. Nephew
b. Uncle
c. Father or Uncle
d. Father

Answer: Option C

Explanation:

Madhav is the only son of one of the sons of Varman's father → Either Varman is the father or uncle of Madhav.

23. Introducing a woman, Shashank said, "She is the mother of the only daughter of my son." How that woman is related to Shashank?

- a. Daughter
b. Sister-in-law
c. Wife
d. Daughter-in-law

Answer: Option D

Explanation:

The woman is the mother of Shashank's granddaughter. Hence, the woman is the daughter-in-law of Shashank.

24. If $A + B$ means B is the brother of A; $A \times B$ means B is the husband of A; $A - B$ means A is the mother of B and $A \% B$ means A is the father of B, which of the following relations shows that Q is the grandmother of T?

- a. $Q - P + R \% T$
b. $P \times Q \% R - T$
c. $P \times Q \% R + T$
d. $P + Q \% R - T$

Answer: Option A

Explanation:

$Q - P \rightarrow Q$ is the mother of P

$P + R \rightarrow R$ is the brother of P

Hence, $\rightarrow q$ is the mother of R

$R \% T \rightarrow R$ is the father of T.

Hence, Q is the grandmother of T

25. A3P means A is the mother of P

A4P means A is the brother of P

A9P means A is the husband of P

A5P means A is the daughter of P

Which of the following means that K is the mother-in-law of M?

a. M9N3K4J

b. M9N5K3J

c. K5J9M3N

d. K3J9N4M

Answer: Option B

Explanation:

$M9N \rightarrow M$ is the husband of N

$N5K \rightarrow N$ is the daughter of K

Hence, $\rightarrow M$ is the son-in-law of K

$K3J \rightarrow K$ is the mother of J

Hence, $\rightarrow K$ is a lady

Hence, $\rightarrow K$ is the mother-in-law of M

26. Introducing Neeta, Anil said, 'She is wife of my mother's only son. How is Neeta related to Anil?

a. Mother

b. Wife

c. Sister

d. Daughter-in-law

Answer: B

Explanation:

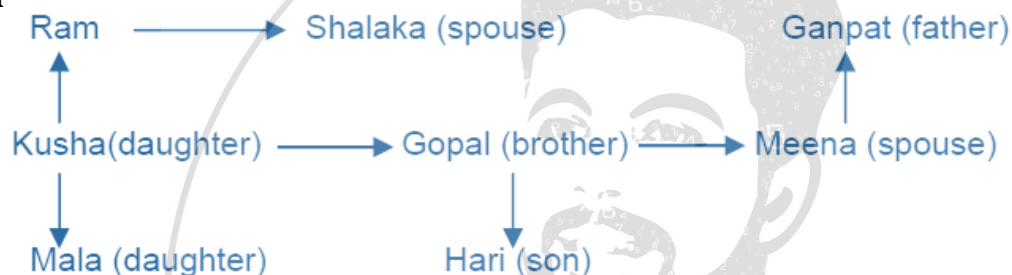
Neeta is the wife of Anil's mother's only son, who is Anil himself. Hence, answer is Neeta is Anil's wife. i.e. **(B) Wife.**

27. 'Ram' is the father of 'Kusha' but 'Kusha' is not his son. 'Mala' is the daughter of 'Kusha'. 'Shalaka' is the spouse of 'Ram'. 'Gopal' is the brother of 'Kusha'. 'Hari' is the son of 'Gopal'. 'Meena' is the spouse of 'Gopal'. 'Ganpat' is the father of 'Meena'. Who is the grand daughter of 'Ram'?

- a. Hari
c. Meena
b. Mala
d. Shalaka

Answer: B

Explanation:



'Mala' is the daughter of 'Kusha' and 'Ram' is the father of 'Kusha'. So, 'Mala' is the granddaughter of 'Ram'. Hence, **answer is (B) Mala.**

28. Pointing to a gentleman, Dinesh said "His only brother is the father of my daughter's father." How is the gentleman related to Dinesh?

- a. Uncle
c. Father
b. Grandfather
d. Brother- in-law

Answer: A

Explanation:

The gentleman's only brother is the father of Dinesh (Dinesh daughter's father is Dinesh himself.). Gentleman is brother of Dinesh's father. Gentleman is Dinesh's uncle. Hence, **answer is (1) Uncle.**

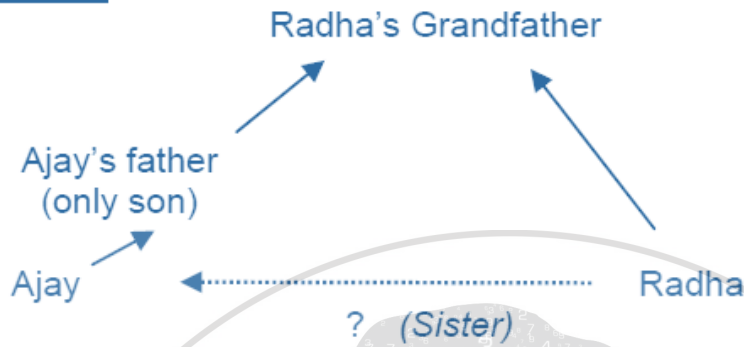
29. Pointing to Ajay, Radha said, "His father is the only son of my grandfather". How is Radha Related to Ajay?

- a. Brother
c. Mother
b. Sister
d. Daughter

Answer: B

Explanation:

Traditional Method



When Radha's Grandfather's only son is Ajay's father, then Ajay's father is also the father of Radha. So Radha is Ajay's sister. Hence, **answer is (B) Sister**.
 We know, 'Only son of my grandfather' means 'my father'. "His father is the only son of my grandfather" thus becomes "His father is my father". So Radha is Ajay's sister. Hence, **answer is (B) Sister**.

30. Lalita said to Tina, "You are the daughter-in-law of the grandmother of my father's only son."

How is Lalita related to Tina?

- a. Aunt
- b. Sister
- c. Mother
- d. Indeterminable

Answer: D

Explanation:

'My father's only son' is my (Lalita's) brother. Tina is daughter-in-law of grandmother of (Lalita's) brother. Tina thus can be their mother (wife of grandmother's only son). However as it is not mentioned that the grandmother has only one son, Tina can be wife of grandmother's other son i.e. Tina could also be their aunt. Hence, answer is (4) Indeterminable.

31. Pointing to a photograph, Amar said, "I have no brother or sister but that man's father is my father's son." Whose photograph was it?

- His son's
- His father's
- His nephew's
- His own

Answer: A

Explanation:

Since Amar has no brother or sister so his father's son is the man himself and so the

man who is talking is the father of the man in the photograph i.e. the man in the photograph is his son. Hence, **answer is (A) His son's.**

32. Looking at the portrait of a man, Ashok said, 'His mother is the wife of my father's son. Brothers and sisters I have none'. At whose portrait was Ashok looking?

His cousin
His uncle

His nephew
His son

Answer: D

Explanation:

My (Ashok's) father's son will be Ashok himself as he has no brother or sister. Ashok's wife is mother of the person in the portrait. The portrait is thus of Ashok's own son. Hence, **answer is (D) His Son.**

33. Pointing to a photograph of a boy Suresh said, "He is the son of the only son of my mother." How is Suresh related to that boy?

Brother
Cousin

Uncle
Father

Answer: D

Explanation:

The boy in the photograph is the only son of the son of Suresh's mother i.e., the son of Suresh. Hence, Suresh is the father of boy.

34. If $A + B$ means A is the mother of B; $A - B$ means A is the brother B; $A \% B$ means A is the father of B and $A \times B$ means A is the sister of B, which of the following shows that P is the maternal uncle of Q?

a. $Q - N + M \times P$
c. $P - M + N \times Q$

b. $P + S \times N - Q$
d. $Q - S \% P$

Answer: C

Explanation:

$P - M \rightarrow P$ is the brother of M

$M + N \rightarrow M$ is the mother of N

$N \times Q \rightarrow N$ is the sister of Q

Therefore, P is the maternal uncle of Q .

Z's cousin.

38. $X - Z$ means X is the mother of Z; $X \times Z$ means X is the father of Z and $X + Z$ means X is the daughter of Z. Now, if $M - N \times T + Q$, then which of the following is not true?

- a. T is N's daughter.
- b. N is wife of Q
- c. M is mother-in-law of Q
- d. Q is wife of N.

Answer: B

Explanation:

$M - N \times T + Q$

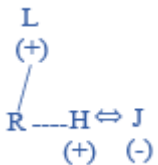
M is the mother of N who is the father of T who is the daughter of Q. So, M is the grandmother of the daughter of Q, i.e., M is the mother-in-law of Q. Hence (B) is not true.

39. If ' $A \times B$ ' means 'B is father of A', ' $A + B$ ' means 'A is wife of B' and ' $A \div B$ ' means 'A is brother of B', then, what is the relation of J with L in ' $J + H \div R \times L$ '?

- a. Daughter
- b. Daughter-in-law
- c. Sister-in-law
- d. Cannot be determined

Answer: B

Explanation:



J is R's brother's wife. L is the father of H and R. Hence, J is daughter-in-law of L.

40. A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D?

- Grandfather
- Grandmother
- Daughter
- Granddaughter

Answer: D) Granddaughter

Explanation:

A is the sister of B and B is the daughter of C. So, A is the daughter of C. Also, D is the father of C. So, A is the granddaughter of D.

41. P is the brother of Q and R. S is R's mother. T is P's father. Which of the following statements cannot be definitely true?

T is Q's father

S is P's mother

P is S's son

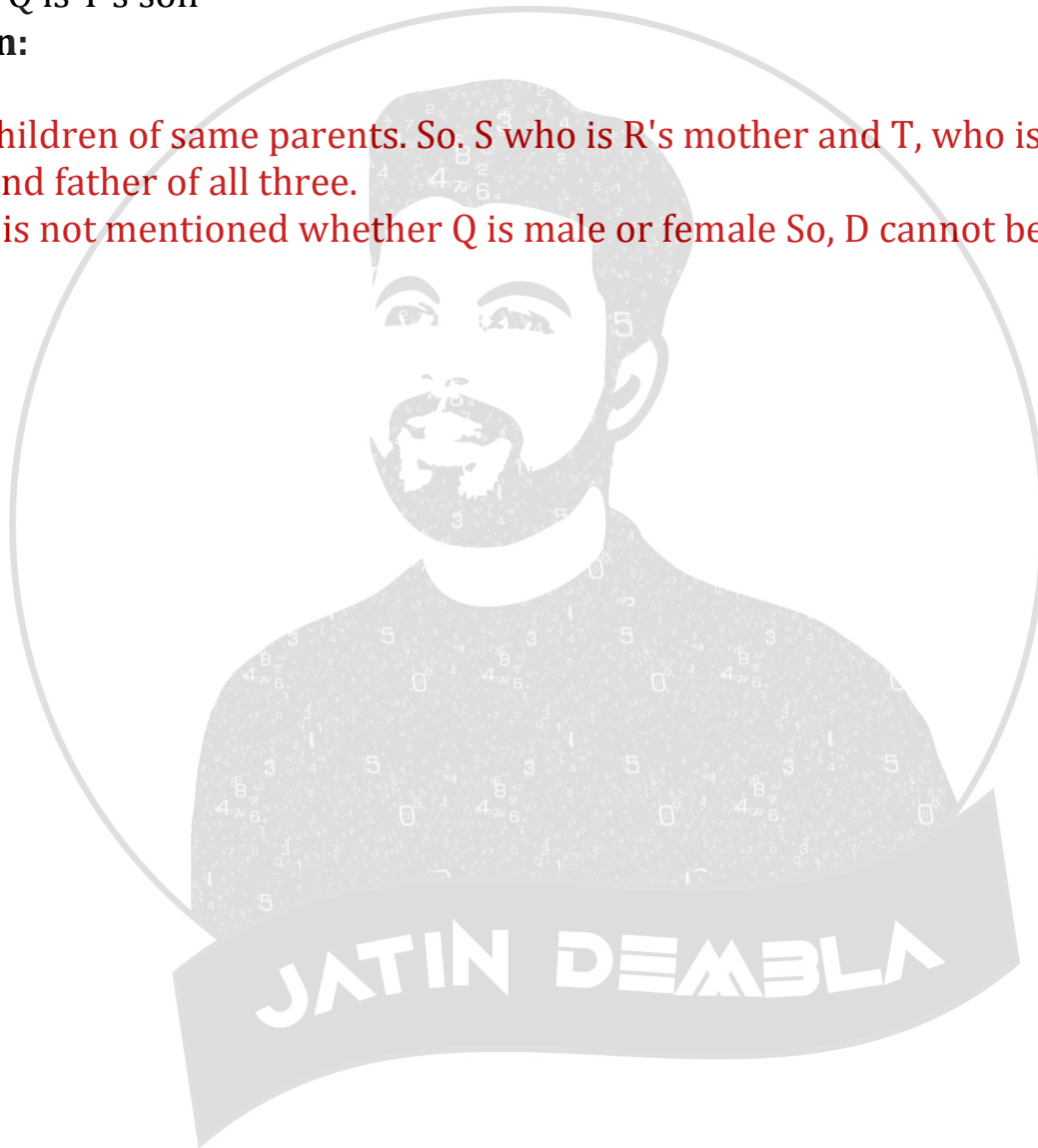
Q is T's son

Answer: D) Q is T's son

Explanation:

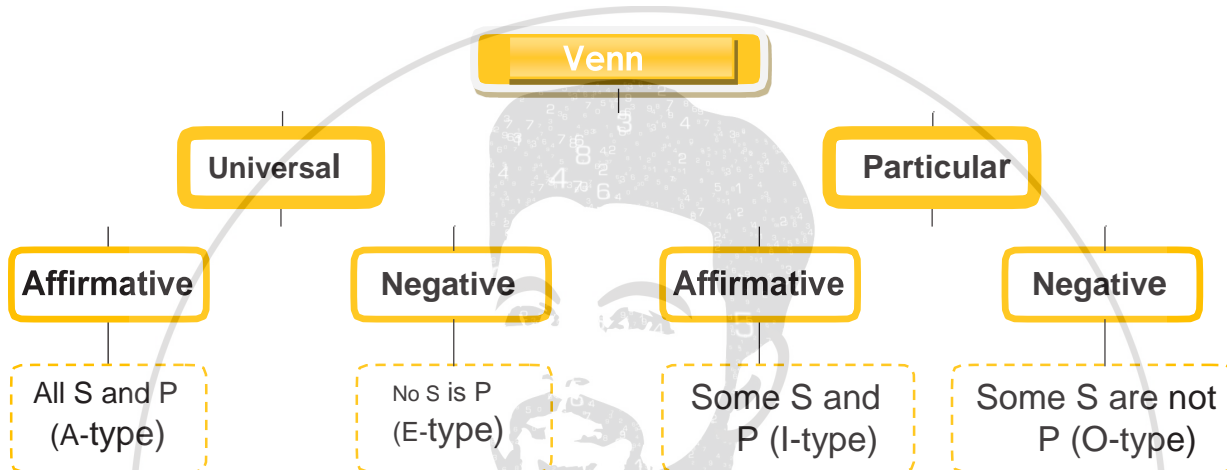
P, Q, R are children of same parents. So, S who is R's mother and T, who is R's father will be mother and father of all three.

However, it is not mentioned whether Q is male or female So, D cannot be definitely true.



CHAPTER 13

SYLLOGISM



SYLLOGISM	'Syllogism' checks basic aptitude and ability of a candidate to derive inferences from given statements using step by step methods of solving problems.	
PROPOSITION	A proposition or premise is grammatical sentence Components of Proposition:	
	Quantifier	The words 'All' 'No' and 'Some' are called quantifiers as they specify a quantity. Keep in mind that 'All' and 'No' are universal quantifiers because they refer to each and every object of a certain set
	Subject	Subject is the part of the sentence something is said about. It is denoted by S.
	Copula	It is that part of a proposition that denotes the relation between subject and predicate.
Predicate	It is that part of a proposition which is affirmed detail about that subject.	

CLASSIFICATION OF PROPOSITION	CATEGORICAL PROPOSITION	• In categorical proposition, there exists a relationship between the subject and the predicate without any condition. It means predicate is either affirmation or denial of the subject unconditionally
	HYPOTHETICAL PROPOSITION	• In a hypothetical proposition, relationship between subject and predicate is asserted conditionally
	DISJUNCTIVE PROPOSITION	• In a disjunctive proposition, the assertion is of alteration

Following are the main rules for solving syllogism problems:

- 1) **All + All = All**
- 2) **All + No = No**
- 3) **All + Some = No conclusion**
- 4) **Some + No = Some Not**
- 5) **Some + Some = No conclusion**
- 6) **No + All = Some not (Reversed)**
- 7) **No + All = Some Not (Reversed)**
- 8) **No + Some = Some Not (Reserved)**
- 9) **No + No = No conclusion**
- 10) **Some Not/ Some not reserved + Anything = No conclusion**
- 11) **If all A are B then we can say - Some B are Not A is a possibility**
- 12) **If Some B are not A then we can say - All A are B is a possibility**
- 13) **If some A are B then we can say All A are B is a possibility. All B are A is a possibility.**
- 14) **No conclusion = Any possibility is true**



1. Statements: Some actors are singers. All the singers are dancers.
Conclusions:

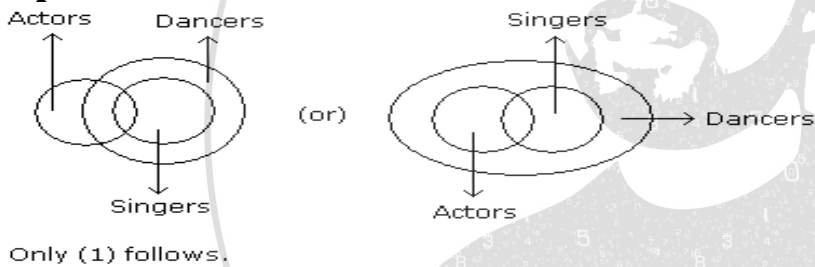
Some actors are dancers.

No singer is actor.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option A

Explanation:



2. Statements: All the harmoniums are instruments. All the instruments are flutes.

Conclusions:

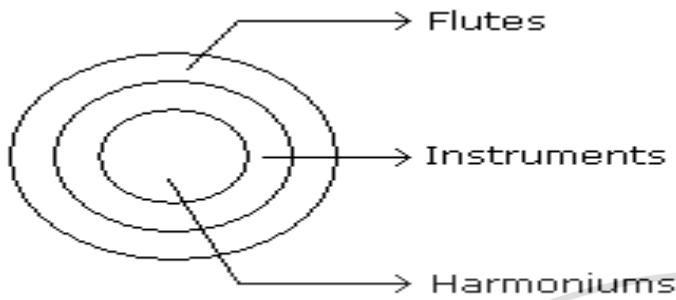
All the flutes are instruments.

All the harmoniums are flutes.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option B

Explanation:



Only (2) follows.

3. Statements: Some mangoes are yellow. Some tixo are mangoes.

Conclusions:

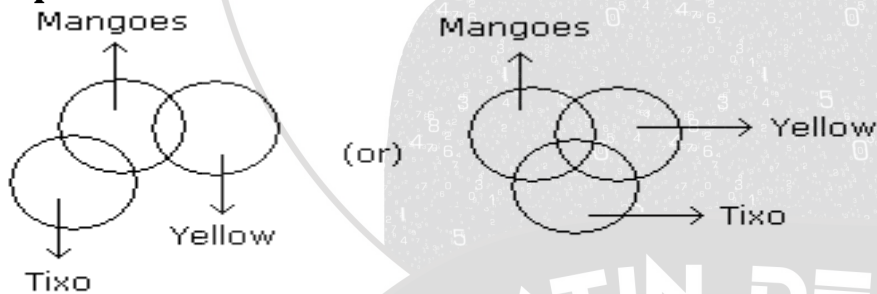
Some mangoes are green.

Tixo is a yellow.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option D

Explanation:



None of the two follows.

4. Statements: Some ants are parrots. All the parrots are apples.

Conclusions:

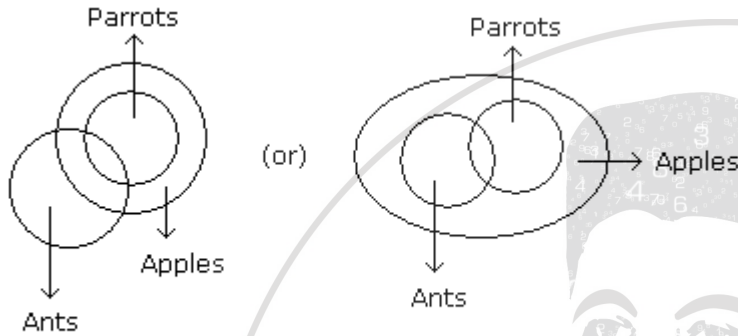
All the apples are parrots.

Some ants are apples.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option B

Explanation:



Only (2) follow.

Statements: Some papers are pens. All the pencils are pens.

Conclusions:

Some pens are pencils.

Some pens are papers.

Only (1) conclusion follows

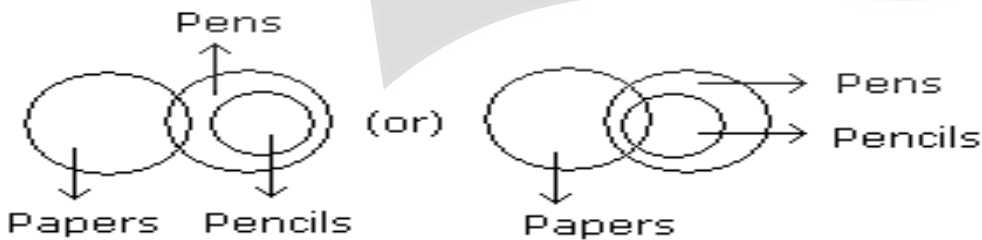
Only (2) conclusion follows

Either (1) or (2) follows

Both (1) and (2) follow

Answer: Option D

Explanation:



Both (1) and (2) follow.

6. Statements: All the actors are girls. All the girls are beautiful.

Conclusions:

All the actors are beautiful.

Some girls are actors.

Only (1) conclusion follows

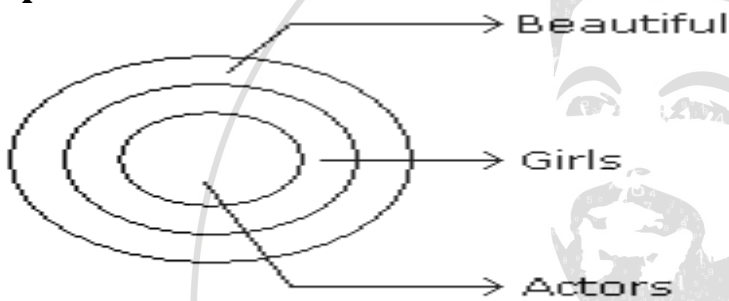
Only (2) conclusion follows

Either (1) or (2) follows

Both (1) nor (2) follows

Answer: Option E

Explanation:



Both (1) and (2) follows.

7. Statements: All the windows are doors. No door is a wall.

Conclusions:

Some windows are walls.

No wall is a door.

a. Only (1) conclusion follows

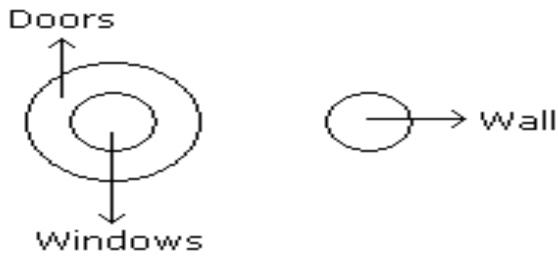
b. Only (2) conclusion follows

c. Either (1) or (2) follows

d. Neither (1) nor (2) follows

Answer: Option B

Explanation:



Only (2) follows.

8. Statements: All cups are books. All books are shirts.

Conclusions:

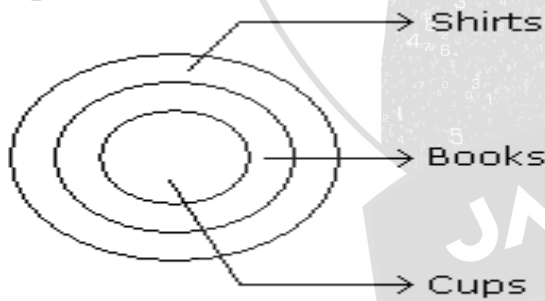
Some cups are not shirts.

Some shirts are cups.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option B

Explanation:



Only (2) follows.

9. Statements: Some cows are crows. Some crows are elephants.

Conclusions:

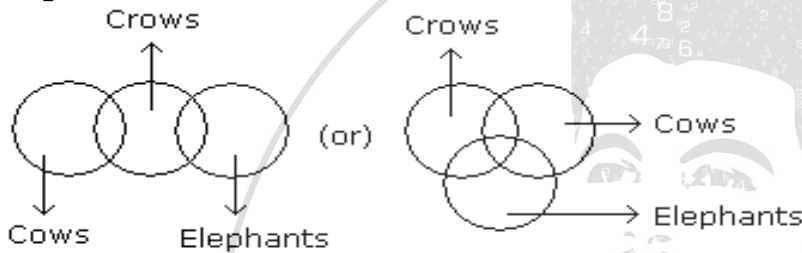
Some cows are elephants.

All crows are elephants.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option D

Explanation:



None of the two follows.

10. Statements: All the pencils are pens. All the pens are inks.

Conclusions:

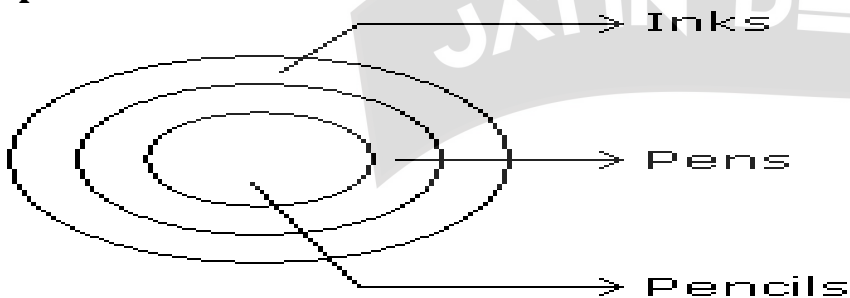
All the pencils are inks.

Some inks are pencils.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Both (1) and (2) follow

Answer: Option D

Explanation:



Both (1) and (2) follow.

11. Statements: Some dogs are bats. Some bats are cats.

Conclusions:

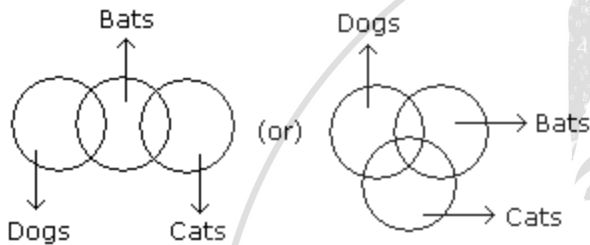
Some dogs are cats.

Some cats are dogs.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option D

Explanation:



None of the two follows.

12. Statements: All the trucks are flies. Some scooters are flies.

Conclusions:

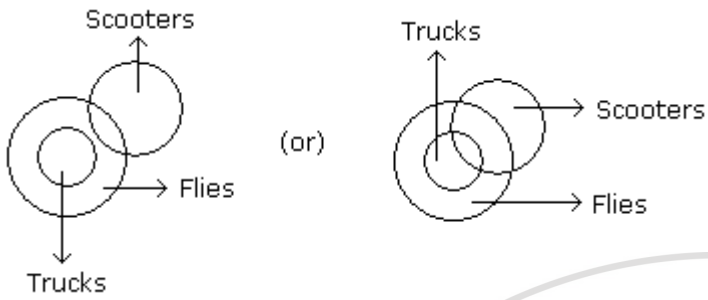
All the trucks are scooters.

Some scooters are trucks.

- a. Only (1) conclusion follows
- b. Only (2) conclusion follows
- c. Either (1) or (2) follows
- d. Neither (1) nor (2) follows

Answer: Option D

Explanation:



Neither (1) nor (2) follows.

13. Statements: All buildings are chalks. No chalk is toffee.

Conclusions:

No building is toffee

All chalks are buildings.

A. Only (1) conclusion follows

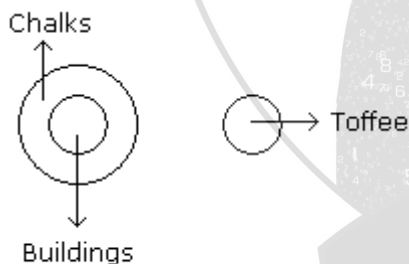
Only (2) conclusion follows

D. Neither (1) nor (2) follows

Either (1) or (2) follows

Answer: Option A

Explanation:



Only (1) follows.

14. Statements: All cars are cats. All fans are cats.

Conclusions:

All cars are fans.

Some fans are cars.

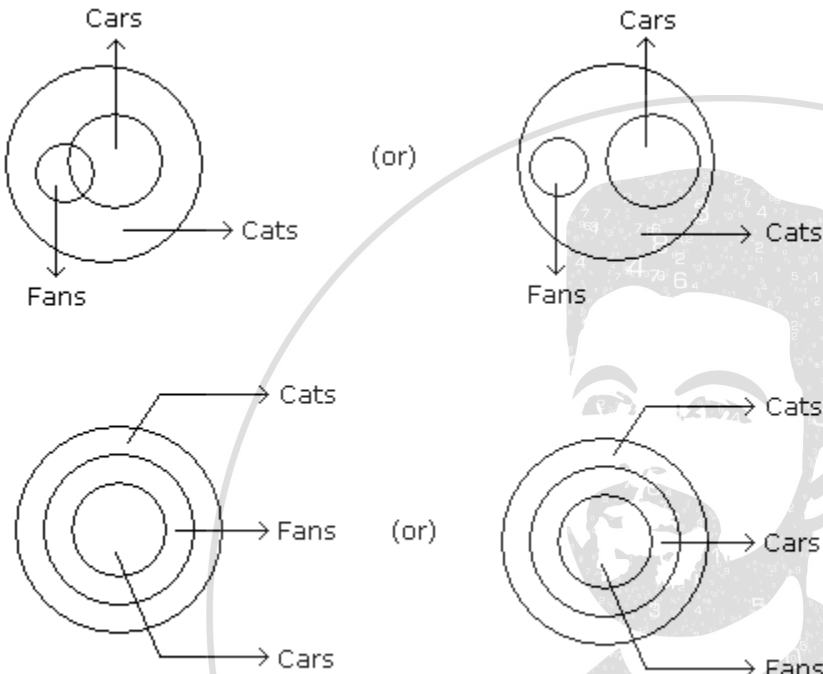
A. Only (1) conclusion follows

Only (2) conclusion follows

C. Either (1) or (2) follows Neither (1) nor (2) follows

Answer: Option D

Explanation:



None of these two follows.

15.. Directions: (15.1 to 15.5): Read the statements carefully and then decide which of the following conclusions does not logically follow.

15.1. Statements:

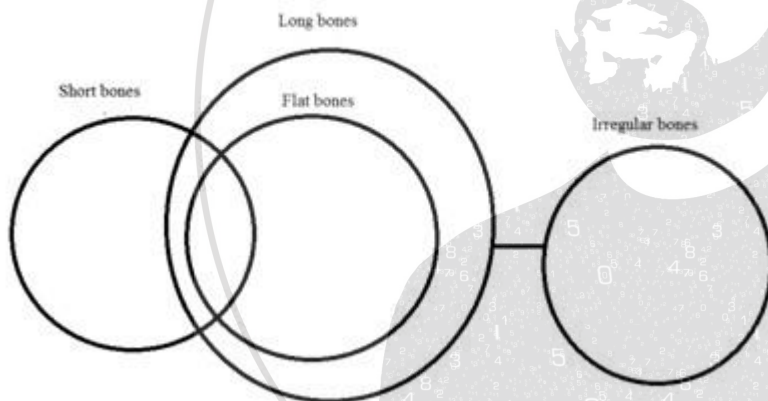
- Some Short bones are flat bones**
- All flat bones are long bones**
- No long bones is irregular bones**

Conclusions:

- i. All irregular bones being short bone is a possibility
 - ii. All short bones being long bones is a possibility
 - iii. All flat bones being short bones is a possibility
 - iv. Some irregular bones are being flat bone is a possibility
- a. Conclusion I does not follow b. Conclusion II does not follow
c. Conclusion III does not follow d. Conclusions IV does not follow

Answer: Option D

Explanation:



15.2. Statements:

All Brills are Barb All barbs are char No char is Dory No

dory is globy Conclusions:

Conclusions:

- i. At least Some barbs are dory
 - ii. Some dory are definitely not char
 - iii. Some brill being not globy is a possibility
- a. Conclusion I does not follow b. Conclusion II does not follow

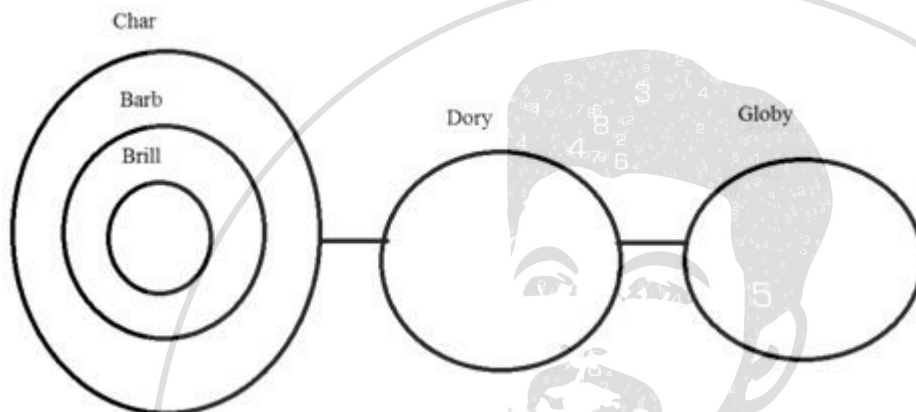
c. Conclusion III does not follow

d. Conclusions I and II does not follow

Answer: Option A

Explanation:

15.3. Statements:



Some Nokia are Redmi

Some Redmi are Samsung

No Samsung is apple

All apple are sony

Conclusions:

- i. Some apple are not redmi
- ii. Frequently Samsung are sony
- iii. Some nokia are Samsung
- iv. No redmi is sony

a. Conclusion I does not follow

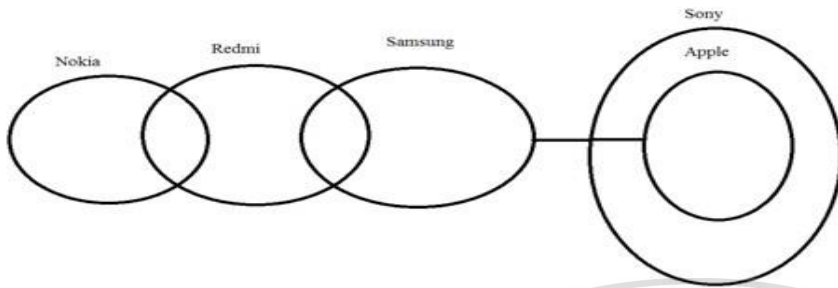
c. Conclusion III does not follow

b. Conclusion II does not follow

d. All conclusions not follows

Answer: Option D

Explanation:



15.4. Statements:

All orange are grapes

Some mango potato

No potato is grape

All apple are orange

Conclusions:

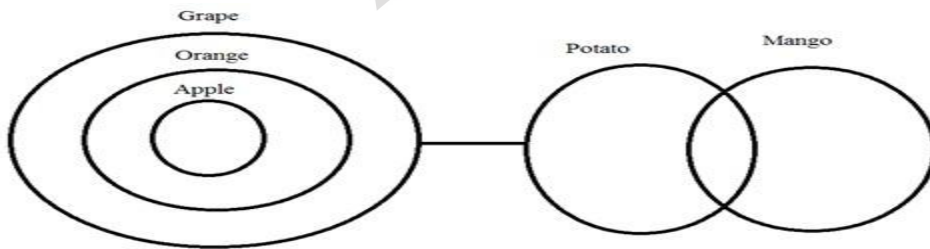
- i. Some mango are not grape
- ii. Some Apple are not mango
- iii. No orange is potato
- iv. No mango is orange

- a. Conclusion I does not follow
- c. Conclusion III does not follow

- b. Conclusion II does not follow
- d. Conclusions II and IV does not follow

Answer: Option D

Explanation:



15.5. Statements:
Some cat are rat

All rat are dog

No dog is bat

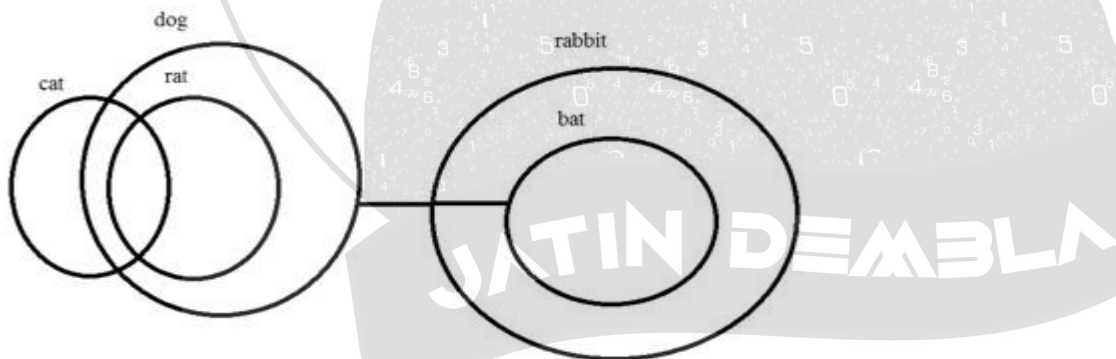
All bat are rabbit

Conclusions:

- i. Some cat are dog
 - ii. Some rat are not rabbit
 - iii. No rat is bat
 - iv. All bat is cat is possibility
- a. Conclusion I does not follow b. Conclusion II does not follow
c. Conclusion III does not follow d. Conclusions IV does not follow

Answer: Option B

Explanation:



16. From the series “Z 5P I J M Q 2 % T @ © UK 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N”. Which of the following is the sixth to the left of the fifteenth from the left end of the given arrangement?

- a. 2
- b. #
- c. %
- d. \$

Answer: Option c

Explanation:

6th to the left of 15th from left = $15 - 6 = 9^{\text{th}}$ from left = %
 Z 5 P I J M Q 2 %

17. From the series "Z 5 P I J M Q 2 % T @ © U K 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N" How many such numbers are there in the given arrangement, each of which is immediately preceded by a consonant and not immediately followed by a letter?
- a. None
 b. One
 c. Two
 d. Three

Answer: Option b

Explanation:

Q2%

18. From the series "Z 5 P I J M Q 2 % T @ © U K 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N" How many such symbols are there in the given arrangement, each of which is immediately followed by a letter but not immediately preceded by a number?
- a. None
 b. One
 c. Two
 d. Three

Answer: Option d

Explanation:

@ © U, G&Z, W\$Y

Z 5 P I J M Q 2 % T @ © U K 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N

19. From the series "Z 5 P I J M Q 2 % T @ © U K 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N" How many such consonants are there in the given arrangement, each of which is immediately preceded by a consonant and immediately followed by a symbol?
- a. None
 b. One
 c. Two
 d. Three

Answer: Option a

20. From the series “Z 5P I J M Q 2 % T @ © UK 5 V 1 W \$ Y 2 B E 6 # 9 D H 8 G & Z N” What should come in the place of question mark (?) in the following series based on the above arrangement? P J Q T © K 1 \$ 2 ?
- a. E#D
 - b. 69D
 - c. 698
 - d. 69H

Answer: Option d

21. Directions: In question below, there are three statements followed by four conclusions numbered I, II, III, and IV. You have to take the given statements to be true even if they seem to be at variance with commonly known facts and then decide which of the given conclusions logically follow(s) from the given statements.

Statements:

All table are laptop.

Some laptops are computer.

All computers are smart phones.

Conclusions:

Some table are computer.

Some table are not computer.

Some laptops are smart phone.

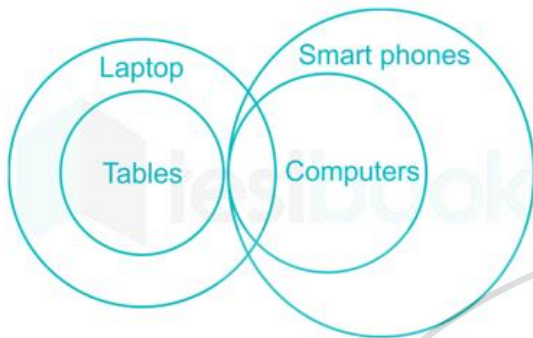
Some laptops are not smart phone.

- a. Either I or II follow
- b. Either III or IV follow
- c. Either I or II and Either III or IV follow
- d. Either I or II and III follow

Answer: Option d

Explanation:

Let us draw Venn diagram for all given statements.



Conclusions:

Some table are computer. ⇒ Not true because it is possible but not definite.
 Some table are not computer. ⇒ Not true because it is possible but not definite.
 Some laptops are smart phone. ⇒ It's, definitely true.
 Some laptops are not smart phone. ⇒ Not true because it is possible but not definite.
 Also according to Venn diagram conclusion (I) and (II) form pairs i.e. either of them must be true in any diagram we can draw.
 Hence, either I or II and III follow.

22. Directions: In question below are two statements followed by two conclusions numbered I and II. You have to take the two given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

Statements:

No picnic is blades.
 Some blades are CD's.

Conclusions:

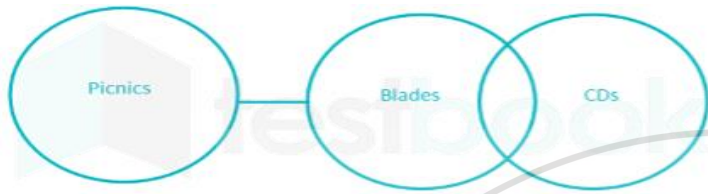
No CD is a picnic.
 Some picnics are definitely not CD's.

- a. Only conclusion I follows
- b. Only conclusion II follows
- c. Either conclusion I or II follows
- d. Neither conclusion I nor II follows

Answer: Option d

Explanation:

Let's draw a least possible Venn diagram using given statements:



Conclusions:

No CD is a picnic: It is possible but not definite.

Some picnics are definitely not CD's: It is possible but not definite.

So, none of the conclusions follow.

23. Directions: In the question below there are three statements followed by three conclusions numbered I, II and III. Read all the conclusions and then decide which of the given conclusions logically follows from the given statements.

Statements:

- All fruits are vegetables.
- All pens are vegetables.
- All vegetables are rains.

Conclusions:

- All fruits are rains.
- All pens are rains.
- Some rains are vegetables.
- a. None follows
- c. Only II and III follow

- b. Only I and II follow
- d. All follow

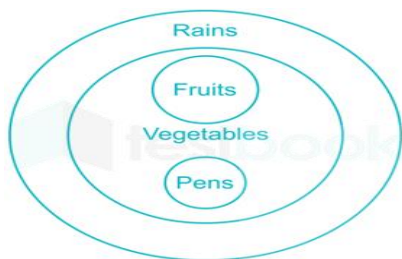
Answer: Option d

Explanation:

Note: Here, a conclusion is definite if it can be shown in a diagram drawn with least -

possibilities. If a conclusion can't be shown in least - possibilities diagram then the conclusion is possible but not definite.

On drawing least - possibilities Venn - diagram:



Conclusions:

All fruits are rains. ⇒ It's, definitely, true.

All pens are rains. ⇒ It's, definitely, true.

Some rains are vegetables. ⇒ It's, definitely, true.

Hence, all conclusions follow.

24. Directions: In the question below is given three statements followed by two conclusions numbered I and II. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read both of the conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

Statements:

Some books are pens.
Some pens are pencils.
Some pencils are buttons.

Conclusions:

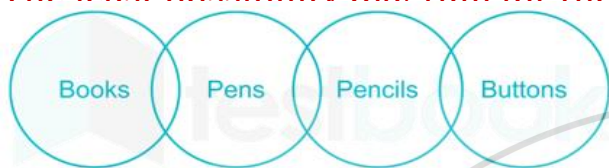
Some buttons are definitely pens.
Some pencils are books.

- | | |
|--|--------------------------------------|
| a. Only conclusions I follows | b. Only conclusions II follows |
| c. Neither conclusion I nor II follows | d. Either conclusion I or II follows |

Answer: Option c

Explanation:

The least possibility diagram for the given statement is follows:



Conclusions:

Some buttons are definitely pens. → Its possible but not definitely true, hence false.

Some pencils are books. → Its possible but not definitely true, hence false.

Hence, neither of the conclusions I nor II follows.

25. Directions: In the question below are two statements followed by two conclusions numbered I and II. You have to take the two given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the two statements disregarding commonly known facts.

Statements:

Some students are classes.

Some classes are Schools.

Conclusions:

At least some schools are students

No school is student

- | | |
|--------------------------------------|--|
| a. Only conclusion I follows | b. Only Conclusion II follows |
| c. Either Conclusion I or II follows | d. Neither Conclusion I nor II follows |

Answer: Option c

Explanation:

The Venn diagrams are as follows:



Conclusions:

- I) At least some schools are students → Its possible but not definitely true, hence false.
- II) No school is student → Its possible but not definitely true, hence false.

Conclusions I & II are complementary to each other.

Hence, either of the conclusion I or II follows.

26. Directions: In the question below are three statements followed by three conclusions numbered I, II and III. You have to take the three given statement to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusion logically follows from the three given statements, disregarding commonly known facts.

Statements:

Some doors are windows.
Some windows are lamps.
All lamps are candles.

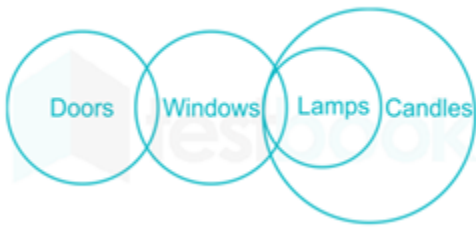
Conclusions:

Some candles being door is a possibility
Some candle are definitely windows
At least some lamps are doors

- | | |
|---------------------|--------------------|
| a. Only I follows | b. Only II follows |
| c. Only III follows | d. I and II follow |

Answer: Option D

Explanation:



Conclusions:

Some candles being door is a possibility → It's possible. Hence possibility is true.

Some candles are definitely windows → It's definitely possible. Hence, true.

At least some lamps are doors → Its possible but not definitely true hence false.

Hence only conclusions I and II follows.

27. **Directions:** In the question below are three statements followed by three conclusions numbered I, II and III. You have to take the three given statement to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusion logically follows from the three given statements, disregarding commonly known facts.

Statements:

Some rivers are hills.

No hill is taxi.

All taxis are buses

Conclusions:

Some buses are rivers

Some rivers are definitely not taxis

No bus is river

a. None follows

c. III follows

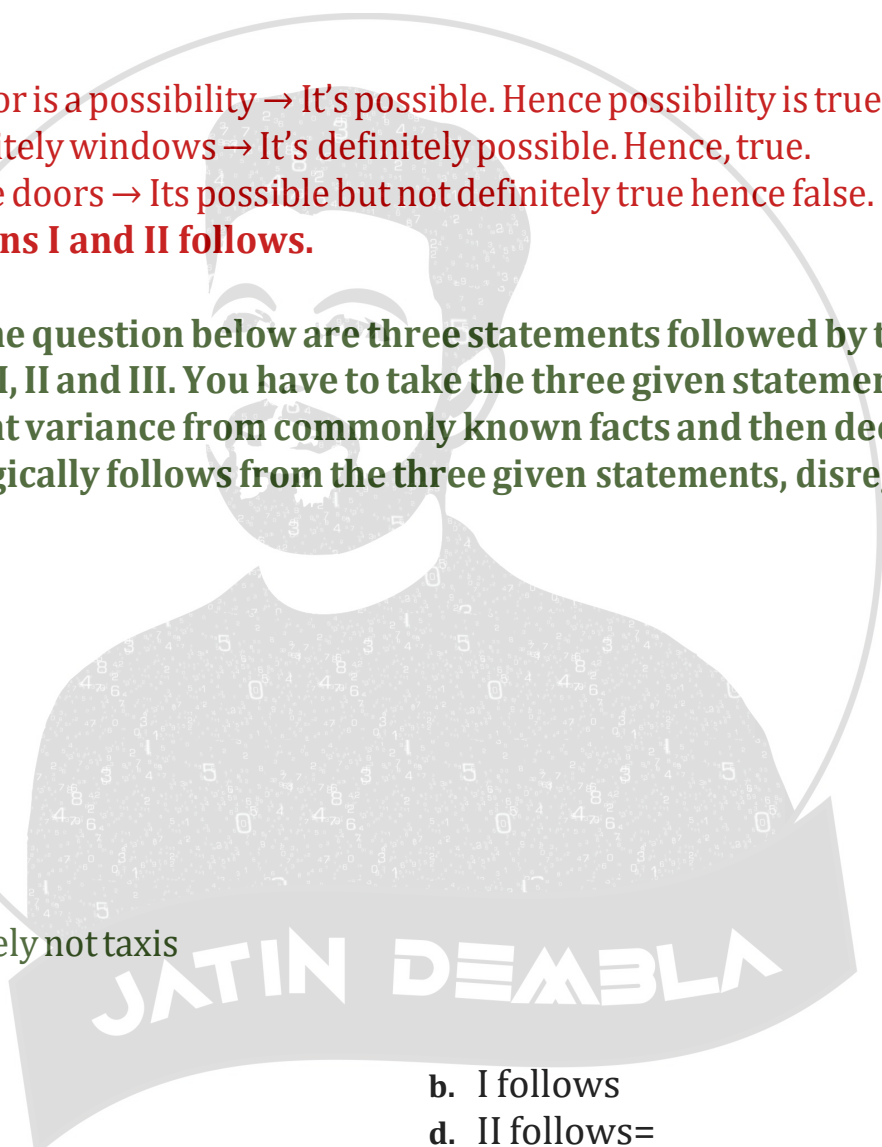
b. I follows

d. II follows=

Answer: Option D

Explanation:

From the statements, the least possible Venn diagram can be drawn as below:





Conclusions:

- I) Some buses are rivers → Its possible but not definitely true, hence false.
- II) Some rivers are definitely not taxis → Some rivers are hills and no hill is taxi, so that much portion of river which is hill will never be taxi. Hence true.
- III) No bus is river → Its possible but not definitely true.



Hence, only conclusion II follows.
 But, since I and III make complementary pair hence ,
 Either III or I also Follow.

So, Statement II and either III or I follows.

28. Directions: In question below are three statements followed by two conclusions numbered I and II. You have to take the three given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusion logically follows from the three statements disregarding commonly known facts.

Statements:

All bulbs are tubes.

Some tubes are knives.

All knives are frames.

Conclusions:

Some frames are tubes.

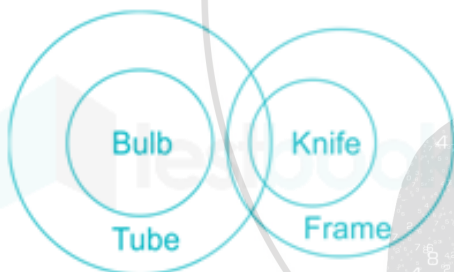
Some knives are bulbs.

- a. Only conclusion I follows.
- b. Only conclusion II follows
- c. Either conclusion I or II follows.
- d. Both conclusions I and II follows

Answer: Option A

Explanation:

The least possibility Venn diagram for the given statements is as follows.



Conclusions:

Some frames are tubes → clearly true.

Some knives are bulbs → it's possible but not definite, hence false.

Hence only conclusion I follows.

29. Directions: In the question below is given three statements followed by two conclusions numbered I and II. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read both of the

conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

Statements:

No toffee is chocolate.

Some chocolates are ice-creams.

All ice-creams are candies.

Conclusions:

No candy is toffee.

Atleast some candies are chocolates.

- a. Only conclusions I follows
- b. Only conclusions II follows
- c. Either conclusion I or II follows
- d. Neither conclusion I nor II follows

Answer: Option B

Explanation:

From the given statements, the least possibility Venn diagram is as follows.



Conclusions:

No candy is toffee. → Its possible but not definitely true, hence false.

Atleast some candies are chocolates. → Its definitely possible. Hence, true.

Hence only conclusion II is follows.

30. **Directions:** In the question below is given three statements followed by two conclusions numbered I and II. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read both of the conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

Statements:

All letters are black.
 All black are blue.
 No blue is green.

Conclusions:

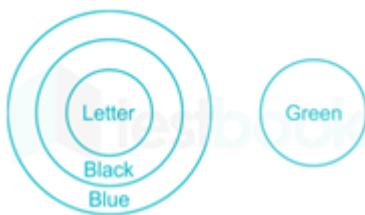
No letter is green.
 Some black are blue.

- a. Only conclusions I follows
- b. Only conclusions II follows
- c. Either conclusion I or II follows
- d. Both conclusions I and II follow

Answer: Option D

Explanation:

From the given statements, the least possibility Venn diagram is as follows.



Conclusions:

No letter is green. → Its definitely possible. Hence, true.

Some black are blue. → As all black are blue is true, so Some black are blue is also true. Hence,

both conclusions I and II follow

31. Statements:

Some Cats are Rats.
All bats are tables.
All Rats are Bats.

Conclusion:

I. Some Cats are bats
II. All bats are rats
III. All tables are cats
All bats are cats

Only I & II follow
Only I & IV follow

Answer: Option A

Explanation:



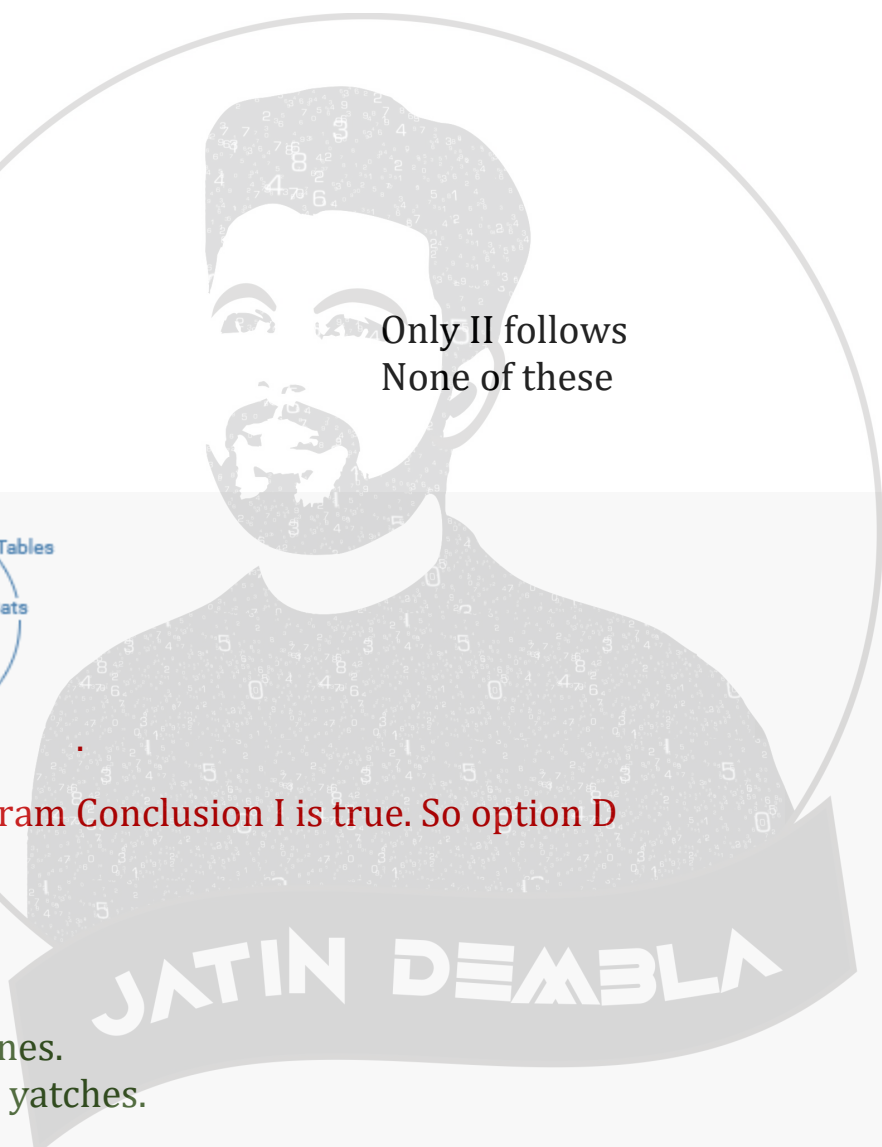
Clearly, from the diagram Conclusion I is true. So option D

32. Statements:

Some ships are boats.
All boats are submarines.
Some submarines are yatches.

Conclusion:

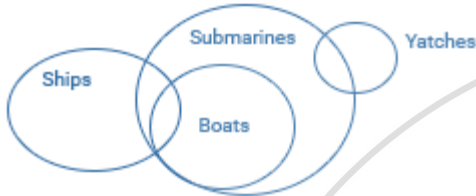
I. Some yatches are boats.
II. Some submarines are boats.
III. Some submarines are ships.
IV. Some yatches are ships



- a. All follow
- b. Only II and III follow
- c. Only III follows
- d. Only IV follows

Answer: Option B

Explanation:



From the diagram we can infer that some submarines are boats and some submarines are ships. So 2nd option.

33. Statements:

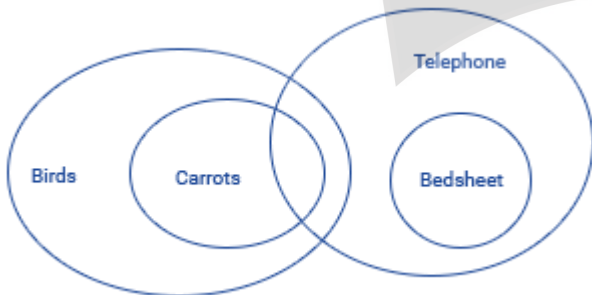
- All Carrots are birds.
- Some telephones are Carrots.
- All bedsheets are telephone.

Conclusion:

- I. All bedsheet are birds
 - II. Some bedsheet are birds
 - III. Some birds are telephone
 - IV. All telephone are birds
- a. Only I follows
 - b. Only II follows
 - c. Only I and III follow
 - d. Only III follows

Answer: Option D

Explanation:



JATIN DEMBLA

The diagram gives all the possibilities. But only conclusion III is true.

34. Statements:

Most CPUs are keyboards.
 No keyboard is a Mouse.
 All Mouses are CPU.

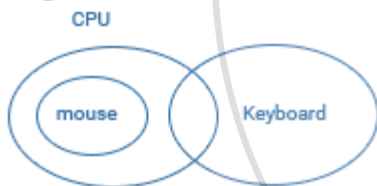
Conclusion:

I. Some keyboards are CPU
 II. All CPU's are Mouse
 III. No Mouse is a keyboard
 Some Mouse are keyboard

- a. Only I follows
- b. Only II and III follow
- c. Only I and III follow
- d. Only II follows

Answer: Option C

Explanation:



Clearly from the diagram, I and III are true.

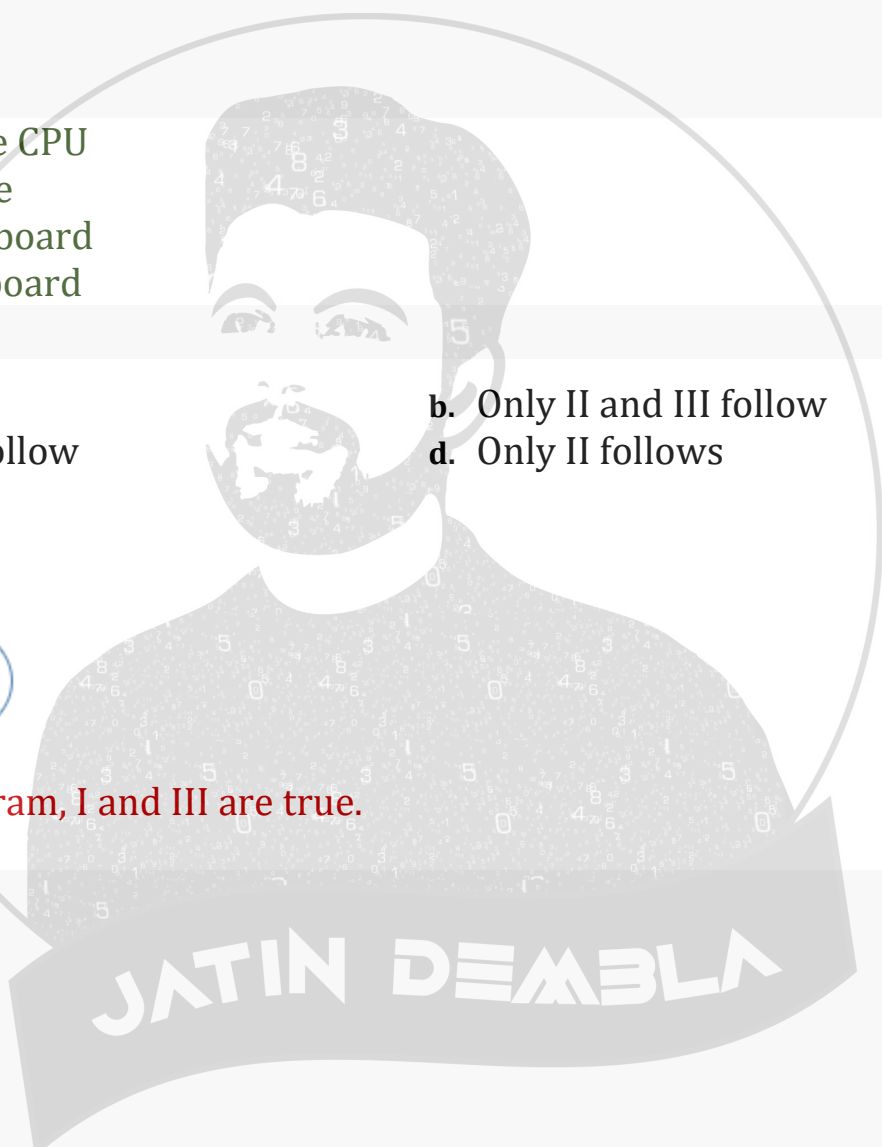
35. Statements:

Samosas are Jalebi.
 All Jalebis are Tikki.
 All Tikkis are Barfi.

Conclusion:

I. All Jalebis are Barfi
 II. All Tikkis are Samosas
 III. All Samosas are Barfi
 IV. All Barfi are Jalebi

- a. Only I and II follow
- b. Only I and III follow
- c. Only II and III follow
- d. All follow



Answer: Option B

Explanation:



Clearly from the diagram, I and III are true.

36. Statements:

Some eyes are ears. Some ears are lungs.

All lungs are hands

Conclusion:

I. Some hands are eyes.

II. Some hands are ears

III. Some lungs are eyes

IV. No hand is eye

a. None follow

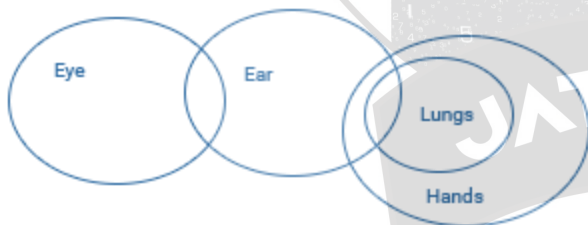
c. Only II follows

b. Only IV follows

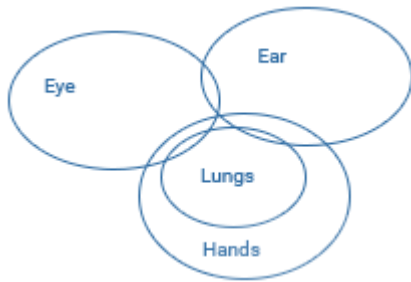
d. Only III follows

Answer: Option C

Explanation:



OR



From the diagram II definitely follows

37. Statements:

- All liquids are solids.
- Some solids are gases.
- All gases are clouds.

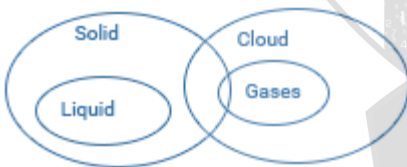
Conclusion:

- I. Some clouds are solids
- II. Some clouds are liquids
- III. Some gases are liquids
- Some solids are clouds

- a. Only I follows
- b. Only III follows
- c. Only IV follows
- d. Only II and IV follow

Answer: Option D

Explanation:.



Clearly from the diagram I and IV are true

38. Statements:

- All Gold are Platinum.
- No Platinum is silver.
- Some Diamonds are silver.

Conclusion:

- I. Some Diamonds are Gold



- II. Some Diamonds are Platinum
- III. Some Gold are Silver
- IV. No Silver is Gold

- e. Only I follows
- g. Only IV follows

- f. Only III follows
- h. Only II and IV follow

Answer: Option C

Explanation:



From the diagram we see all the possibilities. Clearly only IV is true for all possibilities

39. Statements:

- Some messages are whatsapp.
- All Hikes are whatsapp.
- All whatsapp are facebook.

Conclusion:

- I. Some Facebook are messages
- II. All hikes are Facebook
- III. Some messages are hikes
- IV. Some message are Facebook
- a. All follow
- c. Only I, II and IV follow

- b. Only I, II and III follow
- d. Only III and IV follow

Answer: Option C

Explanation:



Clearly from the diagram I, II and IV are true.

40. Statements:

No watch is cycle.

No cycle is Motorbike.

Some auto are motorbike

Conclusion:

I. No Motorbike is watch

II. No motor bike is cycle

III. Some cycles are watches

IV. All Motorbikes are watches

a. None follows

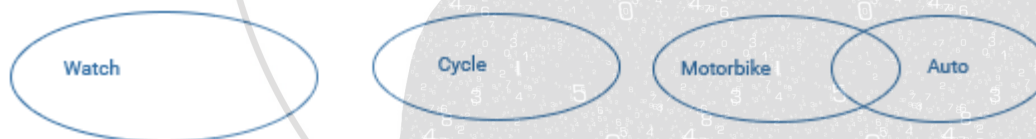
c. Only I and III follow

b. Only I follows

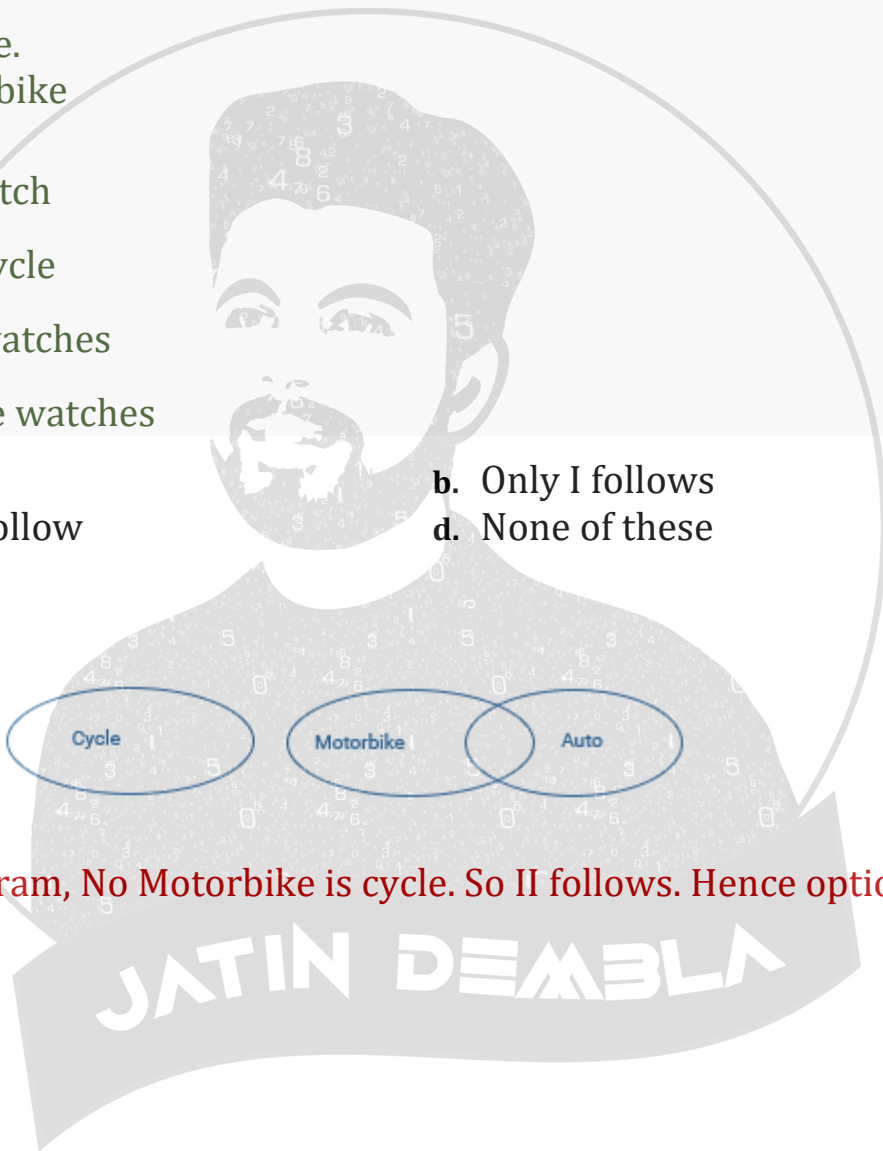
d. None of these

Answer: Option D

Explanation:



Clearly from the diagram, No Motorbike is cycle. So II follows. Hence option 4.

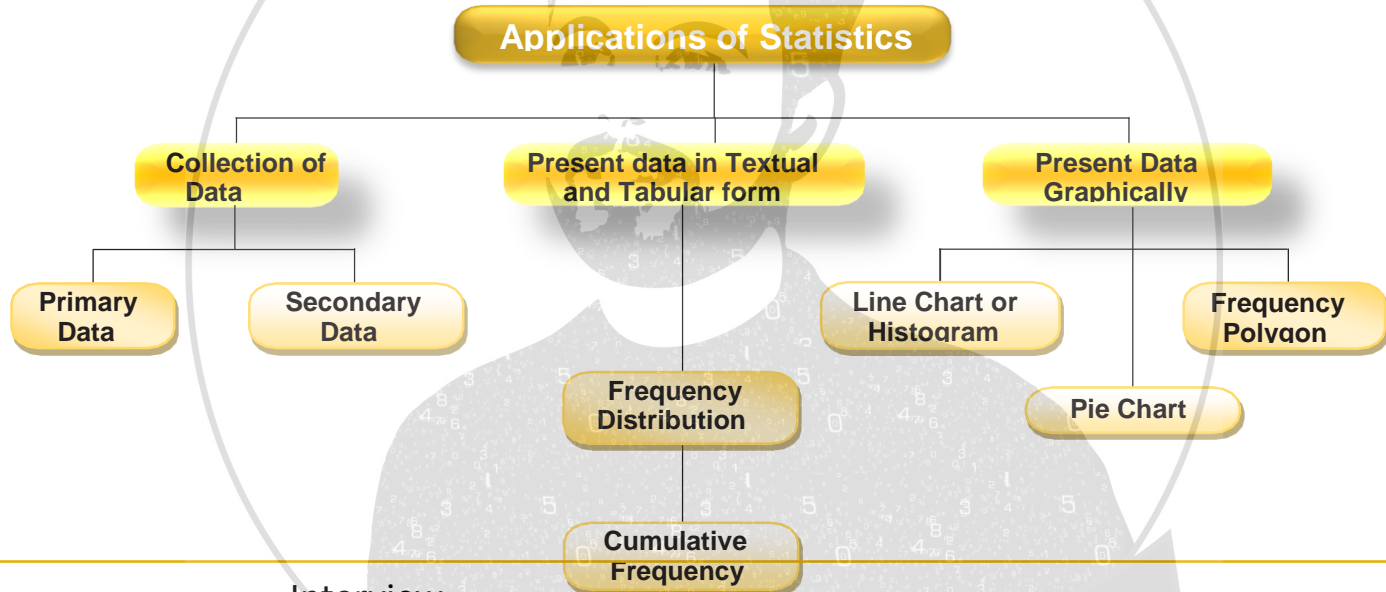




CHAPTER 14

STATISTICAL DESCRIPTION OF DATA

COLLECTION OF DATA



Primary method

- Interview
- Mailed questionnaire
- Observation
- Questionnaires filled and sent by enumerators.

Secondary method

- International sources
- Unpublished sources of various research institutes, researchers
- Government sources
- Private and quasi-government sources

PRESENTATION OF DATA	Classification or Organisation of Data	The process of arranging data on the basis of the characteristic under consideration into a number of groups or classes according to the similarities of the observations.
	Data may be classified as	(i) Chronological or Temporal or Time Series Data; (ii) Geographical or Spatial Series Data; (iii) Qualitative or Ordinal Data; (iv) Quantitative or Cardinal Data.
	Mode of Presentation of Data	(a) Textual presentation (b) Tabular presentation or Tabulation (c) Diagrammatic representation I. Line diagram or Historiogram II. Bar diagram III. Pie chart
FREQUENCY DISTRIBUTION	tabular representation of statistical data, usually in an ascending order, relating to a measurable characteristic according to individual value or a group of values of the characteristic under study	
Class Limit (CL)	Corresponding to a class interval, the class limits may be defined as the minimum value and the maximum value the class interval may contain.	
Class Boundary (CB)	Class boundaries may be defined as the actual class limit of a class interval $LCB = LCL - \frac{D}{2}$	
Mid-point or Mid-value or class mark	Corresponding to a class interval, this may be defined as the total of the two class limits or class boundaries to be divided by 2. Thus, we have $\text{mid-point} = \frac{LCL + UCL}{2}$	

Cumulative Frequency	The cumulative frequency corresponding to a value for a discrete variable and corresponding to a class boundary for a continuous variable may be defined as the number of observations less than the value or less than or equal to the class boundary.
GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION	<ul style="list-style-type: none"> • Histogram or Are a diagram: A histogram is an accurate representation of the distribution of numerical data. It is an estimate of the probability distribution of a continuous variable (quantitative variable) and was first introduced by Karl Pearson. • Frequency Polygon: Frequency polygons are a graphical device for understanding the shapes of distributions. They serve the same purpose as histograms, but are especially helpful for comparing sets of data. Frequency polygons are also a good choice for displaying cumulative frequency distributions. • Ogives or Cumulative Frequency Graph: Cumulative histograms, also known as ogives, are graphs that can be used to determine how many data values lie above or below a particular value in a data set. The cumulative frequency is calculated from a frequency table, by adding each frequency to the total of the frequencies of all data values before it in the data set.
Frequency Curve	<p>A frequency curve is a smooth curve for which the total area is taken to be unity. It is a limiting form of a histogram or frequency polygon.</p> <p>Types of frequency curves namely:</p> <ol style="list-style-type: none"> (a) Bell-shaped curve (b) U-shaped curve (c) J-shaped curve (d) Mixed curve.
STATISTICS	The term statistics is ultimately derived from the New Latin <i>statisticum collegium</i> ("council of state") and the Italian word <i>statista</i> ("statesman" or "politician"). ... Thus, the original principal purpose of Statistik was

$$\left[\left(\frac{10\% \text{ of } 8550}{17\% \text{ of } 8550} \right) \times 100 \right] \%$$

$$(iv) S = \left[\left(\frac{16\% \text{ of } 5700}{17\% \text{ of } 8550} \right) \times 100 \right] \% = 62.75\%.$$

$$(v) T = \left[\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \right) \times 100 \right] \% = 75\%.$$

$$(vi) V = \left[\left(\frac{15\% \text{ of } 5700}{12\% \text{ of } 8550} \right) \times 100 \right] \% = 83.33\%.$$

$$(vii) X = \left[\left(\frac{12\% \text{ of } 5700}{16\% \text{ of } 8550} \right) \times 100 \right] \% = 50\%.$$

Highest of these is 86.67% corresponding to institute R.

3. The number of candidates passed from institutes S and P together exceeds the number of candidates enrolled from institutes T and R together by:

- | | |
|--------|--------|
| a. 288 | b. 279 |
| c. 399 | d. 407 |

Answer: Option C

Explanation:

Required difference = [(16% + 18%) of 5700] - [(8% + 10%) of 8550]

$$= [(34\% \text{ of } 5700) - (18\% \text{ of } 8550)]$$

$$= (1938 - 1539)$$

$$= 399.$$

4 What is the percentage of candidates passed to the candidates enrolled for institutes Q and R together?

- | | |
|-----|-----|
| 68% | 80% |
| 74% | 65% |

Answer: Option B

Explanation:

Candidates passed from institutes Q and R together = $[(13\% + 17\%) \text{ of } 5700]$
 $= 30\% \text{ of } 5700.$

Candidates enrolled from institutes Q and R together = $[(15\% + 10\%) \text{ of } 8550]$
 $= 25\% \text{ of } 8550.$

∴ Required Percentage

$$= \left(\frac{30\% \text{ of } 5700}{25\% \text{ of } 8550} \times 100 \right) \%$$

$$= \left(\frac{30 \times 5700}{25 \times 8550} \times 100 \right) \%$$

$$= 80\%.$$

5. What is the ratio of candidates passed to the candidates enrolled from institute P?

9 : 11

14 : 17

6 : 11

9 : 17

Answer: Option C

Explanation:

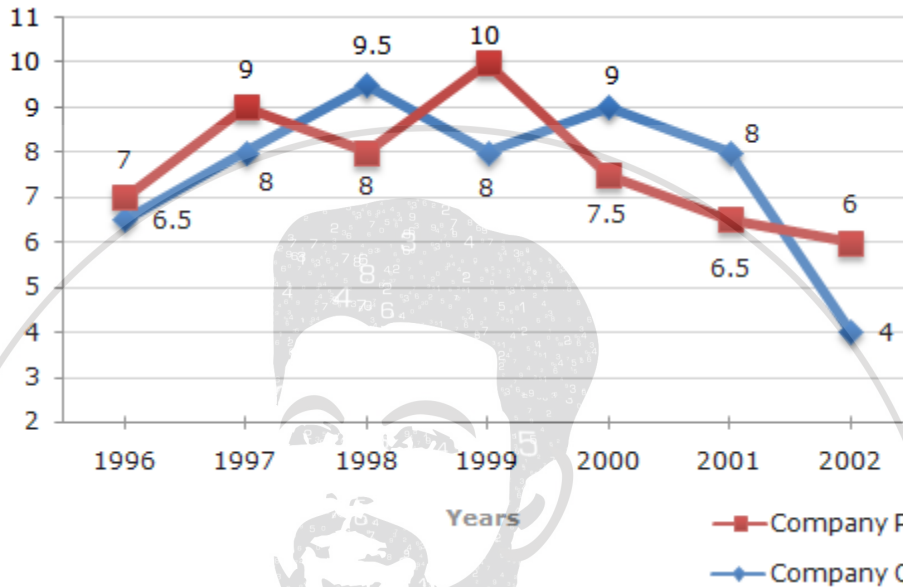
$$\text{Required ratio} = \left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550} \right) = \left(\frac{18 \times 5700}{22 \times 8550} \right) = \frac{6}{11}.$$

Direction (for Q.Nos'. 6 - 10):

Two different finance companies declare fixed annual rate of interest on the amounts invested with them by investors. The rate of interest offered by these companies may differ from year to year depending on the variation in the economy of the country and

the banks rate of interest. The annual rate of interest offered by the two Companies P and Q over the years are shown by the line graph provided below.

Annual Rate of Interest Offered by Two Finance Companies Over the Years.



6. A sum of Rs. 4.75 lakhs was invested in Company Q in 1999 for one year. How much more interest would have been earned if the sum was invested in Company P?

- a. Rs. 19,000
- b. Rs. 14,250
- c. Rs. 11,750
- d. Rs. 9500

Answer: Option D

Explanation:

Difference

$$\begin{aligned}
 &= \text{Rs. } [(10\% \text{ of } 4.75) - (8\% \text{ of } 4.75)] \text{ lakhs} \\
 &= \text{Rs. } (2\% \text{ of } 4.75) \text{ lakhs} \\
 &= \text{Rs. } 0.095 \text{ lakhs} \\
 &= \text{Rs. } 9500.
 \end{aligned}$$

7 If two different amounts in the ratio 8:9 are invested in Companies P and Q respectively in 2002, then the amounts received after one year as interests from Companies P and Q are respectively in the ratio?

- a. 2:3
- b. 3:4
- c. 6:7
- d. 4:3

Answer: Option D

Explanation:

Let the amounts invested in 2002 in Companies P and Q be Rs. $8x$ and Rs. $9x$ respectively.

$$= \text{Rs. (6\% of } 8x)$$

Then, interest received after one year from Company P

$$= \text{Rs. } \frac{48}{100} x.$$

and interest received after one year from Company Q

$$= \text{Rs. (4\% of } 9x)$$

$$= \text{Rs. } \frac{36}{100} x.$$

$$\therefore \text{ Required ratio} = \frac{\left(\frac{48}{100}x\right)}{\left(\frac{36}{100}x\right)} = 4 \frac{=3}{=3}$$

8. In 2000, a part of Rs. 30 lakhs was invested in Company P and the rest was invested in Company Q for one year. The total interest received was Rs. 2.43 lakhs. What was the amount invested in Company P?

Rs. 9 lakhs

Rs. 11 lakhs

Rs. 12 lakhs

Rs. 18 lakhs

Answer: Option D

Explanation:

Let Rs. x lakhs be invested in Company P in 2000, the amount invested in Company Q in 2000 = Rs. $(30 - x)$ lakhs.

Total interest received from the two Companies after 1 year

$$= \text{Rs. } [(7.5\% \text{ of } x) + \{9\% \text{ of } (30 - x)\}] \text{ lakhs}$$

$$= \text{Rs.} \left[2.7 - \left(\frac{1.5x}{100} \right) \right] \text{ lakhs.}$$

$$\therefore \left[2.7 - \left(\frac{1.5x}{100} \right) \right] = 2.43 \Rightarrow x = 18.$$

9. An investor invested a sum of Rs. 12 lakhs in Company P in 1998. The total amount received after one year was re-invested in the same Company for one more year. The total appreciation received by the investor on his investment was?

- a. Rs. 2,96,200
c. Rs. 2,25,600

- b. Rs. 2,42,200
d. None

Answer: Option C

Explanation:

Amount received from Company P after one year (i.e., in 1999) on investing Rs. 12 lakhs in it

$$= \text{Rs.} [12 + (8\% \text{ of } 12)] \text{ lakhs}$$

$$= \text{Rs.} 12.96 \text{ lakhs.}$$

Amount received from Company P after one year on investing Rs. 12.96 lakhs in the year 1999

$$= \text{Rs.} [12.96 + (10\% \text{ of } 12.96)] \text{ lakhs}$$

$$= \text{Rs.} 14.256.$$

Appreciation received on investment during the period of two years

$$= \text{Rs.} (14.256 - 12) \text{ lakhs}$$

$$= \text{Rs.} 2.256 \text{ lakhs}$$

$$= \text{Rs.} 2,25,600.$$

10. An investor invested Rs. 5 lakhs in Company Q in 1996. After one year, the entire amount along with the interest was transferred as investment to Company P in 1997 for one year. What amount will be received from Company P, by the investor?

- a. Rs. 5,94,550

- b. Rs. 5,80,425

c. Rs. 5,77,800

d. Rs. 5,77,500

Answer: Option B

Explanation:

Amount received from Company Q after one year on investment of Rs. 5 lakhs in the year 1996

= Rs. [5 + (6.5% of 5)] lakhs

= Rs. 5.325 lakhs.

Amount received from Company P after one year on investment of Rs. 5.325 lakhs in the year 1997

= Rs. [5.325 + (9% of 5.325)] lakhs

= Rs. 5.80425 lakhs

= Rs. 5,80,425.

Direction (for Q.Nos. 11 - 15):

The following table gives the sales of batteries manufactured by a company over the years.

**Number of Different Types of Batteries Sold by a Company Over the Years
(Numbers in Thousands)**

Year	Types of Batteries					Total
	4AH	7AH	32AH	35AH	55AH	
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

11. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992?

28%

31%

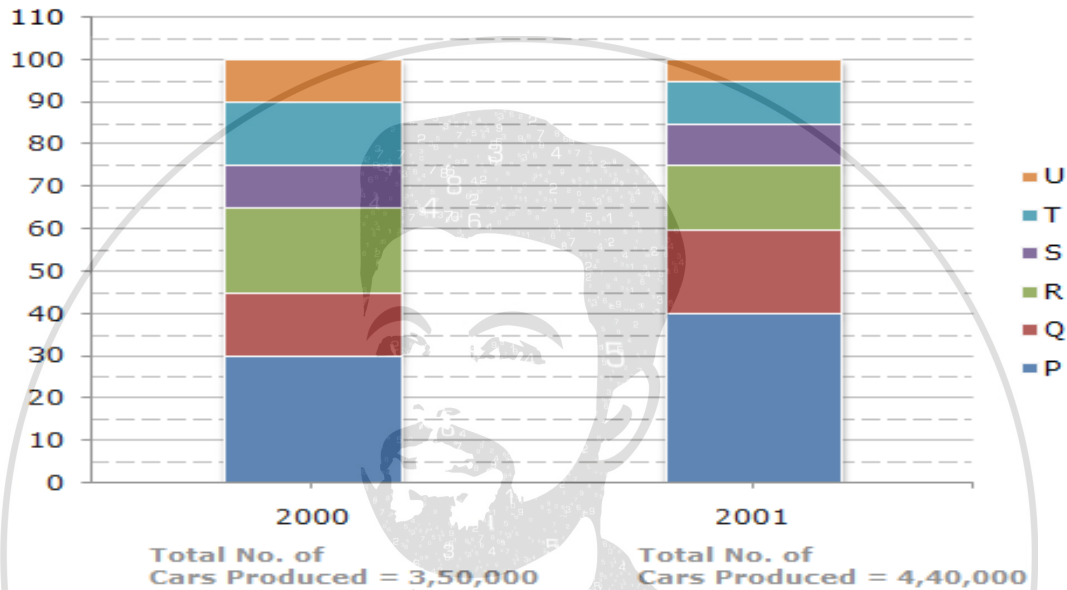
33%

34%

Answer: Option D

The bar graph given below shows the percentage distribution of the total production of a car manufacturing company into various models over two years.

Percentage of Six different types of Cars manufactured by a Company over Two Years



16. What was the difference in the number of Q type cars produced in 2000 and that produced in 2001?

- a. 35,500
- b. 27,000
- c. 22,500
- d. 17,500

Answer: Option A

Explanation:

Total number of Q type cars produced in 2001

$$=(60 - 40)\% \text{ of } 4,40,000 = 88,000.$$

Total number of Q type cars produced in 2000

$$=(45 - 30)\% \text{ of } 3,50,000 = 52,500.$$

$$\therefore \text{Required difference} = (88000 - 52500) = 35,500.$$

17. Total number of cars of models P, Q and T manufactured in 2000 is?

- a. 2,45,000
- b. 2,27,500
- c. 2,10,000
- d. 1,92,500

Answer: Option C

Explanation:

Analysis of the graph:

We shall first determine the number of cars of each model produced by the Company during the two years:

In 2000 : Total number of cars produced = 3,50,000.

$$P = (30 - 0)\% \text{ of } 3,50,000 = 30\% \text{ of } 3,50,000 = 1,05,000.$$

$$Q = (45 - 30)\% \text{ of } 3,50,000 = 15\% \text{ of } 3,50,000 = 52,500.$$

$$R = (65 - 45)\% \text{ of } 3,50,000 = 20\% \text{ of } 3,50,000 = 70,000.$$

$$S = (75 - 65)\% \text{ of } 3,50,000 = 10\% \text{ of } 3,50,000 = 35,000.$$

$$T = (90 - 75)\% \text{ of } 3,50,000 = 15\% \text{ of } 3,50,000 = 52,500.$$

$$U = (100 - 90)\% \text{ of } 3,50,000 = 10\% \text{ of } 3,50,000 = 35,000.$$

In 2001 : Total number of cars produced = 4,40,000.

$$P = (40 - 0)\% \text{ of } 4,40,000 = 40\% \text{ of } 4,40,000 = 1,76,000.$$

$$Q = (60 - 40)\% \text{ of } 4,40,000 = 20\% \text{ of } 4,40,000 = 88,000.$$

$$R = (75 - 60)\% \text{ of } 4,40,000 = 15\% \text{ of } 4,40,000 = 66,000.$$

$$S = (85 - 75)\% \text{ of } 4,40,000 = 10\% \text{ of } 4,40,000 = 44,000.$$

$$T = (95 - 85)\% \text{ of } 4,40,000 = 10\% \text{ of } 4,40,000 = 44,000.$$

$$U = (100 - 95)\% \text{ of } 4,40,000 = 5\% \text{ of } 4,40,000 = 22,000.$$

Total number of cars of models P, Q and T manufacture in 2000

$$= (105000 + 52500 + 52500)$$

$$= 2,10,000.$$

18. If the percentage production of P type cars in 2001 was the same as that in 2000, then the number of P type cars produced in 2001 would have been?

a. 1,40,000

b. 1,32,000

c. 1,17,000

d. 1,05,000

Answer: Option B

Explanation:

If the percentage production of P type cars in 2001

= Percentage production of P type cars in 2000

= 30%.

then, number of P type cars produced in 2001

= 30% of 4,40,000

= 1,32,000.

19. If 85% of the S type cars produced in each year were sold by the company, how many S type cars remain unsold?

a. 7650

b. 9350

c. 11,850

d. 12,250

Answer: Option C

Explanation:

Number of S type cars which remained unsold in 2000 = 15% of 35,000

and number of S type cars which remained unsold in 2001 = 15% of 44,000.

∴ Total number of S type cars which remained unsold

= 15% of (35,000 + 44,000)

= 15% of 79,000

= 11,850

20. For which model the percentage rise/fall in production from 2000 to 2001 was minimum?

a. Q

b. R

c. S

d. T

Answer: Option B

Explanation:

The percentage change (rise/fall) in production from 2000 to 2001 for various models is:

$$\text{For P} = \left[\frac{(176000 - 105000)}{105000} \times 100 \right] \% = 67.62\%, \text{ rise.}$$

$$\text{For Q} = \left[\frac{(88000 - 52500)}{52500} \times 100 \right] \% = 67.62\%, \text{ rise.}$$

$$\text{For R} = \frac{(70000 - 66000)}{66000} \times 100 \% = 5.71\%, \text{ fall.}$$

$$\left[\begin{array}{c} 70000 \\ \end{array} \right]$$

$$\text{For S} = \left[\frac{(44000 - 35000)}{35000} \times 100 \right] \% = 25.71\%, \text{ rise.}$$

$$\text{For T} = \left[\frac{(52500 - 44000)}{52500} \times 100 \right] \% = 16.19\%, \text{ fall.}$$

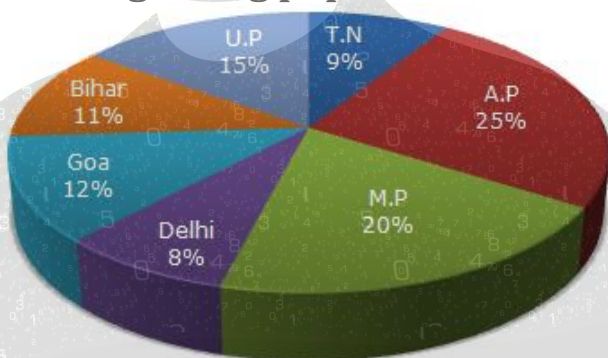
$$\text{For U} = \left[\frac{(35000 - 22000)}{35000} \times 100 \right] \% = 37.14\%, \text{ fall.}$$

∴ Minimum percentage rise/fall in production is the case of model R.

Direction (for Q.Nos. 21 - 24):

Study the following graph and the table and answer the questions given below.

Data of different states regarding population of states in the year 1998



Total population of the given States = 3276000.

States	Sex and Literacy wise Population Ratio			
	Sex		Literacy	
	M	F	Literate	Illiterate
A.P.	5	3	2	7
M.P.	3	1	1	4
Delhi	2	3	2	1
Goa	3	5	3	2
Bihar	3	4	4	1
U.P.	3	2	7	2

Explanation:

$$\text{No. of illiterate people in A.P.} = \left[\frac{7}{9} \text{ of } (25\% \text{ of } 3276000) \right] = 637000.$$

$$\text{No. of illiterate people in M.P.} = \left[\frac{4}{5} \text{ of } (20\% \text{ of } 3276000) \right] = 524160.$$

$$\therefore \text{Total number} = (637000 + 524160) = 1161160.$$

23. What was the number of males in U.P. in the year 1998?

254650

294840

321470

341200

Answer: Option B

Explanation:

$$\text{Number of males in U.P.} = \left[\frac{3}{5} \text{ of } (15\% \text{ of } 3276000) \right]$$

$$= \frac{3}{5} \times \frac{15}{100} \times 3276000$$

$$= 294840.$$

24. If in the year 1998, there was an increase of 10% in the population of U.P. and 12% in the population of M.P. compared to the previous year, then what was the ratio of populations of U.P. and M.P. in 1997?

42 : 55

48 : 55

7 : 11

4 : 5

Answer: Option A

Explanation:

Let x be the population of U.P. in 1997. Then,

Population of U.P. in 1998 = 110% of x = $110x$.

100

Also, let y be the population of M.P. in 1997. Then,

$$\text{Population of M.P. in 1998} = 112\% \text{ of } y = \frac{112}{100} \times y.$$

$$\text{Ratio of populations of U.P. and M.P. in 1998} = \frac{\left(\frac{110}{100} \times x\right)}{\left(\frac{112}{100} \times y\right)} = \frac{110x}{112y}$$

From the pie-chart, this ratio is $\frac{15}{20}$.

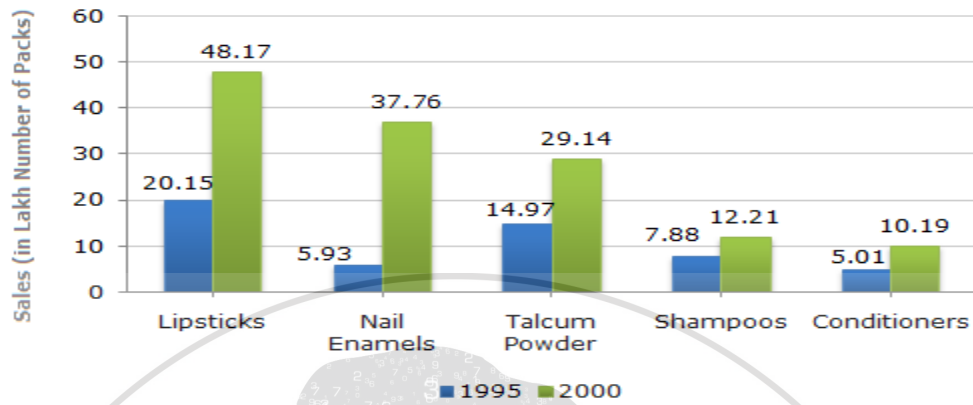
$$\frac{110x}{112y} = \frac{15}{20} \Rightarrow \frac{x}{y} = \frac{15}{20} \times \frac{112}{110} = \frac{42}{55}$$

Thus, ratio of populations of U.P. and M.P. in 1997 = $x : y = 42 : 55$

Direction (for Q.Nos. 25 - 30):

A cosmetic company provides five different products. The sales of these five products (in lakh number of packs) during 1995 and 2000 are shown in the following bar graph.

Sales (in lakh number of packs) of five different products of Cosmetic Company during 1995 and 2000



25. The sales of lipsticks in 2000 was by what percent more than the sales of nail enamels in 2000? (rounded off to nearest integer)

- a. 33%
- b. 31%
- c. 28%
- d. 22%

Answer: Option C

Explanation:

$$\begin{aligned} \text{Required percentage} &= \left[\frac{(48.17 - 37.76)}{37.76} \times 100 \right] \% \\ &= 27.57\% \\ &\approx 28\%. \end{aligned}$$

26. During the period 1995-2000, the minimum rate of increase in sales is in the case of?

- Shampoos
- Talcum powders
- Nail enamels
- Lipsticks

Answer: Option A

Explanation:

The percentage increase from 1995 to 2000 for various products are:

$$\text{Lipsticks} = \left[\frac{(48.17 - 20.15)}{20.15} \times 100 \right] \% = 139.06\%.$$

$$\text{Nail enamels} = (37.76 - 5.93) \times 100 \% = 536.76\%.$$

$$\left[\begin{array}{c} 5.93 \\ \end{array} \right]$$

$$\text{Talcum powders} = \left[\frac{(29.14 - 14.97)}{14.97} \times 100 \right] \% = 94.66\%.$$

$$\text{Shampoos} = \left[\frac{(12.21 - 7.88)}{7.88} \times 100 \right] \% = 54.95\% \approx 55\%.$$

$$\text{Conditioners} = \left[\frac{(10.19 - 5.01)}{5.01} \times 100 \right] \% = 103.39\%.$$

∴ The minimum rate of increase in sales from 1995 to 2000 is in the case of Shampoos.

27. What is the approximate ratio of the sales of nail enamels in 2000 to the sales of Talcum powders in 1995?

7:2

5:2

4:3

2:1

Answer: Option B

Explanation:

$$\text{Required ratio} = \frac{37.76}{14.97} \approx 2.5 = \frac{5}{2}$$

28. The sales have increase by nearly 55% from 1995 to 2000 in the case of?

a. Lipsticks

b. Nail enamels

c. Talcum powders

d. Shampoos

Answer: Option D

Explanation:

The percentage increase from 1995 to 2000 for various products are:

$$\text{Lipsticks} = \left[\frac{(48.17 - 20.15)}{20.15} \times 100 \right] \% = 139.06\%.$$

$$\text{Nail enamels} = (37.76 - 5.93) \times 100 \% = 536.76\%.$$

$$\left[\begin{array}{c} 5.93 \\ \end{array} \right]$$

$$\text{Talcum powders} = \left[\frac{(29.14 - 14.97)}{14.97} \times 100 \right] \% = 94.66\%$$

$$\text{Shampoos} = \left[\frac{(12.21 - 7.88)}{7.88} \times 100 \right] \% = 54.95\% \approx 55\%$$

$$\text{Conditioners} = \left[\frac{(10.19 - 5.01)}{5.01} \times 100 \right] \% = 103.39\%$$

29. The sales of conditioners in 1995 was by what percent less than the sales of shampoos in 1995? (rounded off to nearest integer)

57%

36%

29%

25%

Answer: Option B

Explanation:

$$\begin{aligned} \text{Required percentage} &= \left[\frac{(7.88 - 5.01)}{7.88} \times 100 \right] \% \\ &= 36.42\% \\ &= 36\% \end{aligned}$$

30. Following are the weights in kgs. of 36 BBA students of Khalsa College.

70	73	49	61	47	57	50	59
59	68	45	55	68	56	68	55
70	70	57	44	73	64	49	63
65	70	65	62	73	67	60	50

35. What is the mode of 10, 2, 8, 6, 7, 8, 9, 10, 10, 11 and 10?

- a. 10
c. 14
b. 12
d. 8

Answer: A

Explanation:

Mode = Observation with the highest frequency = 10

36. The mean of the marks in Statistics of 100 students in class X was 72. The mean of marks for boys was 75, while their number was 70. What is the mean of marks of girls in the class?

- a. 35
c. 68
b. 65
d. 86

Answer: B

Explanation:

$$\frac{\text{Total marks of boys}}{\text{Total number of girls}} = \frac{1950}{30} = 65$$

37. Which of the following is true about the mode of a given data?

- a. It may or may not exist for a given data.
c. It is very difficult to compute mode.
b. It is always unique.
d. We cannot calculate mode without the empirical formula.

Answer: A

Explanation:

Mode of a given data may or may not exist sometimes.

$$\text{Range} = 22 - 6 = 16$$

38. The A.M. of 12 observations is 15. If an observation 20 is removed, what is the arithmetic mean of the remaining observations?

14.5
15

13
13.5

Answer: B

Explanation:

The A.M. of 12 observations is 15.

⇒ Sum of 12 observations = $12 \times 15 = 180$

An observation 20 is removed

⇒ Mean of the remaining observations

$$= \frac{180 - 20}{(12 - 1)} = \frac{160}{11} = 14.5$$

39. If for a given data median is 125.6 and mean is 128, find mode.

120.8
108.2

128.0
180.2

Answer: A

Explanation:

Given median = 125.6 and mean = 128. Mode = $3 \text{ Median} - 2 \text{ Mean}$
 $= (3 \times 125.6) - (2 \times 128)$

$= 376.8 - 256$

$= 120.8$

40. What is the arithmetic mean of $a+2$, a and $a-2$?

$a+2$
 $a-2$

a
 $3a$

Answer: B

Explanation:

$$\text{Mean} = \frac{a+2+a+a-2}{3} = \frac{3a}{3} = a$$

41. The mean of 10 numbers is 7. If each number is multiplied by 12, find the mean of new set of numbers.

82

48

78

84

Answer: D

Explanation:

Total of 10 numbers = $10 \times 7 = 70$

If each number is multiplied by 12,

New total = 70×12

\therefore New mean = $\frac{70 \times 12}{10} = 84$

42. The mean of 9, 11, 13, p, 18 and 19 is p. Find the value of 'p'.

12

13

14

15

Answer: C

Explanation:

Given mean = $p = \frac{9+11+16+p+18+19}{6} = p$

$p = 14$

43. What is the value of 'n' if the mean of first 9 natural numbers is $\frac{5n}{9}$?

7

8

9

11

Answer: C

Explanation:

Mean of first 9 natural numbers = $\frac{1+2+\dots+9}{9}$

$\frac{45}{9} = 5$

Given mean of first 9 natural numbers is $\frac{5n}{9}$

$\frac{5n}{9} = 5$

$$n = \frac{9 \times 5}{5} = 9$$

44. In the set above, which is larger: the median, the mean, or the mode?

- a. Mean
b. Median
c. All are equal
d. Mode

Answer: A

Explanation:

Begin by ordering the set from smallest to largest:

6, 7, 8, 8, 9, 10, 11, 12

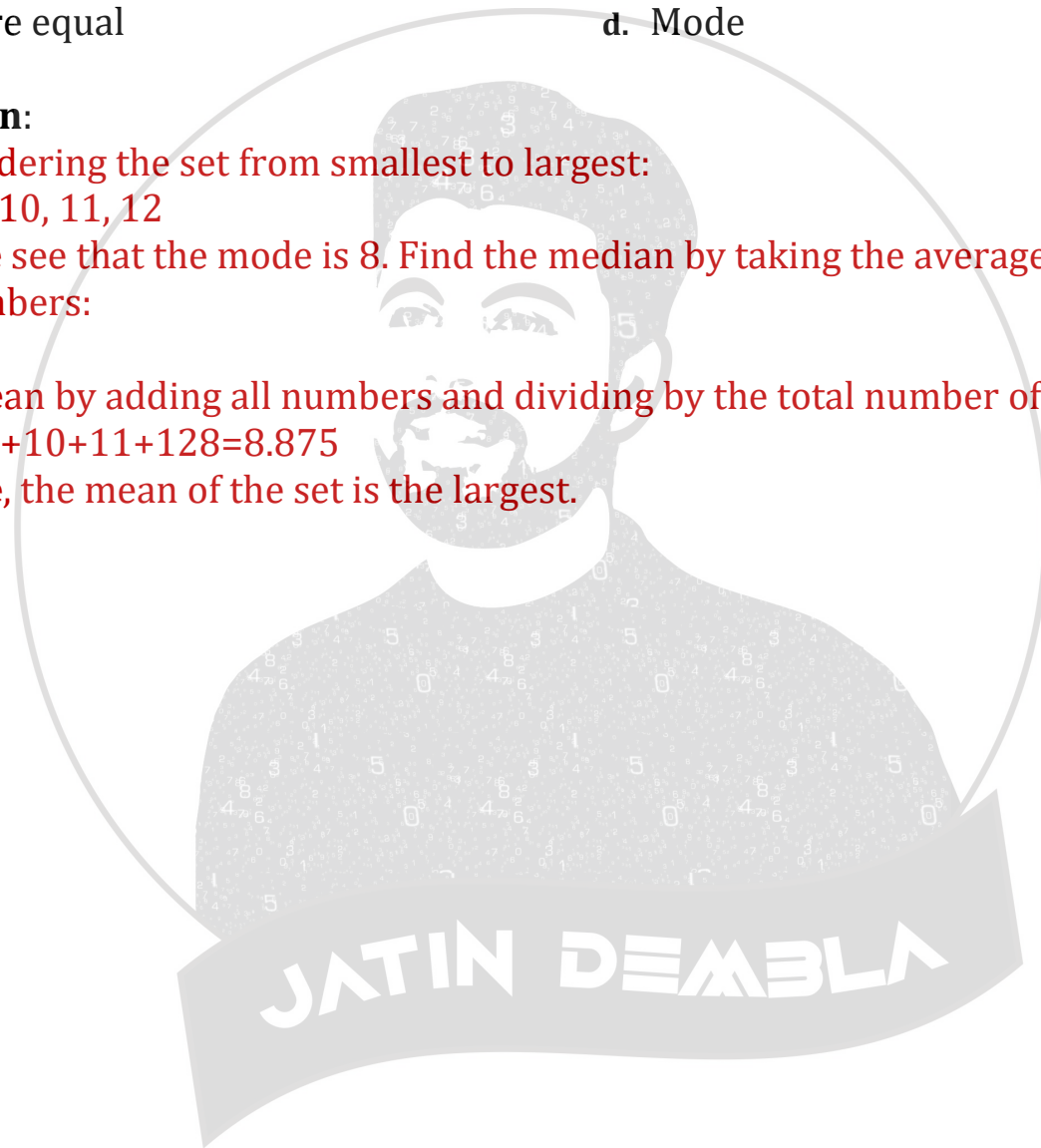
Already, we see that the mode is 8. Find the median by taking the average of the two middle numbers:

$$8 + 9 = 17$$
$$17 \div 2 = 8.5$$

Find the mean by adding all numbers and dividing by the total number of terms:

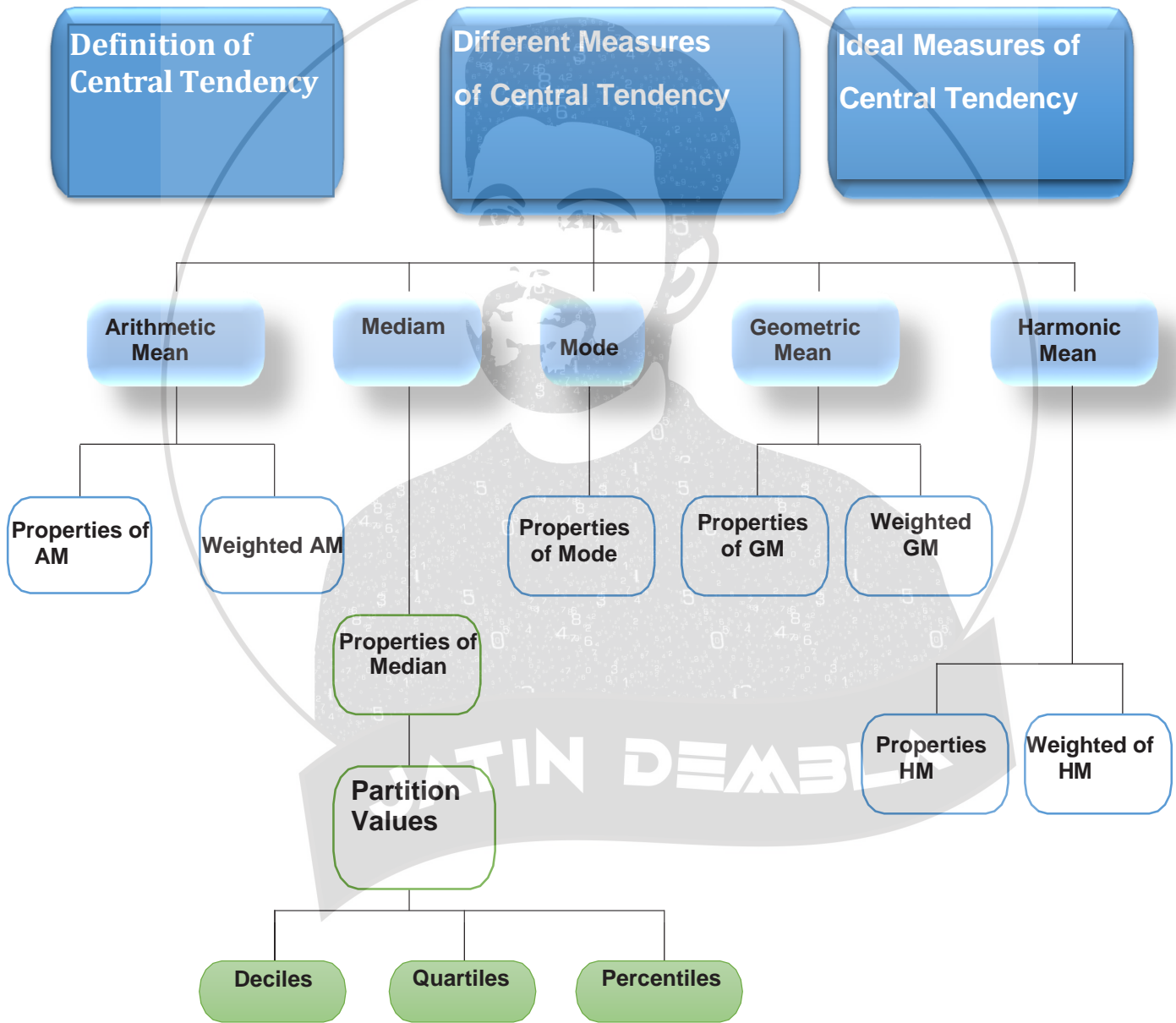
$$6 + 7 + 8 + 8 + 9 + 10 + 11 + 12 = 71$$
$$71 \div 8 = 8.875$$

Of the three, the mean of the set is the largest.



CHAPTER 15

MEASURES OF CENTRAL TENDENCY AND DISPERSION



UNIT I: MEASURES OF CENTRAL TENDENCY

CENTRAL TENDENCY	<p>Tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location, or average.</p>																		
ARITHMETIC MEAN	<p>The AM may be defined as the sum of all the observations divided</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #FFD700;"></th> <th style="background-color: #800080; color: white;"></th> </tr> <tr> <th style="background-color: #FFD700;">Formulas</th> <th style="background-color: #800080; color: white;">No. of Boxes</th> </tr> </thead> <tbody> <tr> <td>Individual Series Direct Method : $= \frac{\sum x}{N}$</td> <td style="text-align: center;">x</td> </tr> <tr> <td>Shortcut Method : $= A + \frac{\sum dx}{N}$</td> <td style="text-align: center;">x dx</td> </tr> <tr> <td>Discrete Series Direct Method : $= \frac{\sum fx}{\sum f}$</td> <td style="text-align: center;">x f fx</td> </tr> <tr> <td>Shortcut Method : $= A + \frac{\sum fdx}{\sum f}$</td> <td style="text-align: center;">x f dx fdx</td> </tr> <tr> <td>Continuous Series Direct Method : $\frac{\sum fx}{\sum f}$</td> <td style="text-align: center;">x f x(M.V) fx</td> </tr> <tr> <td>Shortcut Method : $= A + \frac{\sum fdx}{\sum f}$</td> <td style="text-align: center;">x f x (M.V) dx fdx</td> </tr> <tr> <td>Step Deviation : $= A + \frac{\sum fdx}{\sum f} \times c$</td> <td style="text-align: center;">x f x (M.V) dx dx' fdx'</td> </tr> </tbody> </table> </div> <p>by the number of observations. Thus, if a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$, then the AM of x, to be denoted by \bar{X}, is given by:</p>			Formulas	No. of Boxes	Individual Series Direct Method : $= \frac{\sum x}{N}$	x	Shortcut Method : $= A + \frac{\sum dx}{N}$	x dx	Discrete Series Direct Method : $= \frac{\sum fx}{\sum f}$	x f fx	Shortcut Method : $= A + \frac{\sum fdx}{\sum f}$	x f dx fdx	Continuous Series Direct Method : $\frac{\sum fx}{\sum f}$	x f x(M.V) fx	Shortcut Method : $= A + \frac{\sum fdx}{\sum f}$	x f x (M.V) dx fdx	Step Deviation : $= A + \frac{\sum fdx}{\sum f} \times c$	x f x (M.V) dx dx' fdx'
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Step Deviation : $= A + \frac{\sum fdx}{\sum f} \times c$	x f x (M.V) dx dx' fdx'																		

MEDIAN – PARTITION VALUE

$$\text{Median} = l + \frac{h}{f} \left(\frac{N}{2} - c \right)$$

Where:

l = lower class boundary of the median class

h = Size of the median class interval

f = Frequency corresponding to the median class

N = Total number of observations i.e. sum of the frequencies

c = Cumulative frequency preceding median class.

TYPES OF MEDIAN

Calculation of Quartiles, Deciles and Percentiles

- **For Continuous Series**

1. Q_1 = Size of $N/4^{\text{th}}$ item
2. Q_3 = Size of $3N/4^{\text{th}}$ item
3. D_1 = Size of $N/10^{\text{th}}$ item
4. D_9 = Size of $9N/10$ item
5. P_1 = Size of $N/100^{\text{th}}$ item
6. P_{99} = Size of $99N/100^{\text{th}}$ item

- **Formula to be used in continuous series:**

1. $Q_1 = L_1 + N/4 - c.f*i/f$
2. $Q_3 = L_1 + 3N/4 - c.f*i/f$
3. $D_1 = L_1 + N/10 - c.f*i/f$
4. $D_9 = L_1 + 9N/10 - c.f*i/f$
5. $P_1 = L_1 + N/100 - c.f*i/f$
6. $P_{99} = L_1 + 99N/100 - c.f*i/f$

MODE

Formula of Mode :

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

where,

Z = value of Mode

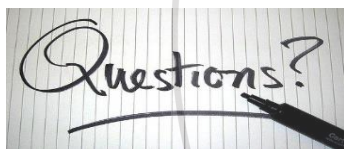
l_1 = lower limit of modal class

f_0 = Frequency of the preceding modal class

f_2 = Frequency of the subsequent modal class or post modal class

i = Class interval of the modal class

GEOMETRIC MEAN & HARMONIC MEAN & WEIGHTED MEAN	<p>Geometric Mean: $GM = \sqrt[n]{\prod_{i=1}^n x_i} = \sqrt[n]{x_1 x_2 x_3 \dots x_n}$</p> <p>Harmonic Mean: $HM = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$</p> <p>Weighted Mean: $WM = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i} = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n}{w_1 + w_2 + w_3 + \dots + w_n}$</p>
RELATIONSHIP BETWEEN MEAN, MEDIAN AND MODE	<p>Mean - Mode = 3(Mean - Median) Mode = 3 Median - 2 Mean</p>
RELATION BETWEEN AM, GM, AND HM	<p>AM > GM > HM</p>



1. Relationship between Mean, Median and Mode

- a. Mean - Mode = 3(Mean - Median) b. Mode = 3 Median - 2 Mean
 c. Both d. None of these

ANSWER : c

EXPLANATION:

If a frequency distribution is positively skewed, the mean is greater than median and median is greater than mode.

2. If median - 20 , and mean-22.5 in a moderately skewed distribution then compute approximate value of mode

- a. 15 b. 20
 c. 25 d. 30

ANSWER: a

EXPLANATION:

Mean - Mode = 3(Mean - Median)

$$22.5 - \text{Mode} = 3(22.5 - 20)$$

$$22.5 - \text{Mode} = 7.5$$

$$\text{Mode} = 22.5 - 7.5$$

$$\text{Mode} = 15$$

3. A numerical value used as a summary measure for a sample, such as sample mean, is known as a

population parameter
sample statistic

sample parameter
population mean

ANSWER: c

EXPLANATION:

If it pertains to sample it is called a statistic, if it pertains to population it is called a parameter.

4. Since the population size is always larger than the sample size, then the sample statistic

- | | |
|---|---|
| a. can never be equal to the population parameter | b. can never be zero |
| c. can never be smaller than the population parameter | d. None of the above answers is correct |

ANSWER: d

EXPLANATION:

Sample statistic will depend upon the sample chosen. It can be less than, greater than, equal to population parameter. It can assume the value of zero.

5. σ^2 ; is an example of a

- | | |
|-------------------------|---------------------|
| a. population parameter | b. sample statistic |
| c. population variance | d. mode |

ANSWER: a

EXPLANATION:

σ^2 is a standard representation for population parameter.

6. The mean of a sample is

- | | |
|---|---|
| a. always equal to the mean of the population | b. always smaller than the mean of the population |
|---|---|

c. computed by summing the data values and dividing the sum by (n - 1)

d. computed by summing all the data values and dividing the sum by the number of items.

ANSWER: d

EXPLANATION:

Mean= Total of sample values/ sample size

7. The sum of the percent frequencies for all classes will always equal

a. one

b. the number of classes

c. the number of items in the study

d. 100

ANSWER: d

EXPLANATION:

If we count the total frequency it is equal to the sample size n. $n/n * 100 = 100$

8. In a five number summary, which of the following is not used for data summarization?

a. the smallest value

b. the largest value

c. the median

d. the 25th percentile

ANSWER: d

EXPLANATION:

the 25th percentile

9. Since the mode is the most frequently occurring data value, it

a. can never be larger than the mean

b. is always larger than the median

c. is always larger than the mean

d. None of the above answers is correct.

ANSWER: d

EXPLANATION:

The mean, median and mode values will be distributed according to the skewness of the distribution. Accordingly mode can be greater than or less than mean or mode.

11. The following table gives the distribution of 100 accidents during seven days of the week in a given month. During a particular month there were 5 Fridays and Saturdays and only four each of other days. Calculate the average number of accidents per day.

Days:	Sun	Mon	Tue	Wed	Thru	Fri	Sat.	Total
Number	20	22	10	9	11	8	20	100

of
accidents:

a. 14

b. 12

c. 17

d. 19

ANSWER: a

EXPLANATION:

Calculation of Number of Accidents per Day

Day	No. of Accidents (X)	No. of Days in Month (f)	Total fX
Sunday	20	4	80
Monday	22	4	88
Tuesday	10	4	40
Wednesday	9	4	36
Thursday	11	4	44
Friday	8	5	40
Saturday	20	5	100
Total	100	N= 30	$\Sigma fX = 428$

= $14.27 = 14$ accidents per day

11. Following are the daily wages in Rupees of a sample of 9 workers: 58, 62, 48, 53, 70, 52, 60, 84, 75. Compute the mean wage.

a. 62.44

b. 62.04

c. 60.44

d. 31.22

ANSWER:

EXPLANATION:

Let x denote the daily wage in rupees.

Then as given, $x_1=58, x_2=62, x_3=48, x_4=53, x_5=70, x_6=52,$

$x_7=60, x_8=84$ and $x_9=75$. Applying (15.1.1) the mean wage is

given by,

$$= \frac{\sum Xi}{n}$$

$$\frac{58 + 62 + 48 + 53 + 70 + 52 + 60 + 84 + 75}{9}$$

$$\frac{562}{9} = 62.44$$

12. Find the AM for the following distribution:

Class Interval	350-369	370-389	390 - 409	410 - 429	430 - 449	450 - 469	470 - 489
Frequency	23	38	58	82	65	31	11

a. 416

b. 416.17

c. 416.71

d. 41.71

ANSWER:

EXPLANATION:

Computation of AM

Class Interval	Frequency(f)	Mid-Value(x)	d = xi - A xi = -419.50	fd
(1)	(2)	(3)	(4)	(5) = (2)X(4)
350 - 369	23	359.50	- 3	- 69
370 - 389	38	379.50	- 2	- 76
390 - 409	58	399.50	- 1	- 58
410 - 429	82	419.50 (A)	0	0
430 - 449	65	439.50	1	65
450 - 469	31	459.50	2	62
470 - 489	11	479.50	3	33
Total	308	-	-	- 43

The required AM is given by

$$\begin{aligned}
 X &= A + \frac{\sum fidi}{N} \times c \\
 &= 419.50 + \frac{(-43)}{308} \times 20 \\
 &= 419.50 - 2.79
 \end{aligned}$$

= 416.71

13. The mean salary for a group of 40 female workers is Rs.5200 per month and that for a group of 60 male workers is Rs.6800 per month. What is the combined mean salary?

- a. 6160
b. 616
c. 6.16
d. 61.6

ANSWER: a

EXPLANATION:

As given $n_1 = 40$, $n_2 = 60$, $x_1 = \text{Rs.}5200$ and $x_2 = \text{Rs.}6800$

hence, the combined mean salary per month is

$$\bar{X} = \frac{n_1x_1 + n_2x_2}{n_1 + n_2}$$

$$\frac{40 \times \text{Rs.}5200 + 60 \times \text{Rs.}6800}{40 + 60}$$

$$= 6160$$

14. The sum of the deviation of a given set of individual observations from the arithmetic mean is always Infinte. The Statement is True or not?

- a. Correct
b. Incorrect
c. Error
d. none

ANSWER: b

EXPLANATION:

According to Mathematical Properties of the Arithmetic Mean: The sum of the deviation of a given set of individual observations from the arithmetic mean is always zero. Symbolically, $\sum (x_i - \bar{x}) = 0$. It is due to this property that the arithmetic mean is characterised as the center of gravity i.e., the sum of positive deviations from the mean is equal to the sum of negative deviations.

15. The mean age of a combined group of men and women is 30 years. If the mean age of the group of men is 32 and that of women group is 27. find out the percentage of men and women in the group.

- a. 30%, 70%
c. 60%, 40%

- b. 20%, 80%
d. 40%, 60%

ANSWER: c

EXPLANATION:

Let us take group of men as first group and women as second group. Therefore. = 32 years. = 27 years, and = 30 years. In the problem, we are not given the number of men and women. We can assume

$N_1 + N_2 = 100$ and therefore. $N_1 = 100 - N_2$

Apply =

$30 = (\text{Substitute } N_1 = 100 - N_2)$

$30 \times 100 = 32(100 - N_2) + 27N_2$ or $5N_2 = 200$

$N_2 = 200/5 = 40\%$

$N_1 = (100 - N_2) = (100 - 40) = 60\%$

Therefore, the percentage of men in the group is 60 and that of women is 40.

16. Median and mode of the wage distribution are known to be Rs. 33.5 and 34 respectively. Find the third missing values.

Wages (Rs.)	No. of Workers
0 - 10	4
10 - 20	16
20 - 30	?
30 - 40	?
40 - 50	?
50 - 60	6
60 - 70	4
Total	230

- a. 6
c. 9

- b. 10
d. 60

ANSWER: d

EXPLANATION:

We assume the missing frequencies as 20 – 30 as x, 30 – 40 as y, and 40 – 50 as $230 - (4 + 16 + x + y + 6 + 4) = 200 - x - y$.

We now proceed further to compute missing frequencies:

Wages (Rs.) X	No. of workers f	Cumulative frequencies Cf
0 – 10	4	4
10 – 20	16	20
20 – 30	x	20 + x
30 – 40	y	20 + x + y
40 – 50	200 – x – y	220
50 – 60	6	226
60 – 70	4	230
	N = 230	

Apply, Median =

$$33.5 =$$

$$y(33.5 - 30) = (115 - 20 - x)10$$

$$3.5y = 1150 - 200 - 10x$$

$$10x + 3.5y = 950 \dots(i)$$

Apply, Mode =

$$34 =$$

$$4(3y - 200) = 10(y - x)$$

$$10x + 2y = 800 \dots(ii)$$

Subtract equation (ii) from equation (i),

$$1.5y = 150, y =$$

Substitute the value of $y = 100$ in equation (i), we get

$$10x + 3.5(100) = 950$$

$$10x = 950 - 350$$

$$x = 600/10 = 60$$

Third missing frequency = $200 - x - y = 200 - 60 - 100 = 40$.

17. Calculate mode from the following data:

Marks	No. of Students
Below 10	4
" 20	6
" 30	24

" 40	46
"50	67
" 60	86
" 70	96
" 80	99
" 90	100

a. 41.3

c. 40.13

b. 40

d. 89

ANSWER: a**EXPLANATION:**

Since we are given the cumulative frequency distribution of marks, first we shall convert it into the normal frequency distribution:

Marks	Frequencies
0-10	4
10- 20	6-4=2
20- 30	24-6=18
30- 40	46-24=22
40-50	67-46=21
50-60	86-67=19
60-70	96-86=10
70-80	99-96=3
80-90	100-99=1

It is evident from the table that the distribution is irregular and maximum chances are that the distribution would be having more than one mode. You can verify by applying the grouping and analysing table.

The formula to calculate the value of mode in cases of bio-modal distributions is:

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean.}$$

Computation of Mean and Median:

Marks	Mid - Value X	Frequency f	Cumulative frequencies Cf	(dx)	fdx
0-10	5	4	4	-4	-16
10- 20	15	2	6	-3	-6
20- 30	25	18	24	-2	-36
30- 40	35	22	46	-1	-22
40-50	45	21	67	0	0
50-60	55	19	86	1	19
60-70	65	10	96	2	20
70-80	75	3	99	3	9
80-90	85	1	100	4	4
		$\Sigma f = 100$			$\Sigma fdx = -28$

Mean =

Median = size of item = = 50th item

Because 50 is smaller to 67 in C.f. column. Median class is 40 – 50

Median =

Median =

Apply, Mode = 3 median – 2 mean

Mode = $3 \times 41.9 - 2 \times 42.2 = 125.7 - 84.6 = 41.3$

18. Find the arithmetic mean of the first 7 natural numbers.

a. 5

b. 6

c. 7

d. 4

5

a. 6

b. 7

c. 4

ANSWER: d

EXPLANATION:

The first 7 natural numbers are 1, 2, 3, 4, 5, 6 and 7.

Let x denote their arithmetic mean.

Then mean = Sum of the first 7 natural numbers/number of natural numbers

$x = (1 + 2 + 3 + 4 + 5 + 6 + 7)/7$

$$= 28/7$$

$$= 4$$

Hence, their mean is 4.

19. The heights of five runners are 160 cm, 137 cm, 149 cm, 153 cm and 161 cm respectively. Find the mean height per runner.

a. 152

b. 150

c. 148

d. 120

ANSWER: a

EXPLANATION:

Mean height = Sum of the heights of the runners/number of runners

$$= (160 + 137 + 149 + 153 + 161)/5 \text{ cm}$$

$$= 760/5 \text{ cm}$$

$$= 152 \text{ cm.}$$

Hence, the mean height is 152 cm.

20. Find the mean of the first five prime numbers.

a. 4.6

b. 6.5

c. 7.8

d. 5.6

ANSWER: d

EXPLANATION:

The first five prime numbers are 2, 3, 5, 7 and 11.

Mean = Sum of the first five prime numbers/number of prime numbers

$$= (2 + 3 + 5 + 7 + 11)/5$$

$$= 28/5$$

$$= 5.6$$

Hence, their mean is 5.6

21. Find the mean of the first six multiples of 4.

a. 12

b. 13

c. 14

d. 15

ANSWER: c

EXPLANATION:

The first six multiples of 4 are 4, 8, 12, 16, 20 and 24.

Mean = Sum of the first six multiples of 4/number of multiples

$$= (4 + 8 + 12 + 16 + 20 + 24)/6$$

$$= 84/6$$

$$= 14.$$

Hence, their mean is 14.

22. If the mean of 9, 8, 10, x, 12 is 15, find the value of x.

- | | |
|----|----|
| 30 | 41 |
| 36 | 63 |

ANSWER: c

EXPLANATION:

Mean of the given numbers = $(9 + 8 + 10 + x + 12)/5 = (39 + x)/5$

According to the problem, mean = 15 (given).

Therefore, $(39 + x)/5 = 15$

$$\Rightarrow 39 + x = 15 \times 5$$

$$\Rightarrow 39 + x = 75$$

$$\Rightarrow 39 - 39 + x = 75 - 39$$

$$\Rightarrow x = 36$$

Hence, $x = 36$.

23. If the mean of five observations x, x + 4, x + 6, x + 8 and x + 12 is 16, find the value of x.

- | | |
|--------|--------|
| a. 154 | b. 54 |
| c. 451 | d. 541 |

ANSWER: c

EXPLANATION:

Mean of the given observations

$$= x + (x + 4) + (x + 6) + (x + 8) + (x + 12)/5$$

$$= (5x + 30)/5$$

According to the problem, mean = 16 (given).

Therefore, $(5x + 30)/5 = 16$

$$\Rightarrow 5x + 30 = 16 \times 5$$

$$\Rightarrow 5x + 30 = 80$$

$$\Rightarrow 5x + 30 - 30 = 80 - 30$$

$$\Rightarrow 5x = 50$$

$$\Rightarrow x = 50/5$$

$$\Rightarrow x = 10$$

Hence, $x = 10$.

$$148 + 153 + 146 + 147 + 154$$

24. The mean of 40 numbers was found to be 38. Later on, it was detected that a number 56 was misread as 36. Find the correct mean of given numbers.

a. 38

b. 26

c. 38.5

d. 89

ANSWER: c

EXPLANATION:

Calculated mean of 40 numbers = 38.

Therefore, calculated sum of these numbers = $(38 \times 40) = 1520$.

Correct sum of these numbers

= $[1520 - (\text{wrong item}) + (\text{correct item})]$

= $(1520 - 36 + 56)$

= 1540.

Therefore, the correct mean = $1540/40 = 38.5$.

25. The mean of the heights of 6 boys is 152 cm. If the individual heights of five of them are 151 cm, 153 cm, 155 cm, 149 cm and 154 cm, find the height of the sixth boy.

a. 157

b. 159

c. 150

d. 89

ANSWER: c

EXPLANATION:

Mean height of 6 boys = 152 cm.

Sum of the heights of 6 boys = $(152 \times 6) = 912$ cm

Sum of the heights of 5 boys = $(151 + 153 + 155 + 149 + 154)$ cm = 762 cm.

Height of the sixth boy

= (sum of the heights of 6 boys) - (sum of the heights of 5 boys)

= $(912 - 762)$ cm = 150 cm.

Hence, the height of the sixth girl is 150 cm.

26. Find the mode of the following set of marks.

Marks	1	2	3	4	5
Frequency	6	7	7	5	3

- a. 2 and 4
- b. 4 and 3
- c. 2 and 3
- d. 2 and 5

ANSWER: c

EXPLANATION:

The marks 2 and 3 have the highest frequency. So, the modes are 2 and 3.

Note: The above example shows that a set of observations may have more than one mode.

27. There are 8 number cards with values 0 – 7. Each time a card is drawn at random and the card value is recorded. The frequency refers to the number of times a value is shown.

Card values	0	1	2	3	4	5	6	7
Frequency	8	12	7	10	12	13	12	10

- a. 75, 5
- b. 5, 79
- c. 80, 89
- d. None

ANSWER: a

EXPLANATION:

a) Mode: 75 kg (highest frequency of 12)

b) Mode: 5 (highest frequency of 13)

28. The following frequency table shows the marks obtained by students in a quiz. Given that 4 is the mode, what is the least value for x?

Marks	1	2	3	4	5	6
Number of students(Frequency)	7	9	10	x	9	11

- a. 12
- b. 10
- c. 3
- d. 6

ANSWER: a

EXPLANATION:

x is as least 12 (if x is less than 12 then 4 will not be the mode)

29. The mean of the following frequency distribution is

Class Interval	Frequency
0 - 10	4

10 - 20	6
20 - 30	10
30 - 40	16
40 - 50	14

a. 25

b. 35

c. 30

d. 31

ANSWER:D**EXPLANATION:**

Class Interval	Mid Point	Freq.	Diff, From (A=25)	fd
0-10	5	4	-20	-80
10-20	15	6	-10	-60
20-30	25	10	0	0
30-40	35	16	10	160
40-50	45	14	20	280
Total	$\Sigma f=50$			$\Sigma fd=300$

$$(\bar{X}) = A + \frac{\Sigma fd}{\Sigma f} = 25 + \frac{300}{50} = 31$$

30. Mean of twenty observations is 15. If two observations 3 and 14 replaced by 8 and 9 respectively, then the new mean will be

a. 14

b. 15

c. 16

d. 17

ANSWER: D**EXPLANATION:**

Mean of 20 observations =15

∴Sum of 20 observations =15×20=300

Replacing 3 and 14 by 8 and 9 will mean that 3+14=17is replaced by 8+9=17

Hence there will be no effect on the sum. It will still remain 300, so the mean will not change and will remain 15.

31.

Factory A	Factory B
No. of wage earners 250	200
Average daily wage Rs. 2.00	Rs.2.50

The average of daily wages for the earners of the two factories combined is

- a. Rs. 2.12
- b. Rs. 2.06
- c. Rs. 2.20
- d. Rs. 2.22

ANSWER: C

EXPLANATION:

$$\begin{aligned} \text{Required average} &= \frac{250 \times 2.00 + 200 \times 2.50}{250 + 200} \\ &= \frac{1000}{450} \\ &= \frac{20}{9} \\ &= \text{Rs. 2.22} \end{aligned}$$

32. The height of 30 boys of a class are given in the following table :

Height in cm	Frequency
120 - 129	2
130 - 139	8
140 - 149	10
150 - 159	7
160 - 169	3

If by joining of a boy of height 140 cm, the median of the heights is changed from M_1 to M_2 then $M_1 - M_2$ in cm is

- a. 0.1
- b. -0.1
- c. 0
- d. 0.2

ANSWER: C

EXPLANATION:

Height In cms	Frequency	Cumulative Frequency	Actual Class limit
120 - 129	2	2	119.5-129.5
130 - 139	8	10	129.5-139.5

140 - 149	10	20	139.5-149.5
150 - 159	7	27	149.5-159.5
160 - 169	3	30	159.5-169.5
n = 30			

Here $n = 30$

$$\therefore \frac{n}{2} + 1 = 15 + 1 = 16$$

$\therefore 16$ is under cumulative frequency 20. So median class be 140-149

$$L_1 = 139.5, L_2 = 149.5, f = 10, n = 30, c = 10.$$

$$\text{Median } M_1 = L_1 + \frac{L_2 - L_1}{f} \left(\frac{n}{2} - c \right)$$

$$= 139.5 + \frac{10}{10} (15 - 10)$$

$$= 139.5 + \frac{10}{10} \times 5 = 144.5$$

If by joining f a boy of height 140 cms, the $n=31, f=11$

$$\therefore \text{Median } M_2 = 139.5 + \frac{149.5 - 139.5}{11} (15.5 - 10)$$

$$= 139.5 + \frac{10}{11} \times 5.5 = 144.5 \text{ cms}$$

$$\text{Then } M_1 - M_2 = 144.5 - 144.5 = 0$$

33. The marks awarded to seven students in a school admission test were:

Mathematics		English
A	55	35
B	45	32
C	75	44
D	15	50
E	10	45
F	40	60
G	06	40

Which subject has the better median value?

Mathematics

English

Both [a] and [b] above

None of the above

ANSWER: B**EXPLANATION:**

The awarded makes in Mathematics and English were arranged in ascending in ascending order separately.

Maths	English
06	32
10	35
15	40
40	44
45	45
55	50
75	60

Hence, English has the better median value.

34. Identify the mode of the given distribution.

Marks	4	5	6	7	8
Number of Students	3	5	10	6	1

a. 7

b. 1

c. 8

d. 6

Answer: d**Explanation:**

Mode is 6 as it has the highest frequency

35. The given data are the times (in minutes), it takes seven students to go to school from their homes.

11	6	22	7	10	6	15
----	---	----	---	----	---	----

Which statement about the data is false?

a. Their median is 11.

b. Their mean is 11.

c. Their range is 16.

d. Their mode is 6.

Answer: A**Explanation:**

Arranging the given data in ascending order, we get, 6, 6, 7, 10, 11, 15, 22

- a. 11.9
c. 76

- b. 9
d. 12

Answer:

Explanation:

To find the median, arrange the numbers from smallest to largest:

4,4,4,4,6,7,9,9,12,12,12,12,12,12,18,76,90

There are 17 numbers in total. Since 17 is an odd number, the median will be the middle number of the set. In this case, it is the 9th number, which is 12.

39. There are 3,500 people in group A and 5,000 people in group B:

Car Type	% in Group A Who Own	% in Group B Who Own
Motorbike	4	9
Sedan	35	25
Minivan	22	15
Van	9	12
Coupe	3	6

What is the median of the number of people in group B who own either a minivan, van, or coupe?

- a. 600
c. 1500

- b. 300
d. 750

Answer: D

Explanation:

Treat the percentages as a list, as we are including every demographic from the 3 vehicle types mentioned. If we do each $0.06(5000)$, $0.12(5000)$, and $0.15(5000)$ we note from observation that the median, or middle value, would have to be the 12% row since the sample size does not change. The question asks for EITHER of the 3 categories, so we can ignore the other two.

$0.12(5000) = 600$ (van) is the median of the 3 categories.

8, 12, 9, 8, 7, 11, 10, 6

40. The grades on a test taken by 1515 students are 50, 70, 87, 95, 100, 34, 56, 76, 43, 88, 92, 76, 82, 45, and 65 respectively. What was the median score for this test?

- a. 73
c. 70

- b. 76
d. 89

Answer: B

Explanation:

To solve this problem, we must be aware of the definition of a median for a set of numbers. The median is defined as the number that is in middle of a set of numbers sorted from smallest to largest. Therefore we must first sort the numbers from largest to smallest.

34,43,45,50,56,65,70,76,76,82,87,88,92,95,100

43,45,50,56,65,70,76,76,81,87,88,82,95

45,50,56,65,70,76,76,81,87,88,82

50, 56, 65, 70, 76, 76, 81, 87, 88

56,65,70,76,76,81,87

65, 70, 76, 76, 81

70, 76, 76

76

Then by slowly eliminating the smallest and the largest numbers we find that the median score for this test is 76.

41. SetA=[-10,4,2,-14,-2]

Quantity A: The mean of SetA

Quantity B: The median of SetA

- | | |
|--|----------------------------------|
| a. Quantity B is greater. | b. Quantity A is greater. |
| c. The relationship cannot be determined | d. The two quantities are equal. |

Answer: a

Explanation:

Begin by reordering the set in numerical order:

SetA=[-10,4,2,-14,-2]

Then becomes

SetA=[-14,-10,-2,2,4]

Since there is an odd number of values, the median is the middle value.

Quantity B: -2

Now, to find the arithmetic mean, take the sum of values divided by the total number of values.

$-14-10-2+2+45$

units are compatible (miles per hour not miles per minute), which means the total 45 minute run time needs to be converted to 0.75 of an hour; therefore (4miles + 3.25 miles/0.75 hour) is the final answer.

44. Find the mode for the following data.

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

- a. 20.22
- b. 19.47
- c. 21.12
- d. 20.14

Answer: B

Explanation:

Since, maximum class frequency is 35, so the mode class is 18-24.

$$\text{Now, Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$18 + \left(\frac{35 - 25}{2 \times 35 - 25 - 18} \right) \times 6$$

$$= 18 + 2.22 = 20.22$$

45. Find the median for the following distribution of workers.

Daily wages	No. of workers	Daily wages	No. of workers
1-3	6	9-11	21
3-5	53	11-13	16
5-7	85	13-15	4
7-9	86	15-17	4

- a. 7.14
- b. 6.84
- c. 5.92
- d. 5.57

Answer: B

Explanation:

Daily wages	No of workers	Cumulative Frequency (cf)
1-3	6	6
3-5	53	59
5-7	85	144
7-9	86	230
9-11	21	251

11-13	16	267
13-15	4	271
15-17	4	275

Here, $n=275$

$$\frac{n}{2} = 137.5$$

Median class 5-7

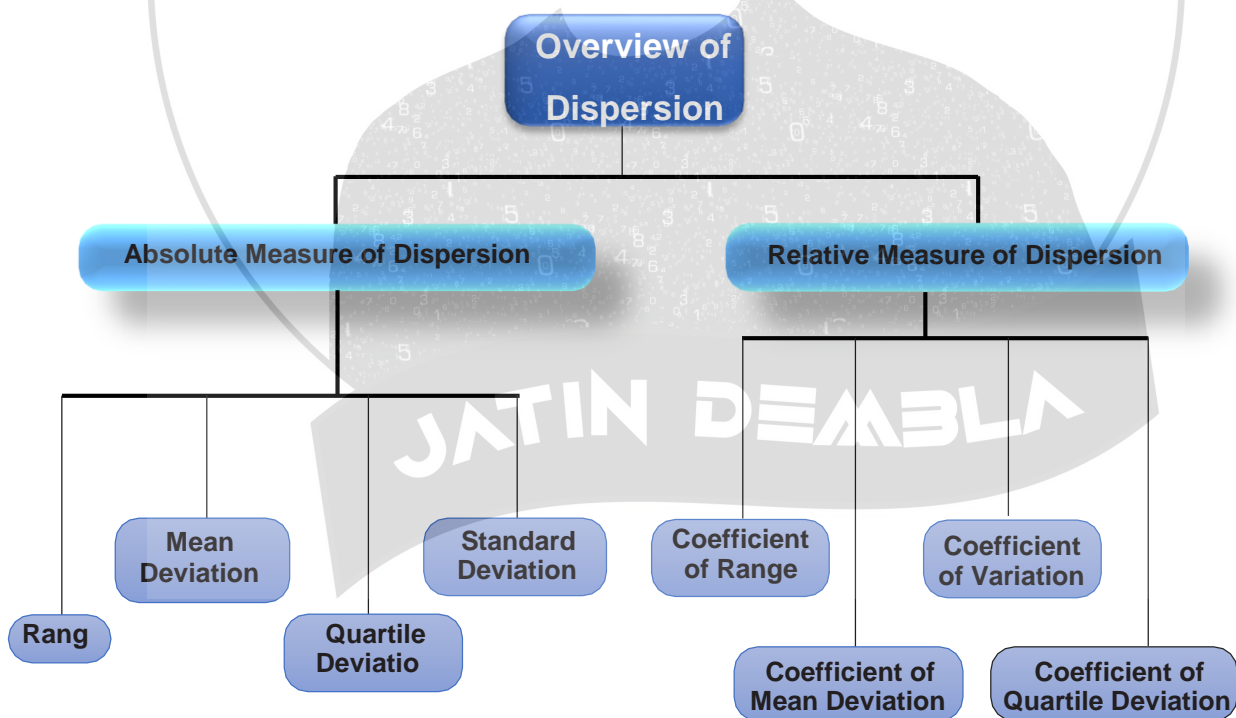
$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f} \right) \times h$$

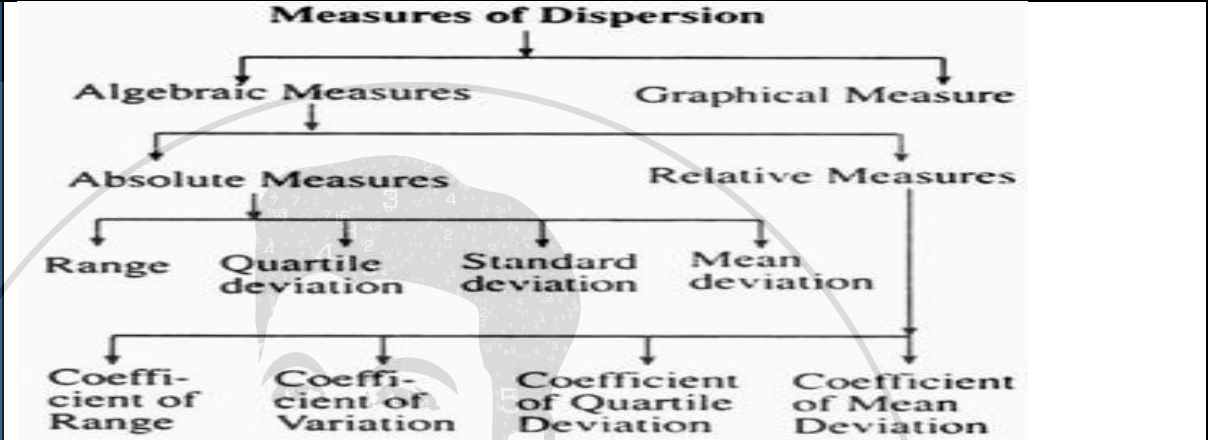
$$= 5 + \left(\frac{137.5 - 59}{85} \right) \times 2 = 5 + \frac{78.5}{85} \times 2$$

$$= 5 + 1.84$$

$$= 6.84$$

UNIT II: DISPERSION



<p>DISPERSION</p>	<p>The amount of deviation of the observations, usually, from an appropriate measure of central tendency. Two distributions may be identical in respect of its first important characteristic i.e. central tendency and yet they may differ on account of scatterness.</p>
<p>CLASSIFICATION OF DISPERSION</p>	
<p>ALGEBRIC MEASURES</p>	<p>Range = L-S</p> $\text{Mean(population)} = \mu = \frac{\sum_{i=1}^k f_i x_i}{n}$ $\text{Standard Deviation(population)} = \sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$ $\text{Variance(population)} = \sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}$
<p>RELATIVE MEASURES</p>	<p>(i) Coefficient of Range</p> $= \frac{\text{Range}}{\text{Highest value} + \text{Lowest value}} \times 100$ <p>(ii) Coefficient of Variation</p> $= \frac{\text{Standard Deviation}}{\text{Mean}} \times 100$ <p>(iii) Coefficient of Quartile Deviation</p> $= \frac{\text{Quartile Deviation}}{\text{Median}} \times 100$ <p>(iv) Coefficient of Mean Deviation</p> $= \frac{\text{Mean Deviation}}{\text{Mean or Median}} \times 100$



1. Following are the wages of 8 workers expressed in rupees: 82, 96, 52, 75, 70, 65, 50, 70. Find the range and also its coefficient.

a. 46,31.51

b. 64, 32

c. 56,76

d. 90, 33

ANSWER: a

EXPLANATION:

The largest and the smallest wages are $L = \text{Rs.}96$ and $S = \text{Rs.}50$ Thus
range = $\text{Rs.}96 - \text{Rs.}50 = \text{Rs.}46$

$$\begin{aligned} \text{coefficient of Range} &= \frac{96 - 50}{96 + 50} \times 100 \\ &= 31.51 \end{aligned}$$

2. What is the coefficient of Range for the following distribution of weights?

Weights in kgs:	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
No. of Students:	12	18	23	10	3

a. 20

b. 21

c. 20.16

d. 40.34

ANSWER: c

EXPLANATION:

The lowest class boundary is 49.50 kgs. and the highest class boundary is 74.50 kgs.
Thus we have

Range = $74.50 \text{ kgs.} - 49.50 \text{ kgs.}$

= 25 kgs.

$$\text{coefficient of Range} = \frac{74.50 - 49.50}{74.50 + 49.50} \times 100$$

$$= \frac{25}{124} \times 100$$

=20.16

=Value of $3\left(\frac{20+1}{4}\right)$ th item

=Value of (15.75)th item =15th item+0.75(16th item-15th item)=1750

$Q_3=1750+3.75=1753.75$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{1753.75 - 1260}{2} = \frac{493.75}{2}$$

=246.875

5. Compute coefficient of variation from the following data:

Age:	under 10	under 20	under 30	under 40	under 50	under 60
No. of persons Dying:	10	18	30	45	60	80

- a. 48.83
- c. 756.34

- b. 89.88
- d. NONE

ANSWER: a

EXPLANATION:

Age in years class Interval	No. of persons dying (f _i)	Mid-value (x _i)	d _i = x _i - 25	f _i d _i	f _i d _i ²
0-10	10	5	- 2	-20	40
10-20	18-10= 8	15	- 1	-8	8
20-30	30-18=12	25	0	0	0
30-40	45-30=15	35	1	15	15
40-50	60-45=15	45	2	30	60
50-60	80-60=20	55	3	60	180
Total	80	-	-	77	303

The AM is given by:

$$\begin{aligned}\bar{X} &= A + \frac{\sum f_i d_i}{N} \times C \\ &= \left(\frac{25 + 77 \times 10}{80} \right) \text{years} \\ &= 34.63 \text{ years}\end{aligned}$$

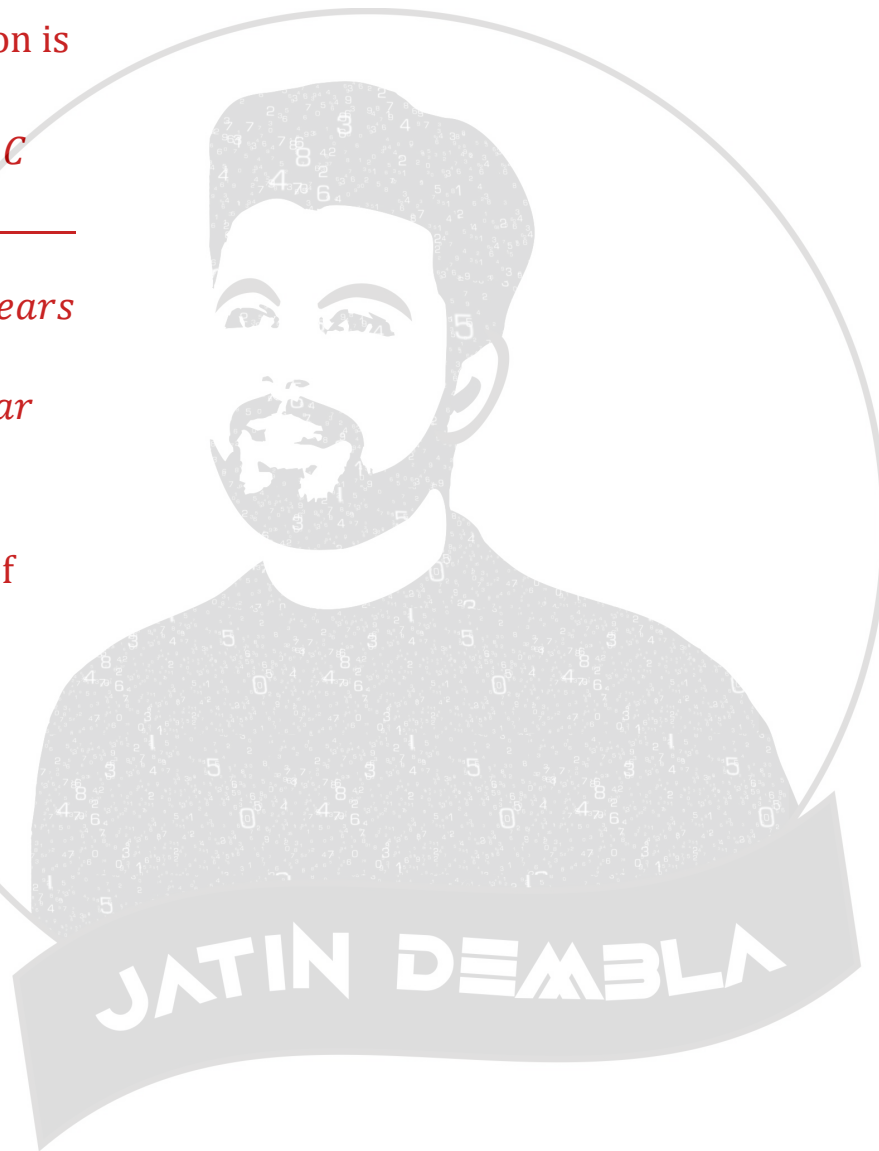
The standard deviation is

$$\begin{aligned}&\sqrt{\frac{\sum f_i d^2}{N} - \left[\frac{\sum f_i d}{N} \right]^2} \times C \\ &\sqrt{\frac{303}{80} - \left[\frac{77}{80} \right]^2} \times 10 \text{ years} \\ &\sqrt{3.79 - 0.93} \times 10 \text{ year}\end{aligned}$$

$$= 16.91 \text{ years}$$

Thus the coefficient of variation is given by

$$\begin{aligned}CV &= \frac{S}{\bar{X}} \times 100 \\ &= \frac{16.91}{34.63} \times 100 \\ &= 48.83\end{aligned}$$



6. What is the mean deviation about mean for the following numbers? 5, 8, 10, 10, 12, 9.

a. 1.74

b. 1.67

c. 1.87

d. 1.47

ANSWER: b

EXPLANATION:

The mean is given by

$$\bar{X} = \frac{5 + 8 + 10 + 10 + 12 + 9}{6}$$

$$= 9$$

Computation of MD about AM

X_i	$X_i - \bar{X}$
5	4
8	1
10	1
10	1
12	3
9	0
Total	10

Thus mean deviation about mean is given by

$$X_i - \bar{X} = \frac{\sum 10}{6} = 1.67$$

7. From the above data calculate coefficient of mean deviation

a. 12.45

b. 123

c. 989

d. None

ANSWER: a

EXPLANATION:

$$\text{coefficient of mean deviation} = \frac{\text{MD about Median}}{\text{Median}} \times 100$$

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	1 6	4	2

a. 10.16

b. 30.69

c. 28

d. 30

Answer: a**Explanation:**

Class	Frequency	Cumulative frequency	Mid-point x_i
0 – 10	6	6	5
10 – 20	7	7 + 6 = 13	15
20 – 30	15	13 + 15 = 28	25
30 – 40	16	28 + 16 = 44	35
40 – 50	4	44 + 4 = 48	45
50 – 60	2	48 + 2 = 50	55
	50		

$$N \sum f_i = 50$$

Median Class $\left(\frac{N}{2}\right)^{th}$ term

$$\left(\frac{50}{2}\right)^{th} \text{ term}$$

25th

In above data, cumulative frequency of class 20-30 is 28 which is slightly greater than 25.

\therefore Median class = 20 – 30

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Where,

l = Lower limits of median class

N = sum of frequencies

f = frequency of median class

C = Cumulative frequency of class before median class

Here, $l = 20, N = 50, C = 13, h = 10, f = 15$

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

$$20 + \frac{\frac{50}{2} - 13}{15} \times 10$$

$$20 + \frac{25 - 13}{15} \times 10$$

$$20 + \frac{12}{15} \times 10$$

$$20 + 8 = 28$$

Finding mean deviation about Median = $\frac{\sum f_i |x_i - M|}{\sum f_i}$

Class	Frequency	Cumulative frequency	Mid-point x_i	$ x_i - M $	$f_i x_i - M $
0 - 10	6	6	5	$ 5 - 28 = 23$	$6 \times 23 = 138$
10 - 20	7	$7 + 6 = 13$	15	$ 15 - 28 = 13$	$7 \times 13 = 91$
20 - 30	15	$13 + 15 = 28$	25	$ 25 - 28 = 3$	$15 \times 3 = 45$
30 - 40	16	$28 + 16 = 44$	35	$ 35 - 28 = 7$	$16 \times 7 = 112$
40 - 50	4	$44 + 4 = 48$	45	$ 45 - 28 = 17$	$4 \times 17 = 68$
50 - 60	2	$48 + 2 = 50$	55	$ 55 - 28 = 27$	$2 \times 27 = 54$
	$\sum f_i = 50$				$\sum f_i x_i - M = 508$

$$\sum f_i = 50 \text{ \& } |x_i - M| = 508$$

$$\therefore \text{Mean Deviation (M)} = \frac{\sum f_i |x_i - M|}{\sum f_i}$$

$$\frac{508}{50} = 10.16$$

11. 5 students obtained following marks in statistics: 20, 35, 25, 30, 15 Find out range and coefficient of range.

a. 20, 0.4

b. 20, 0.5

c. 30, 10

d. 30, 5

Answer: a**Explanation:**

Here,

Highest value (H) = 35Lowest value (L) = 15

Range = Highest value – Lowest value

i.e. $R = H - L$

Substituting the given values in the formula

 $R = 35 - 15 = 20$

Coefficient of Range is as follows:

$$CR = \frac{H-L}{H+L}$$

$$\text{or, } CR = \frac{35-15}{35+15}$$

$$= \frac{20}{50}$$

$$CR = 0.4$$

Hence, the range (R) of the above data is 20 and coefficient of Range (CR) is 0.4

12. Prices of shares of a company were note as under from Monday through Saturday. Find out range and the coefficient of range

Day	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.
Price	200	210	208	160	220	250

a. 20, 0.4

c. 30, 0.65

b. 90, 0.22

d. 30, 5.69

Answer: B**Explanation:**

Here,

Highest value among the prices of shares =

250
Lowest Value among the prices of shares = 160Range (R) = Highest value (H) – Lowest Value (L) or, $R = 250 - 160$ $R = 90$

$$\text{Coefficient of Range (CR)} = \frac{H-L}{H+L}$$

$$\text{or, CR} = \frac{250-160}{250+160}$$

$$= \frac{90}{410}$$

CR = 0.219 or 0.22(Approx).

Hence, the Range (R) of the above data is 90 and Coefficient of Range (CR) is 0.22

13. You know share market is going bullish during the last several months. Collect weekly data on the share price of any two important industries during the past six months. Calculate the range of share prices. Comment on how volatile are the share prices.

- a. Tata Motors shares are more volatile as compared to the prices of Reliance shares.
- b. Tata Motors shares are less volatile as compared to the prices of Reliance shares.
- c. Tata Motors shares are equal as to the prices of Reliance shares.
- d. None of these

Answer: B

Explanation:

Month	Price of shares Tata Motors	Price of shares Reliance
Oct.	325	913.35
Nov.	397	900.25
Dec.	405	750.90
Jan.	415	780.70
Feb.	420	799.25
Mar.	388	850.35

For Tata Motors Highest Value=420 Lowest Value=325

Range (R) = Highest Value (H)– Lowest Value (L) or, $R_1 = 420 - 325$

$$R_1 = 95$$

$$\text{Coefficient of Range (CR)} = \frac{H-L}{H+L}$$

$$\begin{aligned} \text{or, CR} &= \frac{420-325}{420+325} \\ &= \frac{95}{745} = 0.127 \end{aligned}$$

For Reliance

$$\text{Highest Value} = 913.35$$

$$\text{Lowest Value} = 750.90$$

$$\text{Range (R)} = \text{Highest value (H)} - \text{Lowest value (L)} \text{ or, } R_2 = 913.35 - 750.90$$

$$R_2 = 162.45$$

$$\text{Coefficient of Range (CR)} = \frac{H-L}{H+L}$$

$$\begin{aligned} \text{CR} &= \frac{913.35-750.90}{913.35+750.90} \\ &= \frac{162.45}{1664.25} = 0.097 \end{aligned}$$

From the above results we can observe that the price of the Tata Motors shares are less volatile as compared to the prices of Reliance shares.

14. Calculate range and the coefficient of range of the following series:

Marks	10	20	30	40	50	60	70
No. of Students	15	18	25	30	16	10	9

- a. 20, 0.4
c. 60, 0.75

- b. 20, 0.5
d. 30, 5

Answer: C

Explanation:

Here,

$$\text{Highest value} = 70$$

$$\text{Lowest value} = 10$$

$$\text{Range (R)} = \text{Highest value (H)} - \text{Lowest Value (L)}$$

$$= 70 - 10$$

$$= 60$$

$$\text{Coefficient of Range (CR)} = \frac{H-L}{H+L}$$

$$\text{CR} = \frac{70-10}{70+10} = \frac{60}{80} = 0.75$$

Hence, the Range (R) of the above series is 60 and Coefficient of Range (CR) is 0.75

15. Find the variance of the following data:

6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

a. 33

b. 15

c. 10

d. 14

Answer: A

Explanation:

x_i	$d_i = \frac{x_i - 14}{2}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	$\frac{6 - 14}{2} = -4$	$6 - 15 = -9$	$(-9)^2 = 81$
8	$\frac{8 - 14}{2} = -3$	$8 - 15 = -7$	$(-7)^2 = 49$
10	$\frac{10 - 14}{2} = -2$	$10 - 15 = -5$	$(-5)^2 = 25$
12	$\frac{12 - 14}{2} = -1$	$12 - 15 = -3$	$(-3)^2 = 9$
14	$\frac{14 - 14}{2} = 0$	$14 - 15 = -1$	$(-1)^2 = 1$
16	$\frac{16 - 14}{2} = 1$	$16 - 15 = 1$	$(1)^2 = 1$
18	$\frac{18 - 14}{2} = 2$	$18 - 15 = 3$	$(3)^2 = 9$
20	$\frac{20 - 14}{2} = 3$	$20 - 15 = 5$	$(5)^2 = 25$
22	$\frac{22 - 14}{2} = 4$	$22 - 15 = 7$	$(7)^2 = 49$
24	$\frac{24 - 14}{2} = 5$	$24 - 15 = 9$	$(9)^2 = 81$
	$\sum_{i=1}^{10} d_i = 5$		$\sum_{i=1}^{10} (x_i - \bar{x})^2 = 330$

$$\text{Mean } \bar{X} = \text{assumed mean} + \frac{\sum_{i=1}^{10} d_i}{n} \times h$$

Where a = assumed mean = 14

$$d_i = \frac{x_i - a}{h}$$

$$h = \text{Class width} = 8 - 6 = 2$$

$$n = \text{number of observations} = 10$$

$$\text{Mean } \bar{X} = 14 + \frac{5}{10} \times 2 = 15$$

$$\text{Variance } (\sigma^2) = \frac{1}{n} \sum (x_i - \bar{X})^2$$

$$\frac{1}{10} \times 330 = 33$$

16. Find the standard deviation of the following data:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

a. 14

b. 50

c. 62

d. 14.17

Answer: D

Explanation:

Class	Frequency (f_i)	Mid – point (x_i)	$f_i x_i$
30 – 40	3	35	$35 \times 3 = 105$
40 – 50	7	45	$45 \times 7 = 315$
50 – 60	12	55	$55 \times 12 = 660$
60 – 70	15	65	$65 \times 15 = 975$
70 – 80	8	75	$75 \times 8 = 600$
80 – 90	3	85	$85 \times 3 = 255$
90 – 100	2	95	$95 \times 2 = 190$
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3100$

$$\Sigma f_i x_i = 3100$$

$$\Sigma f_i = 50$$

$$\text{Mean } \bar{X} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\frac{3100}{50} = 62$$

$$\text{Variance } (\sigma^2) = \frac{1}{n} \Sigma (x_i - \bar{X})^2$$

$$\frac{1}{50} \times 10050 = 201$$

$$\text{Standard deviation } (\sigma) = \sqrt{201}$$

$$(\sigma) = 14.17$$

17. Estimate coefficient of quartile deviation of the following data:

Sr. No.	1	2	3	4	5	6	7	8	9	10	11
Data	8	9	11	12	13	17	20	21	23	25	27

- a. 3.53
c. 0.689

- b. 0.353
d. 0.591

Answer: B

Explanation:

In order to find the quartile deviation in case of individual series, we need to find out the values of third quartile and first quartile using the following equations:

$$Q_1 = \text{Size of } \left(\frac{N + 1}{4} \right)^{\text{th}} \text{ item}$$

$$Q_1 = \text{Size of } \left(\frac{11 + 1}{4} \right)^{\text{th}} \text{ item}$$

$$Q_1 = \text{Size of } 3^{\text{th}} \text{ item}$$

$$Q_1 = 11$$

$$Q_1 = \text{Size of } 3 \left(\frac{N + 1}{4} \right)^{\text{th}} \text{ item}$$

$$Q_1 = \text{Size of } 3 \left(\frac{11 + 1}{4} \right)^{\text{th}} \text{ item}$$

$$\text{or, } Q_3 = \text{Size of } 9^{\text{th}} \text{ item}$$

$$\text{or, } Q_3 = 23$$

Calculating Quartile Deviation and Coefficient of Quartile Deviation:

$$\text{Quartile Deviation (Q.D.) } \frac{Q_3 - Q_1}{2}$$

$$\text{Q.D. } \frac{23 - 11}{2}$$

$$\text{Q.D. } \frac{12}{2}$$

$$\text{Q.D.} = 6$$

$$\text{Coefficient of Quartile Deviation (Q.D.) } \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{23 - 11}{23 + 11} = \frac{12}{34} = 0.353$$

18. A measure of relative dispersion is given by the:

- a. Co-efficient of variance
c. Quartile deviation
b. Standard deviation
d. Variance

Answer: A

Symbol	Symbol Name	Meaning / definition
$\text{var}(X)$	variance	variance of random variable X
σ^2	variance	variance of population values
$\text{std}(X)$	standard deviation	standard deviation of random variable X
σ_x	standard deviation	standard deviation value of random variable X

Explanation:

Co-efficient of variance: This term is used commonly to mean scatter, Deviation, Fluctuation, Spread or variability of data. ... **Relative Measures of Dispersion: Relative measures of dispersion**, are also known as coefficients of **dispersion**, are **obtained** as ratios or percentages.

19. The _____ is the easiest measure of dispersion to calculate.

- a. Standard Deviation
- b. Range
- c. Mean absolute deviation
- d. Variance

Answer: B

Explanation:

Range is basically the difference between the lowest and highest values.

20. Which of the following symbols represents the standard deviation of the population?

- a. σ^2
- b. μ
- c. σ
- d. \bar{X}

Answer: C

Explanation:

σ

21. The variance can never be

- a. larger than the standard deviation
- b. Negative
- c. Smaller than the standard deviation
- d. Zero

Answer: b

Explanation:

Something (negative or positive number) squared is always a positive number, except

zero squared which is still zero. ... Because the squared deviations are all positive numbers or zeroes, their smallest possible mean is zero. It can't be negative. This average of the squared deviations is in fact variance. Hence, the variance can never be negative.

22. The numerical value of the standard deviation can never be

- a. Negative
- b. Larger than the variance
- c. Zero
- d. None

Answer: A

Explanation:

Standard Deviation formula is computed using squares of the numbers. Square of a number cannot be negative. Hence Standard deviation cannot be negative. Here $(x - \text{mean})$ is squared, so, this cannot be negative, N , number of terms cannot be negative, hence SD cannot be negative.

23. The descriptive measure of dispersion that is based on the concept of a deviation about the mean is

- a. The absolute value of the range
- b. Range
- c. Standard Deviation
- d. Inter quartile range

Answer: C

Explanation:

A measure of dispersion is a numerical value describing the amount of variability present in a data set. The standard deviation (SD) is the most commonly used measure of dispersion. With the SD you can measure dispersion relative to the scatter of the values about their mean.

24. When should measures of location and dispersion be computed from grouped data rather than from individual data values?

- a. Whenever computer packages for descriptive statistics are unavailable
- b. As much as possible since computations are easier
- c. Only when the data are from a population
- d. Only when individual data values are unavailable

Answer: D

Explanation:

Only when individual data values are unavailable should measures of location and dispersion be computed from grouped data rather than from individual data values.

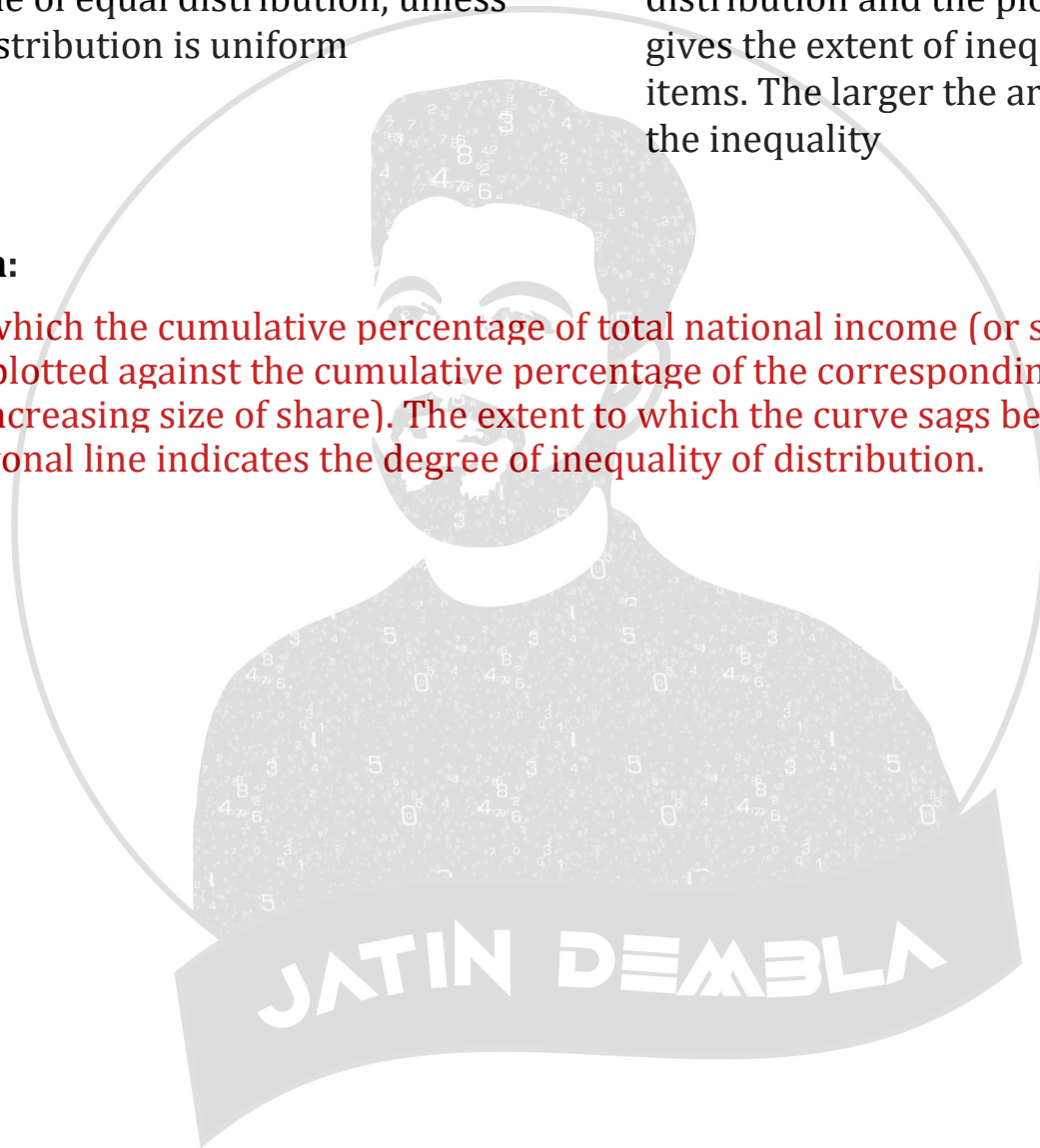
25. Which information is false regarding Lorenz curve

- a. The Lorenz curve devised by Dr. Max O. Lorenz is a graphic method of studying dispersion.
- b. Used this technique to show employment of a group of people
- c. The Lorenz curve always lies below the line of equal distribution, unless the distribution is uniform
- d. The Area between the line of equal distribution and the plotted curve gives the extent of inequality in the items. The larger the area, more is the inequality

Answer: B

Explanation:

A graph on which the cumulative percentage of total national income (or some other variable) is plotted against the cumulative percentage of the corresponding population (ranked in increasing size of share). The extent to which the curve sags below a straight diagonal line indicates the degree of inequality of distribution.



CHAPTER 16

PROBABILITY

PROBABILITY	The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics
RANDOM EXPERIMENT	An experiment is defined to be random if the results of the experiment depend on chance only
EXPERIMENT	An experiment may be described as a performance that produces certain results.
EVENTS	The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types: <ul style="list-style-type: none"> (i) Simple or Elementary, (ii) Composite or Compound
MUTUALLY EXCLUSIVE EVENTS OR INCOMPATIBLE EVENTS:	A set of events A_1, A_2, A_3, \dots is known to be mutually exclusive if not more than one of them can occur simultaneously
EXHAUSTIVE EVENTS	The events A_1, A_2, A_3 , are known to form an exhaustive set if one of these events must necessarily occur.
EQUALLY LIKELY EVENTS OR MUTUALLY SYMMETRIC EVENTS OR EQUI-PROBABLE EVENTS	The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events.
CLASSICAL DEFINITION OF	The probability of occurrence of the event A is defined as the ratio of the number of events Favourable to A to the total number of events. Denoting this by $P(A)$, we have.

**PROBABILITY OR A
PRIOR DEFINITION**

$$P(A) = \frac{\text{No. of equally likely events Favourable to A}}{\text{Total no. of equally likely events}}$$
**REMEMBERANCE POINT
& FORMULA**

(a) The probability of an event lies between 0 and 1, both inclusive.

When $P(A) = 0$, A is known to be an impossible event and when $P(A) = 1$, A is known to be a sure event.

(b) Non-occurrence of event A is denoted by A' or A^C . The event A along with its complementary A' forms a set of mutually exclusive and exhaustive events i.e.,

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

(c) The ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event A and its inverse ratio is known as odds against the event A i.e.,

$$\text{odds in favour of A} = m_A : (m - m_A)$$

$$\text{and odds against A} = (m - m_A) : m_A$$

(d) For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B i.e.,

$$P(A + B)$$

$$P(A + B) = P(A) + P(B)$$

(e) For any $K(> 2)$ mutually exclusive events $A_1, A_2, A_3, \dots, A_K$ the probability that at least one of them occurs is given by the sum of the individual probabilities of the events i.e.,

$$P(A_1 + A_2 + \dots + A_K) = P(A_1) + P(A_2) + \dots + P(A_K)$$

(f) For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B i.e.,

$$P(A + B) = P(A) + P(B) - P(A \cap B)$$

(g) For any three events A, B and C, the probability that at least one of the events occurs is given by

$$P(A + B + C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$+B) - P(A + C) - P(B + C) + P(A + B + C)$$

- (h) For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e.,

$$P(A * B) = P(A) \times P(B/A) \text{ Provided } P(A) > 0$$

- (i) Compound Probability or Joint Probability

$$P(B/A) = \frac{P(B + A) - P(A)}{P(A)} = \frac{P(A + B) - P(A)}{P(A)}$$

GRAPHICAL FORMULA OF PROBABILITY

$$P(A) = \frac{\text{number of favourable events}}{\text{number of total events}}$$

$$P(A) = \frac{n(A)}{n}$$

$$P(B) = \frac{n(B)}{n}$$

$$P(A \cap B) = P(A) P(B)$$

for Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

for non-Mutual Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

for Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$



1. What is the chance of picking a spade or an ace not of spade from a pack of 52 cards?

a. $4/13$

b. $4/14$

c. $15/13$

d. $6/13$

ANSWER: a

EXPLANATION:

A pack of 52 cards contain 13 Spades, 13 Hearts, 13 Clubs and 13 Diamonds. Each of these groups of 13 cards has an ace. Hence the total number of elementary events is 52 out of which 13 + 3 or 16 are favourable to the event A representing picking a Spade or an ace not of Spade. Thus we have

$$P(A) = \frac{16}{52} = \frac{4}{13}$$

2. A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise: 2 ladies.

a. $\frac{140}{429}$

b. $\frac{14}{429}$

a. $\frac{10}{49}$

c. None

ANSWER: a

EXPLANATION:

Since there are altogether 8 + 5 or 13 persons, a committee comprising 7 members can be formed in

$${}^{13}C_7 \text{ or } \frac{13!}{7!6!} \text{ or } \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8!}{7! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \text{ or } 11 \times 12 \times 13 \text{ ways.}$$

When the committee is formed taking 2 ladies out of 5 ladies, the remaining (7-2) or 5 committee members are to be selected from 8 gentlemen. Now 2 out of 5 ladies can be selected in 5C_2 ways and 5 out of 8 gentlemen can be selected in 8C_5 ways. Thus if A denotes the event of having the committee with 2 ladies, then A can occur in ${}^5C_2 \times {}^8C_5$ or 10×56 ways thus,

$$P(A) = \frac{10 \times 56}{11 \times 12 \times 13} = \frac{140}{429}$$

3. What if in above question 2 . 2 ladies be replace by at least 2 ladies?

a. $\frac{92}{429}$

b. $\frac{32}{29}$

c. $\frac{392}{429}$

d. None

ANSWERS: c

EXPLANATION:

Since the minimum number of ladies is 2, we can have the following combinations:

Population:	8G	+	5L
Sample:	2L	+	5G
or	3L	+	4G
or	4L	+	3G
or	5L	+	2G

Thus if B denotes the event of having at least two ladies in the committee, then B can occur in

$${}^5C_2 \times {}^8C_5 + {}^5C_3 \times {}^8C_4 + {}^5C_4 \times {}^8C_3 + {}^5C_5 \times {}^8C_2$$

i.e. 1568 ways.

$$\text{Hence, } P(A) = \frac{1568}{11 \times 12 \times 13} = \frac{392}{429}$$

4. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

a. $\frac{1}{2}$
c. $\frac{8}{15}$

b. $\frac{2}{5}$
d. $\frac{9}{20}$

Answer: Option D

Explanation:

Here, $S = \{1, 2, 3, 4, \dots, 19, 20\}$.

Let E = event of getting a multiple of 3 or 5 = $\{3, 6, 9, 12, 15, 18, 5, 10, 20\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$$

5. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

a. $\frac{10}{21}$
c. $\frac{2}{7}$

b. $\frac{11}{21}$
d. $\frac{5}{7}$

Answer: Option A

Explanation:

Total number of balls = $(2 + 3 + 2) = 7$.

Let S be the sample space.

Then, $n(S)$ = Number of ways of drawing 2 balls out of 7

$$= {}^7C_2$$

$$= \frac{(7 \times 6)}{(2 \times 1)}$$

$$= 21.$$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore n(E)$ = Number of ways of drawing 2 balls out of (2 + 3) balls.

$$= {}^5C_2$$

$$= \frac{(5 \times 4)}{(2 \times 1)}$$

$$= 10.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}$$

6. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

a. $\frac{1}{3}$
b. $\frac{3}{7}$
c. $\frac{7}{19}$

d. $\frac{8}{21}$

Answer: Option A

Explanation:

Total number of balls = $(8 + 7 + 6) = 21$.

event that the ball drawn is neither red nor green

event that the ball drawn is blue.

$$\therefore n(E) = 7.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{21} = \frac{1}{3}$$

7. What is the probability of getting a sum 9 from two throws of a dice?

a. $\frac{1}{6}$
c. $\frac{1}{9}$

b. $\frac{1}{8}$
d. $\frac{1}{12}$

Answer: Option C

Explanation:

In two throws of a dice, $n(S) = (6 \times 6) = 36$.

Let E = event of getting a sum = $\{(3, 6), (4, 5), (5, 4), (6, 3)\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

8. Three unbiased coins are tossed. What is the probability of getting at most two heads?

a. $\frac{3}{4}$
c. $\frac{3}{8}$

b. $\frac{1}{4}$
d. $\frac{7}{8}$

Answer: Option D

Explanation:

Here $S = \{TTT, TTH, THT, HTT, THH, HTH, HHT, HHH\}$

Let E = event of getting at most two heads.

Then $E = \{TTT, TTH, THT, HTT, THH, HTH, HHT\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?

a. $\frac{1}{2}$
c. $\frac{3}{8}$

b. $\frac{3}{4}$
d. $\frac{5}{16}$

Answer: Option B

Explanation:

In a simultaneous throw of two dice, we have $n(S) = (6 \times 6) = 36$.

Then, $E = \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$$\therefore n(E) = 27.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}.$$

10. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is:

a. $\frac{21}{46}$
c. $\frac{1}{50}$

b. $\frac{25}{117}$
d. $\frac{3}{25}$

Answer: Option A

Explanation:

Let S be the sample space and E be the event of selecting 1 girl and 2 boys.

Then, $n(S) =$ Number ways of selecting 3 students out of 25

$$= {}^{25}C_3$$

$$= \frac{(25 \times 24 \times 23)}{(3 \times 2 \times 1)}$$

$$= 2300.$$

$$n(E) = ({}^{10}C_1 \times {}^{15}C_2)$$

$$= \left[10 \times \frac{(15 \times 14)}{(2 \times 1)} \right]$$

$$= 1050.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}$$

11. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

a. $\frac{1}{10}$
c. $\frac{2}{7}$

b. $\frac{2}{5}$
d. $\frac{5}{7}$

Answer: Option C

Explanation:

$$P(\text{getting a prize}) = \frac{10}{(10 + 25)} = \frac{10}{35} = \frac{2}{7}$$

12. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings?

a. $\frac{1}{15}$
c. $\frac{1}{221}$

b. $\frac{25}{57}$
d. $\frac{35}{256}$

Answer: Option c

Explanation:

Let S be the sample space.

$$\text{Then, } n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 2 kings out of 4.

$$\therefore n(E) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$$

13. Two dice are tossed. The probability that the total score is a prime number is:

a. $\frac{1}{6}$

b. $\frac{5}{12}$

c. $\frac{1}{2}$

d. $\frac{7}{9}$

Answer: Option B**Explanation:**Clearly, $n(S) = (6 \times 6) = 36$.

Let E = Event that the sum is a prime number.

Then $E = \{ (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5) \}$

$$\therefore n(E) = 15.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is:

a. $\frac{1}{13}$
c. $\frac{1}{26}$

b. $\frac{2}{13}$
d. $\frac{1}{52}$

Answer: Option C**Explanation:**Here, $n(S) = 52$.

Let E = event of getting a queen of club or a king of heart.

Then, $n(E) = 2$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

15. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:

a. $\frac{1}{22}$
c. $\frac{2}{91}$

b. $\frac{3}{22}$
d. $\frac{2}{77}$

Answer: Option C**Explanation:**

Let S be the sample space.

Then, $n(S)$ = number of ways of drawing 3 balls out of 15

$$= {}^{15}C_3$$

$$= \frac{(15 \times 14 \times 13)}{(3 \times 2 \times 1)}$$

$$= 455.$$

Let E = event of getting all the 3 red balls.

$$\therefore n(E) = {}^5C_3 = {}^5C_2 = \frac{(5 \times 4)}{(2 \times 1)} = 10.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{455} = \frac{2}{91}.$$

16. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is:

a. $\frac{3}{20}$
c. $\frac{47}{100}$

b. $\frac{29}{34}$
d. $\frac{13}{102}$

Answer: Option D

Explanation:

Let S be the sample space.

$$\text{Then, } n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 1 spade and 1 heart.

$$\therefore n(E) = \text{number of ways of choosing 1 spade out of 13 and 1 heart out of 13}$$

$$= ({}^{13}C_1 \times {}^{13}C_1)$$

$$= (13 \times 13)$$

$$= 169.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}.$$

17. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card (Jack, Queen and King only)?

a. $\frac{1}{13}$

b. $\frac{3}{13}$

c. $\frac{1}{4}$

d. $\frac{9}{52}$

Answer: Option B

Explanation:

Clearly, there are 52 cards, out of which there are 12 face cards.

$$\therefore P(\text{getting a face card}) = \frac{12}{52} = \frac{3}{13}.$$

18. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white?

a. $\frac{3}{4}$

b. $\frac{4}{7}$

c. $\frac{1}{8}$

d. $\frac{3}{7}$

Answer: Option B

Explanation:

Let number of balls = $(6 + 8) = 14$.

Number of white balls = 8.

$$P(\text{drawing a white ball}) = \frac{8}{14} = \frac{4}{7}.$$

19. A bag contains 6 white and 4 black balls .2 balls are drawn at random. Find the probability that they are of same colour.

a. $\frac{1}{2}$

b. $\frac{7}{15}$

c. $8/15$ d. $1/9$

Answer: b

Explanation:

Let S be the sample space

Then $n(S) = \text{no of ways of drawing 2 balls out of } (6+4)$
 $= {}^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$

Let E = event of getting both balls of same colour

Then, $n(E) = \text{no of ways (2 balls out of six) or (2 balls out of 4)}$
 $= {}^6C_2 + {}^4C_2 = \frac{6 \times 5}{2 \times 1} + \frac{4 \times 3}{2 \times 1} = 15 + 6 = 21$

Therefore, $P(E) = n(E)/n(S) = 21/45 = 7/15$

20. A problem is given to three students whose chances of solving it are $1/2$, $1/3$ and $1/4$ respectively. What is the probability that the problem will be solved?

a. $1/4$ b. $1/2$ c. $3/4$ d. $7/12$

Answer: c

Explanation:

Let A, B, C be the respective events of solving the problem and \bar{A} , \bar{B} , \bar{C} be the respective events of not solving the problem. Then A, B, C are independent event

∴ A, B, C are independent events

Now, $P(A) = 1/2$, $P(B) = 1/3$ and $P(C) = 1/4$ $P(\bar{A}) = 1/2$, $P(\bar{B}) = 2/3$, $P(\bar{C}) = 3/4$ ∴ $P(\text{none solves the problem}) = P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

∴ A, B, C are Independent] ∴ A, B, C are Independent

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

$$= \frac{1}{4}$$

Hence, $P(\text{the problem will be solved}) = 1 - P(\text{none solves the problem})$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

20. Two cards are drawn at random from a pack of 52 cards. what is the probability that either both are black or both are queen?

a. $52/221$ b. $55/190$ c. $55/221$ d. $19/221$

Answer: c

Explanation:

We have $n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$.

Let A = event of getting both black cards

B = event of getting both queens

$A \cap B$ = event of getting queen of black cards

$$n(A) = 52 \cdot 51 \cdot 152 \cdot 51 \cdot 1 = 26C2 \cdot 26C2 = 325,$$

$$n(B) = 26 \cdot 25 \cdot 126 \cdot 25 \cdot 1 = 4 \cdot 3/2 \cdot 1 = 6 \text{ and}$$

$$n(A \cap B) = 4C2 \cdot 4C2 = 1$$

$$P(A) = n(A)/n(S) = 325/1326;$$

$$P(B) = n(B)/n(S) = 6/1326 \text{ and}$$

$$P(A \cap B) = n(A \cap B)/n(S) = 1/1326$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = (325+6-1) / 1326 = 330/1326 = 55/221$$

21. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is a multiple of

3 or 5?

$$\frac{1}{2}$$

$$\frac{9}{20}$$

Answer: c

Explanation:

Here, $S = \{1, 2, 3, 4, \dots, 19, 20\}$.

Let E = event of getting a multiple of 3 or 5 = $\{3, 6, 9, 12, 15, 18, 5, 10, 20\}$.

$$P(E) = n(E)/n(S) = 9/20.$$

22. Two dice are tossed. The probability that the total score is a prime number is:

a. $\frac{5}{12}$

b. $\frac{1}{6}$

c. $\frac{1}{2}$

d. $\frac{7}{9}$

Answer: a

Explanation:

Clearly, $n(S) = (6 \times 6) = 36$.

Let E = Event that the sum is a prime number.

Then $E = \{(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$

$$n(E) = 15.$$

$$P(E) = n(E)/n(S) = 15/36 = 5/12.$$

23. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1/7) and the probability of wife's selection is (1/5). What is the probability that only one of them is selected?

a. $\frac{2}{7}$

b. $\frac{1}{7}$

c. $\frac{3}{4}$

d. $\frac{4}{5}$

Answer: a

Explanation:

Let A = Event that the husband is selected
and B = Event that the wife is selected.

Then, $P(A) = \frac{1}{7}$ and $P(B) = \frac{1}{5}$.

$\therefore P(\bar{A}) = \left(1 - \frac{1}{7}\right) = \frac{6}{7}$ and $P(\bar{B}) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}$.

\therefore Required probability = $P[(A \text{ and not } B) \text{ or } (B \text{ and not } A)]$

= $P[(A \text{ and } \bar{B}) \text{ or } (B \text{ and } \bar{A})]$

= $P(A \text{ and } \bar{B}) + P(B \text{ and } \bar{A})$

= $P(A) \cdot P(\bar{B}) + P(B) \cdot P(\bar{A}) = \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right) = \frac{10}{35} = \frac{2}{7}$.

24. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:

a. $2/91$

b. $1/22$

c. $3/22$

d. $2/77$

Answer: a

Explanation:

Let S be the sample space.

Then, $n(S)$ = number of ways of drawing 3 balls out of 15
= ${}^{15}C_3 = \frac{15 \times 14 \times 13 \times 2 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 455$.

Let E = event of getting all the 3 red balls.

$n(E) = {}^5C_3 = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 10$.

$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{10}{455} = \frac{2}{91}$.

25. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

a. $2/7$

b. $5/7$

c. $1/5$

d. $1/2$

Answer: a

Explanation:

Total number of outcomes possible, $n(S) = 10 + 25 = 35$

Total number of prizes, $n(E) = 10$

$P(E) = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$

26. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is:

a. $21/46$

b. $1/5$

c. $3/25$ d. $1/50$ A) $21/46$ B) $1/5$ C) $3/25$ D) $1/50$ **Answer: A)****Explanation:**

Let, S - sample space E - event of selecting 1 girl and 2 boys.

Then, $n(S)$ = Number ways of selecting 3 students out of 25

$$= {}^{25}C_3 = 2300.$$

$$n(E) = {}^{10}C_1 \times {}^{15}C_2 = 10 \times 105 = 1050.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}$$

27. What is the probability of getting 53 Mondays in a leap year?a. $1/7$ b. $3/7$ c. $2/7$ d. $2/7$ **Answer: C****Explanation:**

1 year = 365 days . A leap year has 366 days

A year has 52 weeks. Hence there will be 52 Sundays for sure.

$$52 \text{ weeks} = 52 \times 7 = 364 \text{ days}$$

$$366 - 364 = 2 \text{ days}$$

In a leap year there will be 52 Sundays and 2 days will be left.

These 2 days can be:

1. Sunday, Monday
2. Monday, Tuesday
3. Tuesday, Wednesday
4. Wednesday, Thursday
5. Thursday, Friday
6. Friday, Saturday
7. Saturday, Sunday

Of these total 7 outcomes, the favourable outcomes are 2.
Hence the probability of getting 53 days = $2/7$

28. Two dice are thrown together. What is the probability that the sum of the number on the two faces is divided by 4 or 6.

- a. $7/18$
c. $8/18$

- b. $14/35$
d. $7/35$

Answer: a

Explanation:

Clearly, $n(S) = 6 \times 6 = 36$

Let E be the event that the sum of the numbers on the two faces is divided by 4 or 6.

Then, $E = \{(1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (6,2), (6,6)\}$
 $n(E) = 14.$

Hence, $P(E) = n(E)/n(S) = 14/36 = 7/18$

29. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card (Jack, Queen and King only)?

- a. $3/13$
c. $3/52$

- b. $1/13$
d. $9/52$

Answer: a

Explanation:

Clearly, there are 52 cards, out of which there are 12 face cards.

$P(\text{getting a face card}) = 12/52 = 3/13.$

30. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is:

- a. $3/20$
c. $47/100$

- b. $29/34$
d. $13/102$

Answer: d

Explanation:

Let S be the sample space.

Then, $n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

Let E = event of getting 1 spade and 1 heart.

$n(E) = \text{number of ways of choosing 1 spade out of 13 and 1 heart out of 13}$
 $= {}^{13}C_1 \times {}^{13}C_1 = 169.$

$$P(E) = n(E)/n(S) = 169/1326 = 13/102.$$

31. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white?

- a. $3/7$
c. $1/8$

- b. $4/7$
d. $3/4$

Answer: B)

Explanation:

Let number of balls = (6 + 8) = 14.

Number of white balls = 8.

P (drawing a white ball) = $8/14 = 4/7$.

32. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

- $1/2$
 $4/5$

- $3/4$
 $2/5$

Answer: d

Explanation:

$$P(E) = \frac{30}{100} = \frac{3}{10}, P(H) = \frac{20}{100} = \frac{1}{5} \text{ and } P(E \cap H) = \frac{10}{100} = \frac{1}{10}.$$

$$\begin{aligned} P(E \text{ or } H) &= P(E \cup H) \\ &= P(E) + P(H) - P(E \cap H) \\ &= \left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10} \right) = \frac{4}{10} = \frac{2}{5}. \end{aligned}$$

33. If two letters are taken at random from the word HOME, what is the probability that none of the letters would be vowels?

- a. $1/6$
c. $1/3$

- b. $1/2$
d. $1/4$

Answer: A)

Explanation:

P(first letter is not vowel) = 2424

P(second letter is not vowel) = 1313

So, probability that none of letters would be vowels is = $24 \times 13 = 16$

Total number of ways = 5!

Favourable number of ways 2.4!

Hence required probability

$$\frac{2.4!}{5!} = \frac{2}{5}$$

40. A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out 2 socks at random. What is the probability that they match

a. 4/9

b. 5/8

c. 5/9

d. 7/12

Answer: A

Explanation:

Out of 9 socks, 2 can be drawn in 9C_2 ways.

Two socks drawn from the drawer will match if either both are brown or both are blue. Therefore Favourable number of cases is

$${}^5C_2 + {}^4C_2$$

$$\text{Hence the required probability} = \frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$$

41. Ten students are seated at random in a row. The probability that two particular students are not seated side by side is

a. 4/5

b. 3/5

c. 2/5

d. 1/5

Answer: A

Explanation:

Total ways = 10!

Two boys can sit side by side in $2 \times 9!$ ways.

$$\text{So probability} = \frac{2 \times 9!}{10!} = \frac{1}{5}$$

Thus the probability that they are not seated together is $1 - \frac{1}{5} = \frac{4}{5}$

42. A fair coin is tossed 100 times. The probability of getting tails an odd number of times is

a. $\frac{1}{2}$

b. 1/8

c. 3/8

d. None

Answer: A

Explanation:

The total number of cases are 2^{100}

The number of favorable ways ${}^{100}C_1 + {}^{100}C_3 + \dots + {}^{100}C_{99} = 2^{100-1} = 2^{99}$

$$\frac{= 2^{99}}{= 2^{100}} = \frac{1}{2}$$

43. Three cards are drawn at random from a pack of 52 cards. What is the chance of drawing three aces.

- a. $3/5525$
c. $1/5525$

- b. $2/5525$
d. None

Answer: C

Explanation:

Required probability is $\frac{{}^4C_1}{{}^{52}C_3} = \frac{1}{5525}$

44. A bag contains 4 white, 5 red and 6 green balls. Three balls are picked up randomly. The probability that a white, a red and a green ball is drawn is

- A. $15/91$
C. $20/91$

- B. $30/31$
D. $24/91$

Answer: D

Explanation:

Required probability = $\frac{4.5.6}{{}^{15}C_3} = \frac{24}{91}$

45. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4 is

- a. $1/15$
c. $1/5$

- b. $14/15$
d. $4/5$

Answer: D

Explanation:

Total ways = $2! {}^6C_2 = 30$

Favourable cases = $30 - 6 = 24$

Required probability = $\frac{24}{30} = \frac{4}{5}$

Answer: B

Explanation:

There are 10 digits 0,1,2,3,4,5,6,7,8,9.

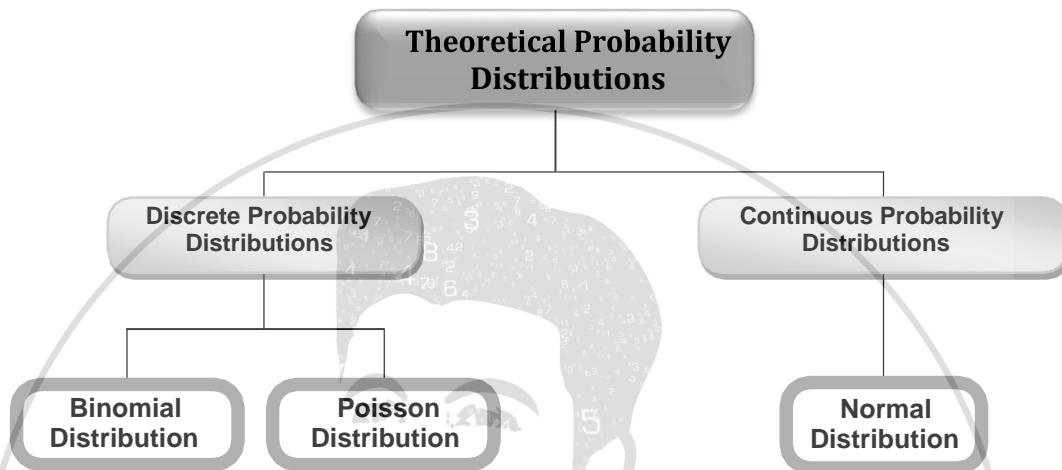
The last two digits can be dialed in $10P_2 = 90$ Ways ,

out of which only one way is Favourable, thus the required probability = $1/90$



CHAPTER 17

THEORETICAL DISTRIBUTIONS



THEORITICAL PROBABILITY	The total probability (i.e. one) is distributed to different mass points in case of a discrete random variable or to different class intervals in case of a continuous random variable
BINOMIAL DISTRIBUTION	One of the most important and frequently used discrete binomial distribution. The binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution such as the normal distribution . This is because the binomial distribution only counts two states, typically represented as 1 (for a success) or 0 (for a failure) given a number of trials in the data
Poisson Distribution	A random variable X is defined to follow Poisson distribution with parameter λ , to be denoted by $X \sim P(\lambda)$ if the probability mass function of x

Poisson Distribution Formula

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

$x = 0, 1, 2, 3, \dots$

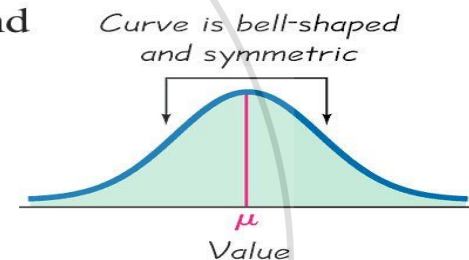
λ = mean number of occurrences in the interval

e = Euler's constant ≈ 2.71828

NORMAL DISTRIBUTIONS**NORMAL DISTRIBUTIONS**

If a continuous random variable has a distribution with a graph that is symmetric and bell-shaped and can be described by the equation

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma \sqrt{2\pi}}$$



we say that it has a **normal distribution**.

Properties of the Normal Distribution

- The normal distribution curve is **bell-shaped**.
- The mean, median, and mode are **equal** and located at the center of the distribution.
- The normal distribution curve is **unimodal** (single mode).
- The curve is **symmetrical** about the mean.
- The curve is **continuous**.
- The curve **never touches the x-axis**.
- The total area under the normal distribution curve is **equal to 1 or 100%**.

a. m^2

b. $m^{1/2}$

c. m

d. $m/2$

Answer: c**Explanation:**

For a discrete probability function, the variance is given by

$$\text{Variance (V)} = \sum_{x=0}^n x^2 p(x) - \mu^2$$

$$P(x) = \frac{e^{-m} m^x}{x!}$$

Where μ is the mean, substitute $\frac{e^{-m} m^x}{x!}$, in the above equation and put $\mu = m$ to obtain

$$V = m.$$

3. The p.d.f of Poisson Distribution is given by

a. $\frac{e^{-m} m^x}{x!}$

b. $\frac{e^{-m} x!}{m^x}$

c. $\frac{m^x e^{-m}}{x!}$

d. $\frac{e^m m^x}{x!}$

Answer: a**Explanation:**

This is a standard formula for Poisson Distribution, it needs no explanation. Even though if you are interested to know the derivation in detail, you can refer to any of the books or source on internet that speaks of this matter.

4. If 'm' is the mean of a Poisson Distribution, the standard deviation is given by

a. $m^{1/2}$

b. m^2

c. m

d. $m/2$

Answer: a**Explanation:**

The variance of a Poisson distribution with mean 'm' is given by $V = m$, hence Standard Deviation = $(\text{variance})^{1/2} = m^{1/2}$.

5. In a Poisson Distribution, the mean and variance are equal

a. True

b. False

c. Can't say

d. not justifiable

Answer: a**Explanation:**

Mean = m
 Variance = m
 \therefore Mean = Variance.

6. In a Poisson Distribution, if mean (m) = e , then $P(x)$ is given by

a. $\frac{e^{-m}m^x}{x!}$
 c. $\frac{m^x e^{-m}}{x!}$

b. $\frac{e^{-m}x!}{m^x}$
 d. $\frac{e^m m^x}{x!}$

Answer: a

Explanation:

Put $m = e$,

$$P(x) = \frac{e^{-m}m^x}{x!}$$

7. Poisson distribution is applied for

- a. Continuous Random Variable
 b. Discrete Random Variable
 c. Irregular Random Variable
 d. Uncertain Random Variable

Answer: b

Explanation:

Poisson distribution along with Binomial Distribution is applied for Discrete Random variable. Speaking more precisely, Poisson Distribution is an extension of Binomial Distribution for larger values 'n'. Since Binomial Distribution is of discrete nature, so is its extension Poisson Distribution.

8. If 'm' is the mean of Poisons Distribution, the $P(0)$ is given by

- a. e^{-m}
 b. e^m
 c. e
 d. m^{-e}

Answer: a

Explanation:

$$P(x) = \frac{e^{-m}m^x}{x!}$$

Put $x = 0$, to obtain e^{-m} .

9. In a Poisson distribution, the mean and standard deviation are equal

- a. True
 b. False

c. Can't say

d. not justifiable

Answer: b**Explanation:**

In a Poisson Distribution,

Mean = m

Standard Deviation = $m^{1/2}$ \therefore Mean and Standard deviation are not equal.**10. For a Poisson Distribution, if mean(m) = 1, then P(1) is**

a. 1/e

b. e

c. e/2

d. Indeterminate

Answer: a**Explanation:**

$$P(x) = \frac{e^{-m} m^x}{x!}$$

Put $m = x = 1$, (given) to obtain 1/e.**11. The recurrence relation between P(x) and P(x + 1) in a Poisson distribution is given by**a. $P(x+1) - m P(x) = 0$ b. $m P(x+1) - P(x) = 0$ c. $(x+1) P(x+1) - m P(x) = 0$ d. $(x+1) P(x) - x P(x+1) = 0$ **Answer: c****Explanation:**

$$P(x) = \frac{e^{-m} m^x}{x!}$$

 $p(x+1) = e^{-1} m^{x+1} / (x + 1)!$ Divide $P(x+1)$ by $P(x)$ and rearrange to obtain $(x+1) P(x+1) - m P(x) = 0$.**12. The mean value for an event X to occur is 2 in a day. Find the probability of event X to occur thrice in a day.**

a. 0.1804

b. 0.1804465

c. 0.18

d. None

ANSWER: b**EXPLANATION:**Mean, $m=2m=2$ Probability of the event to occur thrice, $P(3;2) = e^{-2} \frac{2^3}{3!} = 0.1804465$ **13. A man was able to complete 3 files a day on an average. Find the probability**

15. If the random variable X follows a poisson distribution with mean 3.4 , find $P(X=6)$

- a. 0.071604409
c. 0.0023698

- b. 0.00125948
d. 0.015792

Answer: a

Explanation:

This can be written more quickly as : if $X = \text{Po}(3.4)$

Find $(X = 6)$

Now

$$P(X=6) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \frac{e^{-3.4} (3.4)^6}{6!} \quad (\text{mean, } \lambda = 3.4)$$

= 0.071604409 or 0.072 (to 3 d.p.)

BINOMIAL DISTRIBUTION:

1. In a Binomial Distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by

- a. np
c. p
- b. n
d. np(1-p)

Answer: a

Explanation:

For a discrete probability function, the mean value or the expected value is given by

$$\text{Mean}(\mu) = \sum_{x=0}^n xp(x)$$

For Binomial Distribution $P(x) = {}^n C_x p^x q^{(n-x)}$, substitute in above equation and solve to get

$\mu = np$.

2. In a Binomial Distribution, if p, q and n are probability of success, failure and number of trials respectively then variance is given by

- a. np
c. np²q
- b. npq
d. npq²

Answer: b

Explanation:

c. Small values of 'n'

d. Any value of 'n'

Answer: c

Explanation:

As the value of 'n' increases, it becomes difficult and tedious to calculate the value of ${}^n C_x$.

7. For larger values of 'n', Binomial Distribution

a. loses its discreteness

b. tends to Poisson Distribution

c. stays as it is

d. gives oscillatory values

Answer: b

Explanation:

Where $m = np$ is the mean of Poisson Distribution.

8. In a Binomial Distribution, if $p = q$, then $P(X = x)$ is given by

a. ${}^n C_x (0.5)^n$

b. ${}^n C_n (0.5)^n$

c. ${}^n C_x p^{(n-x)}$

d. ${}^n C_n p^{(n-x)}$

Answer: a

Explanation:

If $p = q$, then $p = 0.5$

Substituting in $P(x) = {}^n C_x p^x q^{(n-x)}$ we get ${}^n C_n (0.5)^n$.

9. Binomial Distribution is a

a. Continuous distribution

b. Discrete distribution

c. Irregular distribution

d. Not a Probability distribution

Answer: b

Explanation:

It is applied to a discrete Random variable, hence it is a discrete distribution

10. 15 dates are selected at random, what is the probability of getting two Sundays?

a. 0.29

b. 34

c. 56

d. 78

ANSWER: a

EXPLANATION:

If X denotes the number at Sundays, then it is obvious that X follows binomial distribution with parameter $n = 15$ and $p =$ probability of a Sunday in a

week = $1/7$ and $q = 1 - p = 6/7$.

Then $f(x) = {}^{15}C_x (1/7)^x \cdot (6/7)^{15-x}$.

for $x = 0, 1, 2, \dots, 15$.

Hence the probability of getting two Sundays

= $f(2)$

= ${}^{15}C_2 (1/7)^2 \cdot (6/7)^{15-2}$

$$\frac{10^5 6^{13}}{7^{15}}$$

= 0.29

11. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that out of 5 workmen, 3 or more will contract the disease?

a. .890

b. .0086

c. .00086

d. NONE

ANSWER: c

EXPLANATION:

Let X denote the number of workmen in the sample. X follows binomial with parameters $n = 5$ and $p =$ probability that a workman suffers from the occupational disease = 0.1

Hence $q = 1 - 0.1 = 0.9$

Thus $f(x) = {}^5C_x \cdot (0.1)^x \cdot (0.9)^{5-x}$

For $x = 0, 1, 2, \dots, 5$.

The probability that 3 or more workmen will contract the disease

$$= P(x \geq 3)$$

$$= f(3) + f(4) + f(5)$$

$$= {}^5C_3 (0.1)^3 (0.9)^{5-3} + {}^5C_4 (0.1)^4 \cdot (0.9)^{5-4} + {}^5C_5 (0.1)^5$$

$$= 10 \times 0.001 \times 0.81 + 5 \times 0.0001 \times 0.9 + 1 \times 0.00001$$

$$= 0.0081 + 0.00045 + 0.00001$$

$$= 0.0086.$$

16. Find the probability of a success for the binomial distribution satisfying the following relation $4 P (x = 4) = P (x = 2)$ and having the parameter n as six.

a. $P \neq 1$

b. $P \neq -1$

c. $P = 1$

d. $P = 0$

ANSWER: b

EXPLANATION:

We are given that $n = 6$. The probability mass function of x is given by

$$f(x) = {}^n C_x p^x q^{n-x} = {}^6 C_x p^x q^{n-x}$$

for $x = 0, 1, \dots, 6$.

Thus $P(x = 4) = f(4)$:

$$= {}^6 C_4 p^4 q^{6-4} = 15 p^4 q^2$$

and $P(x = 2) = f(2)$

$$= {}^6 C_2 p^2 q^{6-2} = 15 p^2 q^4$$

Hence $4 P(x = 4) = P(x = 2)$

$$= 60 p^4 q^2 = 15 p^2 q^4$$

$$= 15 p^2 q^2 (4p^2 - q^2) = 0$$

$$= 4p^2 - q^2 = 0 \text{ (as } p \geq 0, q \geq 0 \text{)}$$

$$= 4p^2 - (1 - p)^2 = 0 \text{ (as } q = 1 - p \text{)}$$

$$= (2p + 1 - p) = 0 \text{ or } (2p - 1 + p) = 0$$

$$= p = -1 \text{ or } p = 1/3 \text{ Thus } p = 1/3 \text{ (as } p \neq -1 \text{)}$$

NORMAL DISTRIBUTION:

1. Normal Distribution is applied for

a. Continuous Random Distribution

b. Discrete Random Variable

c. Irregular Random Variable

d. Uncertain Random Variable

Answer: a

Explanation:

Normal Distribution is applied for Continuous Random Distribution. A discrete probability **distribution** is a probability **distribution** characterized by a probability

mass function. Thus, the **distribution** of a **random** variable X is discrete, and X is called a discrete **random** variable, if, as u runs through the set of all possible values of X .

2. The shape of the Normal Curve is

- a. Bell Shaped
- b. Flat
- c. Circular
- d. Spiked

Answer: a

Explanation:

Due to the nature of the Probability Mass function, a bell shaped curve is obtained.

3. Normal Distribution is symmetric is about

- a. Variance
- b. Mean
- c. Standard deviation
- d. Covariance

Answer: b

Explanation:

Due to the very nature of p.m.f of Normal Distribution, the graph appears such that it is symmetric about its mean.

4. For a standard normal variate, the value of mean is

- a. ∞
- b. 1
- c. 0
- d. not defined

Answer: c

Explanation:

For a normal variate, if its mean = 0 and standard deviation = 1, then its called as Standard Normal Variate. Here, the converse is asked.

5. The area under a standard normal curve is

- a. 0
- b. 1
- c. ∞
- d. not defined

Answer: b

Explanation:

For any probability distribution, the sum of all probabilities is 1. Area under normal curve refers to sum of all probabilities.

6. The standard normal curve is symmetric about the value

Answer: a

Explanation:

It has a theoretical evidence that requires some serious background on several topics
For more details you can refer to any book or website that speaks on the same.

11. In Normal distribution, the highest value of ordinate occurs at

- a. Mean
- b. Variance
- c. Extremes
- d. Same value occurs at all points

Answer: a

Explanation:

This is due the behavior of the pdf of Normal distribution.

12. The shape of the normal curve depends on its

- a. Mean deviation
- b. Standard deviation
- c. Quartile deviation
- d. Quartile deviation

Answer: b

Explanation:

This can be seen in the pdf of normal distribution where *standard deviation* is a variable.

13. The value of constant 'e' appearing in normal distribution is

- a. 2.5185
- b. 2.7836
- c. 2.1783
- d. 2.7183

Answer: d

Explanation:

This is a standard constant.

14. In Standard normal distribution, the value of mode is

- a. 2
- b. 1
- c. 0
- d. Not fixed

Answer: c

Explanation:

In a standard normal distribution, the value of mean is 0 and in normal distribution mean and mode coincide.

Assuming the marks percentage to follow a normal distribution, calculate the mean and standard deviation of marks. If not more than 300 examinees are to fail, what should be the passing marks?

a. 30%

b. 40%

c. 40%

d. 40%

ANSWER: a**EXPLANATION:**

Let X denote the percentage of marks and its mean and S.D. be μ and σ respectively. From the given table, we can write

$P(X < 40) = 0.43$ and $P(X \geq 75) = 0.15$, which can also be written as

$$P\left(z < \frac{40 - \mu}{\sigma}\right) = 0.43 \quad \text{and} \quad P\left(z \geq \frac{75 - \mu}{\sigma}\right) = 0.15$$

The above equations respectively imply that

$$\frac{40 - \mu}{\sigma} = -0.175 \quad \text{or} \quad 40 - \mu = -0.175\sigma \quad \dots (1)$$

and $\frac{75 - \mu}{\sigma} = 1.04 \quad \text{or} \quad 75 - \mu = 1.04\sigma \quad \dots (2)$

Solving the above equations simultaneously, we get $\mu = 45.04$ and $\sigma = 28.81$.

Let X_1 be the percentage of marks required to pass the examination.

Then we have $P(X < X_1) = 0.3$ or $P\left(z < \frac{X_1 - 45.04}{28.81}\right) = 0.3$

$$\therefore \frac{X_1 - 45.04}{28.81} = -0.525 \Rightarrow X_1 = 29.91 \quad \text{or} \quad 30\% \quad (\text{approx.})$$

18. At a petrol station, the mean quantity of petrol sold to a vehicle is 20 litres per day with a standard deviation of 10 litres. If on a particular day, 100 vehicles took 25 or more litres of petrol, estimate the total number of vehicles who took petrol from the station on that day. Assume that the quantity of petrol taken from the station by a vehicle is a normal variate.

a. 333

b. 343

c. 343

d. 567

ANSWER: c**EXPLANATION:**

Let X denote the quantity of petrol taken by a vehicle. It is given that $X \sim N(20, 10)$.

$$\begin{aligned}\therefore P(X \geq 25) &= P\left(z \geq \frac{25 - 20}{10}\right) = P(z \geq 0.5) \\ &= 0.5000 - P(0 \leq z \leq 0.5) = 0.5000 - 0.1915 = 0.3085\end{aligned}$$

Let N be the total number of vehicles taking petrol on that day.

$$\therefore 0.3085 \times N = 100 \text{ or } N = 100/0.3085 = 324 \text{ (approx.)}$$

19. Using the table of areas under the standard normal curve, find the following probabilities :

(i) $P(0 \leq z \leq 1.3)$

(ii) $P(-1 \leq z \leq 0)$

(iii) $P(-1 \leq z \leq 2)$

a. 0.4032, 0.3413, 0.8185

b. 0.4072, 0.4413, 0.8185

c. 0.40456, 0.3456, 0.8155

d. NONE

ANSWER: a

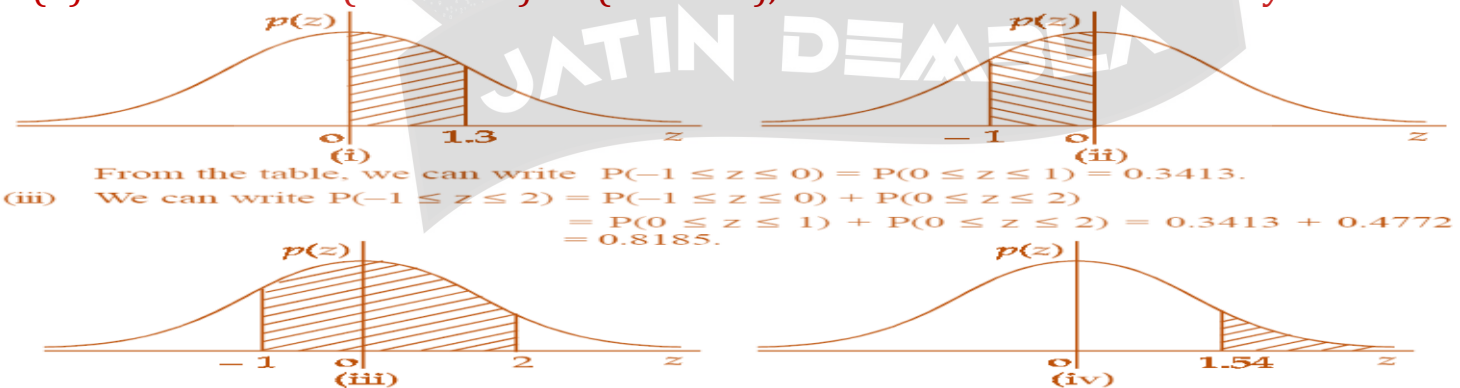
EXPLANATION:

The required probability, in each question, is indicated by the shaded area of the corresponding figure.

a. From the table,

b. (i) we can write $P(0 \leq z \leq 1.3) = 0.4032$.

c. (ii) We can write $P(-1 \leq z \leq 0) = P(0 \leq z \leq 1)$, because the distribution is symmetrical.



20. Determine the value or values of z in each of the following situations:

(i) Area between 0 and z is 0.4495.

(ii) Area between $-\infty$ to z is 0.1401.

a. -1.64, -1.08

b. -1.08, -1.64

c. 1.64, 1.08

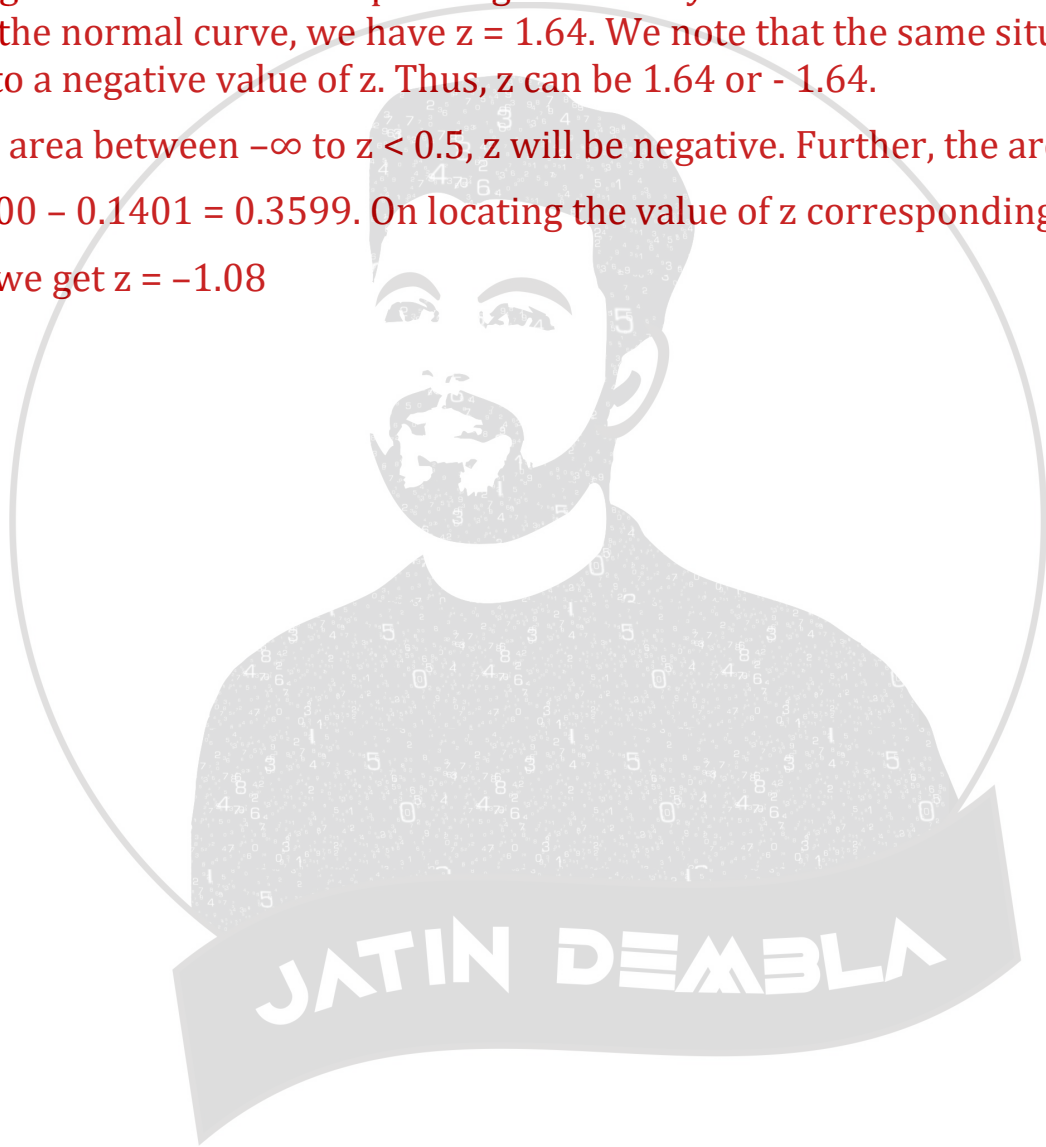
d. -1.64, 1.08

ANSWER: a

EXPLANATION:

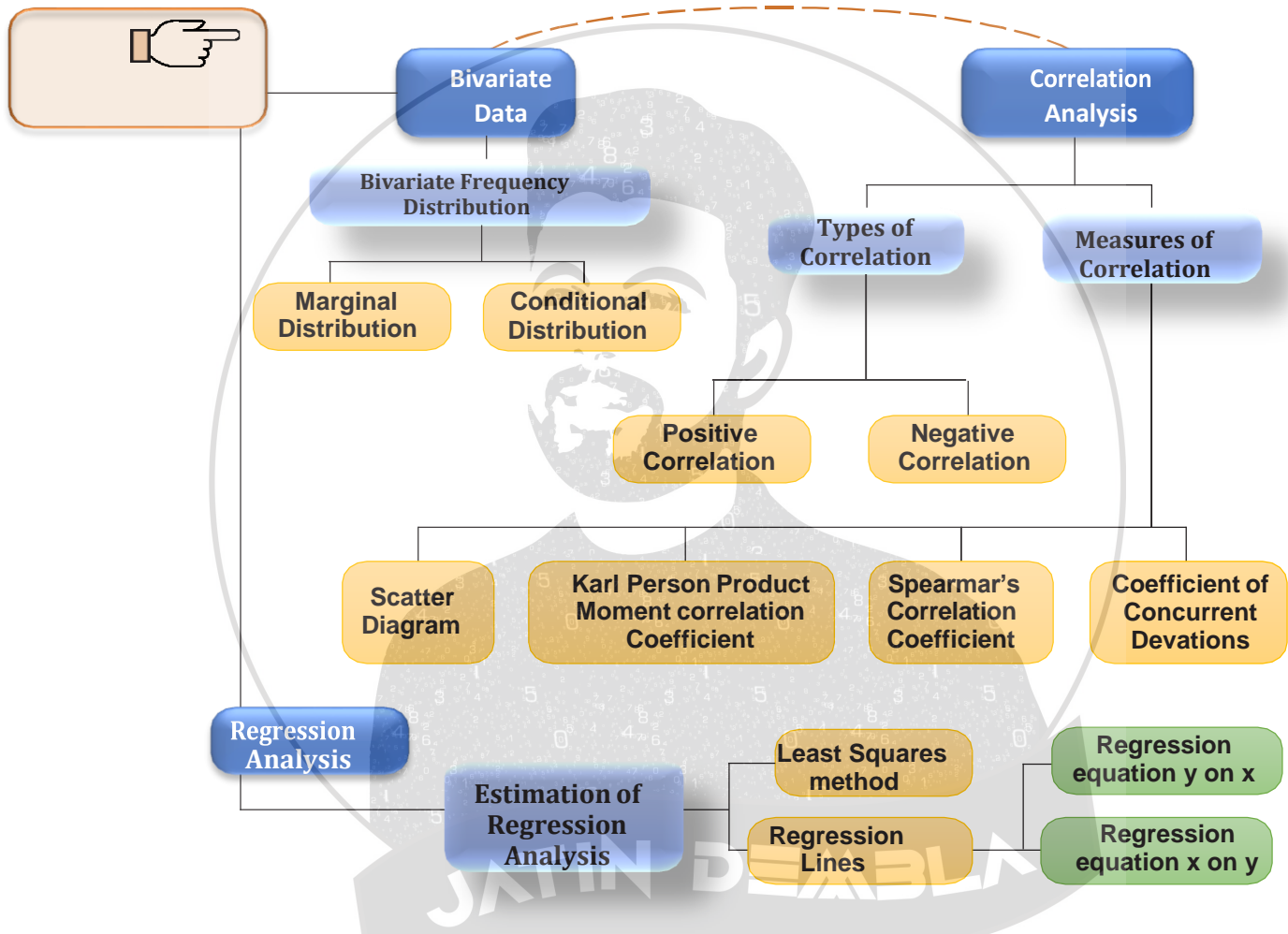
(i) On locating the value of z corresponding to an entry of area 0.4495 in the table of areas under the normal curve, we have $z = 1.64$. We note that the same situation may correspond to a negative value of z . Thus, z can be 1.64 or -1.64.

(ii) Since the area between $-\infty$ to $z < 0.5$, z will be negative. Further, the area between z and 0 = $0.5000 - 0.1401 = 0.3599$. On locating the value of z corresponding to this entry in the table, we get $z = -1.08$



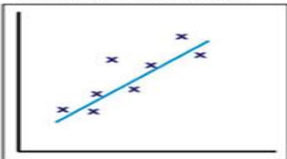
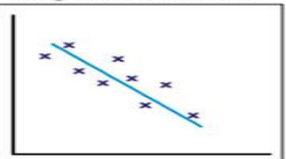
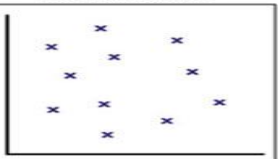
CHAPTER 18

CORRELATION AND REGRESSION

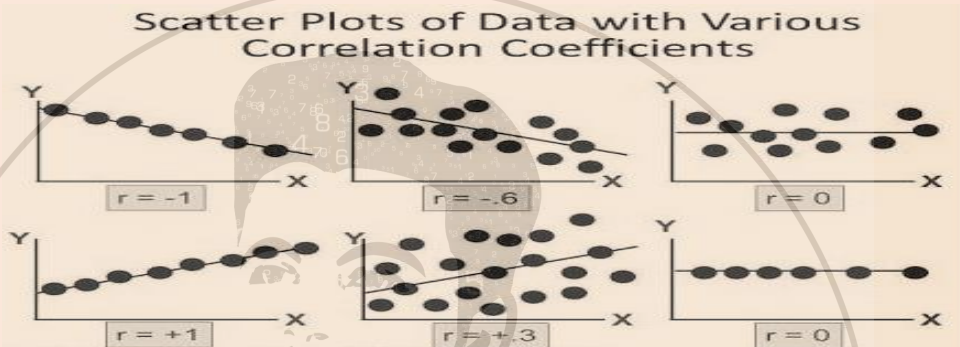


CORRELATION The change in one variable is reciprocated by a corresponding change in the other variable either directly or inversely, then the two variables are known to be associated or correlated

TYPES OF CORRELATION

Positive correlation	Negative correlation	No correlation
		
The points lie close to a straight line, which has a positive gradient.	The points lie close to a straight line, which has a negative gradient.	There is no pattern to the points.
This shows that as one variable increases the other increases .	This shows that as one variable increases , the other decreases .	This shows that there is no connection between the two variables.

MEASURES OF CORRELATION



Rank Correlation

The formula of Spearman's Rank correlation coefficient, is given as:

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

r_s : is the coefficient of rank correlation.

d_i : is the difference in rank between paired values of X and Y, it can be calculated as (rank of X_i - rank of Y_i).

n : is a sample size or the number of pairs values X and Y in the selected sample.

Correlation Coefficient

Correlation coefficient

- A measure of the strength and the direction of a linear relationship between two variables.
- The symbol r represents the sample correlation coefficient.
- A formula for r is

$$r = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

n is the number of data pairs

- The population correlation coefficient is represented by ρ (rho).

Pearson's Correlation Coeff.

Pearson's correlation coefficient between two variables is defined as the covariance of the two variables divided by the product of their standard deviations:

$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$

The above formula defines the *population* correlation coefficient, commonly represented by the Greek letter ρ (rho). Substituting estimates of the covariances and variances based on a sample gives the *sample correlation coefficient*, commonly denoted r :

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}}.$$

$$r_{xy} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{n s_x s_y} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}.$$

The 'coefficient of non-determination' is given by $(1-r^2)$ and can be interpreted as the ratio of unexplained variance to the total variance.

The two lines of regression coincide i.e. become identical when $r = -1$ or 1 or in other words, there is a perfect negative or positive correlation between the two variables under discussion. If $r = 0$ Regression lines are perpendicular to each other. The two lines of regression intersect at the point, where x and y are the variables under consideration.

The regression coefficients remain unchanged due to a shift of origin but change due to a shift of scale.

POINT TO SIGNIFY



1. The table below shows the height, x , in inches and the pulse rate, y , per minute, for 9 people. Find the correlation coefficient and interpret your result

x	68	72	65	70	62	75	78	64	68
y	90	85	88	100	105	98	70	65	72

a. 0.15

b. 0.56

c. -0.15

d. 0.69

ANSWER: C**EXPLANATION:**

You may use the facts that (double check this for practice)

$$\sum x = 622, \quad \sum y = 773, \quad \sum x^2 = 43,206, \quad \sum y^2 = 68,007, \quad \sum xy = 53,336.$$

Calculate the numerator:

$$n \sum (xy) - (\sum x)(\sum y) = 9 \cdot 53336 - 622 \cdot 773 = -782$$

$$\begin{aligned} & \sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2} \\ &= \sqrt{9 \cdot 43206 - (622)^2} \cdot \sqrt{9 \cdot 68007 - (773)^2} \\ &= \sqrt{1970} \cdot \sqrt{14534} = 5350.89 \end{aligned}$$

$$\text{Now, divide to get } r = \frac{-782}{5350.89} = -0.15$$

2. In the previous problem the researcher decides to use data only for adults ages 21 to 60 to compute a correlation coefficient. What value of r should he expect?

a. $r=0$ b. $r \neq 0$ c. $r < 0$ d. $r > 0$ **ANSWER: a****EXPLANATION:**

$r \approx 0$. It is unexpected that mathematical ability and shoe size varies together

3. The following data relate to the test scores obtained by eight

salesmen in an aptitude test and their daily sales in thousands of rupees:

	1	2	3	4	5	6	7	8
scores :	60	55	62	56	62	64	70	54
Sales :	31	28	26	24	30	35	28	24

- a. 48
- c. 4.5

- b. 56
- d. 0.48

ANSWER: D

EXPLANATION:

As

$$b = \frac{24+35}{2} = 30$$

Scores (x_i) (1)	Sales in 1000 (y_i) (2)	$u_i = x_i - 62$ (3)	$v_i = y_i - 30$ (4)	$u_i v_i$ (5) = (3) x (4) (4)	u^2_i (6) = (3) ² (3) ²	v_i^2 (7) = (4) ² (4)
60	31	-2	1	-2	4	1
55	28	-7	-2	14	49	4
62	26	0	-4	0	0	16
56	24	-6	-6	36	36	9
62	30	0	0	0	0	0
64	35	2	5	10	4	25
70	28	8	-2	-16	64	4
54	24	-8	-6	48	64	36
Total	—	-13	-14	90	221	122

Since correlation coefficient remains unchanged due to change of origin, we have

$$= \frac{8 \times 90 - (-13) \times (-14)}{\sqrt{8 \times 221 - (-13)^2} \times \sqrt{8 \times 122 - (-14)^2}}$$

$$= \frac{538}{\sqrt{1768 - 169} \times \sqrt{976 - 196}}$$

$$= 0.48$$

4. If $r = 0.7$; and $n = 64$ find out the probable error of the coefficient of correlation

- a. 0.043
b. 0.43
c. 0.747 , 0.657
d. 0.7

ANSWER: a

EXPLANATION:

$$r = 0.7 ; n = 64$$

$$\text{Probable Error (P.E.)} = 0.6745 \times \frac{1 - (0.7)^2}{\sqrt{64}}$$

$$= (0.6745) \times (0.06375)$$

$$= 0.043$$

5. Compute the Probable Error assuming the correlation coefficient of 0.8 from a sample of 25 pairs of items

- a. 0.0486
b. 0.0456
c. 0.0567
d. 0.0789

ANSWER: a

EXPLANATION:

$$r = 0.8 , n = 25$$

$$\text{P.E.} = 0.6745$$

$$= 0.6745 \times 0.07 = 0.0486$$

7. Difference between Correlation and Causation

- a. The variable mutually influence each other so that neither can be called the cause of other.
b. The correlated variables are influenced by one or more variables.
c. Pure change correlation
d. All

ANSWER: d

EXPLANATION:

The term correlation should not be misunderstood as causation. If correlation exists between two variables, it must not be assumed that a change in one variable is the

a. -0.97

b. 0.97

c. 0.89

d. -0.89

ANSWER: a**EXPLANATION:**

$$r = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y}$$

$$\text{Or } r = \frac{\text{cov}(x, y)}{\sqrt{\text{vary}(x) \cdot \text{vary}(y)}}$$

$$= \frac{-16.5}{\sqrt{2.89 \times 100}}$$

$$= -0.97$$

11. Two random variables have the regression lines $3x + 2y = 26$ and $6x + y = 31$. The coefficient of correlation between x and y is :

a. -0.25

b. 0.5

c. 0.5

d. 0.25

Answer: (c)**Explanation:**

The regression lines $3x + 2y = 26$ and $6x + y = 31$ are given

Let first equation be y on x and second be x on y respectively Therefore, $3x + 2y = 26$

$$26 = y = \left(\frac{-3}{2}\right)x + \left(\frac{26}{2}\right)$$

$$\therefore b_{yx} = -3/2$$

$$\text{and } 6x + y = 31$$

$$= x = \left(\frac{-1}{6}\right)x + \left(\frac{31}{6}\right)$$

$b_{yx} = 1/6$ Now

$r^2 = b_{yx} \cdot b_{xy}$

$$= \left(\frac{-3}{2}\right) \times \left(\frac{-1}{6}\right)$$

$$= 0.25$$

$$r = -0.5$$

$$r_{xy} = \frac{(42075)}{450 \times 12 \times 16} = 0.49$$

Now regression equation of Y on X

$$y - \bar{y} = \frac{r\sigma_y(x - \bar{x})}{\sigma_x}$$

$$= y - 48 = \frac{0.49 \times 16}{12} (x - 40)$$

$$= y = 0.65x + 22$$

When $x=50$, then

$$Y = 0.65 \times 50 + 22 = 54.5$$

15. For 10 pairs of observations, number of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation?

a. $\sqrt{0.2}$

c. $1/3$

b. $-\sqrt{0.2}$

d. $-1/3$

Answer: d

Explanation:

Here $C = 4$, $N = 10$, So

$$n = N - 1 = 10 - 1 = 9$$

$$rc \pm \sqrt{\frac{\pm(2c - n)}{n}}$$

$$rc \pm \sqrt{\frac{\pm(2 \times 4 - 9)}{9}}$$

Here $(2c - n)$ is negative, so negative sign is taken at both the places so,
 $rc = (-1)/3$

16. Karl Pearson's formula :

a. $\frac{[N \sum xy - (\sum x)(\sum y)]}{\sqrt{[N \sum x^2 - (\sum x)^2]}}$

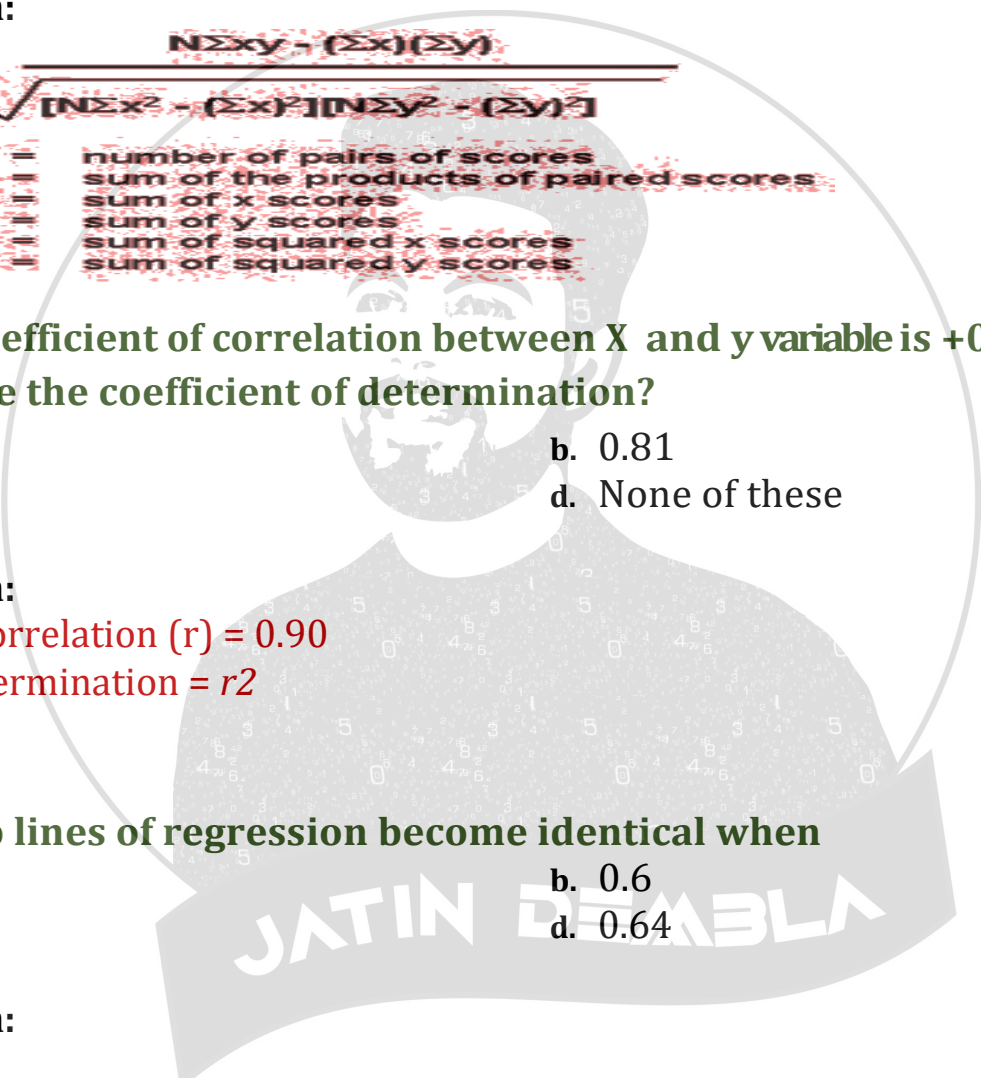
$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

c. Either a or b

b.
d. none

Answer: b

Explanation:



$$r = \frac{N \sum xy - (\sum x)(\sum y)}{\sqrt{[N \sum x^2 - (\sum x)^2][N \sum y^2 - (\sum y)^2]}}$$

Where:

- N = number of pairs of scores
- $\sum xy$ = sum of the products of paired scores
- $\sum x$ = sum of x scores
- $\sum y$ = sum of y scores
- $\sum x^2$ = sum of squared x scores
- $\sum y^2$ = sum of squared y scores

17. If the coefficient of correlation between X and y variable is +0.90 then what will be the coefficient of determination?

- a. 0.30
- b. 0.81
- c. 0.94
- d. None of these

Answer: (b)

Explanation:

If Coeff. of Correlation (r) = 0.90
 Coeff. of Determination = r²
 = (0.90)²
 = 0.81

18. The two lines of regression become identical when

- a. 0.4
- b. 0.6
- c. 0.36
- d. 0.64

Answer: c

Explanation:

If r = 0.6
 Then Coeff. of determination = r²
 = (0.6)²
 = 0.36

19. The two regression lines passing through

- a. Represent means
c. (a) and (b)
- b. Represent S.Ds
d. None of these.

Answer: a

Explanation:

The two Regression lines passing through or (Intersect) at their means

20. The regression equation x and y is $3x + 2y = 100$, the value of b_{xy}

a. $-\frac{2}{3}$

b. $-\frac{3}{2}$

c. $\frac{2}{3}$

d. $\frac{100}{2}$

Answer: a

Explanation:

The regression equation of x & y is

$$3x + 2y = 100$$

$$3x + 2y - 100 = 0$$

$$b_{xy} = -\frac{\text{coefficient of } y}{\text{coefficient of } x} = -\frac{2}{3}$$

21. In a beauty contest there were 10 competitors. Rank of these candidates are assigned by two judges A and B. The sum of squares of differences of ranks is 44. The value of rank correlation is

a. 0.70

b. 0.73

c. 0.80

d. 0.60

Answer: b

Explanation:

Sum of squares of differences of ranks ($\sum d^2$) = 44

$$r_R = ?$$

$$r_R = 1 - 6 \frac{\sum d^2}{n(n^2 - 1)}$$

$$1 - \frac{6 \times 4}{10(10^2 - 1)}$$

$$1 - \frac{6 \times 44}{10 \times 99}$$

$$= 1 - 0.267$$

$$= 0.733$$

So, answer be 0.73

22. If two regression lines are $x + y = 1$ and $x - y = 1$ then mean values of x and y will be:

a. 0 and 1

c. 1 and 0

b. 1 and 1

d. none

Answer: c

Explanation:

Given Regression line

$$X + y = 1 \quad \text{--- (1)}$$

$$X - y = 1 \quad \text{--- (2)}$$

$$2x = 2$$

$$\Rightarrow x = \frac{2}{2} = 1$$

$x = 1$ in equation (1) we get

$$1 + y = 1$$

$$Y = 0$$

$$\text{Mean of } x = x = 1$$

$$\text{Mean of } y = y = 0$$

Hence, 1 and 0

23. The coefficient of correlation between x and y is 0.6. If x and y values are multiplied by -1 , then the coefficient of correlation will be

a. 0.6

c. $1/0.6$

b. $1-0.6$

d. -0.6

Answer: a

Explanation:

The coefficient of correlation between X and Y is 0.6. If X and Y values are multiplied by -1 then coefficient of correlation remains unchanged Then are coefficient of correlation will be 0.6.

24. The coefficient of correlation between the temperature of environment and power consumption is always

- a. +ve
- b. -ve
- c. 0
- d. = 1

Answer: a

Explanation:

The coefficient of correlation between the temperature of environment and power consumption is always positive.

25. Out of the following the one which effects the regression coefficient is

- a. Change of origin only
- b. Change of scale only
- c. Change of scale and origin both
- d. Neither a nor b

Answer: b

Explanation:

By shifting the scale, coefficient of regression is changed.

26. When the correlation coefficient r is equal to + 1, all the points in a scatter diagram would be

- a. On a straight line directed from upper left to lower right
- b. On a straight line directed from lower left to upper right
- c. On a straight line
- d. Both (a) and (b)

Answer: b

Explanation:

When the correlation coefficient r is equal to '+1', all the points in a scatter diagram on a straight line directed from lower left to upper Right.

27. In case of "Insurance Companies" profits and the number of claims they have to pay there is _____ correlation.

- a. +ve
c. No relation
- b. -ve
d. none

Answer: b

Explanation:

In case of Insurance Companies Profits and the Number of claims they have to pay there is **Negative** Correlation.

28. If the correlation coefficient between two variables is zero,, then the lines of regression are:

- a. parallel
c. coincide
- b. perpendicular
d. none

Answer: b

Explanation:

If the correlation coefficient b/w two variables is zero, then the lines of regression are **perpendicular**.

29. Three competitors in a contest are ranked by two judges in the order 1,2,3 and 2,3,1 respectively. Calculate the Spearman's rank correlation coefficient.

- a. -0.5
c. 0.8
- b. -0.8
d. 0.5

Answer: a

Explanation:

Rank by 1 st Judge R ₁	Rank by 2 nd Judge R ₂	Diff D= R ₁ - R ₂	D ²
1	2	-1	1
2	3	-1	1
	1	+2	4
			$\sum d^2 = 6$

Here $n = 3$

$$\text{Spearman's Rank Correlation coefficient} = 1 - 6 \frac{\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 6}{3(3^2 - 1)}$$

$$= -0.5$$

30. The strength (degree) of the correlation between a set of independent variables X and a dependent variable Y is measured by

- a. Coefficient of Correlation
- b. Standard error of estimate
- c. Coefficient of Determination
- d. All of the above

Answer: D

Explanation:

The strength (degree) of the correlation between a set of independent variables X and a dependent variable Y is measured through:

- Coefficient of Correlation
- Standard error of estimate
- Coefficient of Determination

31. The percent of total variation of the dependent variable Y explained by the set of independent variables X is measured by

- a. Coefficient of Correlation
- b. Standard error of estimate
- c. Coefficient of Determination
- d. Coefficient of Skewness

Answer: C

Explanation:

The **coefficient of determination** (denoted by R^2) is a key output of regression analysis. . An R^2 of 0 means that the dependent variable cannot be predicted from the independent variable. An R^2 of 1 means the dependent variable can be predicted without error from the independent variable.

31. A coefficient of correlation is computed to be -0.95 means that

- a. The relationship between two variables is weak.
- b. The relationship between two variables is strong and positive
- c. The relationship between two variables is strong and but negative
- d. Correlation coefficient cannot have this value

Answer: C

Explanation:

A coefficient of correlation is computed to be -0.95 means that relationship between two variables is strong and but negative.

32. Let the coefficient of determination computed to be 0.39 in a problem involving one independent variable and one dependent variable. This result means that

- a. The relationship between two variables is negative
- b. The correlation coefficient is 0.39 also
- c. 39% of the total variation is explained by the independent variable
- d. 39% of the total variation is explained by the dependent variable

Answer: C

Explanation:

The coefficient of determination computed to be 0.39 in a problem involving one independent variable and one dependent variable. 39% of the total variation is explained by the independent variable

33. Relationship between correlation coefficient and coefficient of determination is that

- a. The coefficient of determination is the coefficient of correlation squared
- b. The coefficient of determination is the square root of the coefficient of correlation
- c. both are unrelated
- d. both are equal

Answer: B

Explanation:

Coefficient of correlation is "R" value which is given in the summary table in the Regression output. **R square is also called coefficient of determination.** Multiply R times R to get the R square value. In other words Coefficient of Determination is the square of Coefficient of Correlation. R square or coeff. of determination shows percentage variation in y which is explained by all the x variables together. Higher the better. It is always between 0 and 1. It can never be negative – since it is a squared value.

It is easy to explain the R square in terms of regression. It is not so easy to explain the R in terms of regression.

35. For a bivariate data, two times of regression are $40x - 18y = 214$ and $8x - 10y + 66 = 0$, then find the values of x and y

- a. 17 and 13
b. 13 and 17
c. 13 and -17
d. -13 and 17

Answer: b

Explanation:

$$\text{Given: } 40x - 18y = 214 \quad \underline{\hspace{1cm}} \quad (1)$$

$$8x - 10y = -66 \quad \underline{\hspace{1cm}} \quad (2)$$

On solving (1) and (2) we get

$$x = 13 \text{ and } y = 17$$

$$\therefore x = 13 \text{ and } y = 17$$

36. In multiple regression, when the global test of significance is rejected, we can conclude that:

- a. All of the net sample regression coefficients are equal to zero
b. All of the sample regression coefficients are not equal to zero
c. At least one sample regression coefficient is not equal to zero
d. The regression equation intersects the Y-axis at zero.

Answer: C

Explanation:

In multiple regression, when the global test of significance is rejected, we can conclude that at least one sample regression coefficient is not equal to zero

37. Correlation Coefficient values lies between

- a. -1 and +1
b. 0 and 1
c. -1 and 0
d. None

Answer: A

Explanation:

The strength of the linear association between two variables is quantified by the correlation coefficient. The correlation coefficient always takes a value between -1

Now, divide to get $r = \frac{-1075}{1159.66} = -0.93$

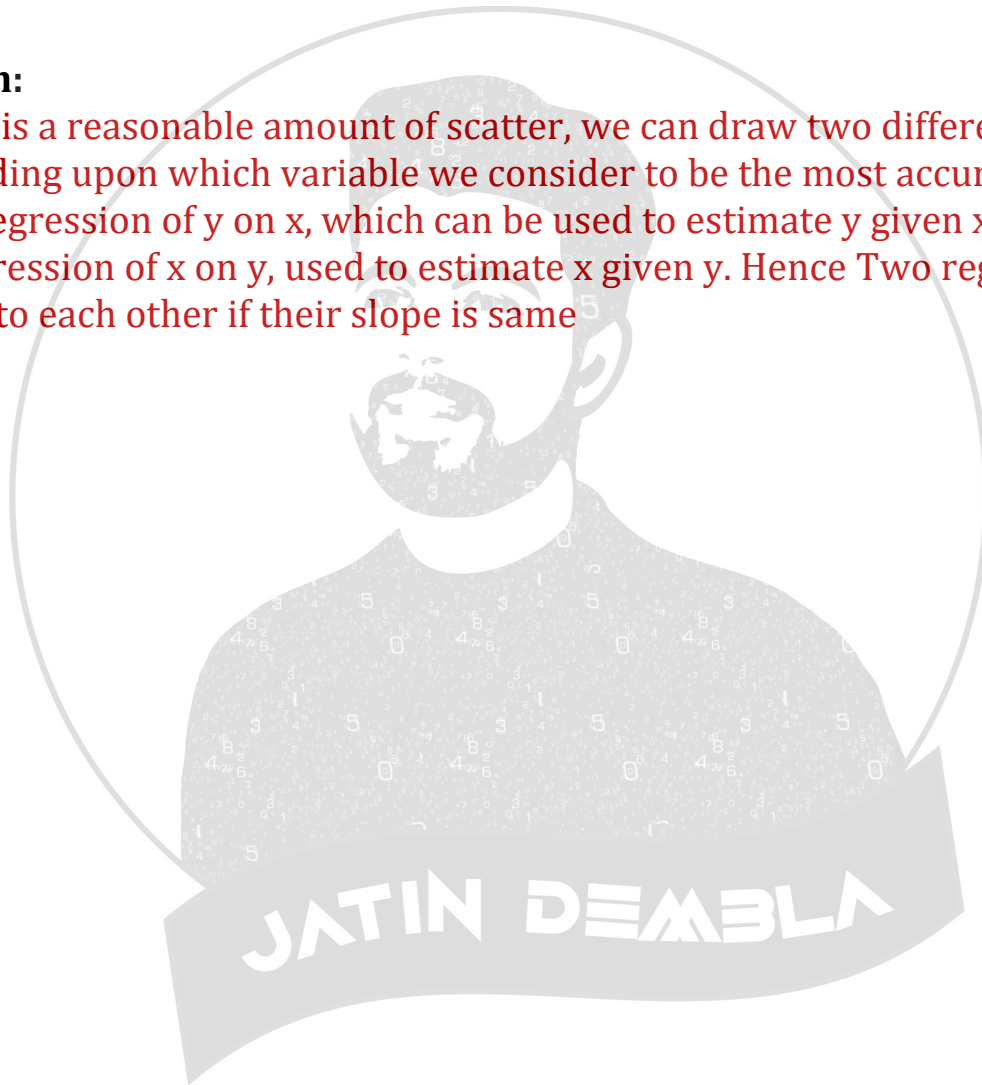
40. Two regression lines are parallel to each other if their slope is

- a. Random
- b. Non Random.
- c. Same
- d. None

Answer: C

Explanation:

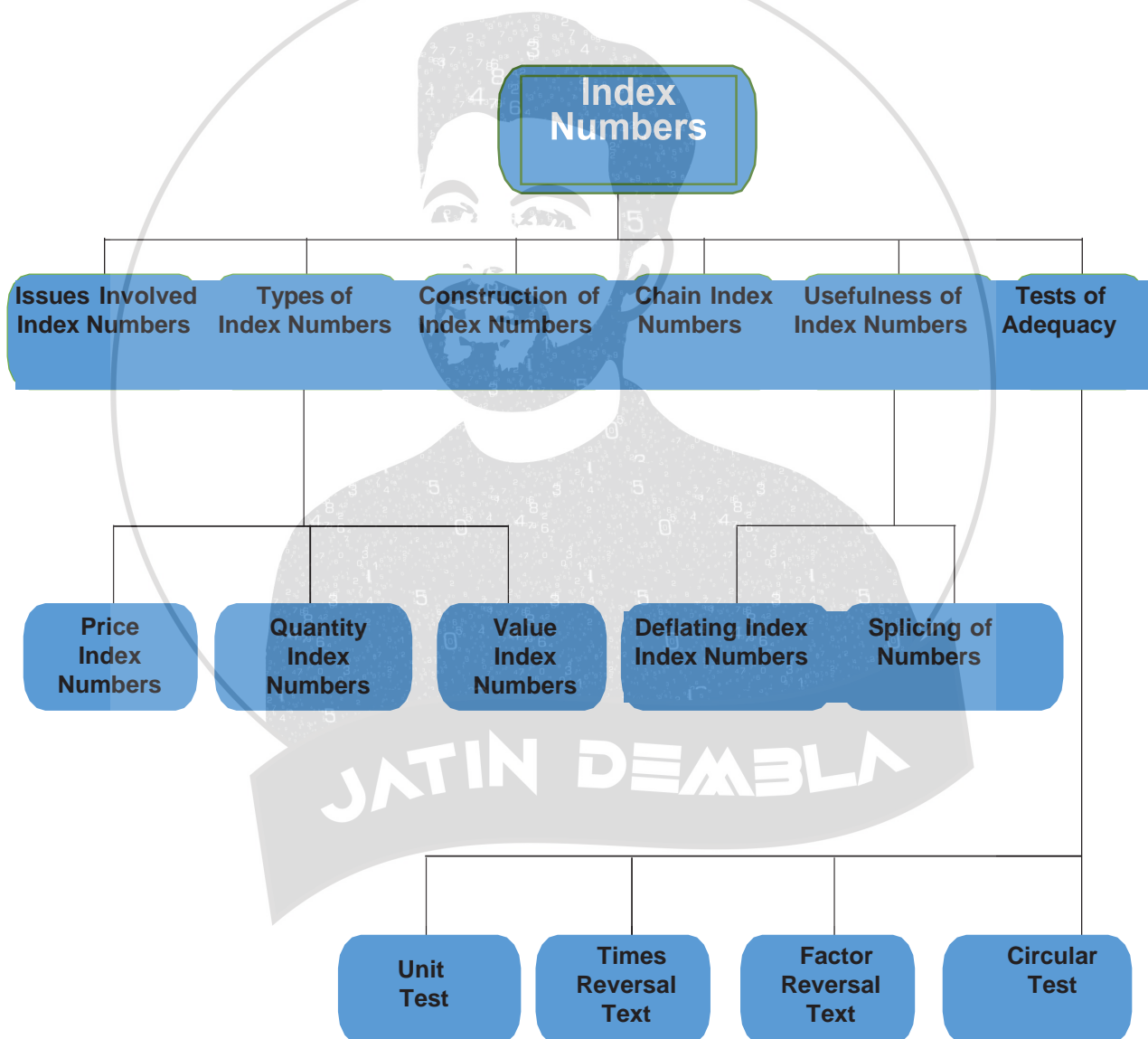
When there is a reasonable amount of scatter, we can draw two different regression lines depending upon which variable we consider to be the most accurate. The first is a line of regression of y on x, which can be used to estimate y given x. The other is a line of regression of x on y, used to estimate x given y. Hence Two regression lines are parallel to each other if their slope is same

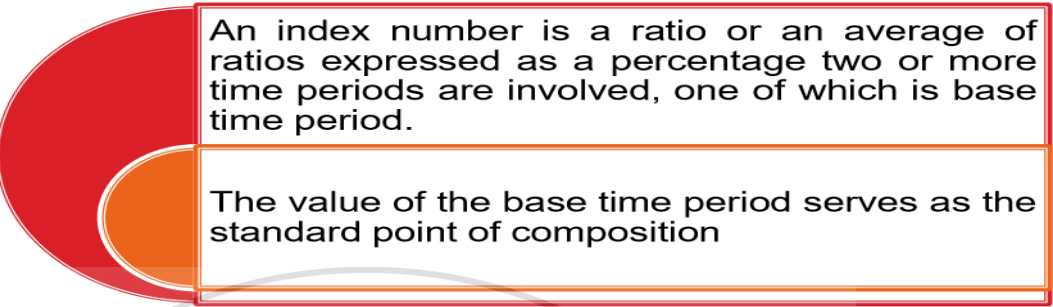
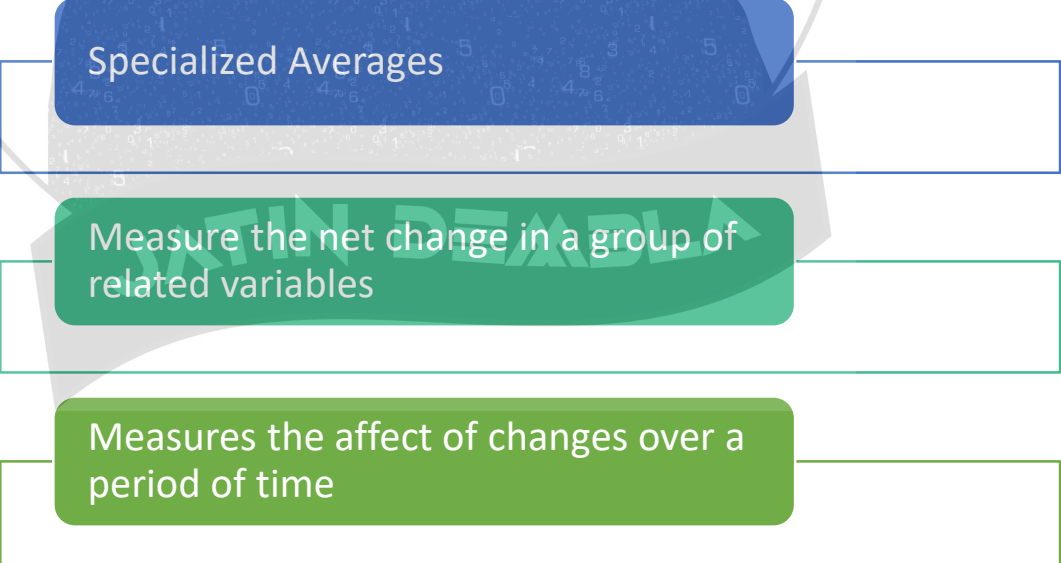


CHAPTER 19

INDEX NUMBER AND TIME SERIES

UNIT - I INDEX NUMBER



<p>INTRODUCTION</p>	 <p>An index number is a ratio or an average of ratios expressed as a percentage two or more time periods are involved, one of which is base time period.</p> <p>The value of the base time period serves as the standard point of composition</p>
<p>ISSUES INVOLVED</p>	<p>Selection of data</p> <hr/> <p>Selection of Base Year</p> <hr/> <p>Types of Formula</p> <hr/> <p>Selection of weights</p> <hr/> <p>The data for Index Numbers</p> <hr/> <p>Choice of Variables</p>
<p>FEATURES OF INDEX NUMBER</p>	 <p>Specialized Averages</p> <p>Measure the net change in a group of related variables</p> <p>Measures the affect of changes over a period of time</p>

<p>METHODS</p>	<pre> graph LR Methods[Methods] --> Simple[Simple] Methods --> Weighted[Weighted] Simple --> Simple_Aggregative[Aggregative] Simple --> Simple_Relative[Relative] Weighted --> Weighted_Aggregative[Aggregative] Weighted --> Weighted_Relative[Relative] </pre>
<p>PRICE RELATIVE</p>	<p>Price relatives are helpful in understanding and interpreting changing economic and business conditions over time.</p> <p>A price relative shows how the current price per unit for a given item compares to a base period price per unit for the same item.</p> <p>A price relative expresses the unit price in each period as a percentage of the unit price in the base period</p>
<p>AGGREGATE PRICE INDEXES</p>	<p>An aggregate price index is developed for the specific purpose of measuring the combined change of a group of items</p> <p>An unweighted aggregate price index in period t,</p>

1

Laspyre's Price index number

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Where

- ▶ P₁=Price of the current year
- ▶ P₀=Price of the base year
- ▶ q₀=Quantity of the base year

2

Paasche's Price index number

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

Where

- ▶ P₁=Price of the current year
- ▶ P₀=Price of the base year
- ▶ q₁=Quantity of the current year

3

Marshall-Edgewohts's Price index number

$$\frac{\sum (q_0 + q_1) \times p_1}{\sum (q_0 + q_1) \times p_0} \times 100$$

Where

- ▶ P₁=Price of the current year
- ▶ P₀=Price of the base year
- ▶ q₀=Quantity of the current year
- ▶ q₁=Quantity of the current year

4

Fisher's Price index number

$$P_{01} = \sqrt{L \times P}$$

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

Where

- ▶ L= Laspyre's Price Index number
- ▶ P=Paachee's Price Index number

5

Weighted Price index number

▶ If Arithmetic Mean is used

$$P_{01} = \frac{\sum PV}{\sum V} \times 100 \quad P = \frac{p_1}{p_0} \times 100$$

Where

- ▶ P₁=Price of Current Year
- ▶ P₀=Price of base year

$$V = P_0 q_0$$

6


Weighted Price index number

▶ If Geometric Mean is used

$$P_{01} = \text{Anti log} \left[\frac{\sum V \log P}{\sum V} \right] \times 100$$

Where

- ▶ P₁=Price of Current Year
- ▶ P₀=Price of base year
- ▶ V=P₀q₀

<p>QUANTITY INDEXES</p>	<p>An index that measures changes in quantity levels over time is called a quantity index.</p> <p>Probably the best known quantity index is the Index of Industrial Production.</p>
<p>QUANTITY INDEXES NUMBERS</p>	<p>1. Simple Aggregate of Quantities = $\frac{\sum Q_n}{\sum Q_0} \times 100$</p> <p>2. The Simple average quantity relatives $\frac{\sum \frac{Q_n}{Q_0}}{N} \times 100$</p> <p>3. Weighted Aggregate Quantity Indices</p> <p>1) With base year weight (Laspyres's Index) $\frac{\sum Q_n P_0}{\sum Q_0 P_0} \times 100$</p> <p>2) With Current year weight (Paasche's Index) $\frac{\sum Q_n P_n}{\sum Q_0 P_n} \times 100$</p> <p>3. Geometric Mean of (1) and (2) $\sqrt{\frac{\sum Q_n P_0}{\sum Q_0 P_0} \frac{\sum Q_n P_n}{\sum Q_0 P_n}} \times 100$</p> <p>4. Base Year average of quantity relatives $\frac{\sum \frac{Q_n}{Q_0} \times (P_0 Q_0)}{\sum P_0 Q_0} \times 100$</p>
<p>VALUE INDEX NUMBER</p>	$\frac{\sum V_n}{\sum V_0} = \frac{\sum P_n Q_n}{\sum P_0 Q_0}$
<p>TEST OF ADEQUACY OF INDEX NUMBERS</p>	 <p>Unit Test</p> <p>Time Reversal Test</p> <p>Factor Reversal Test</p> <p>Circular Test</p>

Unit test

The unit test requires that the formula for constructing an index should be independent of the units in which, or for which, prices and quantities are quoted. All formulae except the simple (un weighted) aggregate index formula satisfy this test.

Time Reversal Test

A method satisfies time reversal test if it gives $P_{01} * P_{10} = 1$

where P_{01} is the price index number for the current year

P_{10} is the index number of the base year, taking current year as the base,

both the indices without the factor 100.

Factor Reversal Test

A method satisfies factor reversal test if it gives

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

where P_{01} is the price index for the current year

Q_{01} is the quantity index for the current year

Fishers index number only satisfies the factor reversal test

Circular test

Circular test is an extension of the time reversal test.

Symbolically, the circular test may be written as

$$P_{01} \cdot P_{12} \cdot P_{23} \dots P_{n-1n} \cdot P_{n0} = 1$$

Circular test satisfies the simple geometric mean of price relatives and weighted aggregate of fixed weights.

CHAIN INDEX	<p>Chain base index numbers is one in which the figures for each are first expressed as percentage of the preceding year. The percentage are chained together by successive multiplication to form a series of chain index, in chain base year index method the base year changes from year to year</p> $\frac{\text{Link relative of current year} \times \text{Chain Index Previous Year}}{100}$
LINK RELATIVE	$\frac{\text{Current year Price Index}}{\text{Immediate previous year price relative}} \times 100$

Splicing and Shifting the Base of Index Numbers

When two or more overlapping series of index numbers are combined into one series, then this process is known as splicing

Splicing

Technique of linking two or more index number series with the same items and a common overlapping year but with different base period in order to form a continuous series

Splicing may be forward or backward

Forward Splicing

Splicing	Index no. of old series	Index no. of New series
Forward Splicing	$=\{100/\text{Overlapping index number of old series}\} * \text{Given index of No. of old series}$	No change

Backward Splicing

Splicing	Index no. of old series	Index no. of New series
Backward Splicing	No change	$=\{\text{Index number of old series}/100\} * \text{Given index No. of new series}$

Index Number using new base

Index Number using new base

Old Index number using old base

X 100

Index number Corresponding new base year

Uses of Index Numbers

1. As the indices are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.
2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.
3. Since many methods are employed for constructing index numbers, the result gives different values and this at times create confusion.

Deflated Time series Using Index Numbers

$$\begin{aligned}
 \text{Deflated Value} &= \frac{\text{Current Value}}{\text{Price Index of the current year}} \text{ or} \\
 &= \text{Current Value} \times \frac{\text{Base Price (Po)}}{\text{Current Price (Pn)}}
 \end{aligned}$$

Limitations of Index Numbers

As we know, our indices are of prices and quantities. The question is: does our index reflect a change in the quality of a product or item?

Apart from quality changes, there are other aspects, that are pertinent while we are interpreting index numbers. We have to ask whether the weights assigned to different items are appropriate.

Methods of Constructing Consumer Price index

Aggregate Expenditure method

Family budget method

Aggregate expenditure method is a weighted aggregated price index where weights are the base period quantities. (Laspyre's Index number)

$$CPI = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Family Budget Method

Weighted Aggregated of price relatives

Index is obtained by taking the average of weighted price relatives and the value weights are ($P_0 q_0$) are used

$$CPI = \frac{\sum \frac{P_1}{P_0} V}{\sum V} \times 100$$

$$\frac{P_1}{P_0} \times 100$$

$$V = P_0 \cdot Q_0$$



1. Construct the following indices by taking 1997 as the base:

(i) simple Aggregative price Index

Items	A	B	C	D	E
Prices Rs. (1997)	6	2	4	10	8
Prices Rs. (1998)	10	2	6	12	12
Prices Rs. (1999)	15	3	8	14	16

a. 140 , 186.67

b. 120.90 , 140.6

c. 140 , 120.90

d. 56 , 420

ANSWER: A

EXPLANATION:

Items	P_0	P_1	P_2	$P_1 = \frac{P_1}{P_0} \times 100$	$P_2 = \frac{P_2}{P_0} \times 100$
A	6	10	15	166.67	250
B	2	2	3	100.00	150
C	4	6	8	150.00	200
D	10	12	14	120.00	140
	$\sum P_0 = 30$	$\sum P_1 = 42$	$\sum P_2 = 56$	$\sum \left(\frac{P_1}{P_0} \times 100 \right) = 686.67$	$\sum \left(\frac{P_2}{P_0} \times 100 \right) = 940$

Simple Aggregative Price Index:

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100 = \frac{42}{30} \times 100 = 140 \quad (\text{For 1998})$$

$$P_{02} = \frac{\sum P_2}{\sum P_0} \times 100 = \frac{56}{30} \times 100 = 186.67 \quad (\text{For 1999})$$

2. A composite price index where the prices of the items in the composite

are weighted by their relative importance is known as the

- a. price relative
- b. CPI
- c. weighted aggregate price index
- d. none of the above

ANSWER: c

EXPLANATION:

Weighted aggregate price index. The ratio of the sum of weighted prices of current and base time periods multiplied by 100 is called weighted aggregate price index. This index is calculated after allocating weights to each commodity on the basis of their relative importance.

3. A weighted aggregate price index where the weight for each item is its current-period quantity is called the
- a. Aggregate index
 - b. Consumer Price Index
 - c. Laspeyres Index
 - d. Paasche Index

ANSWER: D

EXPLANATION:

Paasche index, index developed by German economist Hermann Paasche for measuring current price or quantity levels relative to those of a selected base period. It differs from the Laspeyres index in that it uses current-period weighting.

4. An index that is designed to measure changes in quantities over time is known as the
- a. Quantity index
 - b. Time index
 - c. None of the above
 - d. Paasche index

ANSWER: A

EXPLANATION:

Index numbers. An index number is an economic data figure reflecting price or quantity compared with a standard or base value. The base usually equals 100 and the index number is usually expressed as 100 times the ratio to the base value.

5. Index numbers are expressed in:

- a. Ratios
- b. Squares
- c. Percentages
- d. Combinations

ANSWER: c

EXPLANATION:

Index numbers are values **expressed** as a percentage of a single base figure. For example, if annual production of a particular chemical rose by 35%, output in the second year was 135% of that in the first year. In **index** terms, output in the two years was 100 and 135 respectively. **Index numbers** have no units.

6. Indices calculated by the chain base method are free from:

- a. Seasonal variations
- b. Errors
- c. Percentages
- d. Ratios

ANSWER: a

EXPLANATION:

A value in any specific time period is **based** on the value of the same entity in the preceding period. Changes in values can be compared between sequential time periods. This differs from a fixed **base** index in which values in any period are **based** on the initial value.

7. Consumer price index numbers are obtained by:

- a. Laspeyre's formula
- b. Fisher ideal formula
- c. Marshall Edgeworth formula
- d. Paasche's formula

ANSWER: a

EXPLANATION:

Laspeyres formula. **Laspeyres** suggested this index **formula** in 1871. In case of calculating the price index, assuming that for individual item i , price at the base period to be p_{i0} , at the observation period to be p_{it} , and quantity at the base period to be q_{i0} , the following equation is called "**Laspeyres formula**".

8. The most appropriate average in averaging the price relatives is:

- a. Median
- b. Harmonic mean
- c. Arithmetic mean
- d. Geometric mean

ANSWER: d

EXPLANATION:

Geometric mean index numbers are a multiplicative aggregation of (price or quantity) ratios with their importance exponents/weights derived from one or more observed budget shares. ... This approach is directly inspired by the literature on **index number** theory.

19. The test which is not obeyed by any of the weighted index numbers unless the weights are constant:

- a. Circular test
- b. Time reversal test
- c. Factor reversal test
- d. None of them

ANSWER: a

EXPLANATION:

According to this test the product of price index and quantity index must be equal to the value index. Note: 1. Since Fisher's index number satisfies both time reversal and factor reversal test, it is called an ideal index number. Circular test. It is a generalization of the time reversal test.

20. Index number having upward bias is:

- a. Laspeyres's index
- b. Paasche's index
- c. Fisher's ideal index
- d. Marshall Edgeworth index

ANSWER: b

EXPLANATION:

Paasche index, index developed by German economist Hermann **Paasche** for measuring current price or quantity levels relative to those of a selected base period. It differs from the Laspeyres **index** in that it uses current-period weighting.

21. Marshall Edgeworth price index was proposed by:

- a. One English economist
- b. Two English economist
- c. Three English economist
- d. Many English economist

ANSWER: b

EXPLANATION:

The Marshall-Edge worth index, credited to Marshall (1887) and Edge worth (1925), is a weighted relative of current period to base period sets of prices. This index uses the arithmetic average of the current and based period quantities for weighting. It is considered a pseudo-superlative formula and is symmetric.

22. Paasche's price index number is also called:

- a. Base year weighted
- b. Current year weighted
- c. Simple aggregative index
- d. Consumer price index

ANSWER: b

EXPLANATION:

Paasche index, index developed by German economist Hermann Paasche for measuring current price or quantity levels relative to those of a selected base period. It differs from the Laspeyres index in that it uses current-period weighting.

23. The major groups for whom the consumer price index numbers are constructed in India.

- a. The industrial workers,
- b. The urban non-manual workers and
- c. The urban non-manual workers and
- d. All of the above

ANSWER: D

EXPLANATION:

Consumer price index numbers are having three types:

- (i) The industrial workers,
- (ii) The urban non-manual workers and
- (iii) The agricultural laborers.

24. From the following data construct price index of 1995 taking 1990 as base by using simple Average of price Relative Method:

Commodity	A	B	C	D
Prices in 1990 (Rs.)	60	45	80	25
Prices in 1995 (Rs.)	75	50	70	40

- a. 120.90
- c. 809.56

- b. 12.60
- d. 12.888

Answer: A

Explanation:

Commodity	P ₀	P ₁	$\frac{P_1}{P_0} \times 100$
A	60	75	125
B	45	50	111.11
C	80	70	87.50
TOTAL			$\sum \left(\frac{P_1}{P_0} \times 100 \right) = 483.61$

$$P_{01} = \frac{\sum \left(\frac{P_1}{P_0} \times 100 \right)}{N} = \frac{483.61}{4} = 120.90.$$

25. Calculate weighted aggregative price index from the following data using Laspeyre's method

Base Period		Current Period		
Price	Quantity	Price	Quantity	
A	2	10	4	5
B	5	12	6	10
C	4	20	5	15
D	2	15	3	10

- a. 155.09
- c. 135.26

- b. 12.60
- d. 12.888

Answer: C

Explanation:

Commodity								
A	2	10	4	5	20	40	10	20
B	5	12	6	10	60	72	50	60
C	4	20	5	15	80	100	60	75
					$\sum P_0q_0 = 190$	$\sum P_1q_0 = 257$	$\sum P_0q_1 = 140$	$\sum P_1q_1 = 185$

$$P_{01}^L = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{257}{190} \times 100 = 135.26$$

26. Calculate weighted aggregative price index number from the following data by using Passche's method:

Commodity	Base Year		Current	
	Price	Quantity	Price	Quantity
A	10	30	12	50
B	8	15	10	25
C	6	20	6	30
D	4	10	6	20

- a. 199.79
- c. 135.26

- b. 119.79
- d. 12.888

Answer: b

Explanation:

Commodity	P_0	q_0	P_1	q_1	P_0q_1	P_1q_1
-----------	-------	-------	-------	-------	----------	----------

A	10	30	12	50	500	600
B	8	15	10	25	200	250
C	6	20	6	30	180	180
D	4	10	6	20	80	120
					$\sum P_0q_1 = 960$	$\sum P_1q_1 = 1150$

$$P_{01}^P = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{1150}{960} \times 100 = 119.79$$

28. Calculate Laspeyre's and passche's index for the following data:

Commodity	1970		1990	
	Price	Expenditure	Price	Expenditure
A	8	100	10	90
B	10	60	11	66
C	5	100	5	100
D	3	30	2	24
E	2	8	4	20

a. 109.73 , 107.91

b. 119.79 , 169.56

c. 135.26 , 0.465

d. 135.26 , 0.465

Answer: a

Explanation:

Since we are given the expenditure and price, we can obtain the quantity by dividing expenditure by the price for each commodity

Commodity	Price	Quantity	Price	Quantity	Expenditure	Expenditure	Expenditure	Expenditure
A	8	12.50	10	9	100	125	72	90
B	10	6.0	11	6	60	66	60	66
C	5	20.0	5	20	100	100	100	100
D	3	10.0	2	12	30	20	36	24

$$P_{01}^L = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{327}{298} \times 100 = 109.73$$

$$P_{01}^P = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 = \frac{300}{278} \times 100 = 107.91$$

29 Calculate weighted average of price relative index from the following data:

Items	Weight in % (Rs.)	Base Year	Current Year
		Price (Rs.)	Price (Rs.)
A	40	2	4
B	30	5	6
C	20	4	5
D	10	2	3

- a. 215
- b. 156
- c. 965
- d. 325

Answer: B

Explanation:

Items	W	P ₀	P ₁	R = $\frac{P_1}{P_0} \times 100$	RW
A	40	2	4	$\frac{4}{2} \times 100 = 200$	8000
B	30	5	6	$\frac{6}{5} \times 100 = 120$	3600
C	20	4	5	$\frac{5}{4} \times 100 = 125$	2500
TOTAL					$\sum RW = 15600$

$$P_{01} = \frac{\sum RW}{\sum W} = \frac{15600}{100} = 156$$

30. The monthly per capita expenditure incurred by workers of an industrial center during 1980 and 2005 on the following items are given below. The weights of these items are 75, 10, 5, 6 and 4 respectively. Prepare a weighted index number for cost of living for 2005 with 1980 as base.

Items	Price in 1980	Price in 2005
Food	100	200
Clothing	20	25
Fuel and Lighting	15	20
Misc.	35	65
House Rent	30	40

- a. 185
- c. 165

- b. 156
- d. 325

Answer: a

Explanation:

Item	W	P ₀	P ₁	$R = \frac{P_1}{P_0} \times 100$	RW
Food	75	100	200	200	15000
Clothing	10	20	25	125	1250
Fuel and Lighting	5	15	20	133.33	666.65
House Rent	4	30	40	133.33	799.98
		35	65	185.71	742.84
					$\sum PW = 18459.47$

$$CPI = \frac{\sum RW}{\sum W} = \frac{18459.47}{100} = 184.59 = 185 \text{ (Approx.)}$$

31. An enquiry into the budgets of the middle class families in a certain city gave the following information:

Expenses on Items	Food	Fuel	Clothing	Rent	Misc
	25%	10	20%	15%	
Prices in 2004 (Rs.)	1500	250	750	300	400
Prices in 1995 (Rs.)	1400	200	500	200	250

- a. 165.62
- b. 134.5
- c. 165.60
- d. 325.8

Answer: b

Explanation:

Items	W in %	P ₀ (1995)	P ₁ (2004)	R = $\frac{P_1}{P_0} \times 100$	RW
Food	35	1400	1500	107.14	3750
Fuel	10	200	250	125.00	1250
Clothing	20	500	750	150.00	3000
Rent	15	200	300	150.00	2250
Misc :	20	250	400	160.00	3200

$$CPI = \frac{\sum RW}{\sum W} = \frac{13450}{100} = 134.5$$

32. Calculate the 'Cost of Living Index Number' using family budget method

Commodities	Wheat	Rice	Pulses	Ghee	Sugar	Oil	Fuel	Clothes
Units consumed in	200	50	56	20	40	50	60	40

Price Rs. (In base year)	1.0	3.0	4.0	20.0	2.5	10.0	2.0	15.0
Price Rs. (In current year)	1.2	3.5	5.0	30.0	5.0	15.5	2.5	18.0

- a. 166.62
- b. 136.88
- c. 165.870
- d. 325.8

Answer: b

Explanation:

Commodities	Q_0	P_0	P_1	$R = \frac{P_1}{P_0} \times 100$	$W = P_0 Q_0$	RW
Wheat	200	1.0	1.2	120.00	200	24000
Rice	50	3.0	3.5	116.67	150	17500.5
Pulses	56	4.0	5.0	125.00	224	28000
Ghee	20	20.0	30.0	150.00	400	60000
Sugar	40	2.5	5.0	200.00	100	20000
Oil	50	10.0	15.5	155.00	500	77500
Fuel	60	2.0	2.5	125.00	120	15000
Clothes	40	15.0	18.0	120.00	600	72000
					$\sum W = 2294$	$\sum RW = 314000.5$

$$CPI = \frac{\sum RW}{\sum W} = \frac{314000.5}{2294} = 136.88$$

33 If the salary of a person in the base year is Rs. 4,000 per annum and the current year salary is Rs. 6,000 by how much should his salary rise to maintain the same standard of living if the CPI of the current year is 400?

- a. 10000
- b. 13688

c. 165870

d. 16000

Answer: d**Explanation:**

Salary required in the current year to maintain the same standard of living of base year

$$= \text{Base year Salary} \times \frac{\text{CPI of Current Year}}{\text{CPI of Base Year}} = 4000 \times \frac{400}{100} \text{ Rs. 16,000}$$

Current year salary = Rs. 16,000

The increase in current salary required = 16000 - 6000 = Rs. 10,000.

34. Given the following data:

Year	1995-	1996-	1997-	1998-	1999-	2000-	2001-	2002-
WPI (1993-	121.6	127.2	132.8	140.7	145.7	155.7	161.3	166.8

Calculate the inflation rate of year 1998-99

a. 5.94%

b. 59.89%

c. 4.4%

d. None

Answer: A**Explanation:**

Inflation rate for different years are calculated as:

$$\text{Year 1996-97} = \frac{X_t - X_{t-1}}{X_{t-1}} \times 100 = \frac{127.2 - 121.6}{121.6} \times 100 = 4.6\%$$

$$\text{Year 1997-98} = \frac{X_t - X_{t-1}}{X_{t-1}} \times 100 = \frac{132.8 - 127.2}{127.2} \times 100 = 4.40\%$$

$$\text{Year 1998-99} = \frac{X_t - X_{t-1}}{X_{t-1}} \times 100 = \frac{140.7 - 132.8}{132.8} \times 100 = 5.94\%$$

35. What will be the real wage of the consumer if his money wage is Rs. 10,000 and the cost of living index is 526?

(a) $\frac{\sum p_n}{\sum p_o} \times 100$

(b) $\frac{p_n}{p_{n-1}} \times 100$

(c) $\frac{p_n}{p_o} \times 100$

(d) None

Answer: B**Explanation:**

In the chain index the comparison takes place always between successive calculation periods. In the chain index the change in two calculation periods is used to take forward the index point figure of the desired base period. In the chain index the weights are changed in principle in each calculation period.

39. An index number that can serve many purposes is called:

- | | |
|--------------------------|--------------------------|
| a. General purpose index | b. Special purpose index |
| c. Cost of living index | d. None of them |

Answer: a**Explanation:**

- It is used to measure the changes in the wholesale price level of a country over a period of time.
- It is used to measure the changes in the cost of living of a certain section of the people living in a certain locality.
- It is very much used by the government agencies to formulate policies on different matters viz.

40 Laspeyre's index = 110, Paasche's index = 108, then Fisher's Ideal index is equal to:

- | | |
|--------|--------|
| a. 110 | b. 108 |
| c. 100 | d. 109 |

Answer: d**Explanation:**

$$F = \sqrt{L \times P}$$

So $\sqrt{110 \times 108} = 109$

41. Consumer price index are obtained by:

- | | |
|-------------------------------|---------------------------------|
| a. Paasche's formula | b. Fisher's ideal formula |
| c. Marshall Edgeworth formula | d. Family budget method formula |

Answer: d

Explanation:

A consumer price index (CPI) measures changes in the price level of market basket of consumer goods and services purchased by households. The CPI is a statistical estimate constructed using the prices of a sample of **representative** items whose prices are collected periodically.

42. Which of the following formula satisfy the time reversal test?

$$(a) p_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$$

$$(b) p_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$$

$$(c) p_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

(d) None

Answer: c

Explanation:

Factor **reversal test**. **Time reversal test**. This test is proposed by Irving Fisher. According to him, an index number (**formula**) should be such that when the base year and current year are interchanged (**reversed**) the resulting index number should be the reciprocal of the earlier.

43. Simple average of relatives is equal to

$$(a) \frac{\bar{p}_n}{\bar{p}_0} \times 100$$

$$(b) \frac{\sum p_n}{\sum p_0} \times 100$$

$$(c) \sum \left(\frac{p_n}{p_0} \right) \times 100$$

$$(d) \frac{1}{N} \sum \left(\frac{p_n}{p_0} \right) \times 100$$

Answer: d

Explanation:

In case of un weighted **average of relatives**, price **relative** of each commodity is first calculated and then **average (mean, median or geometric mean)** of these price **relatives** for all the commodities is taken **average of relatives** can be calculated by taking **arithmetic mean, geometric mean** or median as **average**.

44. Link relative of current year is equal to:

$$(a) \frac{\text{Price of the current year}}{\text{Price of the base year}} \times 100$$

$$(b) \frac{\text{Price of the base year}}{\text{Price in the preceding year}} \times 100$$

$$(c) \frac{\text{Price in the current year}}{\text{Price in the preceding year}} \times 100$$

$$(d) \frac{\text{Price in the preceding year}}{\text{Price in the current year}} \times 100$$

Answer: c

Explanation:

This **method** of finding the seasonal indices in the form of the chain **relatives** was

$$(c) \frac{\text{Price in the current year}}{\text{Price in the preceding year}} \times 100$$

developed by Prof. Karl Pearson, and hence, this **method** is also known as the Pearson **method** of seasonal variation. Hence is correct answer

45. Marshall Edge worth price index was proposed by:

a. One English economist

b. Two English economist

c. Three English economist

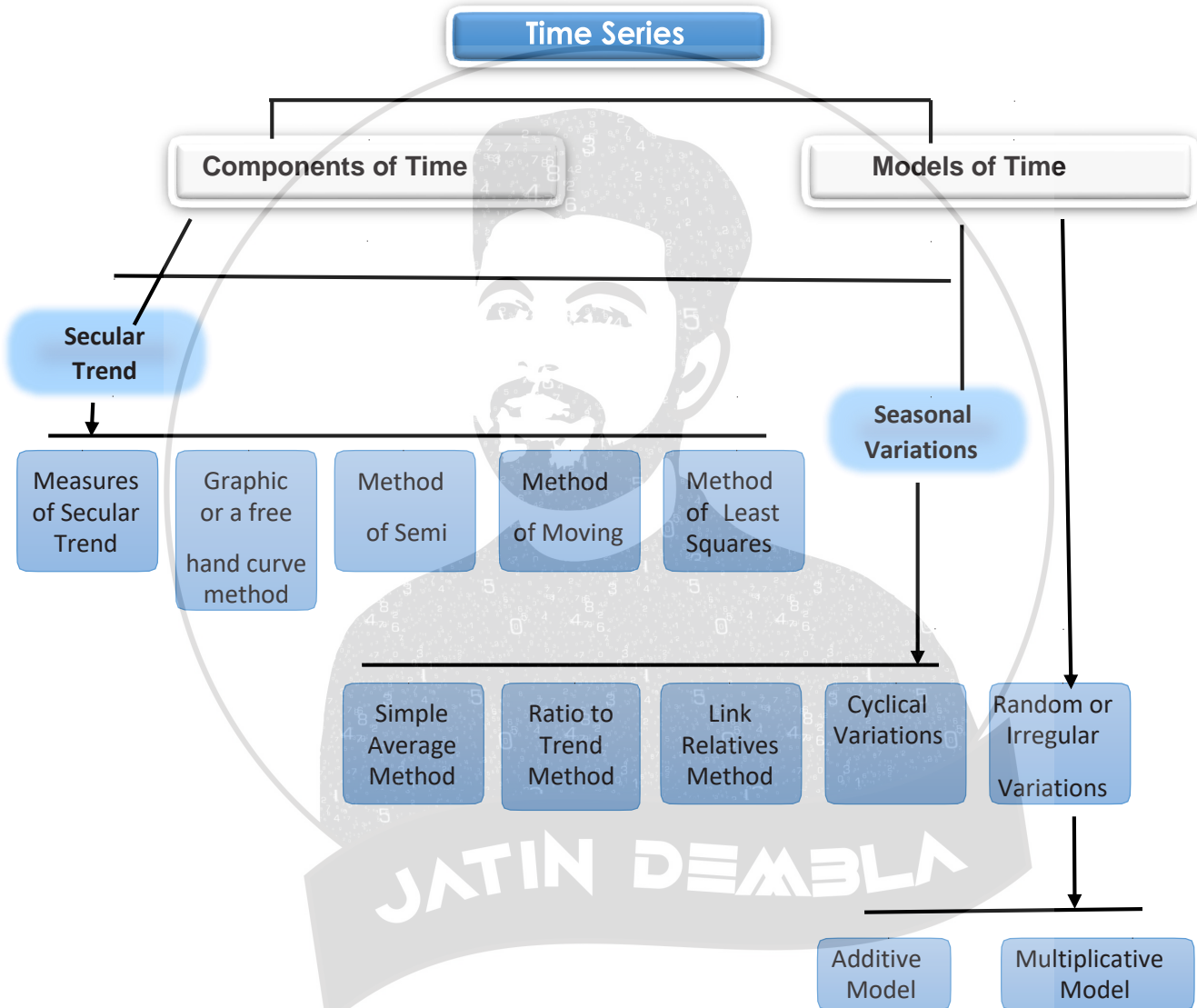
d. Many English economist

Answer: b

Explanation:

The Marshall-Edgeworth index, credited to Marshall (**1887**) and Edgeworth (1925), is a weighted relative of current period to base period sets of prices. This index uses the arithmetic average of the current and based period quantities for weighting. It is considered a pseudo-superlative formula and is symmetric.

UNIT – II: TIME SERIES



TIME SERIES	<p>A time series may be determined by eliminating the computed trend values from the given data set. It may done using additive model or multiplicative model.</p> <p>A time series is set of measurements on a variable taken over some period of time, it has four components.</p>
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	<ul style="list-style-type: none"> (a) Trend (b) Seasonal variations (c) Cyclical variations (d) Irregular variations
MODEL	<p>There are two models of time series</p> <ul style="list-style-type: none"> (a) Additive Model (b) Multiplicative Model
METHODS	<p>Trends can be measured in the following measures</p> <ul style="list-style-type: none"> (a) Free hand curve method (b) (b) Semi-averages method (c) Moving averages method (d) Least squares method
SEASONAL VARIATIONS	<p>Measured in any of the following methods:</p> <ul style="list-style-type: none"> (a) simple averages (b) Ratio to trend method (c) Ratio to Moving averages (d) Link relative method
SIGNIFICANCE	<ul style="list-style-type: none"> 1) Time Series is useful in forecasting future values. 2) Time series data can be deseasonalised by eliminating the effect of seasonal variations from it. 3) Irregular component in a time series is measured as a residue after eliminating all other Fluctuations from data.



1. The tendency of trend to increase or decrease or stagnate over a long period of time is called

- a. Periodic Variation
- b. Cyclic Variation
- c. Secular Trend
- d. Random Variation

ANSWER: c

EXPLANATION:

The tendency of trend to increase or decrease or stagnate over a long period of time is called **Secular Trend**.

2. The equation $Y = a + bx$ is used to get the value of

- a. Parabolic Trend
- b. Exponential Trend
- c. Linear Trend
- d. None of the above

ANSWER: c

EXPLANATION:

The equation $Y = a + bx$ is used to get the value of Linear Trends

3. The trend equation for annual sale of product is $Y = 120 + 36x$ with Year 1990 as origin. The annual sales for year 1992 will be-

- a. 156
- b. 192
- c. 120
- d. None of the above

ANSWER: b

EXPLANATION:

Given= $Y = 120 + 36x$ (Annual Sales Equation)

ORIGIN 1990

Annual sale for 1992=?

Put we get $x=2$ in equations for 1992 sales

$$Y = 120 + 36 \times 2$$

$$Y = 192$$

4. The technique of estimating the probable value of phenomenon a future date is called:

- a. Interpolation
- b. Interpolation
- c. Forecasting
- d. Probability

ANSWER: c

EXPLANATION:

Forecasting is to predict or estimate the probable value of phenomenon at a future date.

5. Which of the following is forecasting on the basis of past data?

- a. Trend projection
- b. Index number
- c. Both
- d. Correlation

ANSWER: b

EXPLANATION:

Forecasting on past data basis INDEX NO.

6. "Occurrence of floods" falls under which type of variations?

- a. Seasonal variation
- b. Simple variation
- c. Cyclic variation
- d. Random variation

ANSWER: d

EXPLANATION:

Random variation do not reveal any regular pattern of movement. These variations are caused by random factors such as strikes, floods, fire, famines, etc.

7. Which of the following is a general form of exponential trend?

- a. $y = a + bt$
- b. $y = a - b$
- c. $y_t = a \times b^t$
- d. $y_t = a + bt + ct^2$

ANSWER: c

EXPLANATION:

The equation of exponential trend $y_t = a \times b^t$

8. How the data is arranged in a Time Series Analysis?

- a. In descending order of their magnitude
- b. Arranged chronologically
- c. In ascending order of their magnitude.
- d. Arranged abruptly

Answer: b

Explanation:

Data in a time series analysis is arranged **chronologically**.

9. For a time series, interval can be:

- a. Year
- b. Month
- c. Week
- d. Any of these

ANSWER: d

EXPLANATION:

For a time series, interval can be created year or monthly or weekly too.

10. Seasonal and cyclic variations are the types of:

- a. Secular Trend
- b. Random Variations
- c. Irregular Variations
- d. Oscillatory Variations

ANSWER: d

EXPLANATION:

These oscillations are mostly observed in economics data and the periods of such oscillations are generally extended from five to twelve years or more. These oscillations are associated with the well-known business cycles.

11. Which of these is not a method of measurement of trend?

- a. Graphic method
- b. Calculative method
- c. Method of moving averages
- d. Method of least squares

ANSWER: b

EXPLANATION:

If a straight line is fitted to the data it will serve as a satisfactory trend, perhaps the most accurate method of fitting is that of least squares.

The formula for a straight-line trend can most simply be expressed as $Y_c = a + bX$

12. Methods of Measuring Trend?

- a. Free hand curve method
- b. Average method
- c. Geographical method
- d. None

ANSWER: a

EXPLANATION:

Trend can be determined: (i) free hand curve method; (ii) moving averages method; (iii) semi averages method; and (iv) least-squares method.

13. A time series consists of the following ____ components or elements?

- a. 5
- b. 4
- c. 7
- d. 8

ANSWER: b

EXPLANATION:

A time series consists of the following four components or elements:

1. Basic or Secular or Long-time trend;
2. Seasonal variations;
3. Business cycles or cyclical movement

4. Erratic or Irregular fluctuations.

14. Which of these is a method of least square?

- a. Linear Trend
- b. Exponential Trend
- c. Parabolic Trend
- d. All of the above.

ANSWER: d

EXPLANATION:

There will be many straight lines which can meet the first condition. Among all different lines, only one line will satisfy the second condition. It is because of this second condition that this method is known as the method of least squares.

15. Additive model of time series is

- a. $O = T + S + C + I$
- b. $O = TSCI$
- c. $O = a + bx$
- d. none

ANSWER: a

EXPLANATION:

$$O = T \times S \times C \times I$$

where O refers to original data,
 T refers to trend.

S refers to seasonal variations,

C refers to cyclical variations and

I refers to irregular variations.

This is the most commonly used model in the decomposition of time series. This model is called Additive model.