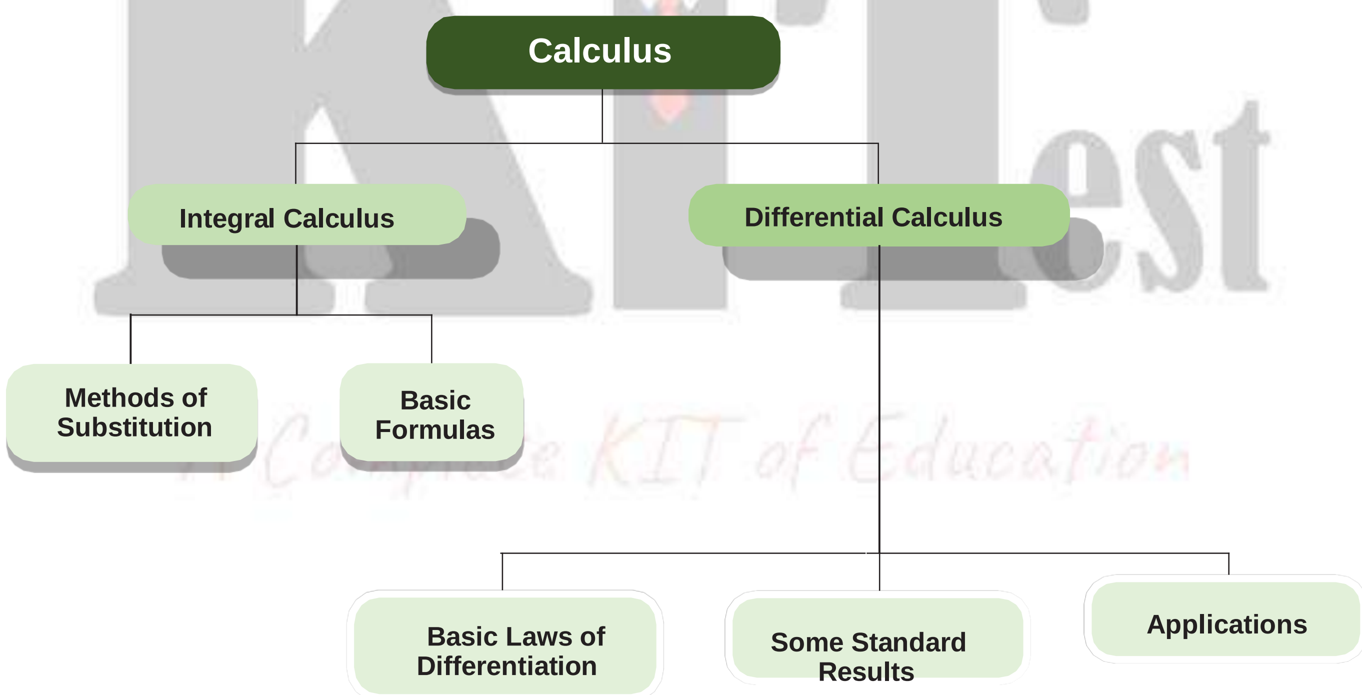
CHAPTER - 8
BASIC CONCEPTS OF DIFFERENTAL AND INTEGRAL CALCULUS

(A) DIFFERENTIAL CALCULUS



INTRODUCTION	Differentiation is one of the most important fundamental operations in calculus. Its theory primarily depends on the idea of limit and continuity of function.
DERIVATIVE OR DIFFERENTIALCOEF	Let y = f(x) be a function. If h be the small increment in x and the

FICIENT	corresponding increment in y or f(x) be y = f(x+h) – f(x)

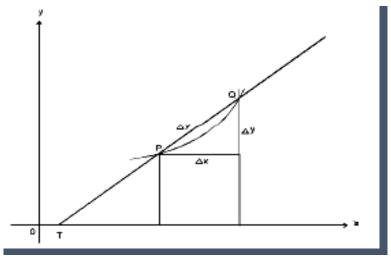
STANDARD FORMULAS

$$\frac{d}{dx}(a)=0$$
$$\frac{d}{dx}(x)=1$$
$$\frac{d}{dx}(au)=a\frac{du}{dx}$$
$$\frac{d}{dx}(u+v-w)=\frac{du}{dx}+\frac{dv}{dx}-\frac{dw}{dx}$$
$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$
$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{1}{v}\frac{du}{dx}-\frac{u}{v^2}\frac{dv}{dx}$$
$$\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$$
$$\frac{d}{dx}(\sqrt{u})=\frac{1}{2\sqrt{u}}\frac{du}{dx}$$
$$\frac{d}{dx}\left(\frac{1}{u}\right)=-\frac{1}{u^2}\frac{du}{dx}$$
$$\frac{d}{dx}\left(\frac{1}{u^n}\right)=-\frac{n}{u^{n+1}}\frac{du}{dx}$$
$$\frac{d}{dx}[f(u)]=\frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u]=\frac{d}{dx}[\log_e u]=\frac{1}{u}\frac{du}{dx}$$
$$\frac{d}{dx}[\log_a u]=\log_a e\frac{1}{u}\frac{du}{dx}$$
$$\frac{d}{dx}e^u=e^u\frac{du}{dx}$$
$$\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$$
$$\frac{d}{dx}(u^v)=vu^{v-1}\frac{du}{dx}+\ln u\ \ u^v\frac{dv}{dx}$$
$$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$$
$$\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$$
$$\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$$
$$\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$$
$$\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$$
$$\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$$

IMPLICIT FUNCTIONS	A function in the form f(x, y) = 0. For example, $x^2y^2 + 3xy + y = 0$ where y cannot be directly defined as a function of x is called an implicit function of x.
PARAMETRIC EQUATION	When both the variables x and y are expressed in terms of a parameter (a third variable), the involved equations are called parametric equations. For the parametric equations x = f(t) and y = h(t) the differential coefficient $\frac{dy}{dx}$
LOGARITHMIC	The process of finding out derivative by taking

DIFFERENTIATION	logarithm in the first instance is called logarithmic differentiation.

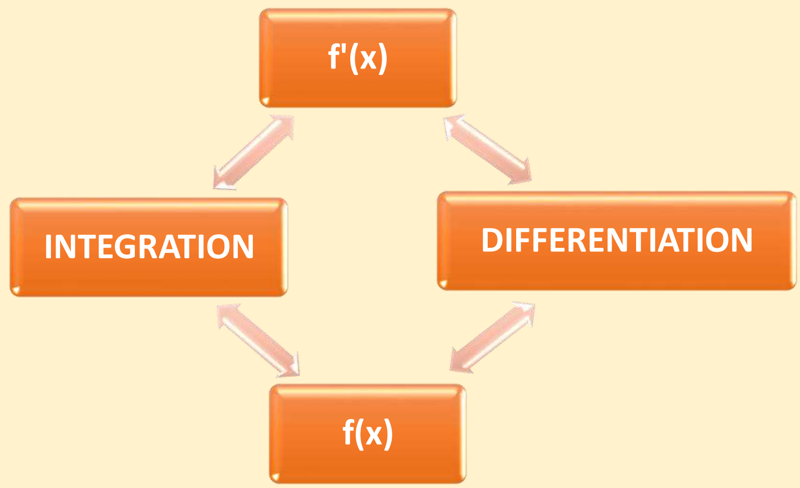


GEOMETRIC INTERPRETATION OF THE DERIVATIVE

COST FUNCTION	TOTAL COST CONSISTS OF TWO PARTS (I) VARIABLE COST (II) FIXED COST. **AVERAGE COST (AC OR C)**: $\dfrac{\textbf{Total Cost } C(X)}{\textbf{Output } \bar{X}}$ **AVERAGE VARIABLE COST (AVC)**: $\dfrac{Variable\ Cost\ \ V(X)}{Output\ \bar{X}}$ **AVERAGE FIXED COST (AFC)**: $\dfrac{Fixed\ Cost\ \ F(X)}{Output\ \bar{X}}$
MARGINAL COST	If C(x) the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units
REVENUE FUNCTION	Revenue, R(x), gives the total money obtained (Total turnover) by selling units of a product. If x units are sold at P per unit, then R(x) = P.X
	Profit P(x), the difference of between total

PROFIT FUNCTION	revenue R(x) and total Cost C (x).

(B) INTEGRAL CALCULUS

INTEGRATION	 Integration is the reverse process of differentiation.
DEFINITEINTEGRATION	**DEFINITION** The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by $$\int f(x)\,dx.$$ The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

Differentiation Formulas:

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(ax) = a$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(\cos x) = -\sin x$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$
7. $\frac{d}{dx}(\cot x) = -\csc^2 x$
8. $\frac{d}{dx}(\sec x) = \sec x \tan x$
9. $\frac{d}{dx}(\csc x) = -\csc x(\cot x)$
10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = (\ln a)a^x$
13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integration Formulas:

1. $\int 1\,dx = x + C$
2. $\int a\,dx = ax + C$
3. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
4. $\int \sin x\,dx = -\cos x + C$
5. $\int \cos x\,dx = \sin x + C$
6. $\int \sec^2 x\,dx = \tan x + C$
7. $\int \csc^2 x\,dx = -\cot x + C$
8. $\int \sec x(\tan x)\,dx = \sec x + C$
9. $\int \csc x(\cot x)\,dx = -\csc x + C$
10. $\int \frac{1}{x}dx = \ln|x| + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C \ a > 0, a \neq 1$
13. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$
14. $\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$
15. $\int \frac{1}{|x|\sqrt{x^2-1}}dx = \sec^{-1} x + C$

Properties of Definite Integral

Assuming f and g are continuous functions

$$\int_a^b f(x)dx = \int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b c\,dx = c(b-a),\ \text{where c is any constant}$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx,\ \text{where c is any constant}$$

$$\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Question 1

Find an expression for y given $\frac{dy}{dx} = 7x^5$

(a) 6 (b) 2

(c) 3 (d) 5

Answer: a

Explanation:

$\frac{dy}{dx} = 7x^5 \rightarrow dy = 7x^5dx$

Integrating both sides, we have

$\int dy = \int 7x^5dx \rightarrow y = \frac{7x^6}{6} + c$

Question 2

Find an expression for y given $\frac{dy}{dx} = x^{-\frac{3}{4}}$

(a) $\frac{2}{3}$

(b) $\frac{1}{4}$

(c) $\frac{5}{4}$

(d) None

Answer: b

Explanation:

$\frac{dy}{dx} = x^{-3/4}$

Y = $\frac{x^{-3/4+1}}{-\frac{3}{4}+1} = \frac{x^{1/4}}{1/4}$

Y = $4x^{1/4}$

Question 3

-12x^{-4} = $\int -12x^{-4}$ - dx solve it;

(a) 6

(b) 2

(c) 3

(d) 4

Answer: d

Explanation:

dy = $\int -12x^{-4}$ dx

= -12 $\int x^{-4}dx$ Use $\int ax^n dx = a\int x^n dx = \frac{ax^{n+1}}{n+1}$+c

= +$\left(\frac{-12x^{-3}}{-3}\right)$+c n = -4, n + 1 = -4 + 1 = -3

Y = 4x^{-3} + c Simplifying fraction, $\frac{-12}{3}$ = 4

Question 4

Given f '(x) = $\frac{1}{2}x^{\frac{1}{3}}$ - $\frac{1}{4}x^{\frac{1}{4}}$ + π, find f (x)

(a) 6

(b) 2

(c) 3

(d) None

Answer: d

Explanation:

$\frac{1}{2}x^{\frac{1}{3}} - \frac{1}{4}x^{\frac{1}{4}} + \pi$

$\int \frac{1}{2} x^{\frac{1}{3}}$ dx - $\int \frac{1}{4} x^{\frac{1}{4}}$dx + $\int \pi dx$

$\frac{1}{2} \int x^{\frac{1}{3}}$ dx - $\frac{1}{4} \int x^{\frac{1}{4}} dx + \pi \int dx$

= $\frac{\frac{1}{2}x^{\frac{4}{3}}}{\frac{4}{3}}$ - $\frac{\frac{1}{4}x^{\frac{5}{4}}}{\frac{5}{4}}$ + $\frac{\pi x}{2}$ + c

= $\frac{3x^{\frac{4}{3}}}{8}$ - $\frac{1}{4} \times \frac{4}{5} x^{\frac{5}{4}}$ + πx + c

Question 5

Given f '(x) = $\int \left(\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x^5}\right)$ dx

(a) -6

(b) 2

(c) -4

(d) None

Answer: c

Explanation:

$\int \left\{\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x^5}\right\}$ dx

= $\int \left(\frac{2}{x} + 3x^{-2} + x^{-5}\right)$ dx — write as negative exponence

= $\int \frac{2}{x}$dx + $\int 3x^{-2}$dx + $\int x^{-5}$dx — Use $\int f(x)dx + g(x)dx = \int f(x)dx + \int g(x)dx$

= 2 In |x| + $\frac{3x^{-1}}{-1}$ + $\frac{x^{-4}}{-4}$ + c — Use $\int ax^n dx = a\int x^n dx = \frac{ax^{n+1}}{n+1}$ + c

= 2 In |x| - $\frac{3}{x}$ - $\frac{1}{4x^4}$ + c — Simplify $\frac{3}{-1}$

Question 6

Integrate $\int \frac{3}{x^{\frac{1}{2}}}$ dx

(a) $6\sqrt{x + c}$

(b) $\sqrt{x + c}$

(c) $8\sqrt{x + c}$

(d) $9\sqrt{x + c}$

Answer: a

Explanation:

$\int \frac{3}{x^{\frac{1}{2}}} dx = \int 3x^{-1/2}$

$= \frac{3x^{-1/2+1}}{-\frac{1}{2}+1} + c$

$= \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 6x^{\frac{1}{2}} + c$

$= 6\sqrt{x} + c$

Question 7

Find y as a function of x if $\frac{d^2y}{dx^2}$ = 2x when x = 2, y = 7

(a) $y = \frac{x^3}{3} + c$

(b) $y = \frac{x^2}{3} + c$

(c) $y = \frac{x}{3} + c$

(d) None

Answer: a

Explanation:

$\int 2xdx = 2\int xdx$

$= \left(\frac{2x^{1+1}}{1+1}\right) + c$ Use $\int ax^n\,dx = a\int x^n dx = \frac{ax^{n+1}}{n+1} + c$

$\frac{dy}{dx} = x^2 + c$ Multiply of fraction/simplify

Finding $y = \int \frac{dy}{dx} = \int x^2 dx$

$Y = \frac{x^3}{3} + c$ Use $\int x^n\,dx = \frac{1}{n+1}x^{n-2} + c$

At (2, 7)

$7 = \frac{2^3}{3} + c$ Substituting x = 2 and y = 7 to find c

$C = \frac{21}{8}$

Thus, the function is $y = \frac{x^3}{3} + c$.

Question 8

Integrate $\int \left(w + \frac{1}{w}\right)\left(w - \frac{1}{w}\right)$ dx

(a) $\frac{w^3}{3} + \frac{1}{w}$

(b) $\frac{w^3}{3} + \frac{1}{w}$ + c

(c) $\frac{w}{3} + \frac{1}{w} + c$

(d) None

Answer: b

Explanation:

$\int(\mathrm{w} + \frac{1}{\mathrm{w}})(\mathrm{w} - \frac{1}{\mathrm{w}})\ \mathrm{dw}$

$= \int \left(\mathrm{w}^2 - \frac{1}{\mathrm{w}^2}\right)\mathrm{dw}$ Express the product as a difference of two squares

$= \int \mathrm{w}^2\mathrm{dw} - \int \frac{1}{\mathrm{w}^2}\mathrm{dw}$ Use $\int \mathrm{f(x)dx + g(x)dx} = \int \mathrm{f(x)dx} + \int \mathrm{g(x)dx}$

$= \int \mathrm{w}^2\mathrm{dw} - \int \mathrm{w}^{-2}\mathrm{dw}$ Express in negative exponential form

$= \frac{\mathrm{w}^3}{3} + \frac{1}{\mathrm{w}} + \mathrm{c}$ Use $\int \mathrm{x^n\ dx} = \frac{1}{\mathrm{n+1}}\mathrm{x^{n-1}}$ + c. Simpliify

Question 9

If $\frac{d^2y}{dx^2}$ = 10 – 3x, find $\frac{dy}{dx}$ + c

(a) 10x - $\frac{3}{2}x^2$+c

(b) 10x - $\frac{3}{2}$+c

(c) 10 - $\frac{3}{2}x^2$ + c

(d) none

Answer: a

Explanation:

$\frac{\mathrm{dy}}{\mathrm{dx}} = \int(10 - 3\mathrm{x})\ \mathrm{dx} = \int 10\mathrm{dx} - \int 3\mathrm{xdx}$ Use $\int \mathrm{f(x)dx + g(x)dx} = \int \mathrm{f(x)dx} + \int \mathrm{g(x)dx}$

$= 10\mathrm{x} - \left(\frac{3\mathrm{x}^{1+1}}{1+1}\right) + \mathrm{c}$ Use $\int \mathrm{ax^n dx} = \mathrm{a}\int \mathrm{x^n\ dx} = \frac{\mathrm{ax^{n+1}}}{\mathrm{n+1}}$ + c

$= 10\mathrm{x} - \left(\frac{3\mathrm{x}^2}{2}\right) + \mathrm{c}$ Simplify

$= 10\mathrm{x} - \frac{3}{2}\mathrm{x}^2 + \mathrm{c}$

Question 10

Calculate $\int x^7\, dx$

(a) $\frac{1}{8}x^7$+c

(b) $\frac{1}{7}x^7 + c$

(c) $\frac{1}{8}x^8 + c$

(d) None

Answer: c

Explanation:

$\int x^7 dx = \frac{1}{7+1}x^{7+1}$ +c

Use $\int x^n$dx = $\frac{1}{n+1}x^{7+1}$+c and substitute n =7

= $\frac{1}{8}x^8$ + c

Question 11

If $\int f(x)dx = xe^{-\log|x|}$ + f(x), then f(x) is

(a) 1

(b) 0

(c) ce^x

(d) logx

Answer: c

Explanation:

$\int$ f(x)dx = $xe^{\log\left|\frac{1}{x}\right|}$ + f(x) → $\int$ f(x)dx = $\frac{x}{|x|}$ + f(x)

On differentiating both sides, we get

F(x) = 0 + f '(x) we know

$\frac{d}{dx}(e^x) = e^x$, ∴ f(x) = ce^x

Question 12

If f (t) =$\int_{-t}^{t}\frac{dx}{1+x^2}$, then f' (1) is

(a) 0

(b) $\frac{2}{3}$

(c) -1

(d) 1

Answer: d

Explanation:

Given f(t) - $\int_{-t}^{t}\frac{dx}{1+x^2}$ = $[\tan^{-1}x]_{-t}^{t}$ = 2 $\tan^{-1}$t

Differentiating with respect to f '(t) = $\frac{2}{1+t^2}$

ϱ f ' (1) = $\frac{2}{2}$ = 1

Question 13

The existence of first order partial derivatives implies continuity

(a) True

(b) False

(c) Not sure

(d) Invalid Question

Answer: b

Explanation:

The mere existence cannot be declared as a condition for continuity because the second order derivates should also be continuous.

Question 14

y = ($x^2(1 + x^3)$) find $\frac{dy}{dx}$

(a) $-(2x + 5x^4)\sin(x^2 + x^2)$

(b) $(2x + 5x^4)\sin(x^2 + x^5)$

(c) $(2x + 5x^4)(x^2 + x^5)$

(d) none

Answer: d

Explanation:

$\frac{dy}{dx}$ = -sin $(x^2 + x^5)\frac{d}{dx}(x^2 + x^5)$ using the chain rule

$\frac{dy}{dx}$ = - (sin $(x^2 + x^5))(2x + 5x^4)$ using the basic derivatives

$\frac{dy}{dx}$ = - $(2x + 5x^4)\sin(x^2 + x^5)$ reordering factors

Question 15

If f(x) = x^k and f '(1) = 10, then the value of k is

(a) 10

(b) -10

(c) $\frac{1}{10}$

(d) None

Answer: a

Explanation:

F(x) = x^k

F(1) = f (1) = k × 1

10 = k × 1

K = 10

Question 16

The points of discontinuity of the function, f(x) = $\frac{x^2+2x+5}{x^2-3x+2}$ are

(a) x=0, x=1

(b) x=1, x=2

(c) x=0, x=2

(d) None

Answer: b

Explanation:

f(x) = $\frac{x^2+2x+5}{x^2-3x+2}$

Denominator = 0

$X^2 - 3x + 2 = 0$

(x-1)(x-2) = 0

X= 1, x = 2

Question 17

The gradient of a function is parallel to the velocity vector of the level curve

(a) True

(b) False

(c) Not sure

(d) Invalid questions

Answer: b

Explanation:

The gradient is perpendicular and not parallel to the velocity vector of the level curve.

Question 18

y = (8 + x^3) (x^3 - 8)

(a) $6x^5$

(b) x^5

(c) 6x

(d) None

Answer: a

Explanation:

This problem is solvable as a product but if you realize that you are looking at a difference of two squares, it because very simple.

$Y = (8 + x^3)(x^3 - 8) = x^6 - 64$

$\frac{dy}{dx} = 6x^5$

Question 19

If (x, y, z, t) = xy + zt + x^2 y z t; x = k^3; y = k^2; z = k; t = $\sqrt{k}$

Find $\frac{df}{dt}$ at k = 1

(a) 34

(b) 16

(c) 32

(d) 61

Answer: b

Explanation:

Using chain rule we have

$\frac{df}{dt} = f_x\frac{dx}{dk}\, f_y\frac{dy}{dk} + f_z\frac{dz}{dk} + f_t\frac{dt}{dk}$

$= (y + 2xy\,zt).(3k^2) + (x + x^2zt).(2k) + (t + x^2yt).(1) + (z + x^2yz).(\frac{1}{2\sqrt{k}})$

Put k = 1; we have x=y=z=t=1

9 + 4 + 2 + 1 = 16.

Question 20

If (x, y) = $x^2 + y^3$; x = $t^2 + t^3$; y = $t^3 + t^9$ find $\frac{df}{dt}$ at t=1.

(a) 0

(b) 1

(c) -1

(d) 164

Answer: d

Explanation:

Using chain rule we have

$\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$

x = (2x). (2t + 3t^2) + (3y^2).(3t^2 + 9t^8)

Put t = 1; we have x = 2; y = 2

= 4.(5) + 12.(12) = 164.

Question 21

f(x, y) = x^2 + xyz + z find f_x at (1, 1, 1)

(a) 0 **(b) 1**

(c) 3 **(d) -1**

Answer: c

Explanation:

F_x= 2x + yz

Put (x, y,z) = (1, 1, 1)

F_x = 2 + 1 = 3.

Question 22

Necessary condition of Euler's theorem is

(a) z should be homogenous and of order n

(b) x should not homogeneous but for order n

(c) Should be implicit

(d) should be the function of x and y only

Answer: a

Explanation:

Of x and y of order 'n' then x$\frac{dz}{dx} + y\ \frac{dz}{dy} = nz$"

Answer 'b' is incorrect as z should be homogeneous.

Answer 'c' is incorrect as z should not be implicit.

Answer 'd' is incorrect as z should be the homogeneous function of x and y not non-homogeneous functions.

Question 23

If f(x, y) = $\frac{x+y}{y}$, x$\frac{dx}{dz}$ + y $\frac{dz}{dy}$ =?

(a) 0 (b) 1

(c) 2 (d) 3

Answer: a

Explanation:

Given function f (x, y) = $\frac{x+y}{y}$ can be written as f(x, y) = $\frac{\left[1+\frac{y}{x}\right]}{\frac{y}{x}}$ = x ^{0} f$\left(\frac{y}{x}\right)$,

Hence by Euler's theorem.

x$\frac{dz}{dx}$ + y$\frac{dz}{dy}$ = 0

Question 24

Find the approximate value of $[0.982 + 2.012 + 1.942]^{1/2}$

(a) 1.96 (b) 2.96

(c) 0.04 (d) -0.04

Answer: b

Explanation:

Let f(x, y, z) = $(x^2 + y^2 + z^2)\left(\frac{1}{2}\right)$ ....(1)

Hence, x = 1, y = 2, z = 2, so that dx = -0.02, dy = 0.01, dz = -0.06

From (1)

$\frac{df}{dx} = \frac{x}{f}$

$\frac{df}{dy} = \frac{y}{f}$

$\frac{df}{dz} = \frac{z}{f}$

df = $\frac{df}{dx}$dx + $\frac{df}{dy}dy + \frac{df}{dx}dz$ = $\frac{(xdx+ydy+zdz)}{f}$ = $\frac{-0.02+0.01-0.12}{3}$ = -0.04

$[0.98^2 + 2.01^2 + 1.94^2]^{1/2}$ = f (1, 2, 2) +df = 3 – 0.04 = 2.96

Question 25

f (x,y) = $\frac{x^3+y^3}{x^{99}+y^{98}x+y^{99}}$ find the value of f_y at (x, y) = (0, 1)

(a) 101 (b) -96

(c) 210 (d) 0

Answer: b

Explanation:

Using Euler theorem

$Xf_x + yf_y = n\, f(x, y)$
Substituting x = 0; n = -96 and y = 1 we have
$F_y = -96.\ F(0, 1) = -96.(1 / 1)$
= - 96

Question 26

f (x, y) = $x^3 + xy^2 + 901$ satisfies the Eulers theorem

(a) True
(b) False
(c) Not sure
(d) Invalid questions

Answer: b

Explanation:

The function is not homogenous and hence does not satisfy the condition posed by euler's theorem.

Question 27

For a homogenous function if critical points exist the value at critical points is

(a) 1
(b) equal to its degree
(c) 0
(d) -1

Answer: c

For a homogeneous function if critical points exist the value at critical points is? f(a, b) = 0(a, b) → critical points. nf (a, b) = 0 ⇒ f(a, b) = 0(a, b) → critical points. Explanation: Euler's theorem is nothing but the linear combination asked here, The degree of the homogeneous function can be a real number.

Question 28

$$\lim_{n \to \infty} \left[\frac{n}{1+n^2} + \frac{n}{4+n^2} + \frac{n}{9+n^2} + \dots + \frac{1}{2n}\right]$$ **is equal to**

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) 1
(d) None of these

Answer: d

Explanation:

We have $\lim_{n \to \infty} \left[\frac{n}{1+n^2} + \frac{n}{4+n^2} + \_\_\_\_ + \frac{1}{2n}\right]$

$= \lim_{n \to \infty} \int_{r=1}^{n} \frac{n}{r^2+n^2} = \lim_{n \to \infty} \sum_{r=1}^{n} \frac{n}{n^2\left(1+\frac{r^2}{n^2}\right)}$

$= \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n\left(1+\frac{r^2}{n^2}\right)} - \int_0^1 \frac{dx}{1+x^2}$

$\left\{\text{Applying formula, } \lim_{n \to \infty} \sum_{r=0}^{n-1} \left\{f\left(\frac{r}{n}\right)\right\} \cdot \frac{1}{n} = \int_0^1 f(x)dx\right\}$

$= [\tan^{-1}x]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4}.$

Question 29

For homogenous function with no saddle points we must have the minimum value as

(a) 90

(b) 1

(c) Equal to degree

(d) 0

Answer: d

Explanation:

Substituting $f_x = f_y = 0$ At critical in euler's theorem we have

nf (a, b) = 0 ➔ f(a, b) = 0(a, b) → critical points.

Question 30

The derivates of $f(x) = \int_{x^2}^{x^3} \frac{1}{logt}dt$, (x>0) is

(a) $\frac{1}{3\log x} - \frac{1}{2\log x}$

(b) $\frac{1}{3\log x}$

(c) $\frac{3x^2}{3\log x}$

(d) $(log x)^{-1}$.x (x -1)

Answer: d

Explanation:

We know that

$\frac{d}{dx}\left(\int_a^b f(t)dt\right) = \frac{db}{dx} f(b) - \frac{da}{dx} f(a)$ a and b

Are functions of x $\therefore$ f(x) = $\int_{x^2}^{x^2} \frac{1}{\log t} dt$ ϱ

$F'(x) = \frac{d}{dx}(x^3)\frac{1}{\log x^3} - \frac{d}{dx}(x^2)\frac{1}{\log x^2}$

$= \frac{3x^2}{3\log x} - \frac{2x}{2\log x} = x(x-1)(\log x)^{-1}$

Question 31

The greatest value of the function f(x)= $\int_1^x |t| dt$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is given by ?

(a) $\frac{3}{8}$ (b) $-\frac{1}{2}$

(c) $-\frac{3}{8}$ (d) $\frac{2}{5}$

Answer: c

Explanation:

$f^d(x) = |x| > 0 \forall x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ hence the

Function is increasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and therefore f(x) has

Maximum at the right point of $\left[-\frac{1}{2}, \frac{1}{2}\right]$

ϱ Max f(x) = $f\left(\frac{1}{2}\right) = \int_1^{\frac{1}{2}} |t| dt$ - $-\frac{3}{8}$.

Question 32

For homogenous function the linear combination of rates of independent change along x and y axis is

(a) Integral multiple function value (b) no relation to function value

(c) real multiple of function value (d) depends if the function is a polynomial

Answer: c

Explanation:

Euler's theorem is nothing but the linear combination asked here, The degree of the homogeneous function can be a real number. Hence, the value is integral multiple of real number.

Question 33

$\int_0^{b-c} f^n(x+a)\,dx$ =Homogenous function can be a real number. Hence the value is integral multiple of real number.

(a) f'(a)-f'(b)

(b) f'(b-c+a)-f'(a)

(c) f' (b+c-a) +f'(a)

(d) None of these

Answer: b

Explanation:

$\int_0^{b-c} f^n (x+a)\,dx$

$= [f'(x+a)]_0^{b-c}$ = f '(b-c+a) – f '(a).

Question 34

$\int_0^x \frac{x^3\,dx}{(x^2+4)^2} =$

(a) 0

(b) ∞

(c) 1/2

(d) None of these

Answer: b

Explanation:

$\int_0^{\infty} \frac{x^3\,dx}{(x^2+4)^2} = \frac{1}{2}\int_0^{\infty} \frac{2x^2\,dx}{(x^2+4)^2}$ dx

$= 2\int_0^{\infty} \frac{t}{(t+4)^2}$ dt, [Putting x^2=t]

$=2\int_0^{\infty}\left[\frac{1}{t+4} - \frac{4}{(t+4)^2}\right]$ dt $= \frac{1}{2}\left[\log(t+4) + \frac{4}{t+4}\right]_0^{\infty}$

$= \frac{1}{2}[\log\infty + 0 - (\log 4 + 1)] = \infty$

Question 35

The points of intersection of F1(x) = $\int_2^x (2t-5)dt$ and f_2(x) = $\int_0^x 2t\,dt$, are

(a) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (b) $\left(\frac{2}{3}, \frac{4}{5}\right)$

(c) $\left(\frac{1}{3}, \frac{3}{6}\right)$ (d) $\left(\frac{5}{4}, \frac{5}{7}\right)$

Answer: a

Explanation:

Let $f_1(x) = y_1 = \int_2^x (2t-5)dt$ and

$F_2(X) = y_2 = \int_0^x 2t dt$ now point of intersection means whose those point at which

$x^2 = x^2 - 5x + 6 \Rightarrow x = \frac{6}{5}$ and $y = x^2 = \frac{36}{25}$ thus point of Intersection is $\left(\frac{6}{5}, \frac{36}{25}\right)$

Question 36

The solution of the equation $\frac{x^2 d^2y}{dx^2}$ = In x, when x=1, y=0 and $\frac{dy}{dx}$ = -1

(a) $\frac{1}{2}$(Int)² + Inx (b) $\frac{1}{2}$(Inx)² – Inx

(c) $-\frac{1}{2}$(Inx)² + Inx (d) $-\frac{1}{2}$(Iogx)² – Iogx

Answer: d

Explanation:

$\frac{d^2y}{dx^2} = \frac{logx}{x^2} \Rightarrow \frac{-(logx+1)}{x} + c$

At $\frac{dy}{dx} = -\int \frac{logx+1}{x}dx = -\frac{1}{2}(logx)^2 - logx$

Question 37

The rate of increase of bacteria in a certain culture is proportional to the number present. If it double 5 hours then in 25 hours its number would be

(a) 8 times the original (b) 16 times the original

(c) 32 times the original (d) 64 times the original

Answer: c

Explanation:

Let P_0 be the initial population and let the

Population after t years be P. then $\frac{dp}{dt}$ = KP ➔ $\frac{dP}{P}$ = kdt
On integrating, we have log P = kt + c At t = 0,
P =P_0 ∴ log P_0 = 0 + C, ∴log P = KT + log P_0
Log $\frac{p}{p0}$ = kt when t = 5 hrs, P = 2P_0 ∴
Log $\frac{2P}{P_0}$ = 5K K =$\frac{LOG^2}{5}$: ∴log $\frac{p}{p_0}$ = $\frac{\log^2}{5}$t when
T = 25 hours, we have
Log $\frac{p}{p_0}$ = $\frac{\log^2}{5}$ × 25 = 5 log 2 = log32; ∴ P = $32P_0$.

Question 38

The degree of the $3\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$ is differential equation

(a) 1 (b) 2
(c) 3 (d) 6

Answer: b

Explanation:

$3\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$ on squaring, we
Get 9 $\left(\frac{d^2y}{dx^2}\right)^2$ = { 1 + $\left(\frac{dy}{dx}\right)^2$ }^{3} obviously the
Highest derivates $\frac{d^2y}{dx^2}$ contains a degree 2.

Question 39

The differential equation representing the family of curves y^2 = 2c(x+$\sqrt{c}$), whre c is a positive parameter, is of

(a) Order 1 (b) Order 2
(c) Degree 3 (d) Degree 4

Answer: a

Explanation:

Given family of curves
y^2 = 2c(x+$\sqrt{c}$), ….. (i)
On differentiating both sides, we get
2y$\frac{dy}{dx}$ = 2c (1 + 0) → c = y $\frac{dy}{dx}$

From equation (i), we have

$y^2 = 2y\frac{dy}{dx}\left\{x + \left(y\frac{dy}{dx}\right)^{1/2}\right\}$

➔ $\left(y^2 - 2xy\,\frac{dy}{dx}\right) = 2$

Question 41

The order and degree of the differentiate equations $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}}$ - 4are

(a) 1, $\frac{2}{3}$

(b) 3, 1

(c) 3, 3

(d) 1, 2

Answer: c

Explanation:

To check, order and degree, the given differential equation should be fees from radicals, hence taking cube on both sides,

$\left(1 + 3.\frac{dy}{dx}\right)^2 = \left(4.\frac{d^3y}{dx^3}\right)^3$

Order = 3, degree = 3.

Question 42

The solution of the differential equation y - x$\frac{dy}{dx}$ = a$\left(y^2 + \frac{dy}{dx}\right)$ is

(a) y=c(x+a) (1+ay)

(b) y=c(x+a) (1- ay)

(c) y=c(x-a) (1+ay)

(d) None of these

Answer: b

Explanation:

Y - x$\frac{dy}{dx}$ = a $\left(y^2 + \frac{dy}{dx}\right)$

Y – ay² = (x +a)$\frac{dy}{dx}$ $\frac{dy}{y(1-ay)} = \frac{dx}{x+a}$

On integrating both sides, we get ρ

Logy – log (1 – ay) = log(x + a) + log c

$\frac{y}{(1-ay)}$ = c(x + a) or c(x + a)(1 – ay) = y.

Question 43

Compute the sum of 4 digit numbers which can be formed with four digit 1, 3, 5, 7 if each digit is used once in each engagement:

(a) 106646

(b) 106636

(c) 106666

(d) None of these

Answer: d

Explanation:

The number of arrangements of 4 different digits taken 4 at a time is given by 4_{p_4} = 4! = 24. Al the four digit will occur equal number of times at each of the position, namely ones, tens, hundreds, thousands.

Thus, each digit will occur $\frac{24}{4}$ = 6 times in each of the position. The sum of digits in one's position will be 6 × (1+3+5+7) = 96. Similar is the case in ten's hundred's and thousand's places.

Therefore, the sum will be 96 + 96 × 10 + 96 × 100 = 106656

Question 44

Eight guests have to be seated 4 on each side of a long rectangular table. 2 particular guests desire to sit on one side of the table and 3 on the other side. The number of ways in which the sitting arrangements can be made is:

(a) 1732

(b) 1728

(c) 1730

(d) 1278

Answer: b

Explanation:

Let the two particular guests sit on right side.

So the three particular guests will sit on left side.

So remaining will be 3 people which need to be selected.

From these 3 people 2 will sit on right side and the one will sit on left side.

Total ways of arranging the people will be =

3_{C_2} × 1_{C_1} = 3

Total ways of arranging the people will be =

Selection of remaining × 4! (For arranging people on left side) × 4! (Arranging people on right side) = 3 × 24 × 24 = 3 × 756 = 1728
So in 1728 ways we can arrange them

Question 45

$$\lim_{x \to 0} \frac{2e^{\frac{1}{x}-3x}}{e^{\frac{1}{x}+x}}$$

(a) -3

(b) 0

(c) 2

(d) 9

Answer: C

Let $\frac{1}{x}$ = y if x ➔ 0, y ➔ ∞

$$\lim_{x \to \infty} \frac{2e^{y}-3\frac{1}{y}}{e^{y}+\frac{1}{y}}$$

$= \lim_{x \to \infty}$

$= \frac{2-3\frac{1}{\infty.e^{\infty}}}{1+\frac{1}{\infty.e^{\infty}}}$

$= \frac{2-0}{1+0} = 2$

Question 46

$$\lim_{X \to 1} \frac{x + x^2 + x^3 \ldots\ldots + x^n - n}{x-1}$$

(a) n

(b) $\frac{n(n+1)}{2}$

(c) (n + 1)

(d) n(n +1)

Answer: b

Explanation:

$$\lim_{X \to 1} \frac{x + x^2 + x^3 \ldots\ldots + x^n - n}{x-1}$$ (:)

By L.H. Rule

$= \lim_{x \to 1} \frac{d/dx(x+x^2+x^3 \pm ------ +x^n - n}{d/dx(x-1)}$

$= \lim_{x \to 1} \frac{1+2x+3x^2+ ------- +nx^{n-1})-0}{1-0}$

$= \frac{1+2\times 1+3(i)^2+ ------- +n(1)^{n-1}}{1}$

= 1 + 2 + 3 + ------- n

$= \sum_{n=} \frac{n(n+1)}{2}$

Past Examination Questions

MAY - 2018

Question 1

The value of $\int_1^2 \frac{1-x}{1+x}dx$ is equal to:

(a) $\log\frac{3}{2} - 1$

(b) $2\log\frac{3}{2} - 1$

(c) $\frac{1}{2}\log\frac{3}{2}$

(d) $\frac{1}{2}\log\frac{3}{2} - 1$

Answer: b

Explanation:

$\int_1^2 \left(\frac{1-x}{1+x}\right) dx = \int_1^2 \left(\frac{1}{1+x} - \frac{x}{1+x}\right)dx$

$\int_1^{-2} \frac{1}{1+x} dx - \int_1^{-2} \frac{x}{x+1}dx$

$\int_1^2 \frac{1}{1+x} dx - \int_1^{-2} (\frac{1+x-1}{1-x}) dx$

$\int_1^2 \frac{1}{(1+x)} dx - \int_1^2 \left(\frac{1}{1+x}\right)dx$

$\int_1^2 \frac{1}{1+x} dx - \int_1^2 1 \times dx + \int_1^2 \frac{1}{1+x} dx$

$2 \int_1^2 \frac{1}{1+x} - \int_1^2 1\, dx$

$2[\log(1+x)]_1^2 - [x]_1^2$

2[log(2+1) – log (1 + 1) – [2 -1]

2[log3 – log2]-1

$2 \log\frac{3}{2} - 1$

Question 2

$\int_0^2 \frac{3\sqrt{x}}{\sqrt{x}}$ is equal to

(a) $\frac{\sqrt[2]{2}}{loge^3}$

(b) 0

(c) $\frac{2(3\sqrt{2-1})}{log_e 3}$

(d) $\frac{3\sqrt{2}}{\sqrt{2}}$

Answer: c

Explanation:

$\int_0^2 \frac{3\sqrt{x}}{\sqrt{x}}$ dx

Let $\sqrt{x}$ = t

$\int_0^2 3^{\sqrt{x}} . \frac{1}{\sqrt{x}}dx \frac{1}{2\sqrt{x}}dx = dt$

$\frac{1}{\sqrt{x}}$ dx = 2dt

x	0	2
t	0	$\sqrt{2}$

$\int_0^{\sqrt{2}} 3^t . 2dt$

$\int_0^{\sqrt{2}} 3^t dt$

$2\left[\frac{3^t}{\log 3}\right]_0^{\sqrt{2}}$

$2\left[\frac{3^{\sqrt{2}}}{log3} - \frac{3^0}{\log 3}\right]$

$\frac{2\left(3^{\sqrt{2}} - 3^0\right)}{log_e 3}$

Question 3

The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}$dx is:

(a) 0

(b) 3

(c) 2

(d) 1

Answer: d

Explanation:

$\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}$ dx ……….(1)

I = $\int_0^2 \frac{\sqrt{0+2-x}}{\sqrt{0+2-x}+\sqrt{2-(0+2-x)}}$ dx

$$\left[\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

I = $\int_0^2 \frac{\sqrt{2-X}}{\sqrt{2-X}+\sqrt{X}}$ dx ……….(2)

Apply (1) and (2) we get

2 I = $\int_0^2 \left[\frac{\sqrt{x}}{\sqrt{x+\sqrt{2-x}}} + \frac{\sqrt{2-x}}{\sqrt{2-x+\sqrt{x}}}\right]$ dx

2 I = $\int_0^2 \frac{(\sqrt{x}+\sqrt{2}-x)}{(\sqrt{x}+\sqrt{2}-x)}$ dx

2 I = $\int_0^2 1$ dx

2 I = $[X]_0^2$

2 I = [2 – 0]

2 I = 2 – 1 = 1

Question 4

The cost function for the production of x unit of a commodity is given by C(x) = $2x^3$ +: $5x^2$ + 36x + 15

(a) 3 (b) 2

(c) 1 (d) 4

Answer: a

Explanation:

The cost function given by C(x) = $2x^3 + 15x^2 + 36x + 15$

$\frac{d}{dx}$ c (x) = $6x^2 - 30x + 36$ ………(1)

C(x) = 0

$6x^2 - 30x + 36 = 00$

$6(x^2 - 5x + 6) = 0$

= $x^2-5x+6=0$

= $x^2-3x-2x+6 = 0$

= x(x-3) -2(x-3) = 0

(x-3) (x-2) = 0

X=3, 2

Differentiating equations (2) again w.r.f. 'x'

$C(x) = 12x-30$ Eq …………… (3)
Putting (x = 2) in
$C(x) = 12\times2-30 = -6$
Putting (x=3) in
$C(x) = 12\times3-30 = 6$(+ve) so function is minimum at x=3

NOV - 2018

Question 1

Let $x = at^3$, $y= \frac{a}{t^2}$. Then $\frac{dy}{dx} =$

(a) $\frac{-1}{t^6}$ (b) $\frac{-3a}{t^6}$

(c) $\frac{1}{3at^6}$ (d) None

Answer: d

Explanation:

If $x = at^3$, $y = \frac{a}{t^2}$=at^{-2}

Given $x = at^3$

Different w.r.t. (t)

$\frac{dy}{dx} = \frac{d}{dt}$ $at^3 = a.3t^2 = 3at^2$

and $y=at^{-2}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2at^{-3}}{3at^2} = \frac{-2}{3t^5}$

Question 2

$\int x(x^2+4)^5$ dx is equal to

(a) $(x^2 + 4)^6 + c$ (b) $\frac{1}{12}(x^2 + 4)^6 + c$

(c) $\frac{1}{6}(x^2 + 4)^6 + c$ (d) None

Answer: b

Explanation:

$\int x(x^2+4)^5$ = x

Let $x^2 + 4 = t$

$2xdx = dt$

$Xdx = \frac{dt}{2}$

$\int (x^2 + 4)^5 .xdx$

$\int t^5 = \frac{dt}{2}$

$= \frac{1}{2} \int t^5 dt$

$= \frac{1}{2} \frac{t^6}{6} + c$

$= \frac{1}{12}(x^2 + 4)^6 + c$

Question 3

$xy = 1$ then $y^2 + \frac{dy}{dx} = ?$

(a) 1 (b) 0

(c) 2 (d) None

Answer: b

Explanation:

$Xy = 1$ then $y^2 + \frac{dy}{dx} = ?$

Given $xy = 1$

$Y = \frac{1}{x}$ --------------- (1)

$Y = x^{-1}$

$\frac{dy}{dx} = (-1)x^{-2}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

Question 4

$\int_{-1}^{3} (1 + 3x + x^3)dx$ is equal to

(a) -4 (b) 4

(c) 3 (d) -3

Answer: a

$\int_{-1}^{3}(1 + 3x + x^3)dx$

$\int_{-1}^{3} 1dx + \int_{-1}^{3} 3xdx - \int_{-1}^{3} x^3dx$

$[x]_{-1}^{3}$ +63$\left[\frac{x^2}{2}\right]_{-1}^{3}$ - $\left[\frac{x^4}{4}\right]_{-1}^{3}$

$[3-(-1)]+\frac{3}{2}[(3)^2 - (-1)^2 - \frac{1}{4}[(3)^4 - (-1)^4]$

$(3+1)+\frac{3}{2}[9 - 1] - \frac{1}{4}[81 - 1]$

$4+\frac{3}{2}\times8-\frac{1}{4}\times80$

4+12-20 = -4

MAY - 2019

Question 1

If $2^x - 2^y = 2^{x-y}$ then $\frac{dy}{dx}$ at x = y = 2

(a) 1 (b) 2

(c) 4 (d) 5

Answer: a

Explanation:

$2^x - 2^y = 2^{x-y}$ x = y= 2 $\frac{dy}{dx}$

$2^x .\text{Log}^2 - 2^y. \log^2. \frac{dy}{dx} = 2^{x-y}. \text{Log}^2\left[1 - \frac{dy}{dx}\right]$

$\text{Log}^2[2^x - 2^y.\frac{dy}{dx}] = \text{Log}^2[2^{x-y}\left(1 - \frac{dy}{dx}\right)]$

$2^2 - 2^2.\frac{dy}{dx} = 2^0\left[1 - \frac{dy}{dx}\right]$

$4-4.\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$4-1 = 4\frac{dy}{dx} - \frac{dy}{dx}$

$3 = 3\frac{dy}{dx}$

$\frac{dy}{dx} = 1$

Question 2

If the given cost of function of a commodity is given by C = 150x-5x² + $\frac{x^3}{6}$, where C stands for cost and x stands for output. If the average cost is equal to the marginal cost then the output x = ______

(a) 5 (b) 10

(c) 15 (d) 20

Answer: c

Explanation:

Average cost = $\frac{Totalcost}{output}$

$C = 150x - 5x^2 + \frac{x^3}{6}$

$\frac{c}{output} = \frac{150x}{x} - \frac{5x^2}{x} \frac{\frac{x^3}{6}}{x}$

$C = 150 - 5x + \frac{x^2}{6}$

$\frac{dc}{dx} = -5 + \frac{2x}{6}$

$--5 + \frac{x}{3} = 0$

$75 + x = 0$

X = 15

Question 3

$\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx =$

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) $\frac{3}{2}$

Answer: a

Explanation:

$I = \int_2^3 \frac{\sqrt{X}}{\sqrt{5-X}+\sqrt{X}}$ ....(1)

$I = \int \frac{\sqrt{5-X}}{\sqrt{15-5+X}+\sqrt{5-X}} = \int \frac{\sqrt{5-X}}{\sqrt{X}+\sqrt{5-X}}$ ....(2)

$2I = \int_2^3 \frac{\sqrt{X}}{\sqrt{5-X}+\sqrt{X}} + \int \frac{\sqrt{X}}{\sqrt{5-X}+\sqrt{X}}$

$2I = \frac{\sqrt{5}+\sqrt{5-X}}{\sqrt{5-X}+\sqrt{X}}$

2I = 2

X = 1

Question 4

$\int \log_e(a^x)dx =$

(a) $\log_e a\left[\frac{x^2}{2}\right]+c$

(b) $\log_e a^{\left[\frac{x}{2}\right]+c}$

(c) $x \log_e a^x - x + c$

(d) None of these

Answer: a

Explanation:

$\int \log_e (a^x)\, dx$

By option method: Base method

Differentiate option a

$log_e a^{\left[\frac{x^2}{2}\right]}$

$\frac{1}{a^{\left|\frac{x^2}{2}\right|}} \times a^{\left[\frac{x^2}{2}\right]} . \log^a . \times \frac{2x}{2}$

$= x.\log_e^a$

$= \log \frac{a^x}{e}$

NOV - 2019

Question 1

$\int a^x dx.$

(a) $x^x(1 + \log x)$

(b) $1 + \log x$

(c) x. log x

(d) $\frac{a^x}{\log a}$+c

Answer: d

Explanation:

(d) Since, we know that

$\frac{d}{dx}\frac{a^x}{\log a} = a^x$

$\int a^x dx = \frac{a^x}{\log a}$ + c

Question 2

$\int x.e^x dx.$

(a) e^x (1 + log x)

(b) e^x .x + e^x + c

(c) log x + e^x + c

(d) $\frac{x^2}{e^x}$ + c

Answer: a

Explanation:

(a) $\int x.e^x dx.$

Following I = Inverse
L = Logarithmic
A = Algebraic
T = Trigonometric
E = Exponential

So, x =>I^{st} function

e =>II^{nd} function

x ex x => u

$\int$ I II^{dx} or e^x => v

Property

Since, $\int u.vdx = u.\int vdx$ _ $\int[\frac{d}{dx}(u).\int vdx$]dx

x $\cdot\int e^x dx$ _ $\int \left[\frac{d}{dx}(x.)\int exdx\right]$dx

x ex - $\int|1 \times e^x|$ dx

x e^x – e^x + c

e^x (x - 1) + c

Question 3

$\int (4x+3)^6$ **dx.**

(a) $\frac{1}{28}(4x+3)^7 + c$

(b) $\frac{1}{7}(4x+3)^7 + c$

(c) $\frac{1}{6}(4x+3)^6 + c$

(d) $\frac{4x}{5} + \frac{3}{5} + c$

Answer: a

Explanation:

(a) $\int (4x+3)^6$ dx

Since, $\int x^n dx = \frac{x}{n+1} + c$

$\int (ax+b)^n\, dx = \frac{(ax+b)}{(n+1)} \times \frac{1}{a} + c$

So,

$\int (4x+3)^6$ dx

$= \frac{(4x+3)}{(6+1)} \times \frac{1}{4} + c$

$= \frac{1}{28}(4x+3)^7 + c$

Question 4

$\int 1_{(2x^2-x^3)_{-1} dx}$

(a) $\frac{4}{3}$

(b) 1

(c) 2

(d) $\frac{2}{3}$

Answer: a

Explanation:

(a) $\int_{-1}^{1} (2x^2 - x^3)$ dx

$= \left[2 \times \frac{x^3}{3} \frac{-x^4}{4}\right]^1$

$= \left[\left(\frac{2}{3} \times 1^3 - \frac{1^4}{4}\right) - \left\{\frac{2}{3} \times (-1)^3 - \frac{(-1)^4}{4}\right\}\right]$

$= \left[\left(\frac{2}{3} - \frac{1}{4}\right) - \left(\frac{-2}{3} - \frac{1}{4}\right)\right]$

$= \frac{2}{3} - \frac{1}{4} + \frac{2}{3} + \frac{1}{4}$

$= \frac{4}{3}$

Question 5

$\frac{d}{dx}$**(x. logx)**

(a) x (1 + log x)

(b) 1 + log x

(c) e^x x. log x

(d) x^2 (log x)

Answer: b

Explanation:

(b) $\frac{d}{dx}$(x. log x)

Since $\frac{d}{dx}$ (u, v) = u $\frac{d}{dx}$ (v) + v$\frac{d}{dx}$(u)

So here u => x

V => log x

$\therefore \frac{d}{dx}$ (x. log x)

= x. $\frac{d}{dx}$(log x) + log × $\frac{d}{dx}$(x)

= x$\frac{1}{x}$ + log x × 1

= 1 + log x

Question 6

Differentiate x^x w.r.t x.

(a) x^x (1 + log x)

(b) $\frac{y}{x}$

(c) $\frac{-y}{x}$

(d) y + x^x log x

Answer: a

Explanation:

(a) $\frac{d}{dx}$ (xx) = ?

Net y = xx

Using log both sides

Log y = x log x

On differentiating both sides w.e.t. x

$\frac{1}{y}.\frac{dy}{dx} = x \, x \frac{d}{dx}(\log x) + \log x \, x \frac{d}{dx}(x)$

$\frac{dy}{dx} = y\left[x \times \frac{1}{x} + \log x \times 1\right]$

Question 7

$\int x^2 . e^x dx.$

(a) 2x. e^x (b) $e^x(x^2 - 2x)$

(c) $x^2 . e^x . (2x) + 2$ (d) $e^x(x - 1)$

Answer: C

Explanation:

$\int x^2 e^x dx$

Using I late

a^2=> 1st function (u)

e^x => 2nd function (v)

$\int u.vdx = u.\int\left[\frac{d}{dx}(u).\int vdx\right]dx$

So $\int$ x2 exdx

$x^2 \int e^x dx - \int\left[\frac{d}{dx}(x2)\int vdx\right]dx$

$x^2 e^x dx - \int[2^x . e^x]dx$

$x^2 . e^x - 2x\int x . e^x dx$ -------Equation (1)

$= x .\int e^x - \int \frac{d}{dx}(x). \int e^x dx \, dx$

$= x. e^x - e^x$

$= e^x (x - 1)$ --------Equation (2)

Put Equation (2) in Equation (1)

$x^2 . e^x - 2 e^x (x-1)$

$x^2 . e^x - 2 e^x . x + 2$

NOV - 2020

Question 1

$\int xe^x dx$ is equal

(a) $e^x(x + 1) + c$ (b) $e^x(x + 2) + c$

(c) $e^x(x - 1) + c$ (d) None

Answer: (c)

Explanation:

$$\int xe^x dx$$

= x $\int xe^x\ dx$

= x $\int e^x\, dx - \int \left(\frac{d}{dx}\, x \int e^x\, dx\right) dx$

= x e^x - $\int i.e^x\, dx$

= x e^x - $\int e^x\, dx$

= x e^x - e^x + c

= $e^x\,(x-1)+c$

Question 2

$$\int \mathbf{e^x\,(x\log x+1)\,x^{-1}\,dx} \textbf{ is equal to}$$

(a) $e^x + \ \ c$

(b) $e^x\, log\, x + c$

(c) $\frac{e^x}{log\, x}$

(d) $e^x\,(log\, x)^2 + \ \ c$

Answer: option (b)

Explanation:

I =$\int e^x\,(x\log x+1)\,x^{-1}\,dx$

$$\int e^x \left(\frac{x\, log\, x+1}{x}\right) dx$$

$$\int e^x \left(\frac{x\, log\, x}{x} + \frac{1}{x}\right) dx$$

$$\int e^x \left(\, log\, x + \frac{1}{x}\right) dx$$

$$\int e^x \log x\, dx + \int e^x\, \frac{1}{x}\, dx$$

$$\int \log x\ \ e^x\, dx + \int e^x\, \frac{1}{x}\, dx$$

= log x $\int e^x\, dx + \int \left(\frac{d}{dx}\,(\log x) \int e^x\, dx\right) dx + \int e^x\, \frac{1}{x}\, dx$

= e^x log x + c

Question 3

If y = x (x – 1) (x-2) then dy/dx is:

(a) – 6x	(b) $3x^2 - 6\,x + 2$

(c) 6 x + 4	(d) $3x^2$ - 6x

Answer: Options (b)

Explanation:

Given y = x (x-1) (x-2)

y = x $[x^2 - 3x + 2]$

y = $x^3 - 3x^2 + 2x$

Diff w.r.t. `x'

$\frac{dy}{dx} = 3x^2 - 6x + 2$

Question 4

The average cost function of good s 2Q + 6 + $\frac{13}{Q}$ where Q is the quantity produced. The approx cost At Q is:

(a) 42 (b) 36

(c) 66 (d) 130

Answer: Options (c)

Explanation:

Average cost function (AC) = 2 Q + 6 + $\frac{13}{Q}$

A.C = $\frac{c(Q)}{Q}$

C (Q) = Q (A.C)

= Q (2 Q + 6 + $\frac{13}{Q}$)

C (Q) = 2 Q^2 + 6 Q + 13

Marginal cost function = $\frac{d}{dx}$ c (Q)

= $\frac{d}{dQ}$ ($2Q^2$ + 6Q + 13)

= (4Q + 6)

Putting Q = 15

= 4 × 15 + 6

= 60 + 6

= 66

Question 5

$\int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}$ dx is equal to

(a) 4 (b) 3

(c) 2 (d) None

Answer: Options (b)

Explanation:

$\int_{2}^{8} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}$ dx ________ (1)

Using Property

$I = \int_{2}^{8} \frac{\sqrt{2+8-x}}{\sqrt{2+8-x}+\sqrt{10-(2+8-x)}}$ dx

$I = \int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}$ ________ (2)

Adding eq. (1) & (2)

$2\,I = \int_{2}^{8} \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} + \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}\right)$ dx

$2\,I = \int_{2}^{8} \frac{(\sqrt{x}+\sqrt{10-x})}{(\sqrt{x}+\sqrt{10-x})}\, dx$

$2\,I = \int_{2}^{8} 1.\,dx$

$2\,I = [8]_{2}^{8}$

$2\,I = (8 - 2)$

$2\,I = 6$

$I = 3$

Jan - 2021

Question 1

The cost function of production is given by c(x) = $\frac{x^3}{2} - 15x^2 + 36\,x$ where x, Denotes the number of items produced.
The level of output for which the average cost is minimum are given by respectively.

(a) 10 and 15 (b) 10 and 12
(c) 12 and 15 (d) 15 and 10

Answer: Options (a)

Explanation:

$C(x) = \frac{x^3}{2} - 15x^2 + 36$ x

Marginal cost M(x) = $\frac{d}{dx}$ c(x)

$= \frac{d}{dx}\left(\frac{x^3}{2} - 15x^2 + 36x\right)$

$= \frac{3x^2}{2} - 30x + 36$

If M(x) is Minimum/Maximum

Then. $\frac{d}{dx}$ [m(x)] = 0

$$\frac{d}{dx}\left[\frac{3x^2}{2} - 30x + 36\right] = 0$$

$\frac{3 \times 2x}{3} - 30 = 0$

3x – 30 = 0

3x = 30

X = 10

Marginal cost is minimum at x = 10

And average A (x) = $\frac{c(x)}{x}$

$= \frac{\frac{x^3}{2} - 15x^2 + 36x}{x}$

$= x\,\frac{\left[\frac{x^2}{2} - 15x + 36\right]}{x}$

A (x) = $\frac{x^2}{2} - 15x + 36$

If A (x) is Minimum/maximum

Then $\frac{d}{dx}$ A (x) = 0

$$\frac{d}{dx}\left(\frac{x^2}{2} - 15x + 36\right) = 0$$

$\frac{2x}{x} - 15 = 0$

X – 15 = 0

X = 15

Average cost is Minimum at x = 15

Question 2

$\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx =$

(a) e $\left(\frac{e}{2} - 1\right)$

(b) e (e-1)

(c) a

(d) $e^2\ (e - i)$

Answer: Options (a)

Explanation:

$$\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

Note $\int e^x \, [f(x) + f'(x)]$ dx = e^x f (x) + c

$= \left[e^x \left(\frac{1}{x}\right)\right]_1^2$

$= e^2 \left(\frac{1}{2}\right) - e^1 \left(\frac{1}{1}\right)$

$= \frac{e^2}{2} - e$

$= e\left(\frac{e}{2} - 1\right)$

JULY - 2021

Question 1

The value of $\int_{-2}^{2} f(x)\,dx$, where f(x) = 1+1, x ≤ 0; f(x) = 1-2x, x ≥ 0 is

(a) 20 (b) -2

(c) -4 (d) 0

Answer: Options (b)

Explanation:

$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$

$= \int_{-2}^{0}(1 + x)\,dx + \int_{0}^{2}(1 - 2x)\,dx$

$= \left(x + \frac{x^2}{2}\right)_{-2}^{0} + \left(x - \frac{2x^2}{2}\right)_{0}^{2}$

$= \left[0 + \frac{0^2}{2}\right] - \left[(-2) + \frac{(-2)^2}{2}\right] + [(2-2)^2 - 0]$

= 0- [-2+2] + [-2-0]

Question 2

In a market there are 30 shops to allocate to people if they allocate X shops then their monthly expenses in rupees I given by p (x) = $-8x^2 + 400x - 1000$ Then the number of shops should they allocate minimize the expenses

(a) 0 (b) 30

(c) 25 (d) 10

Answer: Option (c)

Explanation:

$P(x) = -8x^2 + 400x - 1000$

$\frac{d}{dx} p(x) = \frac{d}{dx}(-8x^2 + 400x - 1000)$

$\frac{dp(x)}{dx} = -16x + 400$

For minimize the expenses

$\frac{dp(x)}{dx} = 0$

$-16x + 400 = 0$

$16x = 400$

$x = 25$

Question 3

The cost function C (x) = 125 + 500 x - $x^2 + \frac{x^3}{3}$ 0 $\leq$ 100 and the demand, function for the is given by p (x) = 1500 – x then the marginal profit when 18 items are sold is;

(a) 751 (b) 571

(c) 676 (d) 875

Answer: Option (c)

Question 4

If f(x) = $3._e^{x^4}$ then $f^1(x) - 4x^3 fx + \left(\frac{1}{3}\right) f(0) - f^1(0)$ is equal to:

(a) 0 (b) ex^2

(c) 1 (d) -1

Answer: Option (c)

Explanation:

If f (x) = $3_e^{x^4}$

Diff. $f^1(x) = 3._e^{x^4} . 4x^3 = 4x^3 f(x)$

$\frac{1}{3} f(0) = 3._e^{0^4} = |0| = 1$

$f^1(0) = 3._e^{0^4} 4(0)^3 = 3 \times 0 = 0$

Now $f^1(x) - 4x^3 f(x) + \frac{1}{3}[f(0)] - f^1(0)$

$= \cancel{4x^3} f(x) \cancel{4x^3} f(x) + 1 - 0$

$= 1$