Chapter 9
Differential Equations
Exercise 9.1

Determine order and degree (id defined) of differential equations given in Exercises 1 to 10

Question 1

$$\frac{d^4y}{dx^4} + \sin(y''') = 0$$

Solution:

The given differential equation is,

$$\frac{d^4y}{dx^4} + \sin(y''') = 0$$

⇒ y "' + sin (y"') = 0
The highest order derivative present in the differential equation is y"" , so its order is three. Hence, the given differential equation is not a polynomial equation in its derivatives and so, its degree is not defined.

Question 2

y' + 5y = 0

Solution:

The given differential equation is, y' + 5y = 0
The highest order derivative present in the differential equation is y' , so its order is one.
Therefore, the given differential equation is a polynomial equation in its derivatives.
So, its degree is one.

Question 3

$$\left(\frac{dx}{dt}\right)^4 + 3x\frac{d^2s}{dt^2} = 0$$

Solution:

The given differential equation is,

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$$

The highest order derivative present in the differential equation is a polynomialEquation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$

So, its degree is one.

Question 4

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

Solution:

The given differential equation is,
$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$
The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.
The order is two. Therefore, the given differential equation is not a polynomial.
So, its degree is not defined.

Question 5

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

Solution:

The given differential equation is,
$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$
$\Rightarrow \frac{d^2y}{dx^2} - \cos 3x - \sin 3x = 0$
The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.
The order is two. Therefore, the given differential equation is a polynomial
Equation in $\frac{d^2y}{dx^2}$ and the power is 1.
Therefore, its degree is one.

Question 6

$$(y'''')^2 + (y'')^3 + (y')^4 + y^5 = 0$$

Solution:

The given differential equation is, $(y'''')^2 + (y'')^3 + (y')^4 + y^5 = 0$
The highest order derivative present in the differential equation is y'''.
The order is three. Therefore, the given differential equation is a polynomial
Equation in y''', y'' and y'.
Then the power raised to y''' is 2.
Therefore, its degree is two.

Question 7

y''' + 2y'' + y' = 0

Solution:
The given differential equation is, y''' + 2y'' + y' = 0
The highest order derivative present in the differential equation is y'''.
The order is three. Therefore, the given differential equation is a polynomial
Equation in y'', y'' and y'.
Then the power raised to y'' is 1.
Therefore, its degree is one.

Question 8

y' + y = e^x

Solution:

The given differential equation is, y' + y = e^x
= y, + y - e^x = 0
The highest order derivative present in the differential equation is y''.
The order is one. Therefore, the given differential equation is a polynomial equation in Y'.
Then the power raised to y' is 1.
Therefore, its degree is one.

Question 9

y''' + $(y')^2$ + 2y = 0

Solution:

The given differential equation is, y'' + $(y')^2$ + 2y = 0
The highest order derivative present in the differential equation is y''.
The order is one. Therefore, the given differential equation is a polynomial equation in Y'' and y'
Then the power raised to y' is 1.
Therefore, its degree is one.

Question 10

y''' + 2y' +sin y = 0

Solution:

The given differential equation is, y''' + 2y' + sin y = 0
The highest order derivative present in the differential equation is y''.
The order is two. Therefore, the given differential equation is a polynomial equation in y'' and y'.

Then the power raised to y” is 1.
Therefore, its degree is one.

Question 11

the degree of the differential equation.
$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ **is**

(A) 3
(B) 2
(C) 1
(D) not defined

Solution:

(D) not defined
The given differential equation is,
$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$
The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.
The order Is three. Therefore, the given differential equation is not a polynomial.
Therefore, its degree is not defined.

Question 12

the order of the differential equation
$2x^2\frac{d^2y}{dx^2} - 3\ 3\ \frac{dy}{dx} + y = 0$ **is**

(A) 2
(B) 1
(C) 0
(D) not defined

Solution:

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the given differential equation is,
$2x^2\frac{d^2y}{dx^2} - 3\ 3\ \frac{dy}{dx} + y = 0$
The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.
Therefore, its order is two.

Exercise 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a Solution of the corresponding differential equation:

Question 1

Y = ex + 1: "y" – y' = 0

Solution:

From the question it is given that y = e^x + 1
Differentiating both sides with respect to x, we get,
$\frac{dy}{dx} = \frac{d}{dx}(e^x)$ ………………… Equation 1
Now, differentiating equation (i) both sides with respect to x, we have,
$\frac{d}{dx}(y') \frac{d}{dx}(e^x)$
⇒ y' = e^x
Then,
Substituting the value of y' and y "in the given differential equations, we get,
Y" – y'= e^x - e^x = RHS.
Therefore, the given function is a solution of the given differential equation.

Question 2

y = x^2 + 2x c: y' – 2x – 2 = 0

Solution:

From the question it is given that y = x^2 + 2x + c
Differentiating both sides with respect to x, we get
y' = $\frac{d}{dx}$ (x^2 + 2x + c)
y' = 2x + 2
then,
substituting the values of y' in the given differential equations, we get,
= y' = 2x -2
= 2x + 2 – 2x – 2
= 0
= RHS
Therefore, the given function is a solution is a solution of the given differential equation.

Question 3

y = cos x + c: y' + sin x = 0

Solution:

From the question it is given that y = cos x + c
Differentiating both sides with respect to x, we get,
Y' = $\frac{d}{dx}$ (cosx +c)
Y' = -sinx
Then,
Substituting the value of y' in the given differential equations, we get,

= y' + sinx
= - sinx + sinx
= 0
=RHS
Therefore, the given function is a solution of the given differential equation.

Question 4

y = √ (1 + x²): y' = ((xy)/ (1 + x²))

Solution:

From the question it is given that y = $\sqrt{1+x^2}$
Differentiating both sides with respect to x, we get,
Y' = $\frac{d}{dx}(\sqrt{1+x^2})$
→ y' = $\frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$
By differentiating $(1+x^2)$ we get,
→ y' = $\frac{2x}{2\sqrt{1+x^2}}$
On simplifying we get,
→ y' = $\frac{x}{\sqrt{1+x^2}}$
By multiplying and dividing √ $(1+x^2)$
→ y' = $\frac{x}{1+x^2}$ x $\sqrt{1+x^2}$
Substituting the value of √ $(1+x^2)$
Substituting the value of √ $(1+x^2)$
=>y' = $\frac{x}{1+x^2} \cdot y$
=> y' = $\frac{xy}{1+x^2}$
Therefore, LHS =RHS
Therefore, the given function is a solution of the given differential equation.

Question 5

y = Ax: xy' = y (x ≠ 0)

Solution:

From the questions it is given that y = Ax
Differentiating both sides with respect to x, we get,
y' = $\frac{d}{dx}$(Ax)
y' = A
Then,
Substituting the values of y' in the given differential equations, we get,
= xy'
= x× A
= Ax

= y ... [from the question]

=RHS

Therefore, the given function is a solution of the given differential equation.

Question 6

y = x sinx: xy' = y + x ($\sqrt{(x^2 - y^2)}$) (x ≠ 0 and x>y or x< - y)

Solution:

From the question it is given that y = xsinx

Differentiating both sides with respect to x, we get,

Y' = $\frac{d}{dx}$ (xsinx)

=> = sinx$\frac{d}{dx}$ (x) + x. $\frac{d}{dx}$ (sinx)

=> y' = sinx +xcosx

Then,

Substituting the values of y' in the given differential equations, we get,

LHS = xy' = x(sinx + xcosx)

= xsinx + x^2cosx

From the question substitute y instead of xsinx, we get,

= y + x^2. $\sqrt{1 - sin^2 x}$

= y + $x^2\sqrt{1 - \left(\frac{y}{x}\right)^2}$

= y + x $\sqrt{(y)^2 - (x)^2}$

= RHS

Therefore, the given function is a solution of the given differential equation

Question 7

xy = logy + c: y' = $\frac{y^2}{1-xy}$ (xy ≠ 1)

Solution:

From the question it is given that xy = logy + c

Differentiating both sides with respect to x, we get,

$\frac{d}{dx}$ (xy) = $\frac{d}{dx}$ (logy)

⇒ y. $\frac{d}{dx}$ (x) + x. $\frac{dy}{dx}$ = $\frac{1}{y}\frac{dy}{dx}$

On simplifying, we get.

⇒ y + xy' = $\frac{1}{y}\frac{dy}{dx}$

by cross multiplication,

⇒ y^2 +xyy' = y'

⇒ (x, y −1)y' = -y^2

⇒Y' = $\frac{y^2}{1-xy}$

By comparing LHS and RHS
LHS = RHS
Therefore, the given function is the Solution of the corresponding differential Equation.

Question 8

y – cos y = x: (y sin y + cos y + x) y' = y

Solution:

From the question it is given that y – cosy = x
Differentiating both sides with respect to x, we get,
$\frac{dy}{dx} - \frac{d}{dx}\cos y = \frac{d}{dx}(x)$
=> y' + siny, y' = 1
=> y' (1 +siny) = 1

$$=> y' = \frac{1}{1 + \sin y}$$

Then,
Substituting the values of y' in the given differential equations, we get,
Consider LHS =(ysiny + cosy + x)'
= (ysiny + cosy + y – cosy) × $\frac{1}{1+\sin y}$
= y(1 + siny) × $\frac{1}{1+\sin y}$
On simplifying we get,
= y
= RHS
Therefore, the given functions is the solution of the corresponding differential equation.

Question 9

x + y = tan⁻¹ y: y² + y² + 1 = 0

Solution:

From the question it is given that x + y = $\tan^{-1}$y
Differentiating both sides with respect to x, we get,
$\frac{d}{dx}(x + y) = \frac{d}{dx}(\tan^{-1}y)$
$\Rightarrow 1 + y' = \left[\frac{1}{1+y^2}\right]y'$
By transposing y' to RHS and it becomes – y' and take out y' as common for Both, we get,
$\Rightarrow y' = \left[\frac{1}{1+y^2} - 1\right] = 1$
On simplifying.
$\Rightarrow y' = \left[\frac{1-(1+y^2)}{1+y^2}\right] = 1$

$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right] = 1$

$\Rightarrow y' = \frac{-(1+y^2)}{y^2}$

Then,

Substituting the values of y' in the given differential, we get,

Consider, LHS = $y^2y' + y^2 + 1$

$= y^2\left[\frac{-(1+y^2)}{y^2}\right] + y^2 + 1$

$= -1 - y^2 + y^2 + 1$

= 0

= RHS

Therefore, the given function is the Solution of the corresponding differential equation.

Question 10

$y = \sqrt{a^2 - x^2}$ x ∈ (-a, a): $x + y\frac{dy}{dx} = 0 (y \neq 0)$

Solution:

From the question it is given that $y = \sqrt{a^2 - x^2}$

Differentiating both sides with respect to x, we get,

$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{a^2 - x^2})$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2-x^2}}.\frac{d}{dx}(a^2 - x^2)$

$= \frac{1}{2\sqrt{a^2-x^2}}(-2x)$

$= \frac{-x}{2\sqrt{a^2-x^2}}$

Then,

Substituting the values of y' in the given differential equations, we get,

Consider LHS = $x + y\frac{dy}{dx}$

$= x + \sqrt{a^2 - x^2} \times \frac{-x}{2\sqrt{a^2-x^2}}$

On simplifying we get,

= x – x

= 0

By comparing LHS and RHS

LHS = RHS

Therefore, the given function is the solution of the corresponding differential equation.

Question 11

the number of arbitrary constants in the general Solution of a differential equation of fourth order are:

(A) 0 **(B) 2**

(C) 3 **(D) 4**

Solution:

(D) 4

The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.

Question 12

the number of arbitrary constants in the particular solution of a differential equation of third order are;

(A) 3 **(B)2**

(C) 1 **(D) 0**

Solution:

(D) 0

The solution free from arbitrary constants i.e., the solutions obtained from the general solution by giving particular values to the differential equation.

Exercise 9.3

In each of the Exercise 1 to 5, form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.

Question 1

$\frac{x}{a} + \frac{y}{b} = 1$

Solution:

From the question it is given that $\frac{x}{a} + \frac{y}{b} = 1$

Differentiating both sides with respect to x, we get,

$\frac{1}{a} + \frac{1}{b}\frac{dy}{dx} = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b}y' = 0$ .... [Equation (i)]

Now, differentiating equation (i) both sides with respect x, we get,

$0 + \frac{1}{b}y'' = 0$

$\Rightarrow \frac{1}{b}y'' = 0$

By cross multiplication, we get,

$\Rightarrow y'' = 0$

∴ the required differential equation is y'' = 0

Question 2

$y^2 = a\,(b^2 - x^2)$

Solution:

From the question it is given that $y^2 = a\,(b^2 - x^2\,)$
Differentiating both sides with respect to x, we get,
$2y\frac{dy}{dx}$ = a (2 – 2x)
→ 2yy’ = -2ax
→ yy’ = (-2/2)ax
Now, differentiating equation (i) both sides, we get,
Y’ x y’ + yy’’ = -a
$(y;)^2$ + yy’’ = -a ... [we call as equation (ii)]
Then,
Dividing equation (ii) by (i), we get,
$$\frac{(y')^2 + yy''}{yy'} = \frac{-a}{-ax}$$
⇒$x(y')^2$ + xyy’’ = yy’
Transposing yy’ to LHS it becomes – yy’
⇒xyy’’ + $x(y')^2$ – yy’ = 0
∴ the required differential equation is xyy’’ + $x(y')^2$ – yy’ = 0.

Question 3

y = $ae^{3x} + be^{-2x}$

Solution:

From the question it is given that y = ae^{3x} + be^{-2x} ... [we call it as equation (i)]
Differentiating both sides with respect to x, we get,
y’ = $3ae^{3x}$ - $2be^{-2x}$ ... [equation (ii)]
Now. Differentiating equation (ii) both sides, we get,
y’ = $9ae^{3x}$ + $4be^{-2x}$ ... [equation (iii)]
Then, multiply equation (i) by 2 and afterwards add it to equation (ii),
We have,
⇒ ($2ae^{3x}$ + $2be^{-2x}$) + ($3ae^{3x}$ - $2be^{-2x}$) = 2y +y’
⇒ $5ae^{3x}$ = 2y +y’
⇒ $ae^{3x} = \frac{2y+y'}{5}$
So now, let us multiply equation (ii) by 3 and subtracting equation (ii),
We have
⇒ ($3ae^{3x}$ - $3be^{-2x}$) – ($3ae^{3x}$ - $2be^{-2x}$) = 3y – y’
⇒ $5be^{-2x}$ = 3y - y’
⇒ $be^{-2x} = \frac{3y-y'}{5}$
Substitute the value of ae^{3x} and be^{-2x} in y’’,
Y’’= 9 x $\frac{2y+y'}{5}$ + 4 x $\frac{2y+y'}{5}$
⇒ Y’’ = $\frac{18y+9y'}{5} + \frac{12y-4y'}{5}$
On simplifying we get,

⇒Y” $\frac{30y+5y'}{5}$
⇒Y” 6y + y’
⇒Y” – y’ – 6y = 0
∴ the required differential equation is y” – y’ – 6y = 0.

Question 4

y = e^{2x} (a + bx)

Solution:

From the question it is given that y = e^{2x} (a + b x) .... [equation (ii)]
Differentiating both sides with respect to x, we get,
Y’ = $2e^{2x}$ (a +b x) + e^{2x} xb ... [equation (ii)]
Then, multiply equation (i) by 2 and afterwards subtract it to equation (ii),
We have, ‘
Y’ – 2y = e^{2x}(2a + 2bxb) - e^{2x} (2a + 2bx)
Y’ – 2y = $2ae^{2x} + 2e^{2x}bx + e^{2x}b - 2ae^{2x} - 2bxe^{2x}$
Y’ – 2y = be^{2x} ... [equation (ii)]
Now, differentiating equation (ii) both sides,
We have,
⇒Y” -2y = $2be^{2x}$ ...[equation(iv)]
Then,
Dividing equation (iv) by (iii), we get.
$\frac{y'' - 2y'}{y' - 2y} = 2$
By cross multiplication,
⇒ Y” – 2y’ = 2y’ – 4y
Transposing 2y’ and -4y to LHS it becomes – 2y’ and 4y
⇒ Y” – 4y’ – 4y = 0
∴ the required differential equation is y” – 4y’ – 4y = 0.

Question 5

y = e^x (a cos x + b sin x)

Solution:

From the question it is given that y = e^x (a cos x + b sin x)
... [we call it as equation (i)]
Differentiating both sides with respect to x, we get,
Y” = e^x (a cos x + b sin x) + e^x (-a sin x + b cos x)
y’ = e^x [(a + b) cos x – (a – b) sin x)] ... [equation (ii)]
Now, differentiating equation (ii) both sides,
We have,
Y’ = e^x [(a + b) cos x – (a – b) sin x)] + e^x [- (a + b) sinx – (a – b) cos x)]
On simplifying, we get,

y" = e^x [2bcosx – 2asinx]
y" = $2e^x$ (b cos x – a sin x) ... [equation (iii)]
Now, adding equation (i) and (iii), we get,

$Y + \frac{y}{2} = e^x [(a + b) \cos x - (a - b)\sin x]$

$Y + \frac{y}{2} = y'$

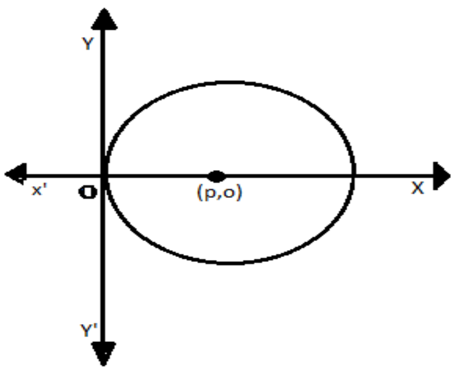
⇒ 2y + y' = 2y'

Therefore, the required differentialis 2y + y' = 2y' = 0

Question 6

From the differential equation of the family of circles touching the y- axis at origin.

Solution:



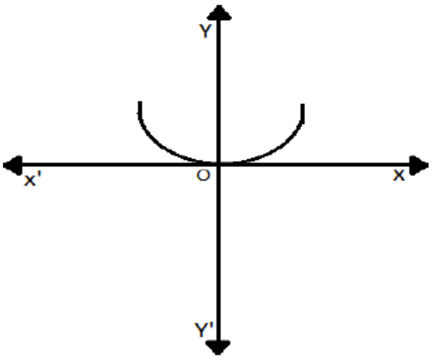
By looking at the figure we can say that the centre of the circle touching the y- axis at origin lies on the x – axis.
Let us assume (p, o) be the centre of the circle.
Hence. It touches the y – axis at origin, its radius is p.
Now, the equation of the circle with centre (p,o) and radius (p) is
⇒ $(x - p)^2 + y^2 = p^2$
⇒ $x^2 + p^2 = 2xp + y^2 = p^2$
Transposing p^2 and – 2xp to RHS then it becomes - p^2 and 2xp
⇒ $x^2 + y^2 = p^2 - p^2 + 2px$
⇒ $x^2 + y^2 = 2px$ ... [equation (i)]
Now, differentiating equation (i) both sides.
We have,
⇒2x + 2yy' = 2p
⇒ x + yy' = p
Now, on substituting the value of 'p' in the equation, we get.
⇒ $x^2 + y^2 = 2(x + yy')x$
⇒ $2xyy' + x^2 = y^2$
Hence, $2xyy' + x^2 = y^2$ is the required differential equation.

Question 7

form the differential equation of the family of parabolas having vertex at origin and axis along positive y – axis.

Solution:

The parabola having the vertex at origin and the axis along the positive y- axis is
x^2 = 4ay ... [equation (i)]



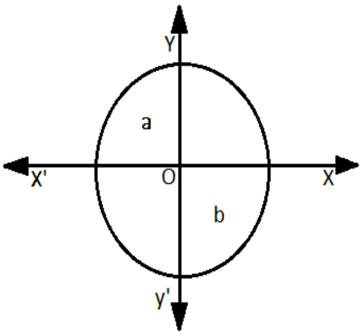
Now, differentiating equation (i) both sides with respect to x,
2x = 4ay’ .... [equation (ii)]
Dividing equation (ii) by equation (i), we get,
$\Rightarrow \frac{2x}{x^2} = \frac{4ay'}{4ay}$
On simplifying, we get,
$\Rightarrow \frac{2}{x} = \frac{y'}{y}$
By cross multiplication,
⇒ xy’ = 2y
Transposing 2y to LHS it becomes = - 2y.
⇒ xy’ – 2y = 0
Therefore, the required differential equation is xy’ – 2y = 0.

Question 8

form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

Solution:

By observing the figure, we can say that, the equation of the family of ellipses having foci on y-axis and the centre at origin.
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...[equation (i)]



Now, differentiating equation (i) both sides with respect to x,

$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0$

$\frac{x}{b^2} + \frac{yy'}{a^2} = 0$ ... [equation (ii)]

Now, again differentiating equation (iI) both sides with respect to x,

$\frac{1}{b^2} + \frac{y'y' + yy''}{a^2} = 0$

On simplifying,

$\frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy'') = 0$

$\frac{1}{b^2} = -\frac{1}{a^2}(y'^2 + yy'')$

Now substitute the value in equation (ii), we get,

$x\left[-\frac{1}{a^2}(y'^2 + yy'')\right] + \frac{yy'}{a^2} = 0$

On simplifying,

$\Rightarrow -x(y')^2 - xyy'' + yy' = 0$

$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

Hence, $xyy'' + x(y')^2 - yy' = 0$ is the required differential equation.

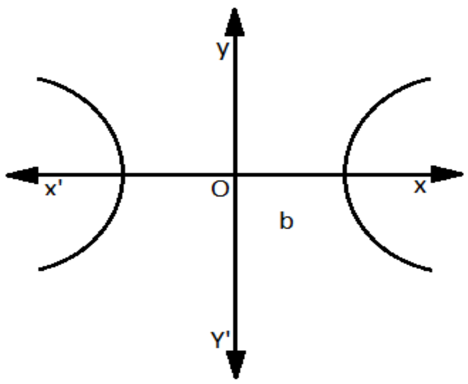
Question 9

form the differential equation of the family of hyperbolas having foci on x- axis and centre at origin.

Solution:

By observing the figure, we can say that, the equation of the family of hyperbolas foci on x – axis and the centre at origin is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .... [equation (i)]



Now, differentiating equation (i) both sides with respect to x,

$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$

$\Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} = 0$ .... [(ii)]

Now, again differentiating equation (ii) both sides with respect to x,

$\frac{1}{a^2} - \frac{y'y'+yy''}{b^2} = 0$

On simplifying,

$\Rightarrow \frac{1}{a^2} - \frac{1}{b^2}(y'^2+yy'') = 0$

$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}(y'^2+yy'')$

Now substitute the value in equation (ii), we get

$\frac{x}{b^2}(y'^2 + yy'') - \frac{yy'}{b^2} = 0$

$\Rightarrow x\, y'^2 + xyy' - yy' = 0$

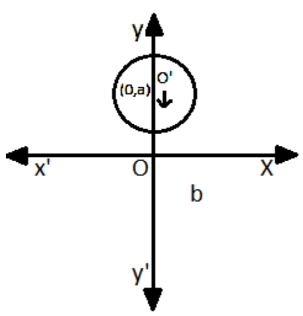
$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

Hence, $xyy'' + x(y')^2 - yy' = 0$ is the required differential equation.

Question 10

form the differential equation of the family of circles having centre on y- axis and radius 3 units.

Solution:



Let us assume the centre of the circle on y – axis be (0, a).
We know that the differential equation of the family of circle with centre at (0, a) and radius 3 is: $x^2 + (y - a)^2 = 3^2$
$\Rightarrow x^2 + (y - a)^2 = 9$ ... [equation (i)]
Now, differentiating equation (i) both sides with respect to x,
$\Rightarrow 2x + 2(y - a) \times y' = 0$ ... [dividing both side by 2]
$\Rightarrow x + y(y - a) \times y' = 0$
Transposing x to the RHS it becomes – x.
$(y - a) \times y' = x$
$Y - a = \frac{-x}{y'}$
Now, substitute the value of (y –a) in equation (i), we get,
$x^2 + \left(\frac{-x}{y'}\right)^2 = 9$
Take out the x^2 as common,
$\Rightarrow x^2\left[1 + \frac{1}{(y')^2}\right] = 9$
On simplifying,
$\Rightarrow x^2((y')^2 + 1) = 9(y')^2$
$\Rightarrow (x^2 - 9)(y')^2 + x^2 = 0$
Hence, $(x^2 - 9)(y')^2 + x^2 = 0$ is the required differential equation.

Question 11

Which of the following differential equations has $y = c_1 e^x + c^2 e^{-x}$ as the general?

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$ (C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx^2} - 1 = 0$

Solution:

(B) $\frac{d^2y}{dx^2} - y = 0$
Explanation:
From the question it is given that $y = c_1 e^x + c_2 e^x$
Now, differentiating given equation both sides with respect to x,
$\frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$ .... [equation (i)]
Now, again differentiating equation (i) both sides with respect to x,
$\frac{d^2y}{dx^2}$ $c_1 e^x + c_2 e^{-x}$
$\frac{d^2y}{dx^2} = y$
$\frac{d^2y}{dx^2} - y = 0$
Hence, $\frac{d^2y}{dx^2} - y = 0$ is the required differential equation.

Question 12

Which of the following differential equations has y = x as one of its particular Solution?

(A) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ **(B)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ **(D)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Solution:

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

Explanation:

From the question it is given that y = x

Now, differentiating given equation both sides with respect to x,

$\frac{dy}{dx} = 1$ ... [equation (i)]

Now, again differentiating equation (i) both sides with respect to x,

$\frac{d^2y}{dx^2} = 0$

then,

substitute the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the given options,

$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy$

$= 0 - (x^2 \times 1) + (x \times x)$

$= -x^2 + x^2$

$= 0$

Exercise 9.4

For each of the differential equations in Exercises 1 to 10. Find the general solution:

Question 1

$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Solution:

Given

$\Rightarrow \frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

We know that 1 – cos x = $\sin^2 (x/2)$ and 1 + cos x = 2 $\cos^2 (x/2)$

Using this formula in above function we get

$\Rightarrow \frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$

We have sin x/cos x = tan x using this we get

$\Rightarrow \frac{dy}{dx} = \tan^2\frac{x}{2}$

From the identity $\tan^2x = \sec^2x - 1$, the above equation can be written as

$\Rightarrow \frac{dy}{dx} = (\sec^2\frac{x}{2} - 1$

Now by rearranging and taking integrals on both sides we get

$\Rightarrow \int dy = \int sec^2 \frac{x}{2} dx - \int dx$

On integrating we get

$\Rightarrow y = 2 \tan^{1}\frac{x}{2} - x + c$

Question 2

$\frac{dy}{dx} = \sqrt{4 - y^2}$ **(-2 < y < 2)**

Solution:

Given

$\Rightarrow \frac{dy}{dx} = \sqrt{4 - y^2}$

On rearranging we get

$\Rightarrow \frac{dty}{\sqrt{4 - y^2}} = dx$

Now taking integrals on both sides,

$\Rightarrow \int \frac{1}{\sqrt{a^2 - a^2}} = \int dx$

We know that,

$\Rightarrow \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}$

Then above equation becomes

$\Rightarrow \text{Sin}^{-1}\frac{y}{2} = x + c$

Question 3

$\frac{dy}{dx} + y = 1$ **(y ≠ 1)**

Solution:

$\Rightarrow \frac{dy}{dx} + y = 1$

On rearranging we get

$\Rightarrow dy = (1 - y) dx$

separating variable by variable separable method we get

$\Rightarrow \frac{dy}{1-y} = dx$

Now by taking integrals on both sides we get

$\Rightarrow \int \frac{dy}{1-y} = \int dx$

On integrating

$\Rightarrow -\log(1 - y) = x + \log c$

$\Rightarrow -\log(1 - y) - \log c = xd$

$\Rightarrow \text{Log}(1 - y) c = e^{-x}$

Above equation can be written as

$\Rightarrow (1 - y) = \frac{1}{c} e^{-x}$

y = 1 + $\frac{1}{e}$ e^{-x}
y = 1 + A e^{-x}

Question 4

sec^2 x tany dx + sec^2 y tan x dy = 0

Solution:

Given
⇒ Sec^2 x tany dx + sex^2 y tan x dy
Dividing both sides by (tan x) (tany) we get
∴ $\frac{sec^2 xtanydx}{tanxtany} + \frac{sec^2 ytanxdy}{tanxtany} = 0$
On simplification we get
⇒ $\frac{sec^2 xdx}{tanx} + \frac{sec^2 ytanxdy}{tanxtany} = 0$
Integrating both sides,
⇒ $\int \frac{sex^2 xdx}{tax} = \int \frac{sec^2 ydy}{tany}$
⇒ Let tan x = t & tan y = u
Then,
Sec^2 x dx = dt&sec^2 y dy = du
By substituting these in above equation we get
∴ $\int \frac{dt}{t} = - \int \frac{du}{u}$
On integrating
⇒ Log t = -log u + log c
Or,
⇒ Log (tan x) = -log (tan y) + log c
⇒ Log tan x = log$\frac{c}{tany}$
⇒ (tan x) (tan y) = c

Question 5

⇒($e^x + e^{-x}$) dy – ($e^x – e^{-x}$) dx = 0

Solution:

Given
($e^x + e^{-x}$) dy – ($e^x – e^{-x}$) dx = 0
On rearranging the above equation, we get
⇒ dy = $\frac{(ex - e-x\ ) dx}{ex + e-x}$
taking integrals both sides,
⇒ $\int dy = \int \frac{(ex - e-x\ ) dx}{ex + e-x}$
Now let ($e^x + e^{-x}$) = t

Then, $(e^x - e^{-x})$ dx = dt

∴ $y = \int \frac{dt}{t}$

On integrating

∴ $\int \frac{dx}{x}$ = log x

So,

⇒ T = log t

Now by substituting the value of t we get

⇒ y = log $(e^x + e^{-x})$ + c

Question 6

$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

Solution:

⇒ $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

Separating variable by variable separable method,

⇒ $\frac{dy}{1+y^2}$ = dx $(1 + x^2)$

Now taking integrals on both sides,

⇒ $\int \frac{dy}{1+y^2} = \int dx + \int x^2 dx$

On integrating we get

⇒ $\tan^{-1} y = x + \frac{x^3}{3} + c$

Question 7

$ylogydx - xdy = 0$

Solution:

Given

Y log y dx – x dy = 0

On rearranging we get

⇒ (y log y) dx = x dy

Separating variables by using variable separable method we get

⇒ $\frac{dx}{x} = \frac{dy}{ylogy}$

Now integrals on both sides,

⇒ $\int \frac{dx}{x} = \int \frac{dy}{ylogy}$

Let logy = t

Then,

⇒ $\frac{1}{y}$ dy = dt

⇒ $log\, x = \int \frac{dt}{t}$

⇒ Log x + log c = log t
Now by substituting the value of t
Log x + log c = log (log y)
Now by using logarithmic formula we get.
⇒ Log c x = log y
⇒ Log y = cx
⇒ Y = e^{cx}

Question 8

$x^5\frac{dy}{dx} = - y^5$

Solution:

Given
⇒ $x^5\frac{dy}{dx} = -y^5$
Separating variables by using variable separable method we get
⇒ $\frac{dy}{y^5} = \frac{-dx}{x^5}$
On rearranging
⇒ $\frac{dy}{y^5} + \frac{dx}{x^5} = 0$
Integrating both sides,
⇒ $\int \frac{dy}{y^5} + \int \frac{dx}{x^5} = a$
Let a be a constant,
⇒ $\int y^{-5}dy + \int x^{-5}\, dx = a$
On integrating we get
⇒ $-4y^{-4} - 4x^{-4} + c = a$
On simplification we get
⇒ $-x^{--4} - y^{--4} = c$
The above equation can be written as
⇒ $\frac{1}{x^4} + \frac{1}{y^4} = c$

Question 9

$\frac{dy}{dx} = \sin^{-1} x$

Solution:

Given
⇒ $\frac{dy}{dx} = \sin^{-1} x$
Separating variables by using variable separable method we get
⇒ $dy = \sin^{-1} x\, dx$
taking integrals on both sides,
⇒ $\int dy = \int sin - 1xdx$

Now to integrate $\sin^{-1}$ x we have to multiply it by 1
To use product rule
$\int u.vdx = u\int vdx - \int \left(\frac{d}{dx}u\right)(\int vdx)$ dx}
Then we get
$\Rightarrow$y = $\int 1.sin - 1xdx$
According to product rule and ILATE rule, the above equation can be written as
$\therefore$ y = $\sin^{-1}$ x $\int 1.dx - \int \frac{dy}{dx}sin^{-1}x)(\int 1.dx)dx\}$
On integrating we get
$\Rightarrow$y = x $\sin^{-1}$ x – $\int \frac{x}{\sqrt{1-x^2}}dx$
Now,
$\Rightarrow$let 1 – x^2 = t
$\Rightarrow$xdx = - $\frac{dt}{2}$
on simplification above equation can be written as
$\Rightarrow$ y = x$\sin^{-1}$ x + $\frac{1}{2}\int t - \frac{1}{2}dt$
$\Rightarrow$ y = x$\sin^{-1}$ x + $\frac{1}{2}\sqrt{t}$ + c
substituting the value of t, we get
$\Rightarrow$ y = x$\sin^{-1}$ x + $\sqrt{1-x^2}$ + c

Question10

e^x tan y dx + (1 - e^x) $\sec^2$ y dy = 0

Solution:

Given
$\Rightarrow$ e^x tan y dx + 1 (1 – e^x) $\sec^2$ y dy = 0
On rearranging above equation can be written as
(1 - e^x) sec^2 y dy = -e^xtany dy = 0
Separating the variables by using variables separable method,
$\Rightarrow \frac{sec^2y}{tan\ y}dy$ = - $\frac{e^x}{1-e^x}$dx
Now by taking integral on both sides, we get
$\Rightarrow \int \frac{sec^2y}{tany}dy = \int \frac{e^x}{1-e^x}$dx
Let tan y t and m1 - e^x = u
Then on differentiating
(sec^2y dy = dt) & (e^xdx = du)
Substituting these in above equation we get
$\therefore \int \frac{dt}{t} = \int \frac{du}{u}$
On integration we get
$\Rightarrow$ log t = log u + log c
Substitution the values of t and u on above equation.
$\Rightarrow$ log (tany) = log (1 - e^x) + log c
$\Rightarrow$ log tany = log c $(1 - e^x)$

By using logarithmic formula above equation can be written as
⇒ tany $(1 - e^x)$

For each of the differential equations in exercises 11 to 14 find a particular solutionSatisfying the given condition

Question 11

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$; y = 1 when x = o

Solution:

Given
$\Rightarrow (x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$
Separating variables by using variable separable method
$\Rightarrow dy = \frac{2x^2+x}{(x+1)(x^2+1)}dx$
taking integrals on both sides, we get
$\Rightarrow \int dy = \int \frac{2x^2+x}{(x+1)(x^2+1)}dx$....1
Integrating it partially using partial fraction method,
$\Rightarrow \frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+1}$
$\Rightarrow \frac{2x^2+x}{(x+1)\,(x^2+1)} = \frac{Ax^2+A(Bx+c)(x+1)}{9x+1)\,(x^2+1)}$
$\Rightarrow 2x^2 + x = Ax^2 + A + Bx + Cx + C$
$\Rightarrow 2x^2 + x = (A + B)x^2 + (B + c)x + A + C$
Now comparing the coefficients of x^2 and x
⇒A + B = 2
⇒B + c = 1
⇒A + c = 0
Solving them we will get the values of A, B, C
A = $\frac{1}{2}$, B = $\frac{3}{2}$, c = -$\frac{1}{2}$
Putting the values of A, B C in 1 we get
$\Rightarrow \frac{2x^2+x}{(x+1)\,(x^2+1)} = \frac{1}{2}\frac{1}{(x+1)} + \frac{1}{2}\frac{3x-1}{2x^2+1}$
Now taking integrals on both sides
$\Rightarrow \int dy = \frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{3x-1}{x^2+1}dx$
On integrating
$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{dx}{x^2+1}$
$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\tan^{-1}x$ 2
For second term
Let $x^2 + 1$ t
Then, 2x dx = dt
$\therefore \frac{3}{4}\int \frac{2x}{x^2+1}dx = \frac{3}{4}\int \frac{dt}{t}$

So, I = $\frac{3}{4}$ log t

Substituting the value of t we get

I = $\frac{3}{4}\log(x^2 + 1)$

Then 2 becomes

$\Rightarrow y = \frac{1}{2}\log(x + 1) + \frac{3}{4}\log(x^2 + 1) - \frac{1}{2}\tan^{-1} x + c$

Taking 4 common

$y = \frac{1}{4}[2\log(x + 1) + 3\log(x^2 + 1)] - \frac{1}{2}\tan^{-1} x + c$

$y = \frac{1}{4}[\log(x + 1)^2 + \log(x^2 + 1)^3] - \frac{1}{2}\tan^{-1} x + c$

$y = \frac{1}{4}[\log(x + 1)^2 + \log(x^2 + 1)^3] - \frac{1}{2}\tan^{-1} x + c$ 3

Now we are given that y = 1 when x = 0

$\therefore\ 1 = \frac{1}{4}[\log(0 + 1)^2 (0^2 + 1)\}] - \frac{1}{2}\tan^{-1} 0 + c$

$1 = \frac{1}{4} \times 0 - \frac{1}{2} \times 0 + c$

Therefore,

C = 1

Putting the values of c in 3 we get

$y = \frac{1}{4}[\log(x + 1)^2 + \log(x^2 + 1)^3] - \frac{1}{2}\tan^{-1} x + c$

Question 12

$x(x^2 - 1)\frac{dy}{dx} = 1$: y = 0 when x = 2

Solution:

Given

$x(x^2 - 1)\frac{dy}{dx} = 1$

separating variables by variable separable method,

$$\Rightarrow dy = \frac{dx}{x(x^2 + 1)}$$

$x^2 + 1$ can be written as (x + 1) (x – 1) we get

$\Rightarrow dy = \frac{dx}{x(x+1)(x-1)}$

Taking integrals on both sides,

$\Rightarrow \int dy = \int \frac{dx}{x(x+1)(x-1)}$...............1

Now by using partial fraction method,

$\Rightarrow \frac{1}{x(x-1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{c}{x-1}$ ...2

$\Rightarrow \frac{1}{x(x+1)(x-1)} = \frac{A(x-1)(x+1)+B(x)(x-1)+c(x)(x+1)}{x(x+1)(x-1)}$

Or

$\frac{1}{x(x+1)(x-1)} = \frac{(A+B+C)x^2+(B-C)x-A}{x(x+1)(x-1)}$

$Nowcomparingtevaluesof A, b, c$

A + B + C = 0

B – c = 0
A = -1
Solving these we will get that B = ½ and c = ½
Now putting the value of A, B, c in 2
$\Rightarrow \frac{1}{x(x+1)(x-1)} = -\frac{1}{x} + \frac{1}{2}\left(\frac{1}{x+\ 1)}\right) + \frac{1}{2}\left(\frac{1}{x-1}\right)$
Now taking integrals we get
$\Rightarrow \int dy = -\int \frac{1}{x} dx + \frac{1}{2}\int \left(\frac{1}{x+1}\right)dx + \frac{1}{2}\int \left(\frac{1}{x\ \ 1}\right)dx$
On integrating
$\Rightarrow y = -\log x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) + \log C$
$\Rightarrow y = \frac{1}{2}\log\left[\frac{c^2(x-1)(x+1)}{x^2}\right]$ ………………3
Now we are given that y = 0 when x = 2
$0 = \frac{1}{2}\log\left[\frac{c^2(x-1)\ (2+1)}{x^2}\right]$
$\Rightarrow \log\frac{3c^2}{4} = 0$
We know $e^0 = 1$ by substituting we get
$\Rightarrow \frac{3c^2}{4} = 1$
$\Rightarrow 3c^2 = 4$
$\Rightarrow c^2 = 4/3$
Now, putting the value of c^2 in 3
Then,
$y = \frac{1}{2}\log\left[\frac{4(x-1)(x+1)}{3x^2}\right]$
$y = \frac{1}{2}\log\left[\frac{4(x^2-1)}{3x^2}\right]$

Question 13

$\cos\left(\frac{dy}{dx}\right) = a\ (a \in R)$: y = 1 when x = 0

Solution:

Given
$\cos\left(\frac{dy}{dx}\right) = a$
On rearranging we get
$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$
$dy = \cos^{-1} a\ dx$
integrating both sides, we get

$$\int dy = \cos^{-1} a \int dx$$

$y = x\cos^{-1} a + c$ … 1
Now y = 1 when x = 0
Then,
1. = 0 $\cos^{-1} a + c$
Hence C = 1

Substituting C = 1 in equation (1), we get;
y= x $\cos^{-1}$ a + 1
(y =-1) /x = $\cos^{-1}$ a
$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$

Question 14

$\frac{dy}{dx}$ = y tan x: y = 1 when x = 0

Solution:

Given
$\frac{dy}{dx}$ = y tan x
Separating variables by variable separable method
$\Rightarrow \frac{dy}{y}$ tan x dx
Taking integrals both sides, we get
$\Rightarrow \int \frac{dy}{y} = \int \tan x\, dx$
On integrating
⇒Log y = - log (cos x) + log c
Using standard trigonometric identity, we get
⇒Log y = log (sec x) + log c
Using logarithmic formula in above equation we get
⇒Log y = log c (sec x)
⇒Y = c (sec x) ...1
Now we are given that y = 1 when x = 0
⇒1 = c (sec 0)
⇒1 = c x 1
⇒C = 1
Putting the value of c in 1
⇒Y = sec x

Question 15

find the equation of a curve passing through the point (0. 0) and whose differential equation is Y' = e^x sin x

Solution:

To find the question of a curve that passes through point (0, 0) and has differential equation y' = e' sin x
So, we need to find the general solution of the given differential equation and the put the given point in to find the value of constant.
So, =>$\frac{dy}{dx}$ – e^x sin x
Separating variables by variable separable method, we get

⇒ dy = e^x sin x dx
integrating both sides,
⇒ $\int dx = \int e^x \sin x dx$……………..…1
Now by using product rule we get
$\int u.vdx = u\int vdx$-$\int \{\frac{d}{dx}u \int vdx\}$ dx
Now let
I = $\int e^x$ sin x dx
⇒ $I = \sin x \int e^x dx - \int (\frac{d}{dx}\sin x \int e^x dx)$
⇒ I = e^x sin x $- \int cosxe^x\, dx$
Now by integrating we get
⇒ I = e^x sin x = [cos x$\int e^x dx + \int sinxe^x\, dx$]
From 1 we have
⇒ I = e^x sin x – e^x cosx – I
Now on simplifying
⇒ 2I = e^x sin x e^x cos x
⇒ 2I = e^x (sin x - cos x)
⇒I = $e^x \frac{(\sin x - \cos x}{2}$
Substituting I in 1 we get
⇒ y = e $^x\frac{(\sin x - coasx\ )}{2} + c$ ………..….2
Now we are given that the curve passes through point (0, 0)
∴ 0 = $e^0\,\frac{(\sin 0 - \cos 0)}{2}$ + c
⇒ 0 = $\frac{1(0-1)}{2}$ + c
⇒ $c = \frac{1}{2}$
Substituting the value of C in 2
⇒ y = e $^x\frac{(\sin x - \cos x)}{2} + \frac{1}{2}$
On rearranging
⇒ 2y = e^x(sin x – cos x) +1
Hence
⇒ 2y – 1 = e^x(sin x - cos x 9)

Question 16

For the differential equation xy $\frac{dy}{dx}$ = (x + 2)(y + 2)
Find the solution curve passing through the point (1, -1).

Solution:

For this question, we need to find the particular solution at point (1, 1) for the Givendifferential equation.
Given differential equation is
⇒ $xy\frac{dy}{dx}$ = (x + 2) (y + 2)

Separating variables by variable separable method, we get

$\Rightarrow \frac{y}{y+2}dy \quad \frac{(x+2)dx}{x}$

Taking integrals both sides, we get'

$\Rightarrow \int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$

$splitting the integrals$

$\Rightarrow \int dy - 2\int \frac{1}{y+2} dy = \int dx + 2\int \frac{1}{x} dx$

⇒y – 2 log (y + 2) = x + 2 log x + c………… …1

Now separating like terms on each side.

⇒ y– x – c = 2log x + 2 log (y + 2)

⇒ y – x – c = log x^2 + log$(y + 2)^2$

Using logarithmic formula, we get

⇒ y – x – c = log {x^2 $(y + 2)^2$} – 1)

Now we are given that the curve passes through (1, -1)

Substituting the values of x and y, to find the value of c

⇒ − 1 – 1 – 1 = log [-1 + $2)^2$}

⇒ - 2 – c = log (1)

We know that log 1 0

⇒ c = -2 + 0

So, c = - 2

Substituting the value of c in 1

y – x – c = log {x^2 $(y + +2)^2$}

y – x + 2 = log {x^2 $(y + 2)^2$}

Question 17

Find the question of a curve passing through the point (0, -2) given that at any point x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

Solution:

We know that slope of a tangent is = $\frac{dy}{dx}$.

So we are given that the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

$y\frac{dy}{dx} = x$

now separating variables by variable separable method.

⇒ y dy = x dx

Taking integrals both sides.

$\Rightarrow \int ydy = \int xdx$

On integrating we get

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

⇒ $y^2 - x^2$ = 2c …1

Now the curve passes through (0, -2).

∴ 4-0 = 2c

⇒ $c = 2$
Putting the value of c in 1 we get
⇒ $y^2 - x^2 = 4$

Question 18

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve given that it passes through (-2, 1).

Solution:

We know that (x, y) is the point of contact of curve and its tangent.
Slope (m1) for line joining (x,y and (-4, -3) is $\frac{y+3}{x+4}$,…………1
Also, we know that slope of tangent of a curve is $\frac{dy}{dx}$,
∴ slope (m2) of tangent = $\frac{dy}{dx}$ …2
Now, according to the question, we can write as
(m2) = 2(m1)
⇒ $\frac{dy}{dx} = \frac{2(y++3)}{x+4}$
Separating variable by variable separable method, we get
⇒ $\frac{dy}{y+3} = \frac{2dx}{x+4}$
Taking integrals on both sides,
⇒ $\int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$
On integrating we get
⇒ $l$ogy + 3) = 2log(x + 4) + log c
Using logarithmic formula above equation can be written as
⇒log(y + 3) = log c(x +4 $)^2$
⇒ y + 3 = c (x + 4 $)^2$ ………. 3
Now, this equation passes through the point (-2, 1).
⇒1 + 3 = c (-2 + 4 $)^2$
⇒4 = 4c
⇒C = 1
Substitute the value of c in 3
⇒ y + 3 = (x + 4 $)^2$

Question 19

The volume of spherical balloon being inflated at a constant rate. Initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of Balloon after t seconds.

Solution:
Let the rate of change of the volume of the balloon be k where is a constant

$$\therefore \frac{dy}{dt} = k$$

$\frac{d}{dt}(\frac{4}{3}\pi r^3)$ = k {volume of sphere = $\frac{4}{3}\pi r^3$}
On differentiating with respect to r we get
⇒ $\frac{4}{3}\pi 3r^3\frac{dr}{dt}$ = k
on rearranging
⇒$4\pi r^2 dr = kdt$
Taking integrals on both sides,
⇒$4\pi \int r^2 dr = k\int dt$
On integrating we get
⇒ $\frac{4\pi r^3}{3}$ =kt + c ………….1
Now, from the question we have
At t = 0, r = 3:
⇒4π x 33 = 3 (k x 0 + c)
⇒108 π = 3c
⇒C = 36π
At t = 3, r = 6:
⇒$4\pi \times 6^3$ = 3 9 k x 3 + c)
⇒K = 84π
Substituting the values of k and c in 1
⇒ $4\pi r^3 = 3(84\pi t + 36\pi)$
⇒$4\pi r^3 = 4\pi(63t + 27)$
⇒ $r^3 = 63t + 27$
⇒r = $\sqrt[3]{63t+27}$
so, the radius of balloon after 1 second is $\sqrt[3]{63t+27}$

Question 20

In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

Solution:

Let t be time, p be principal and r be rate of interest
According the information principal increases at the rate of r% per year.
∴ $\frac{dp}{dt} = \left(\frac{r}{100}\right)p$
Separating variable by variable separable method, we get
⇒ $\frac{dp}{p} = \left(\frac{r}{100}\right)dt$
Taking integrals on both sides,
⇒ $\int \frac{dp}{p} = \frac{r}{100}\int dt$
On integrating we get
⇒Logp = $\frac{rt}{100}$ + k
⇒P = $\frac{\text{rt}}{\text{e}100}$ + k…………1
Given that t = 0, p = 100.
⇒100 = e^k …2

Now, if t = 10. Then p = 2 x 100 = 200
So,
200 = $\frac{rt}{e10}$ +k
200 = $\frac{rt}{e10}e^k$
From 2
200 = $\frac{rt}{e10} x$ 100
$\frac{rt}{e10}$ = 2
$\frac{rt}{e10}$ = log 2= r = 6.93
So, r is 6.93%

Question 21

In a bank principal increase continuously at the rate 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years (0.5 = 1.648).

Solution:

Let p and t be principal and time respectively.
Given that principal increase continuously at rate of 5% per year.
∴ $\frac{dp}{dx} = \left(\frac{5}{100}\right) p$
Separating variable by variable separable method,
⇒ $\frac{dp}{p} = \frac{p}{25}$
taking integrals on both sides,
⇒ $\int \frac{dp}{p} = \frac{1}{20} \int dt$
⇒ $log\, p = \frac{t}{e^{20}} + c$……………1
When t = o, p = 1000
⇒1000 = e^c
At t = 10
⇒ =P = $\frac{1}{e2}$ + c
the above equation can be written as
⇒P = $e^{0.5}$ x e^c
⇒P = 1.648 x 1000 ($e^{0.5}$ = 1.6489)
⇒P = 1648
So, after 10 year the total amount would be Rs. 1648

Question 22

in a culture, the bacteria count is 1, 00,000 The number is increased by 10% in 2 hours. In how many hours will the count reach 2, 00,000, if the rate of growth of bacteria is proportional to the number present?

Solution:

Let y be the number of bacteria at any instant t.
Given that the rate of growth of bacteria is proportional to the number present

$\therefore \frac{dy}{dt} \propto y$

$\Rightarrow \frac{dy}{dt}$ = ky (k is constant)

Separating variables by variable by separable method we get,

$\Rightarrow \frac{dy}{dt}$ = kdt

taking integrals on both sides.

$$\Rightarrow \int \frac{dy}{y} = k \int dt$$

On integrating we get

⇒Log y = k t + c ...1

Let y' be the number of bacteria at t = 0.

⇒Let y; = c

Substituting the value of c in 1

⇒ log y k t + log y '

⇒ Log y – log y' = k t

Using logarithmic formula, we get

$\Rightarrow \log \frac{y}{y'}$ = kt 2

Also, given that number of bacteria increases by 10% in 2 hours.

Therefore,

$\Rightarrow y = \frac{110}{100}y'$

$\Rightarrow \frac{y}{y'} = \frac{11}{10}$ ...3

Substituting this value in 2, we get

$\Rightarrow K \times 2 = \log \frac{11}{10}$

$\Rightarrow K = \frac{1}{2} \log \frac{11}{10}$

So, 2 becomes

$\Rightarrow \frac{1}{2}\log \frac{11}{10} \times t = \log \frac{y}{y'}$

$\Rightarrow t = \frac{2\log \frac{y}{y}}{\log \frac{11}{10}}$ ...4

Now, let the time when number of bacteria increase from 100000 to 200000 bet'

$\Rightarrow y$= 2y' at t = t'

So, from 4, we have

$\Rightarrow t' = \frac{2\log \frac{y}{y;}}{\log \frac{11}{10}} = \frac{2 \log 2}{\log \frac{11}{10}}$

So, bacteria increase from 100000 to 200000 in $\frac{2 \log 2}{\log \frac{11}{10}}$ hours

Question 23

The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = C$ **(B) $e^x + e^y = C$**

(c) $e^{-x} + e^y = C$ **(D) $e^{-x} + e^{-y} = C$**

Solution:

(A) $e^x + e^{-y} = C$
Explanation:
We have
$\Rightarrow \frac{dy}{dx} = e^{x} + ^{y}$
Using laws of exponents, we get
$\Rightarrow \frac{dy}{dx} = e^{x+y}$
Separating variable separable method, we get
$\Rightarrow e^{-y}\, dy = e^x\, dx$
Now taking integrals on both sides
$\Rightarrow \int e^{-y} dy = \int e^{x} dx$
On integrating
$\Rightarrow -e^{-y} = e^x + c$
$\Rightarrow e^x + e^{-y} = -c$
or,
$e^x + e^{-y} = c$
So, the correct option is A.

Exercise 9.5

In each of the Exercise 1 to 10, show that the given differential equation is homogeneous and solve each of them.

Question 1

$(x^2 + x y)\, dy = (x^2 + y^2)\, dx$

Solution:

On rearranging the given equation, we get
$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$
Let $f(x, y) = \frac{x^2+y^2}{x^2+xy}$
Here, substituting x = k x and y = k y
$f(kx, ky) = \frac{(kx)^2 + (ky)^2}{(kx)^2 + kx.ky}$
Taking k^2 common
$= \frac{k^2}{k^2}, \frac{x^2+y^2}{x^2+xy}$
$= k^0, f(x, y)$
Therefore, the given differential equation is homogeneous.
$(x^2 + x y)\, dy = (x^2 + y^2)\, dx$
$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$
To solve it we make the substitution.

y = v x
Differentiating equation with respect to x, we get
$\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$
We have dy/dx, substituting this in above equation
v + x$\frac{dy}{dx} = \frac{x^2 + (vx)^2}{x^2 + x.vx}$
Taking x^2 common
v + x$\frac{dy}{dx} = \frac{x^2 +(1+ v)^2}{x^2 +(1+ v)}$
On simplification we get
v + x$\frac{dy}{dx} = \frac{x^2 +(1+ v)^2}{x^2 +(1+ v)}$
On rearranging the above equation, we get
x$\frac{dy}{dx} = \frac{1 + v^2}{1 + v}$ - v = $\frac{1+v^2 - v - v^2}{1 + v}$
x$\frac{dy}{dx} = \frac{1 - v}{1 + v}$
$\frac{1 - v}{1 + v}$ dv = $\frac{1}{x}$ dx
Taking integrals on both sides,
$\int \frac{1 - v}{1 + v} dv = \int \frac{1}{x} dx$
$\int \left(-1 + \frac{2}{1-v}\right) dv = \int \frac{1}{x} dx$
On integrating we get
- v – 2log [1 – v] = log |x| + log c
Substituting the value of v, we get
$-\frac{y}{x}$ – 2log | 1 - $\frac{y}{x}$ | = log |x| + log C
Using logarithmic formula, we get
$-\frac{y}{x}$ = log $\frac{(x-y)^2}{x^2}$ + log |x| + log C
$-\frac{y}{x}$ = log $\frac{(x-y)^2}{x^2}$. Cx
On rearranging and computing we get
$-\frac{y}{x}$ = log $\frac{(x-y)^2}{x}$ C

$$\frac{C(x - y)^2}{x} = e^{-y/x}$$

c $(x – y)^2 = xe^{-y/x}$

Question 2

$$y' = \frac{x + y}{x}$$

Solution:

y' = $\frac{x + y}{x}$
The above equation can be written as
$\frac{dy}{dx} = \frac{x + y}{x}$
Let f (x, y) = $\frac{x + y}{x}$

Here, putting x = k x and y = k y

$f(kx, ky) = \frac{kx + ky}{kx}$

$= \frac{k}{k}, \frac{x + y}{x}$

$= k^0 . f(x, y)$

Therefore, the given differential equation is homogeneous.

$y' = \frac{x + y}{x}$

Then the above equation can be written as

$\frac{dy}{dx} = \frac{x + y}{x}$

To above it we make the substitution.

y = v x

Differentiating equation with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Now by substituting the value of v we get

$v + x\frac{dv}{dx} = \frac{x + vx}{x}$

On simplification we get

$v + x\frac{dv}{dx} = 1 + v$

On rearranging we get

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{1}{x} dx$

On integrating we get

v = log x + C

Now by substituting the value of v

$\frac{y}{x} = \log x + c$

y = x log x + c x

Question 3

(x – y) dy – (x + y) dx = 0

Solution:

Given (x – y) dy = (x + y) dx

On rearranging above equation, we can write as

$\frac{dy}{dx} = \frac{x + y}{x - y}$

Let $f(x, y) = \frac{x+y}{x-y}$

Now by substituting x = k x and y = k y

$f(kx, ky) \frac{kx + ky}{kx - ky}$

On simplification we get

$f(kx, ky) = \frac{x+y}{x-y}$

$= k^o . f(x, y)$

Therefore, the given differential equation is homogeneous.

$(x - y)\, dy - (x + y)\, dx = 0$

$\frac{dy}{dx} = \frac{x+y}{x-y}$

For further simplification we make the substitution.

$y = v\,x$

Differentiating equation with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Now by substituting the value of dv/dx we get

$v + x\frac{dv}{dx} = \frac{x + vx}{x - vx}$

Taking x as common we get

$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$

On rearranging

$x\frac{dv}{dx} = \frac{1+v}{1-v} - v$

Now taking LCM and computing we get

$x\frac{dv}{dx} = \frac{1 + v - v + v^2}{1-v}$

$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2} dv = \int \frac{1}{x} dx$

Now by splitting the integrals we get

$\int \frac{v}{1+v^2} dv - \int \frac{v}{1+v^2} dv = \int \frac{1}{x} dx$ ……...1

Let, $I_1 = \int \frac{v}{1+v^2} dv$

Put $1 + v^2 = t$

$2v\, dv = dt$

$v\, dv = \frac{1}{2} dt$

Now by applying integral we get

$$\frac{1}{2}\int \frac{1}{t} dt$$

$\frac{1}{2}\log t$

Now by substituting the value of t we get

$\frac{1}{2}\log(1 + v^2)$

From equation 1 we have

$\therefore \tan^{-1}v - \frac{1}{2}\log(1 + v^2) = \log x + c$

Now by substituting the value of v we get

$\tan^{-1}\frac{y}{x} - \frac{1}{2}\log(1 + (\frac{y}{x})^2 = \log x + c$

Onrearranging, we get

$\tan^{-1}\frac{y}{x} = \log x + \frac{1}{2}\log\left(\frac{x^2+y^2}{x^2}\right) + c$

$\tan^{-1}\frac{y}{x} = \frac{1}{2}\left(2\log x + \log\left(\frac{x^2+y^2}{x^2}\right)\right) + C$

Using logarithmic formula, we get

$\tan^{-1}\frac{y}{x} = \frac{1}{2}\left(\log\left(\frac{x^2+y^2}{x^2} \times x^2\right)\right) + C$

$\tan^{-1}\frac{y}{x} = \frac{1}{2}(\log x^2 + y^2) + c$

Question 4

$(x^2 - y^1)dx + 2xy\ dy = 0$

Solution:

The given equation can be written as
$2xy\ dy = -(x^2 - y^2)\ dx$
On rearranging we get
$\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$
Let f (x, y) = $-\frac{x^2 - y^2}{2xy}$
Here, substituting x = k x and y = k y
f (kx, ky) = $-\frac{k^2x^2 - k^2y^2}{2k^2xy}$
Now by common by taking k^2 common
f (kx, ky) = $-\frac{k^2}{k^2}.\frac{x^2 - y^2}{2xy}$
$= k^0.f(x, y)$
Therefore, the given differential equation is homogeneous.
$(x^2 - y^2)dx + 2xy\ dy = 0$
Again, on rearranging
$2xy\ dy = -(x^2 - y^2)dx$
The above equation can be written as
$\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$
To solve above equation and for further simplification we make the substitution.
Y = v x
Differentiating equation with respect to x, we get
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
Now by substituting the value of dy/dx we get
$v + x\frac{dv}{dx} = -\frac{x^2 - v^2x^2}{2x.vx}$
Now taking x^2 as common
$v + x\frac{dv}{dx} = -\frac{x^2(1 - v^2)}{2vx^2}$
On rearranging
$x\frac{dv}{dx} = -\frac{-1 + v^2 - 2v^2}{2v}$
Now taking LCM and computing
$x\frac{dv}{dx} = \frac{-1 + v^2 - 2v^2}{2v}$
On simplification
$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$
Rearranging the above equation, we get

$-\frac{2v}{1+v^2}dv = \frac{1}{x}dx$

Now by multiplying the above equation by negative sign we get

$\frac{2v}{1+v^2}dv = -\frac{1}{x}dx$

Taking integrals on both sides, we get

$\int \frac{2v}{1+v^2}dv = -\int \frac{1}{x}dx$ 1

Let, $i_1 = \int \frac{2v}{1+v^2}dv$

Put $1 + v^2 = t$

2v dv = dt

Vdv = $\frac{1}{2}$dt

Taking integral, we get

$$\int \frac{1}{t}dt$$

Log t

From 1 we have

$\therefore \log(1+v^2) = -\log x + \log c$

Now by substituting the value of v we get

$\log\left(1 + (\frac{y}{x})^2\right) = -\log x + \log c$

By using logarithmic formula, we get

$\log\left(\frac{x^2+y^2}{x^2}\right) = \log\frac{c}{x}$

On simplification

$X^2 + y^2 = Cx$

Question 5

$x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$

Solution:

The given question can be written as

$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$

Let $f(x, y) = \frac{x^2 - 2y^2 + xy}{x^2}$

Now by substituting x = k x and y = k y

$f(kx, ky) = \frac{k^2x^2 - 2k^2y^2 + kxky}{k^2x^2}$

Now by taking k^2 common we get

$f(kx, ky) = \frac{k^2}{k^2}\frac{x^2 - 2y^2 + xy}{x^2}$

$= k^0 . f(x, y)$

Therefore, the given differential equation is homogeneous.

$x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$

On rearranging we get

$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$

To solve above equation and to make simplification easier we make the substitution.
Y = v x
Differentiating above equation with respect to x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now by substituting the value of dy/dx we get

$v + x\frac{dv}{dx} = \frac{x^2-2x^2+x.vx}{x^2}$

On rearranging we get

$v + x\frac{dv}{dx} = \frac{1-2v^2+v}{1}$

$v + x\frac{dv}{dx} = 1 - 2v^2 + v$

On simplification

$x\frac{dv}{dx} = 1 - 2v^2$

By separating the variable using variable separable method,

$\frac{1}{1-2v^2}dv = \frac{1}{x}dx$

Taking integrals on both sides, we get

$\int \frac{1}{1-2v^2}\,dv = \int \frac{1}{x}dx$

The above equation can be written as

$\int \frac{1}{1-(\sqrt{2}v)^2}\,dv = \int \frac{1}{x}dx$

$\int \frac{1}{1^2-(\sqrt{2}v)^2}\,dv = \int \frac{1}{x}dx$

On integrating using standard trigonometric identity we get

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2.1} . \log\left|\frac{1+\sqrt{2}v}{1-\sqrt{2}v}\right| = \log|x| + C$

Now by substituting the value of v we get

$\frac{1}{2\sqrt{2}} \log\left|\frac{1+\sqrt{2}\frac{y}{x}}{1-\sqrt{2}\frac{y}{x}}\right| = \log|x| + c$

On simplification

$\frac{1}{2\sqrt{2}} \log\left|\frac{1+\sqrt{2}v}{1-\sqrt{2}v}\right| = \log|x| + c$

Question 6

$x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

Solution:

The given question can be written as

$xdy = (\sqrt{x^2+y^2} + y)\,dx$

On rearranging the above equation, we get

$\frac{dy}{dx} = \frac{(\sqrt{x^2+y^2}+y)}{x}$

Let $f(x, y) = \frac{(\sqrt{x^2+y^2}+y)}{x}$

Here, putting x = k x and y = k

K (kx, ky) = $\frac{(\sqrt{k^2x^2+k^2y^2}+ky)}{kx}$

Now taking k as common

f (kx, ky) = $\frac{k}{k}.\frac{(\sqrt{x^2+y^2}+y}{x}$

= k^0. F (x, y 0

Therefore, the given differential equation is homogeneous.

xdy – ydx = $\sqrt{x^2 + y^2}dx$

By separating the variables using variable separable method we get

xdy = ($\sqrt{x^2 + y^2} + y$) dx

On rearranging we get

$\frac{dy}{dx} = \frac{(\sqrt{x^2+y^2} +y}{x}$

To solve above equation, we make the substituting

y= v x

Differentiating equation with respect to x, we get

$\frac{dy}{dx}$ = v +x$\frac{dv}{dx}$

On rearranging and substitution, the value of dy/ dx we get

v + x$\frac{dv}{dx} = \frac{\sqrt{x^2+x^2v^2} +vx}{x}$

Taking x as common and computing we get

v + x$\frac{dv}{dx} = \frac{x\sqrt{1+x^2}+vx}{x}$

x$\frac{dv}{dx}$= $\sqrt{1 + v^2}$

Again, separating variables, we get

$\frac{1}{\sqrt{1+v^2}}$ dv = $\frac{1}{x}dx$

Taking integrals on both sides, we get

$\int \frac{1}{\sqrt{1+v^2}}$ dv = $\int \frac{1}{x}dx$

Using $\int \frac{1}{\sqrt{x^2+a^2}}$ + log (x + $\sqrt{x^2 + a^2}$the above equation can be written as

log (v + $\sqrt{1 + x^2}$) = log x + log c

Now by using logarithmic formula we get

$$\log\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) = \log cx$$

On simplifying we get

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$$

Taking LCM

$$\frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x^2}} = cx$$

$$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = cx$$

On rearranging

$$y + \sqrt{x^2 + y^2} = cx^2$$

Question 7

$$\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$$

Solution:

The given question can be written as

$$\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$$

Let $f(x, y) = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$

Now by substituting x = kx and y = ky

$$f(kx, ky) = \frac{\left\{kx\cos\left(\frac{ky}{kx}\right)+k\,y\sin\left(\frac{ky}{kx}\right)\right\}ky}{\left\{ky\sin\left(\frac{ky}{kx}\right)-k\,x\cos\left(\frac{ky}{kx}\right)\right\}kx}$$

Now by taking k^2 as common we get

$$f(kx, ky) = \frac{k^2}{k^2}\frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$$

$= k^0.\ f(x, y)$

Therefore, the given differential equation is homogeneous.

$$\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$$

To solve above equation, we make the substitution.

y = v x

Differentiating equation with respect to x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now by substituting dy/dx value and on rearranging we get

$$v + x\frac{dv}{dx} = \frac{\{x\cos(v)+vx\sin(v)\}vx}{\{vx\sin(v)-x\cos(v)\}x}$$

Taking x as common and simplifying we get

$$v + x\frac{dv}{dx} = \frac{\{\cos(v)+v\sin(v)\}v}{\{v\sin(v)-\cos(v)\}}$$

On rearranging and computing we get

$$x\frac{dv}{dx} = \frac{\{\cos(v)+v\sin(v)\}v}{\{v\sin(v)-\cos(v)\}} - v$$

Taking LCM and simplifying we get

$$x\frac{dv}{dx} = \frac{v\cos(v)+v^2\sin(v)-v^2\sin(v)+v\cos(v)}{v\sin(v)-\cos(v)}$$

$$x\frac{dv}{dx} = \frac{2v\cos(v)}{v\sin(v)-\cos(v)}$$

Separating the variable by using variable separable method we get

$$\frac{v\sin(v)-\cos v}{2v\cos v}dv = \frac{1}{x}dx$$

Now by splitting the numerator we get

$$\frac{v\sin v}{2v\cos v}dv - \frac{\cos v}{2v\cos v}dv = \frac{1}{x}dx$$

On simplification we get

$$\frac{1}{2}\tan v\,dv - \frac{1}{2}.\frac{1}{2}dv = \frac{1}{x}dx$$

Taking integrals on both sides, we get

$\frac{1}{2}\int tanv dv - \frac{1}{2}.\int \frac{1}{v} dv = \int \frac{1}{x}$ dx
On integrating we get
$\frac{1}{2}$ log secv - $\frac{1}{2}$ logv = logx + logk
Using logarithmic formula, we get
Log secv – logv = 2logkx
Now by substituting the value of v we get
Log sec$\left(\frac{y}{x}\right)$– log $\left(\frac{y}{x}\right)$ = 2logkx
Ahain using logarithmic formula we get
Log $\left(\frac{x}{y} sec\left(\frac{y}{x}\right)\right)$ = log $(kx)^2$
On simplification
$\frac{x}{y}$ sec$\left(\frac{y}{x}\right)$ = $k^2 x^2$
We know that sec x = 1/cos x, by using this in above equation we get
$\frac{1}{xycos\left(\frac{y}{x}\right)} = k^2$
On rearranging
xycos $\left(\frac{y}{x}\right) = \frac{1}{k^2}$
Where C is integral constant
C = $\frac{1}{k^2}$
xycos $\left(\frac{y}{x}\right)$ = c

Question 8

$$x\frac{dy}{dx} - y + xsin\left(\frac{y}{x}\right) = 0$$

Solution:

The given question can be written as
$x\frac{dy}{dx} - y - xsin\left(\frac{y}{x}\right)$
On rearranging we get
$\frac{dy}{dx} = \frac{y - xsin\left(\frac{y}{x}\right)}{x}$
Left(x, y) = $\frac{y - xsin\left(\frac{y}{x}\right)}{x}$
Now put x = k x and y – k y
f (kx,ky) = $\frac{Ky - kxsin\left(\frac{ky}{kx}\right)}{kx}$
By taking k as common we get
f (kx,ky) = $\frac{k}{k}.\frac{y - xsin\left(\frac{y}{x}\right)}{x}$
= k^0.f (x, y)
Therefore, the given differential equation is homogeneous.

$x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$

On rearranging the above equation

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

To solve above equation, we make the substitution.

Y = v x

Differentiating equation with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

On rearranging and substituting the value of dy/dx we get

$v + x\frac{dv}{dx} = \frac{vx - x\sin\left(\frac{vx}{x}\right)}{x}$

On simplification we get

$v + x\frac{dv}{dx} = v - \sin v$

$x\frac{dv}{dx} = -\sin v$

Now separating variables by variable separable method, we get

$\frac{1}{\sin v}dv = -\frac{1}{x}dx$

We know that 1/sinx = cosec x then above equation becomes

$\operatorname{cosec} v\, dv = -\frac{1}{x}dx$

Taking integration on both sides, we get

$\int \operatorname{cosec} v\, dv = -\int \frac{1}{x}dx$

On integrating we get

$\log(\operatorname{cosec} v - \cot v) = -\log x + \log c$

Now by substituting the value of v we get

$\log\left(\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x}\right) = \log\frac{c}{x}$

On simplifying we get

$\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x} = \frac{c}{x}$

We know that 1/ sin x = cosec x and cot x = cos x/ sin x then above equation becomes

$\frac{1}{\sin\frac{y}{x}} - \frac{\cos\frac{y}{x}}{\sin\frac{y}{x}} = \frac{c}{x}$

On rearranging we get

$1 - \cot\frac{y}{x} = \frac{c}{x}.\sin\frac{y}{x}$

$x\left(1 - \cot\frac{y}{x}\right) = C\sin\frac{y}{x}$

Question 9

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

Solution:

Given

$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$

The given equation can be written as

xlog $\left(\frac{y}{x}\right)$ dy – 2xdy = - ydx

Taking dy common

$\left(x\log\left(\frac{y}{x}\right) dy - 2x\right)$ dy = - ydx

On rearranging we get

$\frac{dy}{dx} = \frac{-y}{x\log\left(\frac{y}{x}\right)dy - 2x}$

$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$

Let f (x, y) = $\frac{y}{2x - x\log\left(\frac{y}{x}\right)}$

Now put x = k x and y = k y

F(kx,ky) = $\frac{ky}{2kx - kx\log\left(\frac{ky}{kx}\right)}$

Taking K as common

F(kx,ky) = $\frac{y}{2x - x\log\left(\frac{y}{x}\right)}$

= k^0. f (x, y)

Therefore, the given differential equation ais homogeneous.

Ydx + $x\log\left(\frac{y}{x}\right)$dy – 2xdy = 0

$x\log\left(\frac{y}{x}\right)$dy – 2xdy = -ydx

On rearranging

$\frac{dy}{dx} = \frac{-y}{x\log\left(\frac{y}{x}\right)dy - 2x}$

Simplifying we get

$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$

To solve it we make the substitution.

Y = v x

Differentiation equation with respect to x, we get

$\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$

On rearranging and substitution dy/ dx value we get

+ x$\frac{dv}{dx} = \frac{vx}{2x - x\log\left(\frac{vx}{x}\right)}$

On simplification

v+ x$\frac{dv}{dx} = \frac{v}{2 - \log v}$

x$\frac{dv}{dx} = \frac{v}{2 - \log v}$ – v

taking LCM and simplifying we get

x$\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$

x$\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$

by separating the variables using variable separable method we get

$\frac{2 - \log v}{-v + v\log v}$ dv = $\frac{1}{x} dx$

$\frac{2-\log v}{v\,(\log v-1)}\,dv = \frac{1}{x}\,dx$

On simplifying we get

$\frac{1-(\log v-1)}{v\,(\log v-1)}\,dv = \frac{1}{x}\,dx$

Integrating both sides, we get $\int \frac{1}{v\,(\log v-1)}\,dv - \int \frac{1}{v}\,dv = \int \frac{1}{x}\,dx$ ……………1

Let, $i_1 = \int \frac{1}{v\,(\log v-1)}\,dv$

Put, log v – 1 = t

$\frac{1}{v}\,dv = dt$

On integrating

$\int \frac{1}{t}dt$

Log t

Substituting the value of t

Log (log v – 1)

From equation 1 we have

∴ log (log v– 1) – log (v) = log (x) + log (c)

By using logarithmic formula, we get

$\log\left(\frac{(\log v - 1)}{v}\right) = \log(cx)$

$\frac{\log v - 1}{v} = cx$

On simplification we get

$\frac{\log\left(\frac{y}{x}\right)-1}{\frac{y}{x}} = cx$

$\frac{x}{y}\left(\log\left(\frac{y}{x}\right) - 1\right) = cx$

$\log\left(\frac{y}{x}\right) - 1 = cy$

Question 10

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

Solution:

Given question can be written as

$\frac{dy}{dx} = \frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$

Let f (x,y) = $\frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{(1+e^{x/y})}$

Now put x – k x and y = k y

f(kx,ky) = $\frac{-e^{kx/ky}\left(1-\frac{kx}{yk}\right)}{(1+e^{kx/ky})}$

$= \frac{-e^{kx/ky}\left(1-\frac{kx}{yk}\right)}{(1+e^{kx/ky})}$

$K^0 f(x,y)$
Therefore, the given differential equation is homogeneous.
$(1 + e^{x/y})\, dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$
On rearranging
$(1 + e^{x/y})\, dx = -e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy$
$\frac{dy}{dx} = \frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{(1+e^{x/y})}$
To solve above equation, we make the substitution
x = v y
Differentiation above equation with respect to x, we get
$\frac{dy}{dx} = v + y\frac{dv}{dy}$
On rearranging and substituting for dy/ dx value we get
$v + y\frac{dv}{dy} = \frac{-e^{vy/y}\left(1-\frac{vy}{y}\right)}{(1+e^{vy/y})}$
$y\frac{dy}{dx} = \frac{-e^{v}+ve^{v}}{1+e^{v}} - v$
Now taking LCM and simplifying we get
$\Rightarrow y\frac{dy}{dx} = \frac{-e^{v}+ve^{v}-v-ve^{v}}{1+e^{v}}$
The above equation can be written as
$\Rightarrow y\frac{dv}{dy} = -\left[\frac{v+e^{v}}{1+e^{v}}\right]$
$\Rightarrow \left[\frac{v+e^{v}}{1+e^{v}}\right] dv = -\frac{dv}{y}$
Integrating both sides we get
$\text{Log}\,(v + e^{v}) = -\log y + \log C = \log\left(\frac{c}{x}\right)$
Using logarithmic formula, the above equation can be written as
$\Rightarrow \left[\frac{x}{y} + e^{\frac{x}{y}}\right] = \frac{c}{y}$
$\Rightarrow x + ye^{\frac{x}{y}} = C$

For each of the differential equation in Exercise from 11 to 15, find the particular Solution satisfying the given condition:

Question 11

(x+ y) dy + (x –y) dx = 0: y = 1 when x = 1

Solution:

Given
$(x + y)\, dy + (x - y)\, dx = 0$
The above equation can be written as
$\frac{dy}{dx} = -\frac{(x-y)}{(x+y)}$

Let f(x, y) = $-\frac{(x-y)}{(x+y)}$

Now put x = k x and y = sk y

f(kx,ky) = - $\frac{(kx-ky)}{(kx+ky)}$

By taking k common from both numerator and denominator we get

= $\frac{k}{k}$. - $\frac{(x-y)}{(x+y)}$

=k^0, f (x, y)

Therefore, the given differential equation is homogeneous.

(x + y) d + (x –y) dx = 0

Again, above equation can be written as

$\frac{dy}{dx}$ = - $\frac{(x-y)}{(x+y)}$

To solve it we make the substitution.

y = v x

Differentiating above equation with respect to x, we get

$\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$

On rearranging and substituting the value of dy/dx we get

v + x $\frac{dv}{dx}$ = $-\frac{(x-vx)}{(x+vx)}$

Taking x common and simplifying we get

v + x $\frac{dv}{dx}$ = $-\frac{(1-v)}{(1+v)}$

On rearranging

x$\frac{dv}{dx}$ = $-\frac{(1-v)}{(1+v)}$ – v

Taking LCM and simplifying

x$\frac{dv}{dx}$ = $\frac{-1+v-v-v^2}{(1+v)}$

x$\frac{dv}{dx}$ = $\frac{-1-v^2}{(1+v)}$

x$\frac{dv}{dx}$ = $\frac{-(1+v^2)}{(1+v)}$

Then above equation can be written as.

$\frac{1+v}{1+v^2}$ dv = - $\frac{1}{x}$ dx

Taking integrals on both sides, we get

$\int \frac{1+v}{1+v^2} dv = -\int \frac{1}{x} dx$

Splitting the denominator,

$\int \frac{+v}{1+v^2} dv + \int \frac{1+v}{1+v^2} dv = -\int \frac{1}{x} dx$

On integrating we get

$\tan^{-1}$ v + $\frac{1}{2}$ log $(1 + v^2)$ = - log x + c

Now by substituting the value of v we get

$\text{Tan}^{-1}\frac{y}{x} + \frac{1}{2}\log\left(1+(\frac{y}{x})^2\right)$ = - logx + c

y = 1 when x = 1

$\text{Tan}^{-1}\frac{1}{1} + \frac{1}{2}\log\left(1 + \left(\frac{y}{x}\right)^2\right)$ = - log1 + C

The above equation becomes

$\frac{\pi}{4}+\frac{1}{2}2 = 0 + c$

$C = \frac{\pi}{4} + \frac{1}{2}log2$

$\therefore \tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)^2\right) = -\log x + c$

Where, $C = \frac{\pi}{4} + \frac{1}{2}log2$

$\therefore \tan^{-1}\frac{y}{x} + \log\left(1+\left(\frac{y}{x}\right)^2\right)$

$= -\log x + \frac{\pi}{4} + \frac{1}{2}\log 2$

$2\tan^{-1}\frac{y}{x} + \log\left(\frac{x^2+y^2}{x^2}\right)$

$= -2\log x + \frac{\pi}{2} + \log 2$

On simplifying we get

$2\tan^{-1}\frac{y}{x} + \log\left(\frac{x^2+y^2}{x^2}\right) + \log x^2 = \frac{\pi}{2} + \log 2$

$2\tan^{-1}\frac{y}{x} + \log(x^2 + y^2) = \frac{\pi}{2} + \log 2$

The required solution of the differential equation.

Question 12

$x^2dy + (x\,y\;\; y^2)dx = 0$; **y = 1 when x = 1**

Solution:

Given

$x^2\,dy + (x\,y + y^2)\,dx = 0$

On rearranging we get

$\frac{dy}{dx} = -\frac{(xy + y2)}{x^2}$

Let $f(x,y) = -\frac{(xy + y2)}{x^2}$

Now put x = k x and qk y

$f(kx, ky) = -\frac{(kxky + k^2y^2)}{k^2x^2}$

taking k^2 common we get

$= \frac{k^2}{k^2}, = \frac{(xy + y2)}{x^2}$

$= k^0.f(x, y)$

Therefore, the given differential equation is homogeneous.

$x^2\,dy + (x\,y + y^2)\,dx = 0$

$\frac{dy}{dx} = \frac{(xy + y2)}{x^2}$

To solve it we make the substitution,

y = v x

Differentiating above equation with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx} = -\frac{(x.vx + v^2x^2)}{x^2}$

$\frac{dy}{dx} = v + x\frac{dv}{dx} = -\frac{(x.vx + v^2x^2)}{x^2}$

On computing and simplifying

$v + x\frac{dv}{dx} = - v - v^2$

$x\frac{dv}{dx} = - v - v^2 - v$

$x\frac{dv}{dx} = - v\ (v + 2)$

$\int \frac{1}{v\ (v+2)}$ dv $= -\frac{1}{x}$ dx

Taking integrals on both sides, we get

$\int \frac{1}{v\ (v+2)}$ dv $= - \int \frac{1}{x} dx$

Dividing and multiplying above equation by 2 we get

$\frac{1}{2}\int \frac{2}{v\ (v+2)}$ dv $= - \int \frac{1}{x} dx$

Adding and subtracting v to the numerator we get

$\frac{1}{2}\int \frac{2+v-v}{v\ (v+2)}$ dv $= - \int \frac{1}{x} dx$

Now splitting the denominator, we get

$\frac{1}{2}\int \left(\frac{2}{v\ (v+2)} - \frac{v}{v\ (v+2)}\right)$ dv $= - \int \frac{1}{x} dx$

$\frac{1}{2}\int \left(\frac{1}{v} - \frac{v}{v\ (v+2)}\right)$ dv $= - \int \frac{1}{x} dx$

On integrating we get

$\frac{1}{2}$ (logv – log (v + 2)) = - logx log c

Using logarithmic formula,

$\frac{1}{2}\left(log\frac{v}{v+2}\right) = \log\frac{C}{x}$

$\log\left(\frac{\frac{y}{x}}{\frac{y}{x}+2}\right) = 2\log\frac{C}{x}$

$\log\left(\frac{y}{y+2x}\right) = \log\left(\frac{c}{x}\right)^2$

On simplification we get

$\frac{y}{y+2x} = \left(\frac{c}{x}\right)^2$

$\frac{x^2y}{y+2x} = c^2$

y= 1 when x = 1

$C^2 = \frac{1}{1+2} = \frac{1}{3}$

$\therefore \frac{x^2y}{y+2x} = \frac{1}{3}$

$3x^2y = y + 2x$

$y + 2x = 3x^2y$

The required solution of the differential equation.

Question 13

$\left[xsin^2\left(\frac{y}{x}\right) - y\right]$**dx + x dy = 0: y =** $\frac{\pi}{4}$ **when x = 1**

Solution:

given

$\left[xsin^2\left(\frac{y}{x}\right) - y\right]$dx = $-$ xdy

The above equation can be written as

$\left[xsin^2\left(\frac{y}{x}\right) - y\right] = -\, x\frac{dy}{dx}$

On rearranging

$\frac{dy}{dx} = -\frac{\left[xsin^2\left(\frac{y}{x}\right)-y\right]}{x}$

We know f (x, y) = dy/dx using this in above equation we get

f (x, y) = $-\frac{\left[xsin^2\left(\frac{y}{x}\right)-y\right]}{x}$

now put x = k x and y = k y

f (kx, ky) = $-\frac{k\left[xsin^2\left(\frac{ky}{kx}\right)-ky\right]}{kx}$

Taking k as common

$= \frac{k}{k}. - \frac{\left[xsin^2\left(\frac{y}{x}\right)-y\right]}{x}$

= k^0, f(x, y)

Therefore, the given differential equation is homogeneous.

$\left[xsin^2\left(\frac{y}{x}\right) - y\right]$ dx + xdy = 0

On rearranging

$\left[xsin^2\left(\frac{y}{x}\right) - y\right]$ dx = $-$ xdy

$\left[xsin^2\left(\frac{y}{x}\right) - y\right] = -\, x\frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{\left[xsin^2\left(\frac{y}{x}\right)-y\right]}{x}$

To solve it we make the substitution.

y = v x

Differentiating above equation with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

On rearranging and substituting the value of dy/dx we get

$v + x\frac{dv}{dx} = -\frac{\left[xsin^2\left(\frac{vx}{x}\right)-vx\right]}{x}$

$v + x\frac{dv}{dx} = -\frac{\left[xsin^2v-vx\right]}{x}$

$v + x\frac{dv}{dx} = xsin^2v - v$

On computing and simplifying we get

$x\frac{dv}{dx} = -[xsin^2v - v] - v$

$x\frac{dv}{dx} = -\, xsin^2v - v$

$x\frac{dv}{dx} = xsin^2v$

$\frac{1}{sin^2v}dv = -\frac{1}{x}dx$

Taking integrals on both sides, we get

$\int\frac{1}{sin^2v}dv = -\int\frac{1}{x}dx$

$\int cosec^2v dv$ = - log x – log c

On integrating we get

-cot v = log x – log C
cot v = log x + log c
Substituting the value of v we get
$cot\frac{y}{x}$ = log (Cx)
y = $\frac{\pi}{4}$ when x = 1
$cot\frac{\pi/4}{1}$ = log (C.1)
$cot\frac{\pi}{4}$ = logc
1= C
e^1 = C
∴ $cot\frac{y}{x}$ = log (ex)
The required solution of the differential equation.

Question 14

$\frac{dy}{dx}$- $\frac{y}{x}$ + cosec$\left(\frac{y}{x}\right)$ = 0; y = 0 when x = 1

Solution:

Given
$\frac{dy}{dx}$- $\frac{y}{x}$ + cosec$\left(\frac{y}{x}\right)$ = 0
On rearranging we get
$\frac{dy}{dx}$- $\frac{y}{x}$ - cosec$\left(\frac{y}{x}\right)$
Let f (x, y) = $\frac{y}{x}$ - cosec$\left(\frac{y}{x}\right)$
Now put x = k x and y = k y
F(kx, ky) = $\frac{ky}{kx}$ - cosec $\left(\frac{ky}{kx}\right)$
= $\frac{y}{x}$ - cosec$\left(\frac{y}{x}\right)$
= k^0. F (x, y)
Therefore, the given differential equation is homogeneous.
$\frac{dy}{dx}$- $\frac{y}{x}$ + cosec$\left(\frac{y}{x}\right)$ = 0
$\frac{dy}{dx}$- $\frac{y}{x}$ + cosec$\left(\frac{y}{x}\right)$
To solve it we make the substitution.
y = v x
Differentiating above equation with respect to x. we get
$\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$
Rearranging and substituting the value of dy/ dx we get
v+ x$\frac{dv}{dx}$ = $\frac{vx}{x}$ – cosec$\left(\frac{vx}{x}\right)$
On simplification
v + x$\frac{dv}{dx}$ = v – cosecv
x$\frac{dv}{dx}$ = $-cosecv$

$\frac{1}{cosecv}$ dv = - $\frac{1}{x}$dx
Taking integrals on both sides, we get
$\int sinvdv$ = - $\int \frac{1}{x} dx$
On integrating we get
$-cos$ v = - log x + c
Substituting the value of v
-cos$\frac{y}{x}$ = - logx + c
y = 0 when x = 1
$-cos\frac{0}{1}$ = - log1 + c
- 1 = c
∴ - $cos\frac{y}{x}$ = - logx – 1
$cos\frac{y}{x}$ = logx + loge$cos\frac{y}{x}$ = log|ex|
The required solution of the differential equation.

Question 15

2xy + y^2 - $2x^2\frac{dy}{dx}$ = 0; y 2 when x = 1

Solution:

Given
2xy + y^2 - $2x^2\frac{dy}{dx}$ = 0
The above equation can be written as
$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$
Let f (x, y) = $\frac{2xy + y^2}{2x^2}$
Now put x = k x and y = k y
F (kx, ky) = $\frac{2kxky + (ky)^2}{2(kx)^2}$
Taking k^2 common
= $\frac{k^2}{k^2}. \frac{2xy + y^2}{2x^2}$
= k^0.f(x, y)
Therefore, the given differential equation is homogeneous.
2xy + y^2 - $2x^2\frac{dy}{dx}$ = 0
On rearranging
$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$
To solve it we make the substitution.
y = v x
On rearranging and substitution, the value of dy/dx we get
v + x$\frac{dv}{dx} = \frac{2vx^2}{2x^2}$
On computing and simplification, we get

v +x $\frac{dv}{dx} = \frac{2v+v^2}{2}$
v + x $\frac{dv}{dx}$ = v + $\frac{1}{2}v^2$
x $\frac{dv}{dx} = \frac{1}{2}v^2$
$2\frac{1}{2}v^2 dv = \frac{1}{x}$ dx
Taking integration on both sides, we get
$\int 2\,\frac{1}{2}v^2 dv = \int \frac{1}{x}dx$
On integration we get
$-\frac{2}{v}$ = logx + C
Substituting the value of we get

$$-\frac{2}{y/x} = logx + C$$
$$-\frac{2x}{y} = logx + C$$

y =2 we=hen x = 1

$$-\frac{2.1}{2} = log\,1 + C$$
$$-1 = c$$

∴ $-\frac{2x}{y}$ = log x - 1
$\frac{2x}{y}$ = 1 – logx
y = $\frac{2x}{1-log|X|}$, x ≠e, x≠ 0
The required solution of the differential equation

Question 16

A homogeneous differential equation of the from $\frac{dy}{dx}$ = h$\left(\frac{x}{y}\right)$ can be solved by making the substitution.
(A) y = v x **(B) v = y x**
(C) x = v y **(D) x = v**

Solution:

(C) x = v y
Explanation:
Since, $\frac{dy}{dx}$ is given equal to h $\left(\frac{x}{y}\right)$.
Therefore,
h $\left(\frac{x}{y}\right)$ is a function of $\frac{x}{y}$.
Therefore, we shall substitute, x = v y is the answer

Question 17

Which of the following is a homogeneous differential equation?

a. (4x + 6y + 5) dy – (3y + 2x + 4) dx = 0
B. (x, y) dx – (x^3 + y^3) dy = 0
C. (x^3 + $2y^2$) dx + 2xy dy = 0
D. y^2dx + (x^2 – x y y^2) dy = 0

Solution:

D. y^2dx + (x^2 – x y y^2) dy = 0
Explanation:
We have
y^2 dx + (x^2 – xy – y^2) dy = 0
On rearranging
$\frac{dy}{dx} = -\frac{x^2 - xy - y^2}{y^2}$
Let f(x,y) = $-\frac{x^2 - xy - y^2}{y^2}$
Now put x = k x and y = k y
f (kx,ky) = $-\frac{(kx)^2 - kxky - (ky)^2}{(ky)^2}$
= $\frac{k^2}{k^2} . -\frac{x^2 - xy - y^2}{y^2}$
= k^0. f(x, y)
Therefore, the given differential equation is homogenous.

Exercise 9.6

For each the differential equations given in question, find the general solution

Question 1

$\frac{dy}{dx}$ **+ 2y = sinx**

Solution:

Given
$\frac{dy}{dx}$ + 2y = sinx
Given equation in the form of $\frac{dy}{dx}$ + py = Q where, p = 2 and Q = sin x
Now, I.F. = $e^{\int pdx} = e^{\int 2dx}$ = e^{2x}
Thus, the solution of the given differential equation is given by the relation
y (I.F.) = $\int (QxI.F)$dx + C
⇒ $ye^{2x} = \int sinx. e^{2x}\, dx + c$ ... 1
Let I = $\int sinx. e^{2x}\, dx$
Integrating using chain rule we get
⇒ I = sinx $\int e^{2x} dx - \int \left(\frac{d}{dx}(sinx).\ e^{\int 2dx}\right)$ dx

= sinx. $\frac{e^{2x}}{2}$ - $\int \left(cosx.\frac{e^{2x}}{2}\right) dx$

On integrals and computing we get

= $\frac{e^{2x}sinx}{2}$ - $\frac{1}{2}\left[cosx \int e^{2x} - \int \frac{d}{dx}(cosx).\int e^{2x}dx) \, dx\right]$

= $\frac{e^{2x}sinx}{2}$ - $\frac{1}{2}\left[cosx\frac{e^{2x}}{2} - - \int \left[(-sinx).\frac{e^{2x}}{2}\right] dx\right]$

=$\frac{e^{2x}sinx}{2}$ - $\frac{e^{2x}}{2}$ - $\frac{1}{4}\int (sin.e^{2x})\, dx$

Above equation can be written as

= $\frac{e^{2x}}{4}$(2sinx – cosx) - $\frac{1}{4}$1

⇒$\frac{5}{4}$1 = $\frac{e^{2x}}{4}$ (2sinx – cosx)

⇒ I = $\frac{e^{2x}}{5}$ (2sinx –cosx)

Now, putting the value of I in 1, we get

⇒ ye^{2x} = $\frac{e^{2x}}{5}$ (2sinx – cosx) + x

⇒ y$\frac{1}{5}$(2sinx – cosx) + ce^{-2x}

Therefore, the required general solution of the given differential equation is

y = $\frac{1}{5}$(2sinx – cosx) + ce^{-2x}

Question 2

$\frac{dy}{dx}$ **+ 3 y =** e^{-2x}

Solution:

Given

$\frac{dy}{dx}$ + 3 y = e^{-2x}

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p 3 and Q = e^{-2x}

Now, l. F. = $e^{\int pdx} = e^{\int 3dx} = e^{3x}$

Thus, the solution of the given differential equation is given by the relation

y (I.F) = $\int (QxI.F)dx + c$

$$\Rightarrow ye^{3x} = \int (e^{-2x}xe^{2x})\, dx + C$$

$$\Rightarrow ye^{3x} = \int e^{x}dx + c$$

On integrating we get

⇒ ye^{3x} = $e^{x} + c$

⇒ y = e^{-2x}+ Ce^{-3x}

Therefore, the required general solution of the given differential equation is

= e^{-2x} + ce^{-3x}

Question 3

$\frac{dy}{dx} + \frac{y}{x} = x^2$

Solution:

Given

$\frac{dy}{dx} + \frac{y}{x} = x^2$

This is equation in the form of $\frac{dy}{dx}$ + py = q

Where, p = $\frac{1}{x}$ and Q = x^2

Now, I. f. = $e^{\int pdx} = e^{\int \frac{1}{x}dx} = e^{logx} = x$

Thus, the solution of the given differential equation is given by the relation

y (I.F) = $\int (QxI.F)\, dx + C$

⇒ y(x) = $\int (x^2.x)\, dx + c$

⇒ xy = $\int (x^3) dx + C$

On integrating we get

⇒ xy = $\frac{x^4}{4}$ + C

Therefore, the required general solution of the given differential equation is

xy = $\frac{x^4}{4}$ + C

Question 4

$\frac{dy}{dx}$ + (sec x) y = tan x $\left(0 \le x < \frac{\pi}{2}\right)$

Solution:

Given

$\frac{dy}{dx}$ + (sec x) y = tanx

Given equation is in the form of $\frac{dy}{dx}$ + py = Q

Where, p = sec x and Q = tan x)

Now, I.F. = $e^{\int pdx} = e^{\int secxdx} = e^{log(secx + tanx)}$ = secx + tanx

Thus, the Solution of the given differential equation is given by the relation

y (I, F) = $\int (QxI.F.) dx + C$

⇒ y(secx + tanx) = $\int tanx(secx + tanx)\, dx + C$

⇒ y(secx + tanx) = $\int secxtanxdx + \int tan^2 xdx + C$

⇒ y (secx + tanx) = sec x + tan x – x + C

Therefore, the required general Solution of the given differential equation is

y (sec x +tan x) = sec x + tan x – x + C.

Question 5

$cos^2 x\frac{dy}{dx}$ + y = tan x $\left(0 \le x < \frac{\pi}{2}\right)$

Solution:

Given

$\cos^2\frac{dy}{dx} + y = \tan x$

The above equation can be written as

$\Rightarrow \frac{dy}{dx} + \sec^2 x . y = \sec^2 x \tan x$

Given equation is in the form of $\frac{dy}{dx} + py = Q$

Where, $p = \sec^2 x$ and $Q = \sec^2 x \tan X$

Now, I.F. $= e^{\int pdx} = e^{\int \sec^2 x dx} = e^{\tan x}$

Thus, the Solution of the given differential equation is given by the relation

$Y(i.f.) = \int (Q x I.F.)\, dx + C$

$\Rightarrow y. e^{\tan x} = \int e^{\tan x}\, dx + C$ 1

now, let t = tanx

$\Rightarrow \frac{d}{dx}(\tan x) = \frac{dt}{dx}$

$\Rightarrow \sec^2 x = \frac{dt}{dx}$

$\Rightarrow \sec^2 x dx = \frac{dt}{dx}$

thus, the equation 1 becomes,

$\Rightarrow y. e^{\tan x} = \int (e^t . t) dt + c$

$\Rightarrow y. e^{\tan x} = \int (t. e^t)\, dt + c$

Using chain rule for integration we get

$\Rightarrow y. e^{\tan x} = t. \int e^t dt - \int \left(\frac{d}{dt}(t). \int e^t dt\right) dt + c$

$\Rightarrow y. e^{\tan x} = t. e^t - \int e^t dt + c$

On integrating we get

$\Rightarrow te^{\tan x} = (t - 1)\, e^t + c$

$\Rightarrow te^{\tan x} = (\tan x - 1)\, e^{-\tan x} + c$

$\Rightarrow y = (\tan x - 1) + c\, e^{-\tan x}$

Therefore, the required general solution of the given differential equation is

$y = (\tan x - 1) + c\, e^{-\tan x}$

Question 6

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Solution:

Given

$x\frac{dy}{dx} + 2y = x^2 \log x$

the above equation can be written as

$\Rightarrow \frac{dy}{dx} + \frac{2}{x} y = x \log x$

This is equation in the form of $\frac{dy}{dx} + py = Q$

Where, $p = \frac{2}{x}$ and $Q = x \log x$

Now, I.F. = $e^{\int pdx} = e^{\int \frac{2}{x}dx} = e^{2(logx)} = e^{log\,x^2} = x^2$
Thus, the solution of the given differential equation is given by the relation
y (I.F.) = $\int (QxI.F.)$dx + c
$\Rightarrow$ y. $x^2 = \int (xlogx.\ x^2)$dx + c
The above equation becomes
x^2y = $\int (x^3 logx)\,dx$ + c
On integrating using chain rule we get
$\Rightarrow x^2 y$ = logx $\int x^3 dx - \int \left[\frac{d}{dx}\ (logx0.\ \int x^3 dx\right] dx + c$
$\Rightarrow x^2 y$ = logx. $\frac{x^4}{4}$ - $\int \left(\frac{1}{x}.\ \frac{x^4}{4}\right) dx + c$
$\Rightarrow x^2 y = \frac{x^4 logx}{4}$ - $\frac{1}{4}\int x^3\ dx + c$
Integrating and simplifying we get
$\Rightarrow x^2 y = \frac{x^4 logx}{4}$ - $\frac{1}{4}.\frac{x^4}{4}$ + c
$\Rightarrow x^2 y = \frac{1}{16}x^4\ (4x - 1\ )$ + c
$\Rightarrow y = \frac{1}{16}x^2$ (4logx – 1) + Cx^{-2}
Therefore, the required general solution of the given differential equation
y = $\frac{1}{16}x^2\ (4logx - 1)\ + cx^{-2}$

Question 7

x log x$\frac{dy}{dx}$ + y = $\frac{2}{x}logx$

Solution:

Given
x log x$\frac{dy}{dx}$ + y = $\frac{2}{x}logx$
the above equation can be written as

$$\Rightarrow \frac{dy}{dx} + \frac{y}{xlogx} = \frac{2}{x^2}$$

The given equation is in the form of $\frac{dy}{dx}$ + py = Q
Where, p = $\frac{1}{xlogx}$ and Q = $\frac{2}{x^2}$
Now, I.F. = $e^{\int pdx} = e^{\int \frac{1}{xlogx}dx} = e^{log\,(logx\,) = log\ x}$
Thus, the Solution of the given differential equation is given by the relation
y (I.F.) = $\int (QxI.F.)\ dx$ + C
$\Rightarrow$ y.logx = $\int \left[\frac{2}{x^2}, logx\right]$dx + C 1
Now, $\int \left[\frac{2}{x^2}, logx\right]$ dx = 2 $\int \left(logx, \frac{1}{x^2}\right) dx$
On integrating using chain rule we get
= 2 $\left[logx. \int \frac{1}{x^2} dx - \int \left\{\frac{d}{dx}(logx). \int \frac{1}{x^2} dx\right\} dx\right]$
= 2 $\left[logx \left(-\frac{1}{x}\right) - \int \int (\frac{1}{x}. (-\frac{1}{x}))\ dx\right]$

= $2\left[-\frac{log}{x}+\int\frac{1}{x^2}dx\right]$
= $2\left[-\frac{log}{x}-\frac{1}{x}\right]$
= $-\frac{2}{x}$ (1 + logx)
Now, substituting the value in 1, we get,
⇒ y, logx = $-\frac{2}{x}$ (1 + logx) + C
Therefore, the required general solution of the given differential equation is
y, logx = $-\frac{2}{x}$ (1 + logx) + C

Question 8

(1 + x^2) dy +2xy dx = cot x dx (x ≠ 0)

Solution:

Given
(1 + x^2) dy +2xy dx = cotxdx
The above equation can be written as
⇒ $\frac{dy}{dx}+\frac{2xy}{(1+x^2)}=\frac{cotx}{1+x^2}$
The given equation is in form of $\frac{dy}{dx}$ + py = Q
Where, p = $\frac{2x}{(1+x^2)}$ and Q = $\frac{cotx}{(1+x^2)}$
Now, I. f. = $e^{\int pdx}$ =$e^{\int\frac{2x}{(1+x^2)}dx}$ = $e^{log\,(1+x^2)}$ = $1+x^2$
Thus, the solution of the given differential equation is given by the relation
y (I. F) = $\int QxI\,.F\,)\,dx+c$
⇒ y. $(1+x^2\;=)\int\left[\frac{cotx}{1+x^2}\,(1+\,x^2)\right]$ dx + c
⇒ y. $(1+x^2)=\int cotxdx+c$
On integrating we get
⇒ y. (1 +x^2) = log|sinx| + c
Therefore, the required general solution of the given differential equation is
y(1 + x^2) = log|sinx| + c

Question 9

x$\frac{dy}{dx}$ + y – xy cot x = 0 (x ≠)

Solution:

Given
x$\frac{dy}{dx}$ + y – xy cot x = 0
The above equation can be written as
⇒ x$\frac{dy}{dx}+y(1+xcotx)=x$

⇒ $x\frac{dy}{dx} + \left(\frac{1}{x} + cotx\right)y = 1$

The given equation is in the form of $\frac{dy}{dx}$ + px = Q

Wherefore, p = $\frac{1}{x}$ + cotx and o = 1

Now, I.F = $e^{\int pdx} = e^{\int\left(\frac{1}{x}+cotx\right)dy} = e^{logx + log(sinx)} = e^{log((xsinx)}$ = xsinx

Thus, the solution of the given differential equation is given by the relation

x (I.F.) = $\int(QxI.F.)$dy + c

⇒ y(sinx) = $\int[1 + xsinx]\,dx + c$

⇒ y(xsinx0 = $\int[xsinx]dx + c$

By splitting the integrals, we get

⇒ y(xsinx) = x $\int sinxdx - \int\left[\frac{d}{dx}(x).\int sinxdx\right] + c$

⇒ y(sinx) = x(-cosx) - $\int 1\,.(-\,cosx)\,dx$ + c

on integrating we get

⇒ $y\,(x\,sin\,x) = -x\,cos\,x + sin\,x + c$

⇒ y = $\frac{-xcosx}{xsinx} + \frac{sinx}{xsinx} + \frac{c}{xsinx}$

⇒ y = -cotx + $\frac{1}{x}$ + $\frac{c}{xsinx}$

therefore, the required general solution of the given differential equation is

y = -cotx + $\frac{1}{x}$ + $\frac{c}{xsinx}$

Question 10

(x + y) $\frac{dy}{dx}$ = 1

Solution:

Given

(x + y)$\frac{dy}{dx}$ = 1

The above equation can be written as

⇒ $\frac{dy}{dx} = \frac{1}{x+y}$

⇒ $\frac{dx}{dy}$ = x + y

⇒ $\frac{dx}{dy} - x = y$

The given equation is in the form of $\frac{dy}{dx}$ + px = Q

Wherefore, p = -1 and Q = y

Now, l. F. = $e^{\int pdy} = e^{\int -dy} = e^{-y}$

Thus, the solution of the given differential equation is given by the relation

x (L.F.) = $\int(QxI.F.)dy + c$

⇒ $xe^{-y} = \int[y\,.e^{-y}]dy + c$

xe^{-y} = y $\int e^{-dy} - \int\left[\frac{d}{dy}(y)\int e^{-y}dy\right]dy\ + c$

$xe^{-y} = y(e^{-y}) - \int(-e^{-y})dy + c$

On integrating and computing we get
⇒ xe^{-y} = - ye^{-y} + $\int e^{-y}dt + C$
⇒ $xe^{-y} = -ye^{-y} - e^{-y} + C$
⇒ $x = - y - 1 + Ce^{y}$
⇒ $x + y + 1 = Ce^{y}$
Therefore, the required general of the given differential equation is
$x + y + 1 = Ce^{y}$

Question 11

$y\,dx + (x - x^2)\,dy = 0$

Solution:

Given
$ydx + (x - y^2)\,dy = 0$
The above equation can be written as
⇒ $ydx = (y^2 - x)\,dy$
⇒ $\frac{dx}{dy} = \frac{(y2 - x)}{y} = y - \frac{x}{y}$
on simplifying we get
⇒ $\frac{dx}{dy} + \frac{x}{y} = y$
The above equation is in the form of $\frac{dy}{dx} + px = Q$
Where, $p = \frac{1}{y}$ and $Q = y$
Now, I.F. = $e^{\int pdf} = e^{\int \frac{dy}{y}} = e^{logy} = y$
Thus, the solution of the given differential equation is given by the relation
x (I.F.) = $\int (QxI.F.)dy + C$
⇒ x.y = $\int [y.y]dy + C$
⇒ x.y = $\int y^2 dy + C$
On integrating we get
⇒ x.y = $\frac{y^3}{3} + C$
⇒ xy = $\frac{y^3}{3} + \frac{c}{y}$
Therefore, the required general solution if the given differential equation is
xy = $\frac{y^3}{3} + \frac{c}{y}$

Question 12

$$(x + 3y2)\frac{dy}{dx} = y\,(y > 0)$$

Solution:

Given

$$(x + 3y2)\frac{dy}{dx} = y$$

On rearranging we get

$\frac{dy}{dx} = \frac{3}{x+3y^2}$

$\frac{dy}{dx} = \frac{x+3y^2}{y} = \frac{x}{y}$ +3y

On simplification

$\frac{dy}{dx} - \frac{x}{y}$ = 3y

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = -1/y and Q = 3y

Now, I.F. = $e^{\int pdf} = e^{-\int\frac{dy}{y}} = e^{-logy} = e^{log(\frac{1}{y})} = \frac{1}{y}$

Thus, the solution of the given differential equation is given by the relation

X(I.F.) = $\int(QxI.F.)dy + C$

X.$\frac{1}{y}$ = $\int\left[3y.\frac{1}{y}\right]dy + C$

On integrating we get

$\frac{x}{y}$ = 3y + C

X = $3y^2$ + Cy

Therefore, the required general solution of the given differential equation is x = $3y^2$ cy.

For each of the differential equations given in exercises 13 to 15, find a particular solution satisfying the given condition:

Question 13

$\frac{dy}{dx}$ + 2ytanx = sin x, y = 0 when x = $\frac{\pi}{3}$

Solution:

Given

$\frac{dy}{dx}$ + 2ytanx = sinx

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = 2 tanx and Q =sin x

Now, if. = $e^{\int pdx} = e^{\int 2tanxdx} = e^{2log(secx)} = e^{log(sec^2x)} = sec^2x$

Thus, the solution of the given differential equation is given by the relation:

y (I.F.) = $\int(QxI.F.)\,dx + c$

⇒ y. $(sec^2x) = \int[sinx.sec^2x]dx + c$

⇒ y.$(sec^2x) = \int[secx.tanx]dx + c$

On integrating we get

⇒ y. $(sec^2x) = secx$ + C 1

Now, it is given that y = 0 at x $\frac{\pi}{3}$

$0 \times sec^2\frac{\pi}{3}$ = sec$\frac{\pi}{3} + c$

⇒ 0 = 2 + c

⇒ C = -2

Now, substitution the value of c = -2 in 1, we get
⇒ y (sec^2x) = secx – 2
⇒ y = cos x – $2cos^2x$
Therefore, the required general solution of the given differential equation is
y = cos x - $2cos^2x$

Question 14

$(1+x^2)\frac{dy}{dx}$ + 2xy = $\frac{1}{1+x^2}$; y = 0 when x = 1

Solution:

Given
$(1+x^2)\frac{dy}{dx}$ + 2xy = $\frac{1}{1+x^2}$;
⇒ $\frac{dy}{dx}+\frac{2xy}{(1+x^2)}=\frac{1}{(1+x^2)}$
The given equation is in the form of $\frac{dy}{dx}$ + p y = Q
Where, p = $\frac{2x}{(1+x^2)}$and Q = $\frac{1}{(1+x^2)^2}$
Now, I. F. $e^{\int pdx} = e^{\int \frac{2x}{91+x^2 dx}} = e^{log(1+x^2)}$ = 1 + x^2
Thus, the solution of the given differential equation is given by the relation
y(I.F.) = $\int(QxI.F.0dx$ + C
⇒ y. $(1+x^2)$ = $\int\left[\frac{1}{(1+x^2)^2}(1+\ x^2)\right]dx+c$
⇒ y. $(1+x^2)$ = $\int\frac{1}{(1+x^2)}dx+c$
On integrating we get
⇒ y. $(1+x^2)$ = $\tan^{-1}$ x + c ...1
Now, it is given that y = 0 at x = 1
0 = $\tan^{-1}$ 1 + c
C = -$\frac{\pi}{4}$
Now, substitution the value of c = $-\frac{\pi}{4}$(1), we get
y. $(1+x^2)$ = $\tan^{-1x}-\frac{\pi}{4}$

Question 15

$\frac{dy}{dx}$ – 3y cot x = sin 2x; y = 2 when x = $\frac{\pi}{2}$

Solution:

Given
$\frac{dy}{dx}$ - 3 y cotx = sin 2x
This is equation in the form of $\frac{dy}{dx}$ + p y = Q
Where, p = -3cot x and Q = sin 2x

Now, I.F. = $e^{\int pdx}$ = $e^{-3\int cotxdx}$ = $e^{-3\log|sinx|}$ = $e^{\log\left|\frac{1}{\sin^3 x}\right|}$ = $\frac{1}{\sin^3 x}$

Thus, the solution of the given differential equation is given by the relation

Y (I. F.) = $\int (QxI.F)$.dx + c

Y. $\frac{1}{\sin^3 x}$ = $\int \left[sin2x.\frac{1}{\sin^3 x}\right] dx + c$

Y $cosec^3 x$ = 2 $\int (cotxcosecx)dx + c$

On integrating we get

Y $cosec^3$x = 2cosecx + c

Y = $-\frac{2}{cosec^2 x} + \frac{3}{cosec^3 x}$

Y = $-sin^2 x$ + $Csin^3 x$.......1

Now, it is given that y = 2 when x = $\frac{\pi}{2}$

Thus, we get

= -2 + c

C = 4

Now, substituting the value of C = 4 in 1, we get,

y = $-2sin^2 x$ + $4sin^3 x$

y = $4sin^3 x$ - $2sin^2 x$

Therefore, the required general solution of the given differential equation is

y = $4sin^3 x$ - $2sin^2 x$

Question 16

Find the equation of a curve passing though the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinated=s of the point.

Solution:

Let F (x, y) be the curve passing though origin and let (x, y) be a point on the curve

We know the slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$

According to the given conditions, we get

$\frac{dy}{dx}$ = x + y

On rearranging we get

$\frac{dy}{dx}$ – y = x

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = -1 and Q = x

Now, I. F. = $e^{\int pdx}$ = $e^{\int(-1)dx}$ = e^{-x}

Tus, the solution of the given differential equation is given by the relation:

Y(I.F.) = $\int (QxI.F.)dx + c$

ye^{-x} = $\int xe^{-x}dx + c$ 1

Now, $\int xe^{-x}dx = x\int e^{-x}dx - \int\left[\frac{d}{dx}(x).\int e^{-x}dx\right]dx$

On integrating

= x(e^{-x}) - $\int(-e^{-x})$dx

= x(e^{-x}) + $-e^{-x}$)

$= -e^{x}$(x + 1)
Thus, from equation 1, we get
ye^{-x} = $-e^{-x}$(x + 1) + c
$y = -(x+1) + Cx^{x}$
$x + ym + 1 = Ce^{x}$ 2
Now, it is given that curve passes through origin.
Thus, equation 2 becomes
1 = c
C = 1
Substituting C = 1 in equation 2, we get
X + y – 1 = e^{x}
Therefore, the required general solution of the given differential equation is
X + y – 1 = e^{x}

Question 17

find the equation of a curve passing though the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Solution.

Let F (x, y) be the curve and let (x, y) be a point on the curve
We know the slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$
According to the given conditions, we get,
$\frac{dy}{dx}$ + 5 = x + y
On rearranging we get we get
⇒ $\frac{dy}{dx}$ – y = x – 5
This is equation in the form of $\frac{dy}{dx}$ + py = Q
Wherefore, p = -1 and Q = x – 5
Now, I.F.= $e^{\int pdx} = e^{(-1)dx} = e^{-x}$
Thus, the solution of the given differential equation is given by the relation:
Y (I. F.) = $\int (QxI.F.)\, dx + c$
⇒ $ye^{-x} = \int (x-5)e^{-x}dx + c$ 1
Now, $\int (x-5)\, e^{-x}$dx =(x – 5) $\int e^{-x}dx$ - $\int \left[\frac{d}{dx}(x-5). \int e^{-x}dx\right] dx$
= (x – 5) (e^{-x}) - $\int(-e^{-x})$dx
On integrating we get
= (x – 5) (e^{-x}) + (- e^{-x})
= (4 – x)e^{-x}
Thus, from equation 1, we get,
⇒ ye^{-x} = (4 – x) e^{-x} + C
⇒ y = 4 – x + Ce^{x}
⇒ x + y – 4 = Ce^{x}
Thus, equation (2) becomes:
0 + 2 – 4 = C e^{0}

⇒ - 2 = C
⇒ C = -2
Substituting C = -2 in equation (2), we get,
$x + y - 4 = -2e^x$
$\Rightarrow y = 4 - x\ 2e^x$
Therefore, the required general solution of the given differential equation is
$Y = 4 - x - 2e^x$

Question 18

the Integrating Factor of the differential equator $x\frac{dy}{dx} - y\ 2x^2$ is

(A) e^{-x}
(B) e^{-y}
(C) $1/x$
(D) x

Solution:

C. 1/x
Explanation:
Given
$x\frac{dy}{dx} - y = 2x^2$
On simplification we get
$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$
This is equation in the form of $\frac{dy}{dx} + py = Q$
Where, p = -1/x and Q = 2x
Now, I.F. $= e^{\int pdf} = e^{\int \frac{1}{x}dx} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{2}$
Hence the answer is 1/x

Question 19

The Integrating factor of the differential equation
$(1 - y^2)\frac{dx}{dy} + yx = ay\ (-1 < y < 1)$ is

(A) $\frac{1}{y^2-1}$
(B) $\frac{1}{\sqrt{y^2-1}}$
(C) $\frac{1}{1-y^2}$
(D) $\frac{1}{\sqrt{1-y^2}}$

Solution:

(D) $\frac{1}{\sqrt{1-y^2}}$
Explanation:
Given
$(1 - y^2)\frac{dy}{dx} + yx = ay$
On rearranging we get
$\Rightarrow \frac{dy}{dx} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = $\frac{y}{1-y^2}$ and Q = $\frac{a}{1-y^2}$

Now, I.F. = $\frac{y}{1-y^2}$ and Q = $\frac{a}{1-y^2}$

$= \frac{1}{\sqrt{(1-y^2)}} e^{\int pdf} = e^{\int \frac{y}{1-y^2} dy} = e^{\frac{1}{2}\log(1-y^2)} = e^{\log\left[\frac{1}{\sqrt{(1-y^2)}}\right]}$

<u>MISCELLANEOUS EXERCISE</u>

<u>Question 1</u>

For each of the differential equations given below, indicate its order and degree (if defined) .

(i) $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$

(ii) $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$

(iii) $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$

Solution:

(i) Given

$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$

On rearranging we get

$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x = 0$

We can see that the highest order derivative present in the differential is $\frac{d^2y}{dx^2}$

Thus, its order is two. it is polynomial equation in $\frac{d^2y}{dx^2}$

The highest power raised to $\frac{d^2y}{dx^2}$ is 1.

Therefore, its degree is one.

(ii) $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$

The above equation can be written as

$\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x = 0$

We can see that the highest order derivative present in the differential is $\frac{dy}{dx}$ thus, its order is one. It is polynomial equation in $\frac{dy}{dx}$

The highest power raised to $\frac{dy}{dx}$ is 3.

Therefore, its degree is three.

(iii) Given

$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$

The above equation can be written as

$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x = 0$

We can see that the highest order derivative present in the differential is $\frac{d^4y}{dx^4}$

Thus, its order is four. the given differential equation is not a polynomial equation.
Therefore, its degree is not defined.

Question 2

for each of the exercise given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

(i) $xy = a e^x + be^{-x} + x^2$: $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$

(ii) $y = e^x (a \cos x + b \sin x)$: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(iii) $y = x \sin 3x$: $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$

(iv) $x^2 = 2y^2 \log y$: $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Solution:

(i) given $x y = a e^x + b e^{-x} + x^2$

Now, differential both sides with respect to x, we get,

$\frac{dy}{dx} = a\frac{d}{dx}(e^x) + b\frac{d}{dx}(e^{-x}) + \frac{d}{dx}(x^2)$

$\Rightarrow \frac{dy}{dx} = ae^x - be^{-x} + 2x$

Now, again differentiating above equation both sides with respect to x, we get,

$\frac{d}{dx}(y') = \frac{d}{dx}(ae^x - 2x)$

$\Rightarrow \frac{d^2y}{dx^2} = ae^x + be^{-x} + 2$

Now, substituting the values of $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ in the given differential equation,

We get,

We have

LHS $= x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2$

$= x(ae^x + be^{-x} + 2) + 2(ae^x - be^{-x} + 2) - x(ae^x + be^{-x} + x^2) + x^2 - 2$

$= (ae^x + be^{-x} + 2) + 2(ae^x - be^{-x} + 2) - x(ae^x + be^{-x} + x^2) + x^2 - 2$

$= 2ae^x - 2be^{-x} + x^2 + 6x - 2$

$\neq 0$

$\Rightarrow$LHS $\neq$ RHS

Therefore, the given function is not the solution of the corresponding differential equation.

(ii) Given $y = e^x (a \cos x + b \sin x) = ae^x \cos x + b e^x \sin x$

Now, differential both sides with respect to x, we get,

$\frac{dy}{dx} = a\frac{d}{dx}(e^x\cos x) + b\frac{d}{dx}(e^x\sin x)$

$\Rightarrow \frac{dy}{dx} = a(e^x \cos x - e^x \sin x) + b.(e^x\sin x + e^x\cos x)$

On rearranging we get

$\Rightarrow \frac{dy}{dx}$ = (a + b) e^x cosx + (b – a) e^xsinx

Now, again differentiating both sides with respect to x, we get,

$\frac{d^2y}{dx^2}$ = (a + b). $\frac{d}{dx}$ (e^xcosx) + (b – a) $\frac{d}{dx}$ (e^xsinx)

Taking common

= (a + b). [e^x cosx – e^x sinx] + (b – a) [e^x sinx + e^x sin x + ecosx]

Simplifying we get

= e^x [acosx – asinx + bcosx – bsinx + bsinx + bcos x – asinx – acos x]

= [2e^x (bcosx – asinx)]

Now, substituting the values of $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ In the given differential equation,

We get,

LHS = $\frac{d^2y}{dx^2}$ + 2 $\frac{dy}{dx}$ + 2y

=2e^x (b cows x – a sin x) – 2e^x [(a + b) cos x + (b – a) sin x] + 2e^x (a cos x + b sin x)

= e ^{x} [(2b – 2a- 2b + 2a) cos x] + e^x [9 2a – 22b + 2a + 2bsinx]

= 0 RJS

Therefore. The given function is the solution of the corresponding differential equation.

(iii) It is given that y = xsin 3x

Now, differential both sides with respect to x, we get,

$\frac{dy}{dx}$ = $\frac{d}{dx}$ (xsin3x) = sin3x + x. cos3x.3

$\frac{dy}{dx}$ = sin3x + 3xcos3x

Now, again differential both sides with respect to x,we get,

$\frac{d^2y}{dx^2}$ = $\frac{d}{dx}$ (sin3x) + 3 $\frac{d}{dx}$ (xcos3x)

$\Rightarrow \frac{d^2y}{dx^2}$ = 3xos3x + 3 [cos3x + x (-sin3x). 3]

On simplifying we get

$\Rightarrow \frac{d^2y}{dx^2}$ = 6cos3x – 9xsin3x

Now, substituting the value of $\frac{d^2y}{dx^2}$ in the LHS of the given differential equation.

We get,

$\frac{d^2y}{dx^2}$ + 9y – 6cos3x

= (6. Cos3x – 9xsin3x) + 9xsin3x – 6cos3x

= 0 = RHS

Therefore, the given function is the solution of the corresponding differential equation.

(iv) given $x^2 = 2y^2$ log y

Now, differentiating both sides with respect to x, we get

2x = 2. $\frac{d}{dx}(y^2 logy)$

Using product rule, we get

x$\left[2y.logy.\frac{dy}{dx} + y^2\frac{1}{y}.\frac{dy}{dx}\right]$

x = $\frac{dy}{dx}$ (2ylogy + y)

$\Rightarrow \frac{dy}{dx} = \frac{x}{y(1+2logy)}$

Now, substituting the value of $\frac{dy}{dx}$ in the LHS of the given differential equation,

We get

$(x^2 + y^2)\frac{dy}{dx}$ – xy = $(2y^2 logy + y^2).\frac{x}{y(1+2logy)} - xy$
= y^2 (1 + 2logy). $\frac{x}{y(1+2logy)}$ – xy
= xy – xy
= 0
Therefore, the given function is the corresponding differential equation.

Question 3

From the differential equation representing the family of curves given by (x – a)^2 + $2y^2 = ma^2$ where a is an arbitrary constant.

Solution:

Given (x - a)^2 + $2y^2$ = a^2
$\Rightarrow x^2 + a^2 - 2ax + 2y^2 = a^2$
$\Rightarrow$ $2y^2$ = 2ax - x^2 ………………… 1
Now, differential both sides with respect to x, we get
2y$\frac{dy}{dx}$ = $\frac{2a-2x}{2}$
On simplifying we get
$\Rightarrow \frac{dy}{dx} = \frac{a-x}{2y}$
$\Rightarrow \frac{dy}{dx} = \frac{2ax-2x^2}{4xy}$ …………………… 2
so, equation (1), we get
2ax = $2y^2 + x^2$
On substituting this value in equation 2, we get
$\frac{dy}{dx} = \frac{2y^2+x^2-2x^2}{4xy}$
$\Rightarrow \frac{dy}{dx} = \frac{2y^2-x^2}{4xy}$
Therefore, the differential equation of the family of curves is given as
$\frac{dy}{dx} = \frac{2y^2-x^2}{4xyj}$

Question 4

Prove that that $x^2 - y^2$ = c $(x^2 + y^2)^2$ is the general solution od differential equation $(x^3 - 3xy^2)$ dx = $(y^{3-3xy^2y})dx, wherecisparameter.$

solution:

Given $(x^3 - 3xy^2)$ dx = $(y^3 - 3x^2y)$dy
On rearranging we get
$\Rightarrow \frac{dy}{dx} = \frac{x^2-3xy^2}{y^3-3x^2y}$ …………..1
Now, let us take y = vx further simplification
On differentiating we get

$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, substituting the value of y and dv / dx in equation 1 we get

$v + x\frac{dv}{dx} = \frac{x^3 - 3x\,(vx)^2}{(vx)^3 - 3x^2\,(vx)}$

Taking common and simplifying we get

$\Rightarrow v + x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v}$

$+x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v$

Taking LCM and simplifying we get

$X\frac{dv}{dx} = \frac{1-3v^2-v(v^3-3v)}{v^2-3v}$

$X\frac{dv}{dx} = \frac{1-3v^2}{v^2-3v}$

$\left(\frac{v^3-3v}{1-3v^4}\right)dv = \frac{dx}{x}$

On integrating both sides we get,

$\int\left(\frac{v^3-3v}{1-3v^4}\right)dv = \log x + \log C'$ ……………..2

Splitting the denominator.

$\int\left(\frac{v^3-3v}{1-3v^4}\right)dv = \int\frac{v^3}{1-v^4}dv - 3\int\frac{vdv}{1-v^4}$

$\int\left(\frac{v^3-3v}{1-3v^4}\right)dv = I_1 - 3I_2$, where $I_1 = \int\frac{v^3}{1-v^4}dv$ and $I_2 = \int\frac{vdv}{1-v^4}$ ………….3

Let $1 - v^4 = t$

On differentiating we get

$\frac{d}{dv}(1 - v^4) = \frac{dt}{dy}$

$-4v^3 = \frac{dt}{dv}$

$V^2\,dv = -\frac{dt}{4}$

Now, $I_2 = \int\frac{vdv}{1-v^4} = \int\frac{vdv}{1-(v^2)2}$

Let $v^2 = p$

Differentiating above equation with respect to v

$\frac{d}{dv}(v^2) = \frac{dp}{dv}$

$2v = \frac{dp}{dv}$

$Vdv = \frac{dp}{2}$

Using these things, we get

$\therefore I_2 = \frac{1}{2}\int\frac{dp}{1-p^2} = \frac{1}{2\,x\,2}\log\left|\frac{1+p}{1-p}\right| = \frac{1}{4}\left|\frac{1+v^2}{1-v}\right|$

Now, substituting the value of $|_2$ and $|_2$ in equation (3), we get,

$\int\left(\frac{v^3-3y}{1-v^4}\right)dv = -\frac{1}{4}\log(1 - v^4) - \frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right|$

Thus, equation (2), becomes,

$\Rightarrow -\frac{1}{4}\log(1 - v^4) - \frac{3}{4}log\left|\frac{1+v^2}{1-v^2}\right| = \log C'x$

$\Rightarrow \frac{(1+v^2)^4}{(1-v^2)^2} = (C'\,x)^{-4}$

Computing and simplifying we get

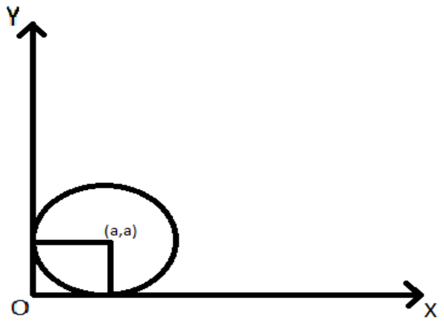
$\Rightarrow \frac{(1+\frac{y^2}{x^2})^4}{(1-\frac{y^2}{x^2})^2} = \frac{1}{C'^4 x^4}$
$\Rightarrow (x^2 - y^2)^2 = C'^4 (x^2 + y^2)^4$
$\Rightarrow (x^2 - y^2)^2 = C'^4 (x^2 + y^2)$
$\Rightarrow (x^2 - y^2) = C (x^2 - y^2)$, where $C = C'^2$
Therefore, the result is proved.

Question 5

Form the differential equation od the family of circles the first quadrant which touch the coordinate axes.

Solution:

We know that the equation of a circle in the first quadrant with centre (a, a) and radius a which touches the coordinate axes is $(x -)^2 + (y - a)^2 = a^2$ ……….1
Now, differentiating above equation with respect to x, we get
2(x-a) + 2(y-a0 dy /dx = 0
⇒ (x – a) + (y – a) y' = 0
On multiplying we get
⇒ x – a + yy' – ay' = 0
⇒ x + yy' –a (1 + y) = 0



Therefore, from above equation we have
$\Rightarrow a = \frac{x+yy'}{1+y'}$
Now, substituting the value of a in equation 1, we get
$\left[x - \left(\frac{x+yy'}{1+y'}\right)\right]^2 + \left[y - \left(\frac{x+yy'}{1+y'}\right)\right]^2 = \left(\frac{x+yy'}{1+y'}\right)^2$
Taking LCM and simplifying we get
$\Rightarrow \left[\frac{(x-y)y'}{1+y'}\right]^2 + [\left[\frac{y-x}{1+y'}\right]^2 = \left(\frac{x+yy'}{1+y'}\right)^2$
$\Rightarrow (x - y)^2 . y'^2 + (x - y +)^2 = (x + yy')^2$
$\Rightarrow (x-y)^2[1 + (y'0^2] = (x + yy')^2$
Therefore, the required differential equation of the family of circles is
$(x - y)^2[1 + (y')^2] = (x + yy')2$

Question 6

find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

Solution:

Given

$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

On rearranging we get

$\Rightarrow \frac{dy}{dx} = - \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = - \frac{dyx}{\sqrt{1-y2}}$

On integrating, we get, +

$\text{Sin}^{-1}y = \sin^{-1} x + c$

$\Rightarrow \text{Sin}^{-1}y = \sin^{-1} x + c$

Question 7

Show that general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by (x + y + 1) = (1 – x – y + 1) = A(1 – x – y – 2xy), where A is parameter.

Solution:

Given

$\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1}$

On rearranging

$\Rightarrow \frac{dy}{dx} = - \left(\frac{y^2+y+1}{x^2+x+1}\right)$

Separating the variables using variable separable method we get

$\Rightarrow \frac{dy}{y^2+y\ 1} = \frac{-dx}{x^2+x+1}$

$\Rightarrow \frac{dy}{y^2+y+1} + \frac{dx}{x^2+x+1} = 0$

Taking integrals on both sides, we get

$\int \frac{dy}{y^2+y+1} + \int \frac{dx}{x^2+x+1} = C$

$\Rightarrow \int \frac{dy}{(y+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] = C$

$\tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right] + \tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right] = C$

on integrating we get

$\frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] + \frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] = C$

$\tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right] + \tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right] = C$

Using $\tan^{-1}$ formula we get

$\Rightarrow \tan^{-1}\left[\dfrac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1\frac{2y+1}{\sqrt{3}}.\frac{2x+1}{\sqrt{3}}-}\right] = \frac{\sqrt{3}}{2}c$

$\Rightarrow \tan^{-1}\left[\dfrac{\frac{2y+2y+2}{\sqrt{3}}}{1-\left(\frac{4xy\ +2x\ +2y\ +1}{3}\right)}\right] = \frac{\sqrt{3}}{2}C$

Computing and simplifying we get

$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y\ 1)}{3-4xy-2x-2y-1}\right] = \frac{\sqrt{3}}{2}C$

$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y\ 1)}{2(1-\ x-y\ -2xy)}\right] = \frac{\sqrt{3}}{2}C$

$\Rightarrow \frac{2\sqrt{3}(x+y\ 1)}{2(1-\ x-y\ -2xy)} = \tan\ \left(\frac{\sqrt{3}}{2}c\right)$

Let $\tan\left(\frac{\sqrt{3}}{2}c\right) = B$

Then,

$X + y + 1 = \frac{2B}{\sqrt{3}}(1 - x - y - 2xy)$

Now, let $A = \frac{2B}{\sqrt{3}}$ is a parameter, then, we get

$X + y + 1 = A\ (1 - x - y - 2xy)$

Question 8

find the equation of the curve passing through the point (0,π/4) whose differential equation is sin x cos y dx + cos x sin y dy = 0

Solution:

Given sin x cos y dx + cos x sin y dy = 0

Dividing the given equation by cos x cos y we get

$\Rightarrow \frac{sinxcosydx\ \ +cosxsinydy}{cosxcosy} = 0$

On simplification we get

Tan x dx + tan y dy = 0

So, on integrating both sides, we get,

Log (sec x) + log (sec y) = log C

Using logarithmic formula, we get

Log (sec x sec y) log C

Sec x sec y = C

The curve passes through point (0, π/4)

Thus,$1 \times \sqrt{2} = C$

$\Rightarrow C = \sqrt{2}$

On substituting $C = \sqrt{2}$ in equation (1), we get,

Sec x sec y = $\sqrt{2}$

$\Rightarrow$ Secx.$\frac{1}{cosy} = \sqrt{2}$

$\Rightarrow$ Cosy = $\frac{secx}{\sqrt{2}}$

Therefore, the required equation of the curve is cosy = $\frac{secx}{\sqrt{2}}$

Question 9

Find the particular solution of the differential equation $(1 + e^{2x})$ dy + $(1 + y^2)$ e^x dx = 0, given that y = 1 when x = 0.

Solution:

Given $(1 + e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$
Separating the variable using variable separable method we get
$\Rightarrow \frac{dy}{1+y^2} + \frac{e^x dx}{1+e^{2x}} = 0$
On integrating both sides, we get,
$\tan^{-1} y + \int \frac{e^x dx}{1+e^{2x}} = C$ ……………….1
let $e^x = t$
$\Rightarrow e^{2x} = t^2$
on differentiating we get
$\Rightarrow \frac{d}{dx}(e^x) = \frac{dt}{dx}$
$\Rightarrow e^x = \frac{dt}{dx}$
$\Rightarrow e^x\, dx = dt$
substituting the value in equation (1), we get,
$\text{Tan}^{-1} y + \int \frac{dt}{1+t^2} = C$
$\Rightarrow \tan^{-1} y + \tan^{-1}y + \tan^{-1}(e^x) = C$ ……………..2
Now, y = 1 at x = 0
Therefore m, equation (2) becomes,
$\text{Tan}^{-1} 1 + \tan^{-1} 1 = C$
$\Rightarrow \frac{\pi}{4} + \frac{\pi}{4} = C$
$\Rightarrow C = \frac{\pi}{4}$
Substituting c = π/4 in (2), we get,
$\text{Tan}^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{4}$

Question 10

Solve the differential equation $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy\ (y \neq 0)$

Solution:

Given
$ye^{\frac{x}{y}}\, dx = \left(xe^{\frac{x}{y}} + y^2\right)dy$
On rearranging we get
$\Rightarrow ye^{\frac{x}{y}}\frac{dx}{dy} = xe^{\frac{x}{y}} + y^2$
Taking common

$\Rightarrow e^{\frac{x}{y}}\left[y.\frac{dy}{dx} - x\right] = y^2$

$\Rightarrow e^{\frac{x}{y}}\frac{\left[y.\frac{dy}{dx}-x\right]}{y^2} =$ ……………1

Let $e^{\frac{x}{y}} = z$

Differentiating it with respect to y, we get,

$\frac{d}{dy}\left(e^{\frac{x}{y}}\right) = \frac{dz}{dy}$

$\Rightarrow e^{\frac{x}{y}}, \frac{d}{dy}\left(\frac{x}{y}\right) = \frac{dz}{dy}$

$\Rightarrow e^{\frac{x}{y}}, \left[\frac{y.\frac{dx}{dy}-x}{y^2}\right] = \frac{dz}{dy}$ ………….2

From equation (1) and equation (2), we have

$\frac{dz}{dy} = 1$

$\Rightarrow$ dx = dy

On integrating both sides, we get,

Z = y + c

$\Rightarrow e^{\frac{x}{y}} = y + c$

Question 11

Find a particular solution of the differential equation (x – y) (dx + dy) = dx – dy, Given that y = - 1, when x = 0. (Hint: put x –y = t)

Solution:

Given (x – y) (dx + dy) = dx – dy

$\Rightarrow$ (x – y + 1) dy = (1 – x + y) dx

On rearranging we get

$\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$

$\Rightarrow \frac{dy}{dx} = \frac{1-(x+y)}{x-(y+1)}$ …………… 1

Let x – y = t

Differentiating above equation with respect to x we get

$\Rightarrow \frac{d(x-y)}{dx} = \frac{dt}{dx}$

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

Now, let us substitute the value of x-y and $\frac{dy}{dx}$ in equation (1), we get,

$1 - \frac{dt}{dx} = \frac{1-t}{1+t}$

On rearranging we get

$\Rightarrow \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t}\right)$

$\Rightarrow \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$

Computing and simplifying we get

$\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$

$\Rightarrow \left(\frac{1+t}{t}\right)$ dt = 2dx

$\Rightarrow \left(1 + \frac{1}{t}\right)$dt = 2dx ……………2

On integrating both sides, we get,

t + log |t| = 2x + C

⇒ (x – y) + log |x – y| = 2x +C

⇒ Log |x – y| = x + y + c …………..3

Now, y = -1 at x = 0

Then, equation (3), we get,

Log 1 = 0 – 1 + c

⇒ C = 1

Substituting C = 1 in equation (3) we get,

Log | x – y| = x + y + 1

Therefore, a particular solution of the given differential equation is log | x – y| = x + y + 1

Question 12

Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1\ (x \neq 0)$

Solution:

Given

$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1$

On rearranging we get

$\Rightarrow \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = $\frac{1}{\sqrt{x}}$ and Q = $\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Now, I.F. = $e^{\int pdx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

Thus, the solution of the given differential equation is given by the relation

Y (I.F>) - $\int (QxI.F.)$ dx +C

$$\Rightarrow Ye^{2\sqrt{x}} = \int \left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} x e^{2\sqrt{x}}\right) dx + C$$

$$\Rightarrow Ye^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}} dx + C$$

On integrating we get

$\Rightarrow Ye^{2\sqrt{x}}$ = 2√x + C

Question 13

Find a particular Solution of the differential equation $\frac{dy}{dx}$ + y cot x = 4x cosec x(x ≠ 0), given that y = 0 when x = π/2

Solution:

Given

$\frac{dy}{dx}$ + ycotx = 4xcosecx

Given equation is in the form of $\frac{dy}{dx}$ + py = Q

Where, p = cot x and Q = 4x cosec x

Now, I.f. = $e^{\int pdx}$ = $e^{\int cotxdx}$ = $e^{log|sinx|}$ = sinx

Thus, the solution of the given differential equation is given by the relation

Y(I.F.) = $\int (QxI.F.)dx + c$

Y sin x = $\int 2xcosecxdx + c$

4 $\int xdx + c$

On integrating we get

= 4.$\frac{x^2}{2}$ + C

⇒ y = sinx = $2x^2$ + c ...1

Now, y = 0 at x = $\frac{\pi}{2}$

Therefore, equation (1) we get,

0 = 2 ×$\frac{\pi^2}{4}$ + C

⇒ C = $\frac{\pi^2}{4}$

Now, substituting C = $\frac{\pi^2}{4}$ in equation (1), we get,

Y sin x = $2x^2$ - $\frac{\pi^2}{4}$

Therefore, the required particular solution of the given differential equation is

⇒Y sin x = $2x^2$ – $\frac{\pi^2}{4}$

Question 14

Find a particular solution of the differential equation, (x + 1) $\frac{dy}{dx}$ = $2e^{-y}$ – 1 Given that y = 0 when x = 0.

Solution:

Given

(x + 1) $\frac{dy}{dx}$= $2e^{-y}$ – 1

On rearranging we get

⇒ $\frac{dy}{2e-y-1} = \frac{dx}{x+1}$

⇒ $\frac{e^y dy}{2-e^y} = \frac{dx}{x+1}$

On integrating both sides, we get,

$\int \frac{e^y\,dy}{2-e^y}$ = log |x + 1| + log c 1

Let 2 – e^y = t

$\therefore \frac{d}{dy}(2 - e^y) = \frac{dt}{dy}$

$\Rightarrow -e^y = \frac{dt}{dy}$

$\Rightarrow e^v dt = -dt$

Substituting value in equation (1), we get,

$\int \frac{-dt}{t}$ = log |x + 1| + logc

On integrating we get

⇒ - log |t| = log | C (x + 1)|

⇒ - log | 2 – e^v | = log | C (x + 1) |

$\Rightarrow \frac{1}{2-e^y} = C\,(x+1)$

$\Rightarrow 2 - ey = \frac{1}{c(x+1)} \quad \ldots\ldots 2$

Now, at x = 0 and y = 0, equation (2) becomes,

$\Rightarrow 2 - 1 = \frac{1}{c}$

⇒ C = 1

Now, substituting the value of C I equation (2), we get,

$\Rightarrow 2 - e^y = \frac{1}{(x+1)}$

$\Rightarrow e^y = 2 - \frac{1}{(x+1)}$

$\Rightarrow e^y = \frac{2x+2-1}{(x+1)}$

$\Rightarrow e^y = \frac{2x+1}{(x+1)}$

$\Rightarrow y = \log\left|\frac{2x+1}{x+1}\right|, (x \neq -1)$

Therefore, for required particular solution of the given different equation is

$\Rightarrow y = \log\left|\frac{2x+1}{x+1}\right|, (x \neq -1)$

Question 15

the population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

Solution:

Let the population at any instant (t) be y.

Now it is given that the rate of increase of population is proportional to the number of inhabitants at any instant.

$$\therefore \frac{dy}{dx} ay$$

$\Rightarrow \frac{dy}{dt}$ = ky

Where K is proportionality constant.

$\Rightarrow \frac{dy}{y}$ = kdt

now, integrating both sides, we get,

log y = k t + C 1

According to given conditions,

In the year 1999, t = 0 and y = 20000

⇒ log 20000 = C 2

Also, in the year 2004, t = 5 and y = 25000

Log 25000 = 5k + log 20000

$\Rightarrow 5k = \log\left(\frac{25000}{20000}\right) = log(\frac{5}{4})$

$\Rightarrow K = \frac{1}{5}log(\frac{5}{4})$....3

Also, in the year 2009, t = 10

Now, substituting the values of t, K and c in equation (1), we get

$\text{Log } y = 10 \text{ x} \frac{1}{5}log(\frac{5}{4}) + \log(20000)$

$\Rightarrow \text{Log } y = \log\left[20000\ x(\frac{5}{4})^2\right]$

$\Rightarrow Y = 20000 \text{ x} \frac{5}{4} \text{ x} \frac{5}{4}$

⇒ Y = 31250

Therefore, the population of the village in 2009 will be 31250.

Question 16

The general solution of the differential equation $\frac{ydx-xdx}{y}$ = 0 is

(A) xy = C **(B)x = Cy²**

(C) y = Cx **(D)y =Cx²**

Solution:

C. y = Cx

Explanation:

Given question is

$\Rightarrow \frac{ydx - xdx}{y} = 0$

On rearranging we get

$\Rightarrow \frac{1}{x}dx - \frac{1}{y}dy = 0$

Integrating both sides, we get,

Log |x| - log |y| = log K

$\Rightarrow \text{Log } |\frac{x}{y}| = \log k$

$\Rightarrow \frac{x}{y} = k$

$\Rightarrow y = \frac{1}{k}x$

$\Rightarrow Y = Cx$ where $C = \frac{1}{k}$

Question 17

The general solution of a differential equation of the type $\frac{dy}{dx} + P_1x = Q_1$ is

(A) y $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dy})$ dy + C

(B) y $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dx})$ dx + C

(C) x $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dy})$ dy + C

(D) x $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dx})$ dx + C

Solution:

(C) x $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dy})$ dy + C

Explanation:

The integrating factor of the given differential equation $\frac{dy}{dx} + P_1 x = Q_1$ is $e^{\int p_1 dy}$.

Thus, the general solution of the differential equation is given by,

X (I.F.) = $\int(QxI.F.)dx + c$

x $e^{\int p_1 dy} = \int(Q_1 e^{\int p_1 dy})$ dy + C

Question 18

The general solution of the differential equation e^x dy + (y e^x + 2x) dx = 0 is

(A) x ey + x^2 = C

(B) x ey + y^2 = C

(C) y ex + x^2 = c

(D) y ey + x^2 = c

Solution:

C. y ex + x^2 = c

Explanation:

Given e^xdy + (ye^x + 2x) 2x) dx = 0

On rearranging we get

$\Rightarrow e^x\frac{dy}{dx} + ye^x + 2x = 0$

$\Rightarrow \frac{dy}{dx} + y = - 2xe^{-x}$

This is equation in the form of $\frac{dy}{dx}$ + py = Q

Where, p = 1 and Q = - $2xe^{-x}$

Now, I.F. = $e^{\int pdx} = e^{\int dx} = e^x$

Thus, the solution of the given differential equation is given by the relation

y (I.F.) = $\int(QxI.F.)dx + c$

$\Rightarrow ye^x = \int(-2xe^{-x}.e^x)$ dx + C

$\Rightarrow ye^x = \int 2xdx + C$

on integrating we get

$ye^x = -x^2 + C$

$ye^x + x^2 + C$