

Chapter – 7

Integrals

Exercise 7.1

Question 1

Find an anti-derivative (or integral) of the following functions by the method of inspection.

$\sin 2x$

$\cos 3x$

e^{2x}

$(ax + b)^2$

$\sin 2x - 4e^{3x}$

1. $\sin 2x$ **Solution:-**

The anti-derivative of $\sin 2x$ is a function of x where derivative is $\sin 2x$

We know that,

$$\frac{d}{dx} (\cos 2x) = -2 \sin 2x$$

We get,

$$\sin 2x = -\frac{1}{2} \frac{d}{dx} (\cos 2x)$$

On further calculations of $\sin 2x$ is $-1/2 \cos 2x$

$$\sin 2x = \frac{d}{dx} \left(-\frac{1}{2} \cos 2x \right)$$

Hence, the anti derivative of $\sin 2x$ is $-1/2 \cos 2x$

2. $\cos 3x$ **Solution:-**

The anti-derivative of $\cos 3x$ is a function of x whose derivative is $\cos 3x$

We know that,

$$\frac{d}{dx} (\sin 3x) = 3 \cos 3x$$

We get,

$$\cos 3x = \frac{1}{3} \frac{d}{dx} (\sin 3x)$$

On further calculation, we get

$$\cos 3x = \frac{d}{dx} \left(\frac{1}{3} \sin 3x \right)$$

Hence, the anti derivative of $\cos 3x$ is $1/3 \sin 3x$

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3. e^{2x} **Solution:-**

The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x}

We know that,

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

On further calculation, we get

$$e^{2x} = \frac{1}{2} \frac{d}{dx}(e^{2x})$$

Hence, the anti derivative of e^{2x} is $\frac{1}{2} e^{2x}$

4. $(ax + b)^2$ **Solution:-**

The anti-derivative of $(ax + b)^2$ is the function of x whose derivatives is $(ax + b)^2$

We Know that.

$$\frac{d}{dx}(ax + b)^3 = 3a(ax + b)^2$$

On Further multiplication, we get

$$(ax + b)^2 = \frac{1}{3a} \frac{d}{dx}(ax + b)^3$$

Hence,

$$(ax + b)^2 = \frac{d}{dx} \left(\frac{1}{3a} (ax + b)^3 \right)$$

Thus, the anti derivative of $(ax + b)^2$ is $\frac{1}{3a} (ax + b)^3$

5. $\sin 2x - 4e^{3x}$

Solution:-

The anti-derivative of $(\sin 2x - 4e^{3x})$ is the function of x whose derivative of $(\sin 2x - 4e^{3x})$

We know that,

$$\frac{d}{dx} \left(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right) = \sin 2x - 4e^{3x}$$

Hence, the anti derivative of $(\sin 2x - 4e^{3x})$ is $(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x})$

Find the following integrals in Exercise 6 to 20:

6. $\int (4e^{3x} + 1) dx$ **Solution:-**

We get,

$$= 4 \int e^{3x} dx + \int 1 dx$$

On further calculation, we obtain,

$$= 4 \left(\frac{e^{3x}}{3} \right) + x + c$$

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Therefore,

$$= \frac{4}{3}e^{3x} + x + c$$

7. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$

Solution: –

We get,

$$= \int x^2 dx - \int 1 dx$$

Hence,

$$= \frac{x^3}{3} - x + C$$

8. $\int (ax^2 + bx + c) dx$

Solution:-

By taking the terms separately, we get,

$$= a \int x^2 dx + b \int x dx + c \int 1 dx$$

On Further calculation, we obtain,

$$= a \left(\frac{x^3}{3}\right) + b \left(\frac{x^2}{2}\right) + cx + C$$

So, we get,

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

9. $\int (2x^2 + e^x) dx$

Solution:-

By taking the terms separately, we get,

$$= 2 \int x^2 dx + \int e^x dx$$

On further calculation, we get,

$$= 2 \left(\frac{x^3}{3}\right) + e^x + C$$

Therefore,

$$= \frac{2}{3}x^3 + e^x + C$$

10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

Solution:-

We get,

$$= \int \left(x + \frac{1}{x} - 2\right) dx$$

By taking the terms separately, we get,

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$$= \int x dx + \int \frac{1}{x} dx - 2 \int 1. dx$$

Hence, we get,

$$= \frac{x^2}{2} + \log |x| - 2x + C$$

$$11. \int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

Solution:-

We get,

$$= \int (x + 5 - 4x^{-2}) dx$$

By taking the terms separately, we get,

$$= \int x dx + 5 \int 1. dx - 4 \int x^{-2} dx$$

On further calculation, we obtain,

$$= \frac{x^2}{2} + 5x - 4 \left(\frac{x^{-1}}{-1} \right) + C$$

Hence, we get,

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

$$12. \int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

Solution:-

We get,

$$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx$$

On further calculation, we get,

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3 \left(x^{\frac{3}{2}} \right)}{\frac{3}{2}} + \frac{4 \left(x^{\frac{1}{2}} \right)}{\frac{1}{2}} + C$$

So,

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

Hence,

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

$$13. \int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

Solution:-

By dividing, we get,

$$= \int (x^2 + 1) dx$$

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By taking the terms separately, we get,

$$= \int x^2 dx + \int 1 dx$$

Therefore, we obtain

$$= \frac{x^3}{3} + x + C$$

$$14. \int (1 - x)\sqrt{x} dx$$

Solution:-

We get,

$$= \int (\sqrt{x} - x^{\frac{3}{2}}) dx$$

On further calculation, we get,

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$$

So,

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

Hence, we get,

$$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$$

$$15. \int \sqrt{x} (3x^2 + 2x + 3) dx$$

Solution:-

We get,

$$= \int (3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}) dx$$

By taking the terms separately, we get,

$$= 3 \int x^{\frac{5}{2}} dx + 2 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx$$

On further calculation, we get

$$= 3 \left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right) + 2 \left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right) + 3 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

Therefore, we get,

$$= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

$$16. \int \sqrt{x} (3x^2 + 2x + 3) dx$$

Solution:-

By taking the terms separately, we get,

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$$= 2 \int x dx - 3 \int \cos x dx + \int e^x dx$$

On further calculation, we get,

$$= \frac{2x^2}{2} - 3(\sin x) + e^x + C$$

Hence, we get,

$$= x^2 - 3 \sin x + e^x + C$$

$$17. \int (2x^3 - 3 \sin x + 5\sqrt{x}) dx$$

Solution:-

By taking the terms separately, we get,

$$= 2 \int x^2 dx - 3 \int \sin x dx + 5 \int x^{\frac{1}{2}} dx$$

On further calculation, we get,

$$= \frac{2x^3}{3} - 3(-\cos x) + 5 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

Therefore, we get

$$= \frac{2}{3} x^3 + 3 \cos x + \frac{10}{3} x^{\frac{3}{2}} + C$$

$$18. \int \sec x (\sec x + \tan x) dx$$

Solution:-

On Multiplication, we get,

$$= \int (\sec^3 x + \sec x \tan x) dx$$

By taking separately, we get,

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

We get,

$$= \tan x + \sec x + C$$

$$18. \int \sec x (\sec x + \tan x) dx$$

Solution:-

On multiplication, we get,

$$= \int (\sec^3 x + \sec x \tan x) dx$$

By taking separately, we get,

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

We Get,

$$= \tan x + \sec x + C$$

$$19. \int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

Solution:-

We get,

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

So,

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

We get,

$$= \int \tan^2 x dx$$

On further calculation, we get,

$$= \int (\sec^2 x - 1) dx$$

By taking separately, we get,

$$= \int \sec^2 x dx - \int 1 dx$$

Therefore, we get,

$$= \tan x - x + C$$

20. $\int \frac{2-3\sin x}{\cos^2 x} dx$

Solution:-

By Separating the terms, we get,

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$$

On further calculation, we get,

$$= \int 2\sec^2 x dx - 3 \int \tan x \sec x dx$$

Hence, we obtain,

$$= 2 \tan x - 3 \sec x + C$$

Choose the correct answer in Exercise 21 and 22

21. The anti-derivation of $(\sqrt{x} + \frac{1}{\sqrt{x}}) dx$ equals

(1/3) $x^{1/2}$ + (2) $x^{1/2}$ + C

(2/3) $x^{2/3}$ + (1/2) x^2 + C

(2/3) $x^{3/2}$ + (2) $x^{1/2}$ + C

(3/2) $x^{3/2}$ + (1/2) $x^{1/2}$ + C

Solution:-

Given

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

We get,

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$$= \int x^{\frac{3}{2}} dx + \int x^{-\frac{1}{2}} dx$$

On further calculation, we get,

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Therefore, we get,

$$= \frac{2}{5} x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C$$

Here, the correct answer is option (C)

22. If $d/dx f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

$$x^4 + \frac{1}{x^2} - \frac{129}{8}$$

$$x^3 + \frac{1}{x^4} + \frac{129}{8}$$

$$x^4 + \frac{1}{x^3} + \frac{129}{8}$$

$$x^3 + \frac{1}{x^4} - \frac{129}{8}$$

Solution:-

Given

$$d/dx f(x) = 4x^3 - \frac{3}{x^4}$$

The anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

Hence,

$$F(x) = \int 4x^3 - \frac{3}{x^4} dx$$

By taking separately, we get,

$$F(x) = 4 \int x^3 dx - 3 \int (x^{-4}) dx$$

we get,

$$f(x) = 4 \left(\frac{x^4}{4} \right) - 3 \left(\frac{x^{-3}}{-3} \right) + C$$

Now we get,

$$F(x) = x^4 + \frac{1}{x^3} + C$$

Also, $f(2) = 0$

By substituting $x = 2$, we get,

$$f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$16 + \frac{1}{8} + C = 0$$

On further calculation, we get,

$$C = - \left(16 + \frac{1}{8} \right)$$

By taking L.C.M, we get,

$$C = \frac{-129}{8}$$

$$\text{Hence, } f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

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Therefore, the correct answer is option (A).

Exercise 7.2

Integrate the function is Exercise 1 to 37:

1. $\frac{2x}{1+x^2} + x^2$

Solution:-

Let us take $1 + x^2 = t$

So, we get,

$$2x \, dx = dt$$

$$\int \frac{2x}{1+x^2} \, dx$$

We get,

$$= \int \frac{1}{t} \, dt$$

On further calculation, we get,

$$= \log|t| + C$$

Now, substituting $t = 1 + x^2$ we get,

$$= \log|1 + x^2| + C$$

$$= \log(1 + x^2) + C$$

2. $(\log x)^{\frac{2}{x}}$

Solution:-

Let us take,

$$\log|x| = t$$

On differentiating, we get,

$$\frac{1}{x} \, dx = dt$$

$$\int \frac{(\log|x|)^2}{x} \, dx$$

We get,

$$= \int t^2 \, dt$$

On further calculation, we get,

$$= \frac{t^3}{3} + C$$

By substituting $t = \log|x|$ we get,

$$= \frac{(\log|x|)^3}{3} + C$$

3. $\frac{1}{(x+x \log x)}$

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Solution:-

Given

$$\frac{1}{x+x \log x}$$

This can be written as

$$= \frac{1}{x(1+\log x)}$$

Let us take,

$$1 + \log x = t$$

We get,

$$\frac{1}{x} dx = dt$$

So,

$$\int \frac{1}{x(1+\log x)} dx$$

We get,

$$= \int \frac{1}{t} dt$$

On calculating further, we get

$$= \log|t| + C$$

Hence, we get,

$$= \log|1 + \log x| + C$$

4. sin x sin (cos x)**Solution:-**Let us take $\cos x = t$

By differentiating, we get

$$-\sin x dx = dt$$

Now,

$$\int \sin x - \sin(\cos x) dx$$

We obtain,

$$= -\int \sin t dt$$

On further calculation, we get

$$= -[-\cos t] + C$$

$$= \cos t + C$$

By substituting $t = \cos x$, we get

$$= \cos(\cos x) + C$$

5. Sin (ax + b) cos (ax + b)**Solution:-**For more Info Visit - www.KITest.in

Given

$$\sin(ax + b) \cos(ax + b)$$

On integrating the above function, we get

$$\sin(ax + b) \cos(ax + b) = \frac{2 \sin(ax + b) \cos(ax + b)}{2}$$

We obtain,

$$= \frac{\sin 2(ax + b)}{2}$$

$$\text{Let } 2(ax + b) = t$$

We get,

$$2a \, dx = dt$$

We get,

$$\int \frac{\sin 2(ax + b)}{2} dx = \frac{1}{2} \int \frac{\sin t \, dt}{2a}$$

On further calculation, we get,

$$= \frac{-1}{4a} \cos 2(ax + b) + C$$

6. $\sqrt{ax + b}$

Solution:-

Let us take,

$$ax + b = t$$

we get,

$$a \, dx = dt$$

Hence,

$$dx = 1/a \, dt$$

Now,

$$\int (ax + b)^{\frac{1}{2}} dx$$

We get,

$$= \frac{1}{a} \int t^{\frac{1}{2}} dt$$

On further calculation, we get

$$= \frac{1}{a} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

Hence, we get,

$$= \frac{2}{3a} (ax + b)^{\frac{3}{2}} + C$$

$$= \frac{2}{3a} (ax + b)^{\frac{3}{2}} + C$$

7. $x\sqrt{x} + 2$

Solution:-

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Let us take

$$(x + 2) = t$$

We get,

$$dx = dt$$

$$\text{Now, } \int (t - 2)\sqrt{t} dx$$

We get,

$$= \int (t - 2) t\sqrt{t} dt$$

On further calculating, we get

$$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt$$

By taking separately, we get

$$= \int t^{\frac{3}{2}} dt - 2 \int t^{\frac{1}{2}} dt$$

So,

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

By further calculation, we get

$$= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C$$

$$= \frac{2}{5} (x + 2)^{\frac{5}{2}} - \frac{4}{3} (x + 2)^{\frac{3}{2}} + C$$

$$8. x\sqrt{1+2x^2} + 2x^2$$

Solution:-

= Let us take,

$$1 + 2x^2 = t$$

We get,

$$4x dx = dt$$

$$\int x\sqrt{1+2x^2} dx$$

We obtain,

$$= \int \frac{\sqrt{tdt}}{4}$$

So,

$$= \frac{1}{4} \int t^{\frac{1}{2}} dt$$

On further calculation, we get

$$= \frac{1}{4} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{1}{6} (1 + 2x^2)^{\frac{3}{2}} + C$$

$$9. (4x + 2)\sqrt{x^2 + x + 1}$$

Solution:-

Let us take

$$x^2 + x + 1 = t$$

We get,

$$(2x + 1) dx = dt$$

$$\int (4x + 2) \sqrt{x^2 + x + 1} dx$$

We obtain,

$$= \int 2\sqrt{t} dx$$

$$= 2 \int \sqrt{t} dt$$

On further calculation, we get

$$= 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{4}{3} (x^2 + x + 1)^{\frac{3}{2}} + C$$

10. $\frac{1}{(x-\sqrt{x})}$

Solution:-

Given

$$\frac{1}{(x-\sqrt{x})}$$

This can be written as

$$= \frac{2}{\sqrt{x}(\sqrt{x}-1)}$$

Let us take

$$(\sqrt{x} - 1) = t$$

We get,

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx = \int_t^2 dt$$

On further calculation, we get

$$= 2 \log|t| + C$$

Hence, we obtain,

$$= 2 \log |\sqrt{x} - 1| + C$$

11. $\frac{x}{(\sqrt{x}+4)}, x > 0$

Solution:-

Let us take,

$$X + 4 = t$$

We get,

$$dx = dt$$

$$\int \frac{x}{\sqrt{x+4}} dx = \int \frac{(t-4)}{\sqrt{t}} dt$$

So,

$$= \int \left(\sqrt{t} - \frac{4}{\sqrt{t}} \right) dt$$

On further calculation, we get

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4 \left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + C$$

$$= \frac{2}{3} (t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$$

$$= \frac{2}{3} t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3} t^{\frac{3}{2}} (t - 12) + C$$

By substituting $t = x + 4$, we obtain

$$= \frac{2}{3} (x + 4)^{\frac{3}{2}} (x + 4 - 12) + C$$

$$= \frac{2}{3} \sqrt{x+4} (x - 8) + C$$

$$12. (x^3 - 1)^{\frac{1}{3}} x^5$$

Solution:-

Let us take,

$$x^3 - 1 = t$$

We get,

$$3x^2 dx = dt$$

$$\int (x^3 - 1)^{\frac{1}{3}} x^5 dx$$

We get,

$$= \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

We get,

$$= \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

By putting $x^3 - 1 = t$, we obtain

$$= \int t^{\frac{1}{3}} (t + 1) \frac{dt}{3}$$

$$= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right)$$

On further calculation, we get

$$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$$

$$= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + c$$

$$= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C$$

13. $\frac{x^2}{(2+3x^3)^3}$

Solution:-

Let us take,

$$2 + 3x^3 = t$$

We get,

$$9x^2 dx = dt$$

$$\int \frac{x^2}{(2+3x^3)^3} dx$$

So,

$$= \frac{1}{9} \int \frac{dt}{(t)^3}$$

On further calculation, we get

$$= \frac{1}{9} \left[\frac{t^{-2}}{-2} \right] + C$$

$$= \frac{-1}{18} \left(\frac{1}{t^2} \right) + C$$

$$= \frac{-1}{18(2+3x^3)^2}$$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

Solution:-

Let us take,

$$\log x = t$$

We get,

$$\frac{1}{x} dx = dt$$

$$\int \frac{1}{x(\log x)^m} dx:$$

We obtain,

$$= \int \frac{dt}{(t)^m}$$

On further calculation, we get

$$= \left(\frac{t^{-m+1}}{1-m} \right) + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

$$15. \frac{x}{(9-4x^2)}$$

Solution:-

Let us take,

$$9 - 4x^2 = t$$

We get,

$$-8x \, dx = dt$$

Now take,

$$\int \frac{x}{9-4x^2} \, dx$$

So,

$$= \frac{-1}{8} \log \int \frac{1}{t} \, dt$$

By further calculating, we obtain

$$= \frac{-1}{8} \log |t| + C$$

$$= \frac{-1}{8} \log |9 - 4x^2| + C$$

$$16. e^{2x+3}$$

Solution:

Let us take

$$2x + 3 = t$$

We get,

$$2 \, dx = dt$$

Now obtain,

$$= \int e^{2x+3} \, dx$$

We obtain,

$$= \frac{1}{2} \int e^t \, dt$$

On further calculation, we get

$$= \frac{1}{2} (e^t) + C$$

$$= \frac{1}{2} e^{(2x+3)} + C$$

$$17. \frac{x}{e^{x^2}}$$

Solution:-

Let us take

$$x^2 = t$$

We get,

$$2x \, dx = dt$$

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$$\int \frac{x}{e^{x^2}} dx$$

So,

$$= \frac{1}{2} \int \frac{1}{e^t} dt$$

$$= \frac{1}{2} \int e^{-t} dt$$

On further calculation, we get

$$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

$$18. \frac{e^{\tan^{-1}x}}{1+x^2}$$

Solution:-

Let us take

$$\tan^{-1} x = t$$

We get,

$$\frac{1}{1+x^2} dx = dt$$

$$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$$

We obtain,

$$= \int e^t dt$$

By further calculation, we get

$$= e^t + C$$

$$= e^{\tan^{-1}x} + C$$

$$19. \frac{e^{2x}-1}{e^{2x}+1}$$

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Solution:-

By dividing numerator and denominator by e^x , we find

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let us assume,

$$e^x + e^{-x} = t$$

So,

$$(e^x - e^{-x}) dx = dt$$

$$\int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

We get,

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$$= \int \frac{dt}{t}$$

By calculating further, we get

$$= \log|t| + C$$

$$= \log |e^x + e^{-x}| + C$$

$$20. \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

Solution:-

Let us assume,

$$e^x + e^{-2x} = t$$

We get,

$$(2e^{2x} - 2e^{-2x}) dx = dt$$

$$2(e^{2x} - e^{-2x}) dx = dt$$

Now

$$\int \left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \right) dx$$

We get,

$$= \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

On calculating further, we get

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log |e^{2x} + e^{-2x}| + C$$

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log |e^{2x} + e^{-2x}| + C$$

$$21. \tan^2(2x - 3)$$

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Solution:-

$$\tan^2(2x - 3) \sec^2(2x - 3) - 1$$

Let us take,

$$2x - 3 = t$$

We get,

$$2dx = dt$$

Now,

$$\int \sec^2(2x - 3) dx = \int [(\sec^2(2x - 3))] dx$$

By Separating, we obtain

$$= \frac{1}{2} \int (\sec^2 t) dt - \int 1 dx$$

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$$= \frac{1}{2} \int \sec^2 t \, dx - \int 1 \, dx$$

On further calculation, we get

$$= \frac{1}{2} \tan t - x + C$$

$$= \frac{1}{2} \tan(2X - 3) - x + C$$

22. $\sec^2(7 - 4X)$

Solution:-

Let us take,

$$7 - 4x = t$$

We get,

$$-4dx = dt$$

Hence,

$$\int \sec^2(7 - 4x) dx = \frac{-1}{4} \int \sec^2 t \, dt$$

On calculating further, we get

$$= \frac{-1}{4} (\tan t) + C$$

$$= \frac{-1}{4} \tan(7 - 4x) + C$$

23. $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$

Solution:-

Let us take,

$$\sin^{-1} x = t$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int t \, dt$$

We get,

$$= \frac{t^2}{2} + C$$

By substituting $t = \sin^{-1} x$, we get

$$= \frac{(\sin^{-1} x)^2}{2} + C$$

24. $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Solution:-

$$\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x}$$

$$6 \cos x + 4 \sin x$$

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This can be written as

$$3 \cos x + 2 \sin x = t$$

$$(-3 \sin x + 2 \cos x) dx = dt$$

$$\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx = \int \frac{dt}{2t}$$

On further calculation, we get

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log|t| + C$$

Therefore, we get

$$= \frac{1}{2} \log|2 \sin x + 3 \cos x| + C$$

25. $\frac{1}{\cos^2 x (1 - \tan x)^2}$

Solution:-

$$\frac{1}{\cos^2 x (1 - \tan x)^2} = \frac{\sec^2 x}{(1 - \tan x)^2}$$

Let us assume,

$$(1 - \tan x) = t$$

$$-\sec^2 x dx = dt$$

$$\int \frac{\sec^2 x}{(1 - \tan x)^2} dx = \int \frac{-dt}{t^2}$$

We get,

$$= - \int t^{-2} dt$$

$$= + \frac{1}{t} + C$$

Therefore, we get

$$= \frac{1}{(1 - \tan x)} + C$$

26. $\frac{\cos \sqrt{x}}{\sqrt{x}}$

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Solution:-

Let us take,

$$\sqrt{x} = t$$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \int \cos t dt$$

By further calculation, we get

$$= 2 \sin t + C$$

$$= 2 \sin \sqrt{x} + C$$

27. $\sqrt{\sin 2x} \cos 2x$ **Solution:-**

Let us take,

$$\sin 2x = t$$

$$2 \cos 2x \, dx = dt$$

$$\Rightarrow \int \sqrt{\sin 2x} \cos 2x \, dx = \frac{1}{2} \int \sqrt{t} \, dt$$

On further calculation, we get

$$= \frac{1}{2} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{1}{3} t^{\frac{3}{2}} + C$$

BY substituting $t = \sin 2x$, we get

28. $\frac{\cos x}{\sqrt{1+\sin x}}$ **Solution:-**

Let us take,

$$1 + \sin x = t$$

$$\cos x \, dx = dt$$

$$\int \frac{\cos x}{\sqrt{1+\sin x}} \, dx = \int \frac{dt}{\sqrt{t}}$$

By further calculation, we get

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1 + \sin x} + C$$

29. $\cot x \log \sin x$ **Solution:-**

Take

$$\log \sin x = t$$

By differentiation we get

$$\frac{1}{\sin x} \cdot \cos x \, dx = dt$$

So we get

$$\cot x \, dx = dt$$

Integrating both sides

$$\int \cot x \log \sin x \, dx = \int t \, dt$$

We get

$$= \frac{t^2}{2} + C$$

Substituting the value of t

$$= \frac{1}{2} (\log \sin x)^2 + C$$

30. $\frac{\sin x}{1+\cos x}$

Solution:-

Take $1 + \cos x = t$

By differentiation

$$-\sin x \, dx = dt$$

By integrating both side

$$\int \frac{\sin x}{1+\cos x} \, dx = \int -\frac{dt}{t^2}$$

So, we get

$$= -\log |t| + C$$

Substituting the value of t

$$= -\log |1 + \cos x| + C$$

31. $\frac{\sin x}{(1+\cos x)^2}$

Solution:-

Take $1 + \cos x = t$

By Differentiation

$$-\sin x \, dx = dt$$

Integrating both sides

$$\int \frac{\sin x}{(1+\cos x)^2} \, dx = \int -\frac{dt}{t^2}$$

We get

$$= -\int t^{-2} \, dt$$

It can be written as

$$= \frac{1}{t} + C$$

32. $\frac{1}{1+\cot x}$

Solution:-

It is given that

$$I = \int \frac{1}{1+\cot x} dx$$

We can write it as

$$= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$

Multiply and divide by 2 in numerator and denominator

$$= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$$

It can be written as

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

On further calculation

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

We get

$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Take $\sin x + \cos x = t$

By Differentiation

$$(\cos x - \sin x) dx = dt$$

We get

$$I = \frac{x}{2} + \frac{1}{2} \int \frac{-dt}{t}$$

By Integration

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

Substituting the value of t

$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

33. $\frac{1}{1-\tan x}$

Solution:-

It is given that

$$I = \int \frac{1}{1-\tan x} dx$$

We can write it as

$$= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx$$

By taking LCM

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$

Multiply and divide by 2 in numerator and denominator

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$

On further calculation

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

We get

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$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Take $\cos x - \sin x = t$

By differentiation

$$(-\sin x - \cos x) dx = dt$$

We get

$$I = \frac{x}{2} + \frac{1}{2} \int \frac{-dt}{t}$$

By integration

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

Substituting the value of t

$$= \frac{x}{2} - \frac{1}{2} \log|\cos x + \sin x| + C$$

34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Solution:-

It is given that

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$

By multiplying $\cos x$ to both numerator and denominator

$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$

On further calculation

$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$

So we get

$$= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$$

Take $\tan x = t$

We get $\sec^2 x dx = dt$

$$I = \int \frac{dt}{\sqrt{t}}$$

By integration we get

$$= 2\sqrt{t} + C$$

Substituting the value of t

$$= 2\sqrt{\tan x} + C$$

35. $\frac{(1+\log x)^2}{x}$

Solution:-

Consider

$$1 + \log x = t$$

So we get

$$\frac{1}{x} dx = dt$$

Integrating both sides

$$\int \frac{(1 + \log x)^2}{x} dx = \int t^2 dt$$

We get

$$= \frac{t^3}{3} + C$$

Substituting the value of t

$$= \frac{(1 + \log x)^3}{3} + C$$

36. $\frac{(x+1)(x+\log x)^2}{x}$

Solution:-

It is given that

$$= \frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right) (x + \log x)^2$$

We can write it as

$$= \left(1 + \frac{1}{x}\right) (x + \log x)^2$$

Consider $x + \log x = t$

By differentiation

$$= \left(1 + \frac{1}{x}\right) dx = dt$$

Integrating both sides

$$\int \left(1 + \frac{1}{x}\right) (x + \log x)^2 dx = \int t^2 dt$$

So we get

$$= \frac{t^3}{3} + C$$

Substituting the value of t

$$= \frac{1}{3} (x + \log x)^3 + C$$

37. $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Solution:-

It is given that

$$\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

Consider $x^4 = t$

We get $4x^3 dx = dt$

$$\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin(\tan^{-1} t)}{1+t^2} dt \quad \dots(1)$$

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Similarly take $\tan^{-1} t = u$

By differentiation we get

$$\frac{1}{1+t^2} dt = du$$

Using equation (1) we get

$$\int \frac{x^3 \sin^{-1}(\tan^{-1} x^4)}{1+x^8} = \frac{1}{4} \int \sin u \, du$$

By integration

$$= \frac{1}{4} (-\cos u) + C$$

Substituting the value of u

$$= \frac{-1}{4} \cos(\tan^{-1} t) + C$$

No substituting the value of t

$$= \frac{-1}{4} \cos(\tan^{-1} x^4) + C$$

Choose the correct answer in Exercises 38 and 39.

38. $\int \frac{10x^9 + 10^x \log_e 10 dx}{z^{10} + 10^z}$ equal's

- a) $10^x - x^{10} + C$
- b) $10^x + x^{10} + C$
- c) $(10^x - x^{10})^{-1} + C$
- d) $\log(10^x + x^{10}) + C$

Solution:-

Take $x^{10} + 10^x = t$

Differentiating both sides

$$(10x^9 + 10^x \log_e 10) dx = dt$$

Integrating both sides we get

$$\frac{10x^9 + 10^x \log_e 10 dx}{z^{10} + 10^z} = \int \frac{dt}{t}$$

So we get

$$= \log t + C$$

Substituting the value of t

$$= \log(10^x + x^{10}) + C$$

Therefore, D is the correct answer.

39. $\int \frac{dx}{\sin^2 \cos^2 x}$ equals

- a) $\tan x + \cot x + C$
- b) $\tan x + \cot x + C$
- c) $\tan x \cot x + C$
- d) $\tan x - \cot 2x + C$

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Solution:-

It is given that

$$I = \int \frac{dx}{\sin^2 \cos^2 x}$$

We can write it as

$$= \int \frac{1}{\sin^2 \cos^2 x} dx$$

Here we get

$$= \int \frac{\sin^2 x + \cos^4 x}{\sin^2 x \cos^2 x} dx$$

By separating the terms

$$= \int \frac{\sin^2 x}{\sin^2 \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 \cos^2 x}$$

We get

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

By integration

$$= \tan x - \cot x + C$$

Therefore, B is the correct answer.

Exercise 7.3**1. $\sin^2(2x + 5)$** **Solution:-**

We have,

By standard trigonometric identity, $\sin^2 x = (1 - \cos 2x)/2$

$$\sin^2(2x + 5) = \frac{1 - \cos 2(2x+5)}{2} = \frac{1 - \cos(4x+10)}{2}$$

Taking integration on both sides, we get,

$$= \int \sin^2(2x + 5) dx = \int \frac{1 - \cos(4x+10)}{2} dx$$

Splitting the integrals,

$$= \frac{1}{2} \int 1. dx - \frac{1}{2} \int \frac{1 - \cos(4x+10)}{2} dx$$

Splitting the integrals,

$$= \frac{1}{2} \int 1. dx - \frac{1}{2} \int \cos(4x + 10) dx$$

$$= \frac{1}{2} x - \frac{1}{2} \int \cos(4x + 10) dx$$

On integrating, we get,

$$= \frac{1}{2} x - \frac{1}{2} \cos(4x + 10) dx$$

On integrating, we get,

$$= \frac{1}{2} x - \frac{1}{2} \left(\frac{\sin(4x+10)}{4} \right) + C$$

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$$= \frac{1}{2} x - \frac{1}{8} \sin(4x + 10) + C$$

2. $\sin 3x \cos 4x$

Solution:-

By standard trigonometric identify $\sin A \cos B = \frac{1}{2} (\sin(A+B) + \cos(A-B))$

$$\int \sin 3x \cos 4x dx = \frac{1}{2} \int \sin(3x + 4x) + \sin(3x - 4x) dx$$

On simplifying,

$$= \frac{1}{2} \int \{\sin 7x + \sin(-x)\} dx$$

$$= \frac{1}{2} \int \{\sin 7x - \sin x\} dx$$

Splitting the integrals, we have,

$$= \frac{1}{2} \int \sin 7x dx - \frac{1}{2} \int (-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

3. $\cos 2x \cos 4x \cos 6x$

Solution:-

By standard trigonometric identify $\cos A \cos B = \frac{1}{2} \{\cos(A+B) + \cos(A-B)\}$

$$\int \cos 2x \cos 4x \cos 6x dx = \int \cos 2x \left[\frac{1}{2} \{\cos(4x + 6x) + \cos(4x - 6x)\} \right] dx$$

$$= \frac{1}{2} \int \{\cos 2x \cos 10x + \cos 2x \cos(-2x)\} dx$$

We know that, $\cos(-x) = \cos x$,

$$= \frac{1}{2} \int \{\cos 2x \cos 10x + \cos^2 2x\} dx$$

Again, by standard trigonometric identify $\cos A \cos B = \frac{1}{2} \{\cos(A+B) + \cos(A-B)\}$ and $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{2} \int \left[\frac{1}{2} \{\cos(2x + 10x) + \cos(2x - 10x)\} + \left(\frac{1 + \cos 4x}{2} \right) \right] dx$$

On simplifying, we get,

$$= \frac{1}{4} \int \cos 12x + \cos 8x + 1 + \cos 4x dx$$

By integrating,

$$= \frac{1}{4} \left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4} \right] + C$$

4. $\sin^3(2x + 1)$

Solution:-

Given, $\sin^3(2x + 1)$

By splitting,

$$= \int \sin^3(2x + 1) dx = \int \sin^2(2x + 1) \cdot \sin(2x + 1) dx$$

We know that, $\sin^2 x = 1 - \cos^2 x$

$$= \int (1 - \cos^2(2x + 1)) \sin(2x + 1) \cdot \sin(2x + 1) dx$$

Let us assume $\cos(2x + 1) = t$

Then,

$$\Rightarrow -2\sin(2x + 1) dx = dt$$

$$\Rightarrow \sin(2x + 1) dx = \frac{-dt}{2}$$

$$\begin{aligned} \sin^3(2x + 1) &= \frac{-1}{2} \int (1 - t^2) dt \\ &= \frac{-1}{2} \left\{ t - \frac{t^3}{3} \right\} \end{aligned}$$

Now substitute the value 't' in equation,

$$\begin{aligned} &= \frac{-1}{2} \left\{ \cos(2x + 1) - \frac{\cos^3(2x + 1)}{3} \right\} \\ &= \frac{-\cos(2x + 1)}{2} + \frac{\cos^3(2x + 1)}{6} + c \end{aligned}$$

5. $\sin^3 x \cos^3 x$

Solution:-

Given, $\int \sin^3 x \cos^3 x \cdot dx$

By splitting the given function,

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$

We know that, $\sin^2 x = 1 - \cos^2 x$

$$= \int \cos^3 x (1 - \cos^2 x) \sin x \cdot dx$$

So, let us assume $\cos x = t$

Then,

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\sin^3 x \cos^3 x = -\int t^3 (1 - t^2) dt$$

$$= -\int (t^3 - t^5) dt$$

On integrating we get,

$$= -\left\{ \frac{t^4}{4} - \frac{t^6}{6} \right\} + C$$

Now substitute the value 't' in equation,

$$= -\left\{ \frac{\cos^4 x}{4} - \frac{\cos^6 x}{6} \right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

6. $\sin x \sin 2x \sin 3x$

Solution:-

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By standard trigonometric identify $\sin A \sin B = -1/2 \{ \cos (A+B) - \cos (A-B) \}$

$$\int \sin x \sin 2x \sin 3x dx = \int \sin x \cdot \frac{1}{2} [\{ (2x - 3x) - \cos(2x + 3x) \}] dx$$

On simplifying, we get,

$$= \frac{1}{2} \int \{ \sin x \cos(-x) - \sin x \cos 5x \} dx$$

We know that, $\cos(-x) = \cos x$,

$$= \frac{1}{2} \int \{ \sin x \cos x - \sin x \cos 5x \} dx$$

Splitting the integrals, by using $\sin 2x = 2 \sin x \cos x$, we get,

$$= \frac{1}{2} \int \frac{\sin 2x}{2} dx - \frac{1}{2} \int \sin x \cos 5x dx$$

On integrating the first tem, and substituting $\sin A \cos B = \frac{1}{2} \{ \sin (A+B) + \sin (A-B) \}$

$$= \frac{1}{4} \left[\frac{-\cos 2x}{2} \right] - \frac{1}{2} \int \left\{ \frac{1}{2} \sin (x + 5x) + \sin(x - 5x) \right\} dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4} \int (\sin 6x + \sin(-4x)) dx$$

Computing and simplifying, we get,

$$= \frac{-\cos 2x}{8} - \frac{1}{4} \left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4} \right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8} \left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4} \right] + C$$

$$= \frac{1}{8} \left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x \right]$$

7. $\sin 4x \sin 8x$

Solution:-

By standard trigonometric identify $\sin A \sin B = -1/2 \{ \cos (4x - 8x) - \cos (A - B) \}$

Then,

$$\int \sin 4x \sin 8x dx = \int \left\{ \frac{1}{2} \cos(4x - 8x) - \cos(4x + 8x) \right\} dx$$

$$= \frac{1}{2} \int (\cos(-4x)) - \cos 2x dx$$

We know that, $\cos(-x) = \cos x$,

$$= \frac{1}{2} \int \{ \cos 4x - \cos 12x \} dx$$

On integrating we get,

$$= \frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right] + C$$

8. $\frac{1-\cos x}{1+\cos x}$

Solution:-

By standard trigonometric identify, we have,

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$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$$

We know that, $(\sin x/\cos x) = \tan x$

$$= 2 \tan^2 \frac{x}{2}$$

Also, we know that, $\tan^{-1}x = \sec x$

$$= \left(\sec^2 \frac{x}{2} - 1 \right)$$

Integrating both the sides, we get,

$$\therefore \int \frac{1-\cos x}{1+\cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2 \tan \frac{x}{2} - x + c$$

9. $\frac{\cos x}{1+\cos x}$

Solution:-

By standard trigonometric identify, we have,

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$$

We know that, $(\sin x / \cos x) = \tan x$ and takeout $\frac{1}{2}$ as common, we get

$$= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right]$$

Integrating both the sides, we get,

$$\int \frac{\cos x}{1+\cos x} dx = \int \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right] dx$$

Using standard trigonometric identify $\tan^2 x + 1 = \sec^2(x)$

$$= \frac{1}{2} \int \left[2 - \sec^2 \frac{x}{2} \right] dx$$

On integrating we get,

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$$= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C$$

$$= x - \tan \frac{x}{2} + C$$

10. $\sin^4 x$

Solution:-

By splitting the given function, we get,

$$\sin^4 x = \sin^2 x \sin^2 x$$

By standard trigonometric identify, we have, $\sin^2 x = (1 - \cos 2x)/2$

$$= \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 - \cos 2x}{2} \right)$$

$$= \frac{1}{4} (1 - \cos 2x)^2$$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$, we get,

$$= \frac{1}{4} [1 + \cos^2 2x - 2\cos 2x]$$

From the standard trigonometric identify, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 4x}{2} \right) - 2\cos 2x \right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right]$$

On simplifying, we get,

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right]$$

Integrating on both the sides,

$$\int \sin^4 x dx = \frac{1}{4} \int \left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right] dx$$

$$= \frac{1}{4} \left[\frac{3}{2} x + \frac{1}{2} \left(\frac{\sin 4x}{4} \right) - \frac{2\sin 2x}{2} \right] + C$$

By simplifying,

$$= \frac{1}{8} \left[3x + \left(\frac{\sin 4x}{4} \right) - 2\sin 2x \right] + C$$

$$= \frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + c$$

11. $\cos^4 2x$

Solution:-

By splitting the given function,

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$$\cos^4 2x = (\cos^2 2x)^2$$

By standard trigonometric identify, we have, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \left(\frac{1 + \cos 4x}{2}\right)^2$$

On Simplifying, we get,

$$= \frac{1}{4} [1 + \cos^2 4x - 2 \cos 4x]$$

By standard trigonometric identify, we have, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 8x}{2}\right) + 2 \cos 4x\right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{1}{2} \cos 8x + 2 \cos 4x\right]$$

By simplifying,

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{1}{2} \cos 8x + 2 \cos 4x\right]$$

Integrating both side,

$$\int \cos^4 2x dx = \int \left[\frac{3}{8} + \frac{1}{8} \cos 8x + \frac{1}{2} \cos 4x\right] dx$$

$$= \frac{3x}{8} + \frac{1}{64} \sin 8x + \frac{1}{8} \sin 4x + C$$

12. $\frac{\sin^2 x}{1 + \cos x}$

Solution:-

By standard trigonometric identify, we have,

$$\frac{\sin^2 x}{1 + \cos x} = \frac{(2 \sin \frac{x}{2} \cos \frac{x}{2})^2}{2 \cos^2 \frac{x}{2}}$$

$$= \frac{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

On simplifying, we get,

$$= 2 \sin^2 \frac{x}{2}$$

From the standard trigonometric identify, we have, $1 - \cos x = 2 \sin^2 \frac{x}{2}$

$$= 1 - \cos x$$

On integrating both the sides, we get,

$$\int \frac{\sin^2 x}{1 + \cos x} dx = \int (1 - \cos x) dx$$

$$= x - \sin x + C$$

13. $\frac{\cos 2x - \cos 2a}{\cos x - \cos a}$

Solution:-

By using the trigonometric identify i.e,

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

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So, we have,

$$\frac{\cos 2x - \cos 2a}{\cos x - \cos a} = \frac{-2 \sin \frac{2x+2a}{2} \sin \frac{2x-2a}{2}}{-1 \sin \sin \frac{x+a}{2} \sin \sin \frac{x-a}{2}}$$

By simplifying we get,

$$= \frac{\sin(x+a) \sin(x-a)}{\sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}$$

Then,

From the identify $\sin 2x = 2 \sin x \cos x$, we have

$$= \frac{\left[2 \sin\left(\frac{x+a}{2}\right) \cos\left(\frac{x+a}{2}\right)\right] \left[2 \sin\left(\frac{x-a}{2}\right) \cos\left(\frac{x-a}{2}\right)\right]}{\sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}$$

On simplifying, we get,

$$= 4 \cos\left(\frac{x+a}{2}\right) \cos\left(\frac{x-a}{2}\right)$$

By using the trigonometry identify $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$, we have

$$= 2 \left[\cos\left(\frac{x+a}{2} + \frac{x-a}{2}\right) + \cos\left(\frac{x+a}{2} - \frac{x-a}{2}\right) \right]$$

$$= 2[\cos(x) + \cos a]$$

$$= 2 \cos x + 2 \cos a$$

Then,

Integrating on both the sides,

$$\int \therefore \frac{\cos 2x - \cos 2a}{\cos x - \cos a} dx = \int (2 \cos x + 2 \cos a) dx$$

We have,

$$= 2 [\sin x + 2 \cos a] + C$$

14. $\frac{\cos x - \sin x}{1 + \sin 2x}$

Solution:-

Given = $\frac{\cos x - \sin x}{1 + \sin 2x}$

By using the standard trigonometric identify, $(1 + \sin 2x) = \sin^2 x + \cos^2 x + 2 \sin x \cos x$.

Then,

$$= \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x}$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Now,

Let us assume that $\sin x + \cos x = t$

Ans also, $(\cos x - \sin x) = dt$

Integrating on both the sides and substitute the value of $(\cos x - \sin x) dx$ and $(\sin x + \cos x)$ we get,

$$\begin{aligned} = \int \frac{\cos x - \sin x}{1 + \sin 2x} dx &= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx \\ &= \int \frac{dt}{t^2} \\ &= -t^{-1} + C \end{aligned}$$

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$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

15. $\tan^3 2x \sec 2x$ **Solution:-**

By splitting the given function, we have,

$$\tan^2 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$

From the standard trigonometric identity, $\tan^2 2x = \sec^2 2x - 1$,

$$= (\sec^2 2x - 1) \tan 2x \sec 2x$$

By multiplying, we get,

$$= (\sec^2 2x \times \tan 2x \sec 2x) - (\tan 2x \sec 2x)$$

Integrating both sides,

$$\int \tan^3 2x \sec 2x dx = \int \sec^2 2x \tan 2x \sec 2x dx - \int \tan 2x \sec 2x dx$$

$$= \int \sec^2 2x \tan 2x \sec 2x dx - \frac{\sec 2x}{2} + C$$

Then,

Let us assume $\sec 2x = t$

And also assume $2 \sec 2x \tan 2x dx = dt$

$$\int \tan^3 2x \sec 2x dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$

On simplifying we get,

$$= \frac{t^2}{6} - \frac{\sec 2x}{2} + C$$

$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

16. $\tan^4 x$ **Solution:-**

By splitting the given function, we have,

$$\tan^4 x = \tan^2 x \times \tan^2 x$$

Then,

From trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$$= (\sec^2 x - 1) \tan^2 x$$

By multiplying we get,

$$= \sec^2 x \tan^2 x - \tan^2 x$$

Again by using trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$$= \sec^2 x \tan^2 x - (\sec^2 x + 1)$$

$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

Now, integrating on both sides we get,

$$\int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx - \int 1 dx$$

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$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C$$

Then let us assume $\tan x = t$

And also assume $\sec^2 x dx = dt$

$$\int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

$$\int \tan^4 x dx = \frac{1}{3} = \frac{1}{3} \tan^3 x - \tan x + x + c$$

17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:-

By splitting up the given function,

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

By simplifying, we get,

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

We know that, $(\sin x / \cos x) = \tan x$ and $(1 / \cos x) = \sec x$

Again, we have $(\cos x / \sin x) = \cot x$ and $(1 / \sin x) = \operatorname{cosec} x$

$$= \tan x \sec x + \cot x \operatorname{cosec} x$$

Integrating on the both sides, we get

$$\begin{aligned} \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx &= \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx \\ &= \sec x - \operatorname{cosec} x + c \end{aligned}$$

18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Solution:-

By using the standard trigonometric identify, $2\sin^2 x = (1 - \cos 2x)$

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x} = \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x}$$

By simplification, we get,

$$= \frac{1}{\cos^2 x}$$

We know that, $(1 / \cos^2 x) = \sec^2 x$

$$= \sec^2 x$$

Integration on both sides, we get,

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx$$

$$= \tan x + C$$

19. $\frac{1}{\sin x \cos^3 x}$

Solution:-

For further simplification, the given function can be written as,

$$\frac{1}{\sin x \cos^3 x} = \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x}$$

Divide both numerator and denominator by $\cos^2 x$

$$= \tan x \sec^2 x + \frac{\frac{2}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}$$

On simplification, we get,

$$= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}$$

By applying the integrals we get,

$$\int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

Let us assume that, $\tan x = t$,

Then, $\sec^2 x dx = dt$

By substituting above values we get,

$$\int \frac{1}{\sin x \cos^3 x} dx = \int t dt + \int \frac{1}{t} dt$$

On integrating,

$$= \frac{t^2}{2} + \log|t| + C$$

Now, by substituting the value of 't' we get,

$$= \frac{1}{2} \tan^2 x + \log|\tan x| + C$$

20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$

Solution:-

We know that, $(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2\sin x \cos x$

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x}$$

Ans also we know that, $\cos^2 x + \sin^2 x = 1$ and $2\sin x \cos x = \sin 2x$,

Then,

$$= \frac{\cos 2x}{1 + \sin 2x}$$

By applying the integrals, we get,

$$\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$$

Let us assume that, $1 + \sin 2x = t$

So, $2\cos 2x dx = dt$

By substituting that, $1 + \sin 2x = t$

$$\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \frac{1}{2} \int \frac{1}{t} dt$$

On integrating,

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$$\frac{1}{2} \log |t| + C$$

Now, by substituting the value of 't' we get,

$$= \frac{1}{2} \log |1 + \sin 2x| + C$$

$$= \frac{1}{2} \log |(\cos x + \sin x)^2| + C$$

$$= \log |\sin x + \cos x| + C$$

21. $\sin^{-1}(\cos x)$

Solution:-

Given, $\sin^{-1}(\cos x)$

Let us assume $\cos x = t$ [equation (i)]

Then, substitute 't' in place of $\cos x$

$$= \sin^{-1}(t)$$

$$\sin x = \sqrt{1 - t^2}$$

So, now differentiating both sides of (i), we get,

$$(-\sin x) dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

By applying the integrals, we get,

$$\int \sin^{-1}(\cos x) dx = \int \sin^{-1}t \left(\frac{-dt}{\sqrt{1-t^2}} \right) = \int \frac{\sin^{-1}t}{\sqrt{1-t^2}} dt$$

Let us assume that, $\sin^{-1} t = v$

$$\frac{dt}{\sqrt{1-t^2}} = dv$$

$$\int \sin^{-1}(\cos x) dx = - \int v dv$$

On integrating,

$$= - \frac{v^2}{2} + C$$

Now, by substituting the value of 'v' and 't', we get,

$$= - \frac{(\sin^{-1}t)^2}{2} + C$$

$$= - \frac{(\sin^{-1}(\cos x))^2}{2} + C$$

.....[equation (ii)]

As we know that,

$$\sin^{-1}x + \cos^{-1}x = \frac{x}{2}$$

22. $\frac{1}{\cos(x-a)\cos(x-b)}$

Solution:-

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Multiplying and dividing by $\sin(a-b)$ to given function, we get,

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right]$$

For further simplification, the given function can be written as,

$$= \frac{1}{\sin(a-b)} \left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} \right]$$

Using $\sin(A-B) = \sin A \cos B - \cos A \sin B$ formula, we get,

$$= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right]$$

We know that, $\sin x / \cos x = \tan x$ by applying this formula we get,

$$= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)]$$

Taking integrals,

$$\int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)} \int [\tan(x-b) - \tan(x-a)] dx$$

On integrating,

$$= \frac{1}{\sin(a-b)} [-\log|\cos(x-b)| + \log|\cos(x-a)|]$$

We know that, $\log(a/b) = \log a - \log b$, using in above equation, we get,

$$= \frac{1}{\sin(a-b)} \left[\log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + C$$

Choose the correct answer in Exercise 23 and 24.

23. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

- a) $\tan x + \cot x + C$
c) $-\tan x + \cot x + C$

- b) $\tan x + \operatorname{cosec} x + C$
d) $\tan x + \sec x + C$

Solution:-

(a) $\tan x + \cot x + C$

By splitting the denominators of given equations,

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

On simplifying we get,

$$= \int (\sec^2 x - \operatorname{cosec}^2 x) dx$$

As we know that,

$$\int \sec^2 x dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x dx = -\cot x + c$$

$$= \tan x + \cot x + C$$

24. $\int \frac{e^x(1+X)}{\cos^2(e^x X)} dx$ equals

- a) $-\cot(e^x) + C$
c) $\tan(e^x) + C$

- b) $\tan(xe^x) + C$
d) $\cot(e^x) + C$

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Solution:-

$$(b) \tan(xe^x) + C$$

Let us assume that, $(xe^x) = t$

Differentiating both sides we get,

$$((e^x \times x) + (e^x + 1)) dx = dt$$

$$e^x(x + 1) = dt$$

Applying integrals,

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

We know that, $(1/\cos^2 t) = \sec^2 t$

$$= \int \sec^2 t \cdot dt$$

$$= \tan t + C$$

Substituting the value of 't',

$$= \tan(e^x x) + C$$

Exercise 7.4

Integrate the function in Exercise 1 to 23.

$$1. \frac{3x^2}{x^6+1}$$

Solution:-

Let us assume that $x^3 = t$

Then, $3x^2 dx = dt$

By applying integrals, we get,

$$\int \frac{3x^2}{x^6+1} dx = \int \frac{dt}{t^2+1}$$

On integrating,

$$= \tan^{-1} t + C$$

No, substitute integrals, we get,

$$= \tan^{-1}(x^3) + C$$

$$2. \frac{1}{\sqrt{1+4x^2}}$$

Solution:-

Take $2x = t$

We get $2x dx = dt$

Integrating both sides

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$$\int \frac{1}{\sqrt{1+a^2}} dt = \log|x + \sqrt{x^2 + a^2}|$$

We get

$$= \frac{1}{2} [\log|t + \sqrt{t^2 + 1}|] + C$$

Substituting the value of t

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 1}| + C$$

3. $\frac{1}{\sqrt{(2-x)^2+1}}$

Solution:-

Take $2 - x = t$

We get $-dx = dt$

Integrating both sides

$$\int \frac{1}{\sqrt{(2-x)^2+1}} dx = - \int \frac{1}{\sqrt{t^2+1}} dt$$

Using the formula

$$\int \frac{1}{\sqrt{(2-x)^2+1}} dx = - \int \frac{1}{\sqrt{t^2+1}} dt$$

Using the formula

$$\int \frac{1}{\sqrt{x^2+a^2}} dt = \log|x + \sqrt{x^2 + a^2}|$$

We get

$$= -\log|t + \sqrt{t^2 + 1}| + C$$

Substituting the value of t

$$= -\log|2 - x + \sqrt{(2 - x)^2 + 1}| + C$$

$$= \log\left|\frac{1}{(2-x) + \sqrt{x^2 - 4x + 5}}\right| + C$$

4. $\frac{1}{\sqrt{9-25x^2}}$

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Solution:-

Take $5x = t$

We get $5dx = dt$

Integrating both sides

$$\int \frac{1}{\sqrt{9-25x^2}} dx = \frac{1}{5} \int \frac{1}{\sqrt{9-t^2}} dt$$

We get

$$= \frac{1}{5} \int \frac{1}{\sqrt{3^2-t^2}} dt$$

On further calculation

$$= \frac{1}{5} \sin^{-1}\left(\frac{t}{3}\right) + C$$

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Substituting the value of t
 $= \frac{1}{5} \sin^{-1} \left(\frac{5x}{3} \right) + C$

5. $\frac{2x}{1+2x^4}$

Solution:-

Take $\sqrt{2} x^2 = t$
 We get $2\sqrt{2} x dx = dt$
 Integrating both sides
 $\int \frac{3x}{1+2x^4} dx = \frac{2}{2\sqrt{2}} \int \frac{dt}{1+t^2}$
 On Further calculation
 $= \frac{3}{2\sqrt{2}} [\tan^{-1}t] + C$
 Substituting the value of t
 $= \frac{3}{2\sqrt{2}} \tan^{-1}(\sqrt{2}x^2) + C$

6. $\frac{x^2}{1-x^6}$

Solution:-

Take $x^3 = t$
 We get $3x^2 dx = dt$
 Integrating both sides
 $\int \frac{x^2}{1-x^6} dx = \frac{1}{3} \int \frac{dt}{1-t^2}$
 On further calculation
 $= \frac{1}{3} \left[\frac{1}{2} \log \left| \frac{1+t}{1-t} \right| \right] + C$
 Substituting the value of t
 $= \frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C$

7. $\frac{x-1}{\sqrt{x^2-1}}$

Solution:-

By separating the terms
 $\int \frac{x-1}{\sqrt{x^2-1}} dx = \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx$ (1)
 Take
 $\int \frac{x}{\sqrt{x^2-1}} dx$

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If $x^2 - 1 = t$ we get $2x dx = dt$

$$\int \frac{x}{\sqrt{x^2-1}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

It can be written as

$$= \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

By integration

$$= \frac{1}{2} [2t^{\frac{1}{2}}]$$

$$= \sqrt{t}$$

Substituting the value of t

$$= \sqrt{x^2 - 1}$$

Using equation (1) we get

$$\int \frac{x-1}{\sqrt{x^2-1}} dx = \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx$$

From formula

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \log |x + \sqrt{x^2 - a^2}|$$

We get

$$= \sqrt{x^2 - 1} - \log |x + \sqrt{x^2 - 1}| + C$$

8. $\frac{x^2}{\sqrt{x^6+a^6}}$

Solution:-

Take $x^3 = t$

We get $3x^2 dx = dt$

Integrating both sides

$$\int \frac{x^2}{\sqrt{x^6+a^6}} dx = \frac{1}{3} \int \frac{dt}{\sqrt{t^2+(a^3)^2}}$$

On further calculation

$$= \frac{1}{3} \log |t + \sqrt{t^2 + a^6}| + C$$

Substituting the value of t

$$= \frac{1}{3} \log |x^3 + \sqrt{x^6 + a^6}| + C$$

9. $\frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

Solution:-

Take $\tan x = t$

We get $\sec^2 x dx = dt$

Integrating both sides

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

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On further calculation

$$= \log|t + \sqrt{t^2 + 4}| + C$$

Substituting the value of t

$$= \log|\tan x + \sqrt{\tan^2 x + x^4}| + C$$

10. $\frac{1}{\sqrt{x^2+2x+2}}$

Solution:-

It is given that

$$\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}} dx$$

Take $x + 1 = t$

We get $dx = dt$

Integrating both sides

$$\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{t^2+1}} dt$$

On further calculation

$$= \log|t + \sqrt{t^2 + 1}| + C$$

Substituting the value of t

$$= \log|(x + 1) + \sqrt{(x + 1)^2 + 1}| + C$$

So we get

$$= \log|(x + 1) + \sqrt{x^2 + 2x + 2}| + C$$

11. $\frac{1}{9x^2+6x+5}$

Solution:-

It is given that

$$\int \frac{1}{9x^2+6x+5} dx = \int \frac{1}{t^2+2^2} dx$$

On further calculation

$$= \frac{1}{3} \left[\frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right) \right] + C$$

Substituting the value of t

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x+1}{2} \right) + C$$

12. $\frac{1}{\sqrt{7-6x-x^2}}$

Solution:-

It is given that

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$$\frac{1}{\sqrt{7-6x-x^2}}$$

We can write it as

$$7 - 6x - x^2 = 7 - (x^2 + 9 - 9)$$

By further calculation

$$= 10 - (x^2 + 6x - 9)$$

We get write it as

$$7 - 6x - x^2 = 7 - (x^2 + 6x + 9 - 9)$$

By further calculation

$$= 16 - (x^2 + 6x - 9)$$

We get

$$= 16 - (x + 3)^2$$

$$= 4^2 - (x + 3)^2$$

Here

$$\int \frac{1}{\sqrt{7-6x-x^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}}$$

Consider $x + 3 = t$

We get $dx = dt$

Integrating both sides

$$\int \frac{1}{(4)^2 - (x+3)^2} dx = \int \frac{1}{(4)^2 - (t)^2} dt$$

We get

$$= \sin^{-1} \left(\frac{1}{4} \right) + C$$

Substituting the value of t

$$= \sin^{-1} \left(\frac{x+3}{4} \right) + C$$

13. $\frac{1}{\sqrt{(x-1)(x-2)}}$

Solution:-

It is given that

$$\frac{1}{\sqrt{(x-1)(x-2)}}$$

We can write as

$$(x - 1)(x - 2) = x^2 - 3x + 2$$

By further calculation

$$= x^2 - 3x + 9/4 - 9/4 + 2$$

We get

$$= \left(x - \frac{3}{2} \right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{3}{2} \right)^2 - \left(\frac{1}{2} \right)^2$$

Here

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$$\int \frac{1}{\sqrt{(x-1)(x-2)}} dx = \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

Consider $x - 3/2 = t$

We get $dx = dt$

Integrating both sides

$$\int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx = \int \frac{1}{\sqrt{t^2 - \left(\frac{1}{2}\right)^2}} dt$$

We get

$$= \log \left| t + \sqrt{t^2 - \left(\frac{1}{2}\right)^2} \right| + C$$

Substituting the value of t

$$= \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2} \right|$$

14. $\frac{1}{\sqrt{8+3x-x^2}}$

Solution:-

It is given that

$$\frac{1}{\sqrt{8+3x-x^2}}$$

We can write it as

$$8 + 3x - x^2 = 8 - \left(x^2 - 3x + \frac{9}{4} - 9/4\right)$$

By further calculation

$$= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$$

Here

$$\frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx$$

Consider $x - 3/2 = t$

We get $dx = dt$

Integrating both sides

$$\int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt$$

We get

$$= \sin^{-1} \left(\frac{t}{\frac{\sqrt{41}}{2}} \right) + C$$

Substituting the value of t

$$= \sin^{-1} \left(\frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}} \right) + C$$

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On further calculation

$$= \sin^{-1} \left(\frac{2x-3}{\sqrt{41}} \right) + C$$

15. $\frac{1}{\sqrt{(x-a)(x-b)}}$

Solution:-

It is given that

$$\frac{1}{\sqrt{(x-a)(x-b)}}$$

We can write it as

$$(x-a)(x-b) = x^2 - (a+b)x + ab$$

By further calculation

$$= x^2 - (a+b)x + \frac{(a+b)^2}{4} - \frac{(a+b)^2}{4} + ab$$

Here

$$= \left[x + \frac{(a+b)}{2} \right]^2 - \frac{(a-b)^2}{4}$$

Integrating both sides

$$\int \frac{1}{\sqrt{(x-a)(x-b)}} dx = \int \frac{1}{\sqrt{\left\{ x - \frac{(a+b)}{2} \right\}^2 - \left(\frac{a-b}{2} \right)^2}}$$

Consider

$$x - \frac{(a+b)}{2} = t$$

We get $dx = dt$

$$\int \frac{1}{\sqrt{\left\{ x - \frac{(a+b)}{2} \right\}^2 - \left(\frac{a-b}{2} \right)^2}} = \int \frac{1}{\sqrt{t^2 - \left(\frac{a-b}{2} \right)^2}} dt$$

It can be written as

$$= \log \left| t + \sqrt{t^2 - \left(\frac{a-b}{2} \right)^2} \right| + C$$

Substituting the value of t

$$= \log \left| x - \frac{(a+b)}{2} + \sqrt{(x-a)(x-b)} \right| + C$$

16. $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Solution:-

Consider

$$4x + 1 = A \frac{d}{dx} (2x^2 + x - 3) + B$$

So we get

$$4x + 1 = A (4x + 1) + B$$

On further calculation

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$$4x + 1 = 4Ax + A + B$$

By equating the coefficients of x and constant term on both sides

$$4A = 4$$

$$A = 1$$

$$A + B = 1$$

$$B = 0$$

Take $2x^2 + x - 3 = t$

By differentiation

$$(4x + 1) dx = dt$$

Integrating both sides

$$\frac{4x+1}{\sqrt{2x^2+x-3}} dx = \int \frac{1}{\sqrt{t}} dt$$

We get

$$= 2\sqrt{t} + C$$

Substituting the value of t

$$= 2\sqrt{2x^2 + x - 3} + C$$

17. $\frac{x+2}{\sqrt{x^2-1}}$

Solution:-

Consider

$$x+2 = A \frac{d}{dx}(x^2 - 1) + B \quad \dots(1)$$

It can be written as

$$x + 2 = A(2x) + B$$

Now equating the coefficients of x and constant term on both sides

$$2A = 1$$

$$A = \frac{1}{2}$$

$$B = 2$$

Using equation (1) we get

$$(x+2) = \frac{1}{2}(2x) + 2$$

Integrating both sides

$$\frac{x+2}{\sqrt{x^2-1}} dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}} dx$$

Separating the terms

$$= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx = \int \frac{2}{\sqrt{x^2-1}} dx \quad \dots\dots (2)$$

Take

$$= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx$$

If $x^2 - 1 = t$ we get $2x dx = dt$

So we get

$$= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

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By Integration

$$= \frac{1}{2} [2\sqrt{t}]$$

$$= \sqrt{t}$$

Substituting the value of t

$$= \sqrt{x^2 - 2}$$

We can write it as

$$\int \frac{2}{\sqrt{x^2-1}} dx = 2 \int \frac{1}{\sqrt{x^2-1}} dx = 2 \log |x + \sqrt{x^2 - 1}|$$

Using equation (2) we get

$$\int \frac{x+2}{\sqrt{x^2-1}} dx = \sqrt{x^2 - 1} + 2 \log |x + \sqrt{x^2 - 1}| + C$$

18. $\frac{5x-2}{1+2x+3x^2}$

Solution:-

Consider

$$5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$$

It can be written as

$$5x - 2 = A(2 + 6x) + B$$

Now equating the coefficient of x and constant term on both sides

$$5 = 6A$$

$$A = 5/6$$

$$2A + B = -2$$

$$B = -11/2$$

Using equation (1) we get

$$5x - 2 = \frac{5}{6}(2 + 6x) + \left(-\frac{11}{2}\right)$$

Integrating both sides

$$\int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{2}}{1+2x+3x^2} dx$$

Squaring the terms

$$= \frac{5}{6} \int \frac{(2+6x)}{1+2x+3x^2} dx - \frac{11}{3} \int \frac{1}{1+2x+3x^2} dx$$

We know that

$$I_1 = \int \frac{2+6x}{1+2x+3x^2} dx \text{ and } I_2 = \int \frac{1}{1+2x+3x^2} dx$$

$$\int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \dots(1)$$

Take

$$I_1 = \int \frac{2+6x}{1+2x+3x^2} dx$$

If $1 + 2x + 3x^2 = t$ we get $(2 + 6x) dx = dt$

So we get

$$I_1 = \int \frac{dt}{t}$$

By integration

$$I_1 = \log|t|$$

Substituting the value of t

$$I_1 = \log|1 + 2x + 3x^2| \quad \dots\dots (2)$$

Take

$$I_1 = \log \int \frac{1}{1+2x+3x^2} dx$$

$$1+2x+3x^2 = 1 + 3(x^2 + 2/3 x)$$

By addition and subtraction of 1/9

$$= 1 + 3\left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9}\right)$$

We get

$$= 1 + 3\left(x + \frac{1}{3}\right)^2 = \frac{1}{3}$$

On further calculation

$$= \frac{2}{3} + 3\left(x + \frac{1}{3}\right)^2$$

Here

$$= 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{2}{9}\right]$$

$$= 3\left[\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2\right]$$

By integration

$$I_2 = \frac{1}{3} \int \frac{1}{\left[\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2\right]} dx$$

So we get

$$= \frac{1}{3} \left[\frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \left(\frac{x + \frac{1}{3}}{\frac{\sqrt{2}}{3}} \right) \right]$$

By taking LCM

$$= \frac{1}{3} \left[\frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) \right]$$

On further calculation

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) \quad \dots\dots(3)$$

Now substituting the equations (2) and (3) in equation (1)

$$\int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6} [\log|1 + 2x + 3x^2|] - \frac{11}{3} \left[\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) \right] + C$$

We get

$$= \frac{5}{6} \log|1 + 2x + 3x^2| - \frac{11}{3} \left[\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) \right] + C$$

19. $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Solution:-

It is given that

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$$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$$

Consider

$$6x+7 = A \frac{d}{dx}(x^2 - 9x + 20) + B$$

It can be written as

$$6x + 7 = A(2x - 9) + B$$

Now equating the coefficient of x and constant term on both sides

$$2A = 6$$

$$A = 3$$

$$-9A + B = 7$$

$$B = 34$$

Using equation (1) we get

$$6x + 7 = 3(2x - 9) + 34$$

Integrating both sides

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} dx = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}} dx$$

Separating the terms

$$= 3 \int \frac{2x-9}{x^2-9x+20} dx + 34 \int \frac{1}{x^2-9x+20} dx$$

We know that

$$I_1 = \int \frac{2x-9}{x^2-9x+20} dx \text{ and } I_2 = \int \frac{1}{x^2-9x+20} dx$$

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} dx = 3I_1 + 34I_2 \quad \dots (1)$$

Take

$$I_1 = \int \frac{2x-9}{x^2-9x+20} dx$$

If $x^2 - 9x + 20 = t$ we get $(2x-9)dx = dt$

So we get

$$I_1 = \frac{dt}{\sqrt{t}}$$

By integration

$$I_1 = 2\sqrt{t}$$

Substituting the value of t

$$I_1 = 2\sqrt{x^2 - 9x + 20} \quad \dots (2)$$

Take

$$I_2 = \int \frac{1}{x^2-9x+20} dx$$

By addition and subtraction of $\frac{81}{4}$

$$x^2 - 9x + 20 = x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

By integration

$$I_2 = \int \frac{1}{\sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

So we get

$$I_2 = \log \left| \left(x - \frac{9}{2} \right) - \sqrt{x^2 - 9x + 20} \right| \quad \dots (3)$$

Now substituting the equations (2) and (3) in equation (1)

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} dx = 3[2\sqrt{x^2 - 9x + 20}] + 34 \log \left[\left(x - \frac{9}{2} \right) - \sqrt{x^2 - 9x + 20} \right] + C$$

We get

$$6\sqrt{x^2 - 9x + 20} + 34 \log \left[\left(x - \frac{9}{2} \right) - \sqrt{x^2 - 9x + 20} \right] + C$$

20. $\frac{x+2}{\sqrt{4x-x^2}}$

Solution:-

Consider

$$X+2 = A \frac{d}{dx} (4x + x^2) + B$$

It can be written as

$$X + 2 = A(4 - 2x) + B$$

Now equating the coefficient of x and constant term in both sides

$$-2A = 1$$

$$A = -1/2$$

$$4A + B = 2$$

$$B = 4$$

Using equation (1) we get

$$(x + 2) = -\frac{1}{2}(4 - 2x) + 4$$

Integrating both sides

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}} dx$$

Separating the terms

$$= -\frac{1}{2} \int \frac{4+2x}{\sqrt{4x-x^2}} dx + 4 \int \frac{1}{\sqrt{4x-x^2}} dx$$

We know that

$$I_1 = \int \frac{4+2x}{\sqrt{4x-x^2}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{4x-x^2}} dx$$

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2} I_1 + 4I_2 \quad \dots (1)$$

Take

$$I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx$$

$$\text{If } 4x - x^2 = t \text{ we get } (4 - 2x) dx = dt$$

So we get

$$I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx$$

$$\text{If } 4x - x^2 = t \text{ we get } (4 - 2x) dx = dt$$

So we get

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t}$$

Substituting the value of t

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$$= 2\sqrt{4x - x^2} \dots\dots (2)$$

Take

$$I_2 = \int \frac{1}{4x-x^2}$$

$$4x - x^2 = -(-4x + x^2)$$

By addition and subtraction of 4

$$4x - x^2 = (-4x + x^2 + 4 - 4)$$

It can be written as

$$= 4-(x - 2)^2$$

By integration

$$I_2 = \int \frac{1}{\sqrt{(2)^2-(x-2)^2}}$$

So we get

$$= \sin^{-1} \left(\frac{x-2}{2} \right) \dots\dots(3)$$

Now substituting the equations (2) and (3) in equation (1)

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2} (2\sqrt{4x - x^2}) + 4 \sin^{-1} \left(\frac{x-2}{2} \right) + C$$

We get

$$= \sqrt{4x-x^2} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) + C$$

21. $\frac{(x+2)}{\sqrt{x^2+2x+3}}$

Solution:-

It is given that

$$\int \frac{(x+2)}{\sqrt{x^2+2x+3}} dx$$

By multiplying and dividing by 2

$$= \frac{1}{2} \int \frac{2(x+2)}{\sqrt{x^2+2x+3}} dx$$

Multiplying the terms

$$\frac{1}{2} \int \frac{2x+4}{\sqrt{x^2+2x+3}} dx$$

Separating the terms

$$= \frac{1}{2} \int \frac{2x+2}{x^2+2x+3} dx + \frac{1}{2} \int \frac{2}{x^2+2x+3} dx$$

We get

$$= \frac{1}{2} \int \frac{2x+2}{x^2+2x+3} dx + \int \frac{2}{x^2+2x+3} dx$$

We know that

$$I_1 = \int \frac{2x+2}{x^2+2x+3} dx + I_2 = \int \frac{1}{x^2+2x+3} dx$$

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} I_1 + I_2 \dots\dots (1)$$

Take

$$I_1 = \int \frac{2x+2}{x^2+2x+3} dx$$

Here $x^2 + 2x + 3 = t$

We get $(2x+ 2) dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t}$$

Substituting the value of t

$$= 2 \sqrt{x^2 + 2x + 3} \quad \dots\dots (2)$$

Take

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

We can write it as

$$x^2 + 2x + 3 = x^2 + 2x + 1 + 2$$

$$= (x + 1)^2 + (\sqrt{2})^2$$

By integration

$$= \log |(x + 1) + \sqrt{x^2 + 2x + 3}| \quad \dots\dots (3)$$

By using equations (2) and (3) in (1) we get

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} \{2\sqrt{x^2 + 2x + 3}\} + \log |(x + 1) + \sqrt{x^2 + 2x + 3}| + C$$

So we get

$$= \sqrt{x^2 + 2x + 3} + \log |(x + 1) + \sqrt{x^2 + 2x + 3}| + C$$

22. $\frac{x+3}{x^2-2x-5}$

Solution:-

Consider

$$(x+3) = A \frac{d}{dx} (x^2 - 2x - 5) + B$$

It can be written as

$$X + 3 = A (2x - 2) + B$$

Now equating the coefficients of x and constant term on both sides

$$2A = 1$$

$$A = \frac{1}{2}$$

$$-2A + B = 3$$

$$B = 4$$

Using equation (1) we get

$$(x + 3) = \frac{1}{2} (2x - 2) + 4$$

Integrating both sides

$$\int \frac{x+3}{x^2-2x-5} dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5} dx$$

Separating the terms

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x-5} dx + 4 \int \frac{1}{x^2-2x-5} dx$$

We know that

$$I_1 = \int \frac{2x-2}{x^2-2x-5} dx \text{ and } I_2 = \int \frac{1}{x^2-2x-5} dx$$

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$$\int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2}I_1 + 4I_2 \quad \dots (1)$$

Take

$$I_1 = \int \frac{2x-2}{x^2-2x-5} dx$$

If $x^2 - 2x - 5 = t$ we get $(2x - 2)dx = dt$

So we get

$$I_1 = \int \frac{dt}{t} = \log|t|$$

Substituting the value of t

$$= \log|x^2 - 2x - 5| \quad \dots (2)$$

Take

$$I_2 = \int \frac{1}{x^2-2x-5} dx$$

We can write it as

$$I_2 = \int \frac{1}{(x^2-2x+1)-6} dx$$

By separating the terms

$$= \int \frac{1}{(x-1)^2 - (\sqrt{6})^2} dx$$

By integration

$$= \frac{1}{2\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| \quad \dots (3)$$

Now substituting the equation (2) and (3) in equation (1)

$$\int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2} \log|x^2 - 2x - 5| + \frac{4}{2\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

We get

$$= \frac{1}{2} \log|x^2 - 2x - 5| + \frac{2}{\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

23. $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Solution:-

Consider

$$5x + 3 = A \frac{d}{dx} (x^2 + 4x + 10) + B$$

It can be written as

$$5x + 3 = A(2x + 4) + B$$

Now equating the coefficients of x and constant term on both sides

$$2A = 5$$

$$A = 5/2$$

$$4A + B = 3$$

$$B = -7$$

Using equation (1) we get

$$5x + 3 = \frac{5}{2}(2x + 4) - 7$$

Integrating both sides

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$$\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}} dx$$

Separating the terms

$$= \frac{5}{2} \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - 7 \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

We know that

$$I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \frac{5}{2} I_1 - 7I_2 \quad \dots(1)$$

Take

$$I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx$$

If $x^2 + 4x + 10 = t$ we get $(2x + 4)dx = dt$

So we get

$$I_1 = \int \frac{dt}{t} = 2\sqrt{t}$$

Substituting the value of t

$$= 2\sqrt{x^2 + 4x + 10} \quad \dots(2)$$

Take

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

We can write it as

$$= \int \frac{1}{\sqrt{(x^2+4x+4)+6}} dx$$

By separating the terms

$$= \int \frac{1}{(x+2)^2+(\sqrt{6})^2} dx$$

By integration

$$= \log|x + 2 + \sqrt{x^2 + 4x + 10}| \quad \dots (3)$$

Now substituting the equation (2) and (3) in equation (1)

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \frac{5}{2} [2\sqrt{x^2 + 4x + 10}] - 7 \log |(x + 2) + \sqrt{x^2 + 4x + 10}| + C$$

We get

$$= 5\sqrt{x^2 + 4x + 10} - 7\log|(x + 2) + \sqrt{x^2 + 4x + 10}| + c$$

Choose the correct answer in Exercise 24 and 25.

24. $\int \frac{dx}{x^2+2x+2}$ equals

- a) $x \tan^{-1}(x+1) + C$
c) $(x+1) \tan^{-1}x + C$

- b) $\tan^{-1}(x+1)+C$
d) $\tan^{-1}x + C$

Solution:-

It is given that

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$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$$

By Separating the terms

$$= \int \frac{1}{(x+1)^2+(1)^2} dx$$

By integrating we get

$$= [\tan^{-1}(x+1)] + c$$

Therefore, B is the correct answer.

25. $\int \frac{dx}{\sqrt{9x-4x^2}}$ Equals

a) $\frac{1}{9} \sin^{-1} \left(\frac{9x-8}{8} \right) + C$

c) $\frac{1}{3} \sin^{-1} \left(\frac{9x-8}{8} \right) + C$

b) $\frac{1}{2} \sin^{-1} \left(\frac{8x-9}{9} \right) + C$

d) $\frac{1}{2} \sin^{-1} \left(\frac{9x-8}{8} \right) + C$

Solution:-

It is given that

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

We can write it as

$$= \int \frac{1}{\sqrt{-4\left(x^2 - \frac{9x}{4}\right)}}$$

By further calculation we get

$$= \int \frac{1}{-4\left(x^2 - \frac{9x}{4} + \frac{81}{64} - \frac{81}{64}\right)} dx$$

Separating the terms we get

$$= \int \frac{1}{\sqrt{-4\left[\left(x - \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2\right]}}$$

On further simplification

$$= \frac{1}{2} \int \frac{1}{\sqrt{\left[\left(\frac{9}{8}\right)^2 - \left(x - \frac{9}{8}\right)^2\right]}} dx$$

Using the formula

$$\int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1} \frac{y}{a} + C$$

$$= \frac{1}{2} \left[\sin^{-1} \left(\frac{x - \frac{9}{8}}{\frac{9}{8}} \right) \right] + C$$

Taking LCM

$$= \frac{1}{2} \sin^{-1} \left(\frac{8x-9}{9} \right) + C$$

Therefore, B is the correct answer.

Exercise 7.5

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Integrate the rational functions in Exercise 1 to 21.

$$1. \frac{x}{(x+1)(x+2)}$$

Solution: –

Consider

$$\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

We get

$$x = A(x+2) + B(x+1)$$

Now by equating the coefficient of x and constant term, we get

$$A + B = 1$$

$$2A + B = 0$$

By solving the equations we get

$$A = -1 \text{ and } B = 2$$

Substituting the values of A and B

$$\frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

By integrating both sides w.r.t.x

$$\int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx$$

So we get

$$= -\log|x+1| + 2\log|x+2| + C$$

We can write it as

$$= \log(x+2)^2 - \log|x+2| + C$$

$$= \log \frac{(x+2)^2}{(x+1)} + C$$

$$2. \frac{1}{(x+3)(x-3)}$$

Solution:-

$$\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$$

We get

$$1 = A(x-3) + B(x+3)$$

Now by equating the coefficient of x and constant term, we get

$$A + B = 1$$

$$-3A + 3B = 0$$

By solving the equations we get

$$A = -1/6 \text{ and } B = 1/6$$

Substituting the value of A and B

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$$\frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

By integrating both sides w.r.t.x

$$\int \frac{1}{(x^2-9)} dx = \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx$$

So we get

$$= -\frac{1}{6} \log |x + 3| + \frac{1}{6} \log |x - 3| + C$$

We can write it as

$$= \frac{1}{6} \log \left| \frac{(x-3)}{(x+3)} \right| + C$$

$$3. \frac{3x-1}{(x-1)(x-2)(x-3)}$$

Solution:-

Consider

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

We get

$$3x - 1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \dots (1)$$

By substituting the value of x in equation (1), we get

$$A = 1, B = -5 \text{ and } C = 4$$

Substituting the values of A, B and C

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

By integrating both sides w.r.t.x

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

So we get

$$= \log |x - 1| - 5 \log |x - 2| + 4 \log |x - 3| + C$$

$$4. \frac{x}{(x-1)(x-2)(x-3)}$$

Solution:-

Consider

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

We get

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \dots (1)$$

By substituting the value of x in equation (1), we get

$$A = 1/2, B = -2 \text{ and } C = 3/2$$

Substituting the value of A, B and C

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} + \frac{2}{(x-2)} + \frac{3}{(x-3)}$$

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By integrating both sides w.r.t.x

$$\int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{2}{2(x-3)} \right\} dx$$

So we get

$$= \frac{1}{2} \log |x - 1| - 2 \log |x - 2| + \frac{3}{2} \log |x - 3| + C$$

5. $\frac{2x}{x^2+3x+2}$

Solution:-

Consider

$$\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

We get

$$2x = A(x+2) + B(x+1) \dots\dots(1)$$

By substituting the value of x in equation (1), we get

$$A = -2 \text{ and } B = 4$$

Substituting the value of A and B

$$\frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

By integrating both sides w.r.t.x .

$$\int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

So we get

$$= 4 \log |x + 2| - 2 \log |x + 1| + C$$

6. $\frac{1-x^2}{x(1-2x)}$

Solution:-

Consider

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{x} + \frac{1}{2} \left(\frac{2-x}{1-2x} \right)$$

We know that

$$\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

We get

$$(2-x) = A(1-2x) + Bx \dots\dots(1)$$

By substituting the value of x in equation (1), we get

$$A = 2 \text{ and } B = 3$$

Substituting the values of A and B

$$\frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

We get

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left\{ \frac{2}{x} + \frac{3}{1-2x} \right\}$$

By integrals both sides w.r.t.x

$$\int \frac{1-x^2}{x(1-2x)} dx = \int \left\{ \frac{1}{2} + \frac{1}{2} \left(\frac{2}{x} + \frac{3}{1-2x} \right) \right\} dx$$

By further calculation

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)} \log|1-2x| + C$$

So we get

$$= \frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + c$$

7. $\frac{x}{(x^2+1)(x-1)}$

Solution:-

We know that

$$\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

It can be written as

$$x = (Ax+B)(x-1) + C(x^2+1)$$

By multiplying the terms

$$x = Ax^2 - Ax + Bx - B + Cx^2 + C$$

Now by equating the coefficient of x^2 , x and constant terms we get

$$A + C = 0$$

$$-A + B = 1$$

$$-B + C = 0$$

By solving the equations

$$A = \frac{1}{2}, B = \frac{1}{2} \text{ and } C = \frac{1}{2}$$

Using Equations (1)

$$\frac{x}{(x^2+1)(x-1)} = \frac{\left(\frac{1}{2}x + \frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{x-1}$$

By integrating both sides w.r.t.x

$$\int \frac{x}{(x^2+1)(x-1)} = \frac{1}{2} \int \frac{x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx$$

We get

$$= -\frac{1}{4} \int \frac{2x}{x^2+1} dx + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x-1| + C$$

Here

$$\int \frac{2x}{x^2+1} dx, \text{ let } (x^2+1) = t$$

We get

$$2x dx = dt$$

Substituting the values

$$\int \frac{2x}{x^2+1} dx = \int \frac{dt}{t}$$

By integrating w.r.t.x

$$= \log|t|$$

Substituting the value of t

$$= \log|x^2 + 1|$$

So we get

$$\int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{4} \log|x^2 + 1| + \frac{1}{2} \tan^{-1}x + \frac{1}{2} \log|x - 1| + C$$

We can write it as

$$= \frac{1}{2} \log|x - 1| - \frac{1}{4} \log|x^2 + 1| + \frac{1}{2} \tan^{-1}x + C$$

8. $\frac{x}{(x-1)^2(x+2)}$

Solution:-

We know that

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

It can be written as

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Taking x - 1 we get

$$B = 1/3$$

Now by equating the coefficient of x^2 and constant terms we get

$$A + C = 0$$

$$-2A + 2B + c = 0$$

By solving the equations

$$A = 2/9 \text{ and } C = -2/9$$

We get

$$\frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2}{9(x+2)}$$

By integrating both sides w.r.t.x

$$\int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9} \int \frac{1}{(x-1)} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{(x+2)} dx$$

Here,

$$= \frac{2}{9} \log|x - 1| + \frac{1}{3} \left(\frac{-1}{x-1} \right) - \frac{2}{9} \log|x + 2| + C$$

By further calculation

$$= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C$$

9. $\frac{3x+5}{x^3-x^2-x+1}$

Solution:-

It is given that

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

We know that

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

It can be written as

$$3x + 5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

We get

$$3x + 5 = A(x^2 - 1) + B(x+1) + C(x^2 + 1 - 2x) \quad \dots (1)$$

By substituting the value of $x = 1$ in equation (1)

$$B = 4$$

Now by equating the coefficient of x^2 and x we get

$$A + C = 0$$

$$B - 2C = 3$$

By solving the equation

$$A = \frac{1}{2} \text{ and } C = \frac{1}{2}$$

We get

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{(x+1)}$$

By integrating both sides w.r.t.x

$$\int \frac{3x+5}{(x-1)^2(x+1)} dx = -\frac{1}{2} \int \frac{1}{x-1} dx + 4 \int \frac{1}{(x-1)^2} dx + \frac{1}{2} \int \frac{1}{(x+1)} dx$$

Here

$$= -\frac{1}{2} \log |x-1| - \frac{4}{(x-1)} + C$$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$

Solution:-

It is given that

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

We know that

$$\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$$

It can be written as

$$(2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$

$$(2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$

We get

$$(2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C) \quad \dots (1)$$

Now by equating the coefficient of x^2 and x we get

$$B = -1/10, A = 5/5 \text{ and } C = -24/5$$

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We get

$$\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} + \frac{1}{10(x-1)} + \frac{24}{5(2x+3)}$$

By integrating both sides w.r.t.x

$$\int \frac{2x-3}{(x^2+1)(2x+3)} dx = \frac{5}{2} \int \frac{1}{(x+1)} dx - \frac{1}{10} \int \frac{1}{x-1} dx - \frac{24}{5} \int \frac{1}{(2x+3)} dx$$

Here

$$= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5 \times 2} \log|2x+3|$$

By further calculation

$$= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{12}{5} \log|2x+3|$$

11. $\frac{5x}{(x+1)(x^2-4)}$

Solution:-

It is given that

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

We know that

$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$$

It can be written as

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \quad \dots (1)$$

By substituting $x = -1, -2$ and 2 in equation (1)

$$A = 5/3, B = -5/2 \text{ and } C = 5/6$$

We get

$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} + \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

By integrating both sides w.r.t.x

$$\int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3} \int \frac{1}{(x+1)} dx - \frac{5}{2} \int \frac{1}{(x+2)} dx + \frac{5}{6} \int \frac{1}{(x-2)} dx$$

By further calculation

$$= \frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C$$

12. $\frac{x^3+x+1}{x^2-1}$

Solution:-

It is given that

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

We know that

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$$\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

It can be written as

$$2x+1 = A(x-1) + B(x+1) \dots (1)$$

By substituting $x = 1$ and -1 in equation (1)

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

We get

$$\frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$$

By integrating both sides w.r.t.x

$$\int \frac{x^3+x+1}{x^2-1} dx = \int x dx + \frac{1}{2} \int \frac{1}{(x+1)} dx + \frac{3}{2} \int \frac{1}{(x-1)} dx$$

By further calculation

$$= \frac{x^2}{2} + \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$$

13. $\frac{2}{(1-x)(1+x^2)}$

Solution:-

We know that

$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$$

It can be written as

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

..... (1)

Now by equating the coefficient of x^2 , x and constant terms

$$A - B = 0$$

$$B - C = 0$$

$$A + C = 2$$

Solving the equations

$$A = 1, B = 1 \text{ and } C = 1$$

We get

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

By integrating both sides w.r.t.x

$$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

Multiplying and dividing by 2 in the second term

$$= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

By further calculation

$$= -\log|x-1| + \frac{1}{2} \log|1+x^2| + \tan^{-1}x + C$$

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$$14. \frac{3x-1}{(x+2)^2}$$

Solution:-

We know that

$$\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

It can be written as

$$3x - 1 = A(x+2) + B \dots (1)$$

Now by equating the coefficient of x and constant terms

$$A = 3$$

$$2A + B = -1$$

Solving the equations

$$B = -7$$

We get

$$\frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} + \frac{7}{(x+2)^2}$$

By integrating both sides w.r.t.x

$$\int \frac{3x-1}{(x+2)^2} dx = 3 \int \frac{1}{(x+2)} dx - 7 \int \frac{x}{(x+2)^2} dx$$

So we get

$$= 3 \log |x + 2| - 7 \left(\frac{-1}{(x+2)} \right) + C$$

By further calculation

$$= 3 \log |x + 2| + \frac{7}{(x+2)} + C$$

$$15. \frac{1}{(x^4-1)}$$

Solution:-

It is given that

$$\frac{1}{(x^4-1)} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x+1)(x-1)(1+x^2)}$$

We know that

$$\frac{1}{(x+1)(x-1)(1+x^2)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)}$$

So we get

$$1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (cx+D)(x^2-1)$$

By multiplying the terms

$$1 = A(x^3 + x - x^2 - 1) + B(x^3 + x + x^2 + 1) + Cx^3 + Dx^2 - Cx - D$$

It can be written as

$$1 = (A + B + C)x^3 + (-A + B + D)x^2 + (A + B - C)x + (-A + B - D) \dots (1)$$

Now by equating the coefficient of x^3, x^2, x and constant terms

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$$A + B + C = 0$$

$$-A + B + D = 0$$

$$A + B - C = 0$$

$$-A + B - D = 1$$

Solving the equations

$$A = -1/4, B = 1/4, C = 0 \text{ and } D = -1/2$$

We get

$$\frac{1}{(x^4-1)} = \frac{-4}{4(X+1)} + \frac{1}{4(X-1)} + \frac{1}{2(x^2+1)}$$

By integrating both sides w.r.t.x

$$\int \frac{1}{(x^4-1)} dx = -\frac{1}{4} \log |x + 1| + \frac{1}{4} \log |x - 1| - \frac{1}{2} \tan^{-1} x + C$$

So we get

$$= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + c$$

16. $\frac{1}{x(x^n+1)}$

Solution:-

By Multiplying both numerator and denominator by x^{n-1}

$$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^n-1x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$$

Here $x^n = t$ we get

$$nx^{n-1} dx = dt$$

So we get

$$\int \frac{1}{x(x^n+1)} dx = \int \frac{x^{n-1}}{x^n(x^n+1)} dx = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

We know that

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$$

It can be written as

$$1 = A(1+t) + Bt \dots (1)$$

By substituting $t = 0$, in equation (1)

$$A = 1 \text{ and } B = -1$$

We get

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(t+1)}$$

By integrating both sides w.r.t.x.

$$\int \frac{1}{x(x^n+1)} dx = \frac{1}{n} \int \left\{ \frac{1}{t} - \frac{1}{(t+1)} \right\} dx$$

So we get

$$= \frac{1}{n} [\log|t| - \log|1+t|] + C$$

Substituting the value of t

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$$= -\frac{1}{n} [\log|x^n|] - \log|x^n + 1| + c$$

It can be written as

$$= \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C$$

17. $\frac{\cos x}{(1-\sin x)(2-\sin x)}$

Solution:-

It is given that

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Consider

$$\sin x = t$$

By Differentiating w.r.t.x

$$\cos x \, dx = dt$$

Integrating w.r.t.x

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)} \, dx = \int \frac{dt}{(1-t)(2-t)}$$

Here we can write it as

$$\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$$

We get

$$1 = A(2-t) + B(1-t) \dots (1)$$

By substituting $t = 2$ and $t = 1$ in equation (1)

$$A = 1 \text{ and } B = -1$$

$$\frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

Integrating w.r.t.x

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)} \, dx = \int \left\{ \frac{1}{1-t} - \frac{1}{2-t} \right\} dt$$

So we get

$$= -\log|1-t| + \log|2-t| + C$$

It can be written as

$$= \log \left| \frac{2-t}{1-t} \right| + C$$

Substituting the value of t

$$= \log \left| \frac{2-\sin x}{1-\sin x} \right| + C$$

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

Solution:-

We know that

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$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

It can be written as

$$\frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$$

So we get

$$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$$

Multiplying the terms

$$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

Grouping the terms

$$4x^2 + 10 = (A + C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$$

Now by equating the coefficient of x^3 , x^2 , x and constant terms

$$A + C = 0$$

$$B + D = 4$$

$$4A + 3C = 0$$

$$4B + 3D = 10$$

By solving these equations

$$A = 0, B = -2, C = 0 \text{ and } D = 6$$

Substituting the values

$$\frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

We can write it as

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)} \right)$$

Integrating both sides w.r.t.x

$$\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left\{ 1 + \frac{-2}{(x^2+3)} - \frac{6}{(x^2+4)} \right\} dx$$

So we get

$$= \int \left\{ 1 + \frac{-2}{x^2+(\sqrt{3})^2} - \frac{6}{x^2+2^2} \right\}$$

Here

$$= x + 2 \left(\frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \right) - 6 \left(\frac{1}{2} \tan^{-1} \frac{x}{2} \right) + C$$

By further calculation

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + C$$

19. $\frac{2x}{(x^2+1)(x^2+3)}$

Solution:-

It is given that

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Consider $x^2 = t$

So we get

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$$2x \, dx = dt$$

Integrating both sides

$$\int \frac{2x}{(x^2+1)(x^2+3)} \, dx = \int \frac{dt}{(t+1)(t+3)}$$

We can write it as

$$\frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{2}{2(t+3)}$$

Integrating w.r.t t

$$\int \frac{2x}{(x^2+1)(x^2+3)} \, dx = \int \left\{ \frac{1}{2(t+1)} - \frac{2}{2(t+3)} \right\} dt$$

So we get

$$= \frac{1}{2} \log|(t+1)| - \frac{1}{2} \log|t+3| + C$$

It can be written as

$$= \frac{1}{2} \log \left| \frac{t+1}{t+3} \right| + C$$

Substituting the value of t

$$= \frac{1}{2} \log \left| \frac{x^2+1}{x^2+3} \right| + C$$

20. $\frac{1}{x(x^4-1)}$

Solution:-

It is given that

$$\frac{1}{x(x^4-1)}$$

By Multiplying both numerator and denominator by x^3

$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

Integration both sides

$$\int \frac{1}{x(x^4-1)} \, dx = \int \frac{x^3}{x^4(x^4-1)} \, dx$$

Consider $x^4 = t$

So we get $4x^3 \, dx = dt$

We can write it as

$$\int \frac{1}{x(x^4-1)} \, dx = \frac{1}{4} \int \frac{dt}{t(t-1)}$$

$$1 = A(t-1) + Bt \dots\dots (1)$$

Now by substituting $t = 0$ in equation (1)

$$A = -1 \text{ and } B = 1$$

Substituting the values

$$\frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

Integrating w.r.t.t

$$\int \frac{1}{x(x^4-1)} \, dx = \frac{1}{4} \int \left\{ \frac{-1}{t} + \frac{1}{t-1} \right\} dt$$

So we get

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$$= \frac{1}{4} [-\log|t| + \log|t - 1|] + C$$

It can be written as

$$= \frac{1}{4} \log \left| \frac{1}{t-1} \right| + C$$

Substituting the value of t

$$= \frac{1}{4} \log \left| \frac{x^4-1}{x^4} \right| + C$$

21. $\frac{1}{(e^x-1)}$

Solution:-

It is given that

$$\frac{1}{(e^x-1)}$$

Consider $e^x = t$

So we get $e^x dx = dt$

We can write it as

$$\int \frac{1}{e^x-1} dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)} dt$$

So we get

$$\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$1 = A(t-1) + Bt \dots\dots (1)$$

Now by substituting $t = 1$ and $t = 0$ in equations (1)

$$A = -1 \text{ and } B = 1$$

Substituting the values

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

Integrating w.r.t.t

$$\int \frac{1}{t(t-1)} dt = \log \left| \frac{t-1}{t} \right| + C$$

Substituting the value of t

$$= \log \left| \frac{e^x-1}{e^x} \right| + C$$

Choose the correct answer in each of the Exercises 22 and 23.

22. $\int \frac{xdx}{(x-1)(x-2)}$ Equals

a) $\text{Log} \left| \frac{(x-1)^2}{x-2} \right| + C$

b) $\text{Log} \left| \frac{(x-2)^2}{x-2} \right| + C$

c) $\text{Log} \left| \left(\frac{x-1}{x-2} \right)^2 \right| + C$

d) $\text{Log} |(x-1)(x-2)| + C$

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Solution:-

We know that

$$\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

It can be written as

$$x = A(x-2) + B(x-1) \dots\dots (1)$$

Now by substituting $x = 1$ and 2 in equations (1)

$$A = -1 \text{ and } B = 2$$

Substituting the value of A and B

$$\frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

Integrating both sides w.r.t.x

$$\int \frac{x}{(x-1)(x-2)} dx = \int \left\{ \frac{1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

We get

$$= -\log|x-1| + 2\log|x-2| + c$$

We can write it as

$$= \log \left| \frac{(x-2)^2}{x-1} \right| + C$$

Therefore, B is the correct answer.

23. $\int \frac{dx}{x(x^2+1)}$ equals

- a) $\log|x| - \frac{1}{2}\log(x^2+1) + C$
- b) $\log|x| + \frac{1}{2}\log(x^2+1) + C$
- c) $-\log|x| + \frac{1}{2}\log(x^2+1) + C$
- d) $\frac{1}{2}\log|x| + \log(x^2+1) + C$

Solution:-

We know that

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

It can be written as

$$1 = A(x^2+1) + (Bx+C)x \dots\dots (1)$$

Now by equating then coefficient of x^2 , x and constant terms

$$A + B = 0$$

$$C = 0$$

$$A = 1$$

By solving the equations we get

$$A = 1, B = -1 \text{ and } C = 0$$

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Substituting the value of A and B

$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-X}{x^2+1}$$

Integrating both sides w.r.t.x

$$\int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} + \frac{-X}{x^2+1} \right\} dx$$

We get

$$= \log |x| - \frac{1}{2} \log |x^2 + 1| + c$$

Therefore, A is the correct Answer.

Exercise 7.6

Integrate the functions in Exercise 1 to 22.

1. x sinx

Solution:-

It is given that

$$I = \int x \sin x dx$$

Here by taking x as first function and sin x as second function

Now integrating by parts we get

$$I = x \int \sin x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x dx \right\} dx$$

So we get

$$= x (-\cos x) - \int 1. (-\cos x) dx$$

It can be written as

$$= -x \cos x + \sin x + C$$

2. x sin x 3x

Solution:-

It is given that

$$I = \int x \sin 3x dx$$

Here by taking x as first function and 3x as second function

Now integrating by parts we get

$$I = x \int \sin 3x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x dx \right\}$$

So we get

$$= x \left(\frac{-\cos 3x}{3} \right) - \int 1. \left(\frac{-\cos 3x}{3} \right) dx$$

By multiplying the terms

$$= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x dx$$

It can be written as

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$$= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C$$

3. $x^2 e^x$

Solution:-

It is given that

$$I = \int x^2 e^x dx$$

Here by taking x^2 as first function and e^x as second function

Now integrating by parts we get

$$I = x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

So we get

$$= x^2 e^x - \int 2x \cdot e^x dx$$

It can be written as

$$= x^2 e^x - 2 \int x \cdot e^x dx$$

Now integrating by parts we get

$$= x^2 e^x - 2 \left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int e^x dx \right\} dx \right]$$

On further calculation

$$= x^2 e^x - 2[xe^x - \int e^x dx]$$

So we get

$$= x^2 e^x - 2[xe^x - e^x]$$

By multiplying the terms

$$= x^2 e^x - 2xe^x + 2e^x + C$$

Taking the common terms

$$= e^x(x^2 - 2x + 2) + C$$

4. $x \log x$

Solution:-

It is given that

$$I = \int x \log x dx$$

Here by taking x as first function and $\log x$ as second function

Now integrating by parts we get

$$I = \log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx$$

So we get

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

By multiplying the terms

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

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It can be written as

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

5. $x \log 2x$

Solution:-

It is given that

$$I = x \log 2x$$

Here by taking $2x$ as first function and x as second function

Now integrating by parts we get

$$I = \log 2x \int x \, dx - \int \left\{ \left(\frac{d}{dx} 2 \log 2x \right) \int x \, dx \right\} dx$$

So we get

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

By multiplying the terms

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

6. $x^2 \log x$

Solution:-

It is given that

$$I = \int x^2 \log x \, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

So we get

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{dx} \cdot \frac{x^3}{3} dx$$

By multiplying the terms

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

7. $x \sin^{-1} x$

Solution:-

It is given that

$$I = x \sin^{-1} x$$

Here by taking $\sin^{-1} x$ as first function and x as second function

Now integrating by parts we get

$$I = \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx$$

So we get

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

By multiplying the terms

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

Addition and subtraction of 1 in the numerator

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

On further simplification

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

Integrating the terms

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} dx - \frac{1}{\sqrt{1-x^2}} dx \right\}$$

So we get

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} dx + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

By further calculation

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{x}{4} \sqrt{1-x^2} + C$$

8. $x \tan^{-1} x$

Solution:-

We know that

$$I = \int x \tan^{-1} x dx$$

Consider $\tan^{-1} x$ as the first function and x as the second function

Here integrating by parts we get

$$I = \tan^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x dx \right\} dx$$

By further calculation

$$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$$

Multiplying the terms

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

Again integrating by parts

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx$$

So we get

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$

On further simplification

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} (x - \tan^{-1} x) + C$$

We get

$$= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C$$

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9. $x \cos^{-1}x$ **Solution:-**

We know that

$$I = \int x \cos^{-1}x dx$$

Consider $\cos^{-1}x$ as the first function and x as the second function

Here integrating by parts we get

$$I = \cos^{-1}x \int x dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1}x \right) \int x dx \right\} dx$$

By further calculation

$$= \cos^{-1}x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

By adding and subtracting 1 to the numerator

$$= \frac{x^2 \cos^{-1}x}{2} - \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

It can be written as

$$= \frac{x^2 \cos^{-1}x}{2} - \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}} \right) \right\} dx$$

Separating the terms

$$= \frac{x^2 \cos^{-1}x}{2} - \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \left(\frac{-1}{\sqrt{1-x^2}} \right) dx$$

We get

$$= \frac{x^2 \cos^{-1}x}{2} - \frac{1}{2} I_1 - \frac{1}{2} \cos^{-1}x \quad \dots (1)$$

We know that

$$I_1 = \int \sqrt{1-x^2} dx$$

Integrating by parts we get

$$I_1 = x\sqrt{1-x^2} - \int \frac{d}{dx} \sqrt{1-x^2} \int x dx$$

On further calculation

$$I_1 = x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}} \cdot x dx$$

So, we get

$$I_1 = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

Addition and subtraction of 1 to numerator

$$I_1 = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

By separating the terms

$$I_1 = x\sqrt{1-x^2} - \left\{ \int \sqrt{1-x^2} dx + \int \frac{-dx}{\sqrt{1-x^2}} \right\}$$

We get

$$I_1 = x\sqrt{1-x^2} - \{I_1 + \cos^{-1}x\}$$

On further calculation

$$2I_1 = x\sqrt{1-x^2} - \cos^{-1}x$$

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We can write it as

$$I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x$$

Now by substituting the value in equation (1)

$$I = x^2 \frac{\cos^{-1}x}{2} - \frac{1}{2} \left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x \right) - \frac{1}{2}\cos^{-1}x$$

We get

$$= \frac{(2x^2-1)}{4}\cos^{-1}x = \frac{x}{4}\sqrt{1-x^2} + C$$

10. $(\sin^{-1}x)^2$

Solution:-

We know that

$$I = \int (\sin^{-1}x)^2 \cdot 1 \, dx$$

Consider $(\sin^{-1}x)^2$ as the first function and 1 as the second function

Here integrating by parts we get

$$I = (\sin^{-1}x)^2 \int 1 \, dx - \int \left\{ \frac{d}{dx} (\sin^{-1}x)^2 \cdot \int 1 \, dx \right\} dx$$

By further calculation

$$= (\sin^{-1}x)^2 \cdot x = \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot x \, dx$$

Multiplying the terms

$$= x(\sin^{-1}x)^2 + \int \sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx$$

Again integrating by parts

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1}x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

So we get

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right]$$

On further simplification

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - \int 2 \, dx$$

We get

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - 2x + C$$

11. $\int \frac{x \cos^{-1}x}{\sqrt{1-x^2}} dx$

Solution:-

$$I = \int \frac{x \cos^{-1}x}{\sqrt{1-x^2}} dx$$

By multiplying and dividing by -2

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}x \, dx$$

Consider $\cos^{-1} x$ as the first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as the second function

Here integrating by parts we get

$$I = \frac{-1}{2} \left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

By further calculation

$$= \frac{-1}{2} \left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right]$$

Multiplying the terms

$$= \frac{-1}{2} [2\sqrt{1-x^2}\cos^{-1}x + \int 2dx]$$

So we get

$$= \frac{-1}{2} [2\sqrt{1-x^2}\cos^{-1}x + 2x] + C$$

On further Simplification

$$= -[\sqrt{1-x^2}\cos^{-1}x + x] + C$$

12. $x \sec^2 x$

Solution:-

It is given that

$$I = \int x \sec^2 x dx$$

Consider x as the first function and $\sec^2 x$ as the second function

Integrating by parts we get

$$I = x \int \sec^2 x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sec^2 x dx \right\} dx$$

By further calculation

$$= x \tan x - \int 1 \cdot \tan x dx$$

So we get

$$= x \tan x + \log |\cos x| + C$$

13. $\tan^{-1} x$

Solution:-

It is given that

$$I = \int \tan^{-1} x dx$$

Consider $\tan^{-1} x$ as the first function and 1 as the second function

Integrating by parts we get

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 dx \right\} dx$$

By further calculation

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx$$

Multiplying and dividing by 2

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$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

We get

$$= x \tan^{-1} x - \frac{1}{2} \log |1 + x^2| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \log (1 + x^2) + C$$

14. $x (\log x)^2$

Solution:-

It is given that

$I = \int x(\log x)^2$ as the first function and x as the second function

Integrating by parts we get

$$I = (\log x)^2 \int x dx - \int \left\{ \left(\frac{d}{dx} (\log x)^2 \right) \int x dx \right\} dx$$

By further calculation

$$= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

It can be written as

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x dx$$

Now integration by parts

$$I = \frac{x^2}{2} (\log x)^2 - \left[\log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \right]$$

So we get

$$= \frac{x^2}{2} (\log x)^2 - \left[\frac{x^2}{2} \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

On further simplification

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx$$

We get

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C$$

15. $(x^2 + 1) \log x$

Solution:-

Consider

$$I = \int (x^2 + 1) \log x dx$$

It can be written as

$$= \int x^2 \log x dx + \int \log x dx$$

We know that

$$I = I_1 + I_2 \dots \dots (1)$$

Here

$$I_1 = \int x^2 \log x dx \text{ and } I_2 = \int \log x dx$$

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Take

$$I_1 = \int x^2 \log x \, dx$$

Consider $\log x$ as the first function and x^2 as the second function

Now integrating by parts

$$I_1 = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

On further calculation

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

It can be written as

$$= \frac{x^2}{3} \log x - \frac{1}{3} (\int x^3 dx)$$

So we get

$$= \frac{x^2}{3} \log x - \frac{x^3}{9} + C_1 \quad \dots (2)$$

Take

$$I_2 = \int \log x \, dx$$

Consider $\log x$ as the first function and 1 as the second function

Now integrating by parts

$$I_2 = \log x \int 1 \, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \, dx \right\}$$

On further calculation

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

It can be written as

$$= x \log x - \int 1 dx$$

So we get

$$= x \log x - x + C_2 \quad \dots (3)$$

By using equations (2) and (3) in (1) we get

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

We get

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

16. $e^x (\sin x + \cos x)$

Solution:-

Consider

$$I = \int e^x (\sin x + \cos x) dx$$

We know that

$$F(x) = \sin x$$

So we get

$$F'(x) = \cos x$$

Here

$$I = \int e^x \{f(x) + f'(x)\} dx$$

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It can be written as

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$I = e^x \sin x + C$$

17. $\frac{xe^x}{(1+x)^2}$

Solution:-

It is given that

$$I = \int \frac{xe^x}{(1+x)^2} dx$$

We can write it as

$$= \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$$

By addition and subtraction of 1 to the numerator

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

Separating the forms we get

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Consider

$$F(x) = \frac{1}{1+x}$$

By differentiation

$$F'(x) = \frac{-1}{(1+x)^2}$$

So we get

$$\int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$$

We know that

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

We get

$$\int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

18. $e^x \left(\frac{1+\sin x}{1+\cos x} \right)$

Solution:-

It is given that

$$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$$

We can write it as

$$= e^x \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos \frac{x}{2}} \right)$$

Using the formula we can write it as

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$$= \frac{e^x (\sin \frac{x}{2} + \cos \frac{x}{2})^2}{2 \cos^2 \frac{x}{2}}$$

By further simplification

$$= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$$

So we get

$$= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2$$

$$= \frac{1}{2} e^x \left(1 + \tan \frac{x}{2} \right)^2$$

By expanding using formula

$$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

We know that

$$= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

So we get

$$\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \dots\dots (1)$$

Consider $\tan x/2 = f(x)$

By differentiation

$$F'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

Here

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

Using equation (1) we get

$$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$$

19. $e^x \left[\frac{1}{x} - \frac{1}{x^2} \right]$

Solution:-

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It is given that

$$I = \int e^x \left[\frac{1}{x} - \frac{1}{x^2} \right] dx$$

Here if $f(x) = 1/x$ we get

$$F(x) = -1/x^2$$

We know that

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

So we get

$$I = \frac{e^x}{x} + c$$

20. $\frac{(x-3)e^x}{(x-1)^3}$

Solution:-

It is given that

$$\int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} dx = \int e^x \left\{ \frac{x-1-2}{(x-1)^3} \right\} dx$$

By separating the terms

$$= \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx$$

We know that

$$F(x) = \frac{1}{(x-1)^2}$$

By differentiation

$$F'(x) = \frac{2}{(x-1)^2}$$

Here

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

We get

$$\int e^x \left\{ \frac{x-3}{(x-1)^2} \right\} dx = \frac{e^x}{(x-1)^2} + C$$

21. $e^{2x} \sin x$ **Solution:-**

It is given that

$$I = \int e^{2x} \sin x \quad \dots (1)$$

Now integrating by parts we get

$$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) e^{2x} dx \right\} dx$$

So we get

$$I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

We can write it as

$$I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x dx$$

Here again integrating by parts we get

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

So we get

$$I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

On further simplification

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int \frac{e^{2x}}{2} \sin x dx \right]$$

By using equation (1) we get

$$I = \frac{e^{2x} \sin x}{2} - I = \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

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It can be written as

$$I + \frac{1}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

We get

$$\frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

By cross multiplication

$$I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

So we get

$$I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

22. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$

Solution:-

Take $x = \tan \theta$ we get $dx = \sec^2 \theta d\theta$

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right) = \sin^{-1} \left(\frac{2 \tan \theta}{1+\tan^2 \theta} \right)$$

So we get

$$= \sin^{-1}(\sin 2\theta) = 2\theta$$

By integrating both sides w.r.t.x

$$\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = \int 2\theta \cdot \sec^2 \theta d\theta$$

We get

$$= 2 \int \theta \cdot \sec^2 \theta d\theta$$

Now integrating by parts we get

$$2 \left[\theta \cdot \int \sec^2 \theta d\theta - \int \left\{ \frac{d}{d\theta} \theta \right\} \int \sec^2 \theta d\theta \right]$$

On further calculation

$$= 2 [\theta \cdot \tan \theta - \int \tan \theta d\theta]$$

By integrating of second term

$$= 2[\theta \tan \theta + \log|\cos \theta|] + C$$

Now by substituting the value of θ

$$= 2 \left[x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C$$

We get

$$= 2x \tan^{-1} x + 2 \log(1+x^2)^{-\frac{1}{2}} + C$$

It can be written as

$$= 2x \tan^{-1} x + 2 \left[-\frac{1}{2} \log(1+x^2) \right] + C$$

By further calculation

$$= 2x \tan^{-1} x - \log(1+x^2) + C$$

Choose the correct answer in Exercise 23 and 24.

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23. $\int x^2 e^{x^3} dx$ equal to

- a) $\frac{1}{3} e^{x^2} + C$
- b) $\frac{1}{3} e^{x^3} + C$
- c) $\frac{1}{2} e^{x^2} + C$
- d) $\frac{1}{2} e^{x^3} + C$

Solution:-

It is given that

$$I = \int x^2 e^{x^3} dx$$

Take $x^3 = t$ we get

$$3x^2 dx = dt$$

Here

$$I = \frac{1}{3} \int e^t dt$$

By integrating w.r.t.x

$$= \frac{1}{3} e^{x^3} + C$$

Therefore, A is the correct Answer.

24. $\int e^x \sec x (1 + \tan x) dx$ equals

- a) $e^x \cos x + C$
- b) $e^x \sec x + C$
- c) $e^x \sin x + C$
- d) $e^x \tan x + C$

Solution:-

It is given that

$$I = \int e^x \sec x (1 + \tan x) dx$$

Multiplying the terms we get

$$= \int e^x \sec x + \sec x \tan x dx$$

Take $\sec x = f(x)$

So we get $\sec x \tan x = f'(x)$

We know that

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

Here

$$I = e^x \sec x + C$$

Therefore, B is the correct answer.

Exercise 7.7

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Integrate the functions in exercise 1 to 9

1. $\sqrt{4 - x^2}$

Solution:-

Given:

$$\sqrt{4 - x^2}$$

Upon integration we get,

$$\int \sqrt{4 - x^2} dx = \int \sqrt{(2)^2 - (x)^2} dx$$

By using the formula,

$$\int \sqrt{a^2 - x^2} dx = \frac{\pi}{2} \sqrt{a^2 - x^2} \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

So,

$$\begin{aligned} \int \sqrt{4 - x^2} dx &= \frac{\pi}{2} \sqrt{4 - x^2} \frac{4}{2} \sin^{-1} \frac{x}{2} + C \\ &= \frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} + C \end{aligned}$$

2. $\sqrt{1 - 4x^2}$

Solution:-

Given

$$\sqrt{1 - 4x^2}$$

Upon integration we get,

$$\int \sqrt{1 - 4x^2} dx = \int \sqrt{(1)^2 - (2x)^2} dx$$

Let $2x = t$

So,

$$2dx = dt$$

$$dx = dt/2$$

Then,

$$I = \frac{1}{2} \int \sqrt{(1)^2 - (t)^2} dt$$

By using the formula,

$$\int \sqrt{a^2 - x^2} dx = \frac{\pi}{2} \sqrt{a^2 - x^2} \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

So,

$$I = \frac{1}{2} \left[\frac{t}{2} \sqrt{1 - t^2} + \frac{1}{2} \sin^{-1} t \right] + C$$

$$= \frac{t}{4} \sqrt{1 - t^2} + \frac{1}{4} \sin^{-1} t + C$$

$$= \frac{2x}{4} \sqrt{1 - 4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

$$= \frac{x}{2} \sqrt{1 - 4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

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$$3. \sqrt{x^2 + 4x + 6}$$

Solution:-

Given:

$$\sqrt{x^2 + 4x + 6}$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{x^2 + 4x + 6} \, dx \\ &= \int \sqrt{x^2 + 4x + 4 + 2} \, dx \\ &= \int \sqrt{(x+2)^2 + (\sqrt{2})^2} \, dx \end{aligned}$$

By using the formula,

$$\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log|x + \sqrt{x^2 + a^2}| + C$$

So,

$$\begin{aligned} I &= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \frac{2}{2} \log|(x+2) + \sqrt{x^2 + 4x + 6}| + C \\ &= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \log|(x+2) + \sqrt{x^2 + 4x + 6}| + C \end{aligned}$$

$$4. \sqrt{x^2 + 4x + 1}$$

Solution:-

Given:

$$\sqrt{x^2 + 4x + 1}$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{x^2 + 4x + 1} \, dx \\ &= \int (\sqrt{x^2 + 4x + 4} - 3) \, dx \\ &= \int \sqrt{(x+2)^2 - (\sqrt{3})^2} \, dx \end{aligned}$$

By using the formula,

$$= \int \sqrt{(x+2)^2 - (\sqrt{3})^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2 - a^2}| + C$$

So,

$$I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 1} - \frac{3}{2} \log|(x+2) + \sqrt{x^2 + 4x + 1}| + C$$

$$5. \sqrt{1 - 4x - x^2}$$

Solution:-

Given:

$$\sqrt{1 - 4x - x^2}$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{1 - 4x - x^2} \, dx \\ &= \int \sqrt{1 - (x^2 + 4x + 4 - 4)} \, dx \\ &= \int \sqrt{1 + 4 - (x + 2)^2} \, dx \\ &= \int \sqrt{(\sqrt{5})^2 - (X + 2)^2} \, dx \end{aligned}$$

By using the formula

$$\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

So,

$$I = \frac{(x+2)}{2} \sqrt{1 - 4x - x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + C$$

6. $\sqrt{x^2 + 4x - 5}$

Solution:-

Given

$$\sqrt{x^2 + 4x - 5}$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{x^2 + 4x - 5} \, dx \\ &= \int \sqrt{(x^2 + 4x + 4) - 9} \, dx \\ &= \int \sqrt{(x + 2)^2 - (3)^2} \, dx \end{aligned}$$

By using the formula,

$$\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

So,

$$I = \frac{(x+2)}{2} \sqrt{x^2 + 4x - 5} - \frac{9}{2} \log |(x + 2) + \sqrt{x^2 + 4x - 5}| + C$$

7. $\sqrt{1 + 3x - x^2}$

Solution:-

Given:

$$\sqrt{1 + 3x - x^2}$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{1 + 3x - x^2} \, dx \\ &= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)} \, dx \end{aligned}$$

$$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2} dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}^2}{2}\right) - \left(x - \frac{3}{2}\right)^2} dx$$

By using the formula,

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

So,

$$I = \frac{x - \frac{3}{2}}{2} \sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2} \sin^{-1} \left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}} \right) + C$$

$$= \frac{2x - 3}{4} \sqrt{1 + 3x - x^2} + \frac{13}{8} \sin^{-1} \left(\frac{2x - 3}{\sqrt{13}} \right) + C$$

8. $\sqrt{x^2 + 3x}$

Solution:-

Given:

$$\sqrt{x^2 + 3x}$$

Upon integration we get,

$$I = \int \sqrt{x^2 + 3x} dx$$

$$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}} dx$$

$$= \int \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 dx$$

By using the formula

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

So,

$$I = \frac{\left(x + \frac{3}{2}\right)}{2} \sqrt{x^2 + 3x} - \frac{9}{2} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C$$

$$= \frac{(2x+3)}{4} \sqrt{x^2 + 3x} - \frac{9}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C$$

9. $\sqrt{1 + \frac{x^2}{9}}$

Solution:-

Given:

$$\sqrt{1 + \frac{x^2}{9}}$$

Upon Integration we get,

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$$I = \int \sqrt{1 + \frac{x^2}{9}} dx$$

$$= \frac{1}{3} \int \sqrt{9 + x^2} dx$$

$$= \frac{1}{3} \int \sqrt{(3)^2 + x^2} dx$$

By using the formula.

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

So,

$$I = \frac{1}{3} \left[\frac{x}{2} \sqrt{x^2 + 9} + \frac{9}{2} \log |x + \sqrt{x^2 + 9}| \right] + C$$

$$= \frac{x}{6} \sqrt{x^2 + 9} + \frac{3}{2} \log |x + \sqrt{x^2 + 9}| + C$$

10. $\int \sqrt{1 + x^2} dx$ is equal to

- a) $\frac{x}{2} \sqrt{1 + x^2} + \frac{1}{2} \log |(x + \sqrt{1 + x^2})| + C$
- b) $\frac{2}{3} (1 + x^2)^{\frac{3}{2}} + C$
- c) $\frac{2}{3} x (1 + x^2)^{\frac{3}{2}} + C$
- d) $\frac{x}{2} \sqrt{1 + x^2} + \frac{1}{2} x^2 \log |x + \sqrt{1 + x^2}| + C$

Solution:-

Given

$$\int \sqrt{1 + x^2} dx$$

By using the formula,

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

So,

$$\int \sqrt{1 + x^2} dx = \frac{x}{2} \sqrt{1 + x^2} + \frac{1}{2} \log |x + \sqrt{1 + x^2}| + C$$

Hence the correct option is A.

11. $\int \sqrt{x^2 - 8x + 7} dx$ is equal to

- a) $\frac{1}{2} (x - 4) \sqrt{x^2 - 2x + 7} + 9 \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$
- b) $\frac{1}{2} (x + 4) \sqrt{x^2 - 8x + 7} + 9 \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$
- c) $\frac{1}{2} (x - 4) \sqrt{x^2 - 8x + 7} + 3\sqrt{2} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$
- d) $\frac{1}{2} (x - 4) \sqrt{x^2 - 8x + 7} + \frac{9}{2} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$

Solution:-

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Given:-

$$\int \sqrt{x^2 - 8x + 7} dx$$

Upon integration we get,

$$\begin{aligned} I &= \int \sqrt{x^2 - 8x + 7} dx \\ &= \int \sqrt{(x^2 - 8x + 16) - 9} dx \\ &= \int \sqrt{(x - 4)^2 - (3)^2} dx \end{aligned}$$

By using the formula,

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2 - a^2}| + C$$

So,

$$I = \frac{(x-4)}{2} \sqrt{x^2 - 8x + 7} - \frac{9}{2} \log |(x - 4) + \sqrt{x^2 - 8x + 7}| + C$$

Hence the correct option is D.

Exercise 7.8

Evaluate the following definite integrals as limit of sums

1. $\int_a^b x dx$

Solution:-

Given

$$\int_a^b x dx$$

We know that $f(x)$ is continuous in $[a, b]$

Then we have,

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

By substituting the value of h in the above expression we get

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left(\frac{b-a}{n} \right) \sum_{r=0}^{n-1} f \left(a + \frac{(b-a)r}{n} \right)$$

Since, $f(a) = a$

$$= \lim_{n \rightarrow \infty} \left(\frac{b-a}{n} \right) \sum_{r=0}^{n-1} f \left(\frac{(b-a)r}{n} \right) + a$$

By expanding the summation we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{b-a}{n} \right) \left(\frac{(b-a)(n-1)(n)}{2n} + a(n-1) \right)$$

Upon simplification we get,

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{b-a}{n} \right) \cdot \frac{(b-a)(n^2 - n) + 2an^2 - 2an}{2n} \\ &= \lim_{n \rightarrow \infty} \left(\frac{b-a}{n} \right) \cdot \frac{(b+a)n^2 - (b+a)n}{2n} \\ &= \lim_{n \rightarrow \infty} \frac{(b+a)(b-a)n^2 - (b+a)(b-a)n}{2n^2} \end{aligned}$$

On computing we get,

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$$\begin{aligned} & \lim_{n \rightarrow \infty} \left(\frac{(b+a)(b-a)}{2} - \frac{(b+a)(b-a)}{n} \right) \\ &= \frac{(b+a)(b-a)}{2} \\ &= \frac{b^2 - a^2}{2} \end{aligned}$$

2. $\int_0^5 (x + 1) dx$

Solution:-

Given:-

$$\int_0^5 (x + 1) dx$$

We know that $f(x)$ is continuous in $[a, b]$ i.e., $[0, 5]$

Then we have,

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Substituting the value of h in the above expression we get,

$$\int_0^5 (x + 1) dx = \lim_{n \rightarrow \infty} \left(\frac{5}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{5r}{n}\right)$$

Since, $f(a) = a$

$$= \lim_{n \rightarrow \infty} \left(\frac{5}{n} \right) \sum_{r=0}^{n-1} \left(\frac{5r}{n} \right) + 1$$

By expanding the summation we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{5}{n} \right) \left(\frac{5(n-1)(n)}{2n} + (n-1) \right)$$

Upon simplification we get,

$$= \lim_{n \rightarrow \infty} \frac{5}{n} \cdot \frac{5n^2 - 5n + 2n^2 - 2n}{2n}$$

$$= \lim_{n \rightarrow \infty} \frac{5}{n} \cdot \frac{7n^2 - 7n}{2n}$$

$$= \lim_{n \rightarrow \infty} \frac{35n^2 - 35n}{2n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{35}{2} - \left(\frac{35}{2n} \right)$$

$$= \frac{35}{2}$$

3. $\int_2^3 x^2 dx$

Solution:-

Given:

$$\int_2^3 x^2 dx$$

We know that $f(x)$ is continuous in $[a, b]$ i.e., $[2, 3]$

Then we have,

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

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Substituting the value of h in the above expression we get.

$$\int_2^3 (x^2) dx = \lim_{n \rightarrow \infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} f\left(2 + \left(\frac{r}{n}\right)\right)$$

Since, $f(a) = a$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} f\left(2 + \left(\frac{r}{n}\right)\right)^2$$

By expanding the summation we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} \left(\frac{r^2}{n^2} + 4 + \frac{4r}{n}\right)$$

Upon simplification we get,

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{(n-1)(n)(2n-1)}{6n^2} + 4n + \frac{4(n-1)(n)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{(n^2-n)(2n-1)}{6n^2} + 4n + \frac{4(n^2-n)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{2n^3-2n^2-n^2+n}{6n^2} + 4n + \frac{2(n^2-n)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{(2n^3-3n^2+n)+(24n^3)+(12n^3-12n^2)}{6n^2}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{38n^3-15n^2+n}{6n^2}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{38n^3-15n^2+n}{6n^2}\right)$$

On computing we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{38}{6}\right) - \left(\frac{15}{6n}\right) + \left(\frac{1}{6n^2}\right)$$

$$= \frac{38}{6}$$

$$= \frac{19}{3}$$

4. $\int_1^4 (x^2 - x) dx$

Solution:-

Given:

$$\int_1^4 (x^2 - x) dx$$

We know that $f(x)$ is continuous in $[a, b]$ i.e. $[1, 4]$

Then we have,

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Substituting the value of h in the above expression we get,

$$\int_1^4 (x^2 - x) dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} f\left(\left(1 + \frac{3r}{n}\right)\right)$$

Since, $f(a) = a$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(\left(1 + \frac{3r}{n}\right)^2 - \left(1 + \frac{3r}{n}\right)\right)$$

By expanding the summation we get,

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$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(1 + \frac{9r^2}{n^2} + \frac{6r}{n} - 1 - \frac{3r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(\frac{9r^2}{n^2} + \frac{3r}{n}\right)$$

Upon simplification we get,

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n-1)(n)(2n-1)}{6n^2} + \frac{3n(n-1)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n^2-1)(2n-1)}{6n^2} + \frac{3n(n-1)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(2n^3-2n^2-n^2+n)}{6n^2} + \frac{3n(n-1)}{2n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(18n^3-27n^2+9n)+(9n^3-9n^2)}{6n^2}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27n^3-36n^2+9n}{6n^2}\right)$$

On computing we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{81n^3-108n^2+27n}{6n^3}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{81}{6}\right) - \left(\frac{108}{6n}\right) + \left(\frac{27}{6n^2}\right)$$

$$= 27/2$$

5. $\int_{-1}^1 e^x dx$

Solution:-

Given:-

$$\int_{-1}^1 e^x dx$$

We know that f(x) is continuous in [a, b] i.e., [-1, 1]

Then we have,

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Then we have,

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Substituting the value of h in the above expression we get,

$$\int_0^2 (e^x) = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f\left(-1 + \frac{2r}{n}\right)$$

Since, F(a) = a

$$= \log_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e_n^{2r} - 1$$

By expanding the summation we get,

$$= \log_{n \rightarrow \infty} \left(\frac{2}{ne}\right) (e^0 + e^h + e^{2h} + \dots + e^{nh})$$

$$\text{Sum of} = e^0 + e^h + e^{2h} + \dots + e^{nh}$$

Whose sum is:

$$= \frac{e^h(1-e^{nh})}{1-e^h}$$

Upon simplification we get,

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$$= \lim_{n \rightarrow \infty} \left(\frac{2}{ne} \right) \left(\frac{e^h(1-e^{nh})}{1-e^h} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{ne} \right) \cdot \frac{e^h(1-e^{nh})}{\frac{1-e^h \cdot h}{h}}$$

$$\lim_{h \rightarrow 0} \frac{1-e^h}{h}$$

$$= -1$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{ne} \right) \left(\frac{e^h(1-e^{nh})}{-h} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{ne} \right) \left(\frac{e^{\left(\frac{2}{n}\right)} \left(1 - e^{n \times \left(\frac{2}{n}\right)}\right)}{-\frac{2}{n}} \right)$$

[Since, $h = 2/n$]

$$= \frac{e^2 - 1}{e}$$

$$= e - e^{-1}$$

6. $\int_0^4 (x + e^{2x}) dx$

Solution:-

Given:

$$\int_0^4 (x + e^{2x}) dx$$

$$h(x) = \int_0^4 x \cdot dx$$

$$g(x) = \int_0^4 e^{2x} \cdot dx$$

$$\text{So, } f(x) = h(x) + g(x)$$

Now let us solve for $h(x)$

We know that $h(x)$ is continuous in $[0, 4]$

Then we have,

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Substituting the value of h in the above expression we get,

$$\int_0^4 (x) dx = \lim_{n \rightarrow \infty} \left(\frac{4}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{4r}{n}\right)$$

Since $f(a) = a$

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{4r}{n}\right)$$

By expanding the summation we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n} \right) \left(\frac{2(n-1)(n)}{n} \right)$$

Upon simplification we get,

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \cdot \frac{2n^2 - 2n}{n}$$

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$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \frac{2n^2 - 2n}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{8n^2 - 8n}{n^2} \\
 &= \lim_{n \rightarrow \infty} 8 - \left(\frac{8}{n}\right)
 \end{aligned}$$

Now let us solve for g(x)

We know that g(x) is continuous in [0,4]

Then we have,

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}$$

Substituting the value of h in the above expression we get,

$$\int_0^4 f(x)dx = \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{4r}{n}\right)$$

Since, f(a) = a

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \sum_{r=0}^{n-1} \left(\frac{4r}{n}\right)$$

By expanding the summation we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) (e^0 + e^h + e^{2h} + \dots + e^{nh})$$

Sum of = $e^0 + e^h + e^{2h} + \dots + e^{nh}$

Whose sum is:

$$= \frac{e^h(1-e^{nh})}{1-e^h}$$

Upon simplification we get,

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \cdot \frac{e^h(1-e^{nh})}{1-e^h \cdot h}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \left(\frac{e^h(1-e^{nh})}{-h}\right)$$

$$\text{[Since, } \lim_{h \rightarrow 0} \frac{1-e^h}{h} = -1]$$

$$= \lim_{n \rightarrow \infty} \left(\frac{4}{n}\right) \left(\frac{e^{\left(\frac{4}{n}\right)}(1-e^{n \times \left(\frac{4}{n}\right)})}{-\frac{4}{n}}\right)$$

$$\text{[Since, } h = 4/n]$$

$$= (e^8 - 1)$$

On comparing we get,

$$F(x) = h(x) + g(x)$$

$$= 8 + e^8 - 1$$

Exercise 7.9

Evaluate the definite integrals in Exercise 1 to 20.

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$$1. \int_{-1}^1 (x + 1) dx$$

Solution:-

$$\text{Let } I = \int_{-1}^1 (x + 1) dx$$

So,

$$I = \int_{-1}^1 (x + 1) dx$$

On splitting the integrals, we have

$$I = \int_{-1}^1 x dx + \int_{-1}^1 1 \times dx \quad \left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

Applying the limits after integration,

$$I = \left[\frac{x^2}{2} \right]_{-1}^1 + [X]_{-1}^1$$

$$I = \left[\frac{1^2}{2} - \frac{(-1)^2}{2} \right] + [1 - (-1)]$$

$$I = \left[\frac{1}{2} - \frac{1}{2} \right] + [1 + 1] = 0 + 2$$

$$I = 2$$

$$\text{Therefore, } \int_{-1}^1 (x + 1) dx = 2$$

$$2. \int_2^3 \frac{1}{x} dx$$

Solution:-

$$\int_2^3 \frac{1}{x} dx$$

$$\text{Let } I = 2$$

$$I = \int_2^3 \frac{1}{x} dx \quad \left[\int \frac{1}{x} dx = \log x \right]$$

Applying the limits after integration,

$$I = [\log|x|]_2^3$$

$$I = \log|3| - \log|2|$$

$$I = \log \frac{3}{2}$$

Therefore,

$$\int_2^3 \frac{1}{x} dx = \log \frac{3}{2}$$

$$3. \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

Solution:-

$$\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

Splitting the integrals, we have

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$$I = \int_1^2 4x^3 dx - \int_1^2 5x^2 dx + \int_1^2 6x dx + \int_1^2 9 dx$$

$$I = 4 \int_1^2 x^3 dx - 5 \int_1^2 x^2 dx + 6 \int_1^2 x dx + 9 \int_1^2 dx$$

Performing integration separately, we get

$$I = 4 \times \left[\frac{x^{3+1}}{3+1} \right]_1^2 - 5 \times \left[\frac{x^{2+1}}{2+1} \right]_1^2 + 6 \times \left[\frac{x^{1+1}}{1+1} \right]_1^2 + 9 \times \left[\frac{x^{0+1}}{0+1} \right]_1^2$$

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

Applying the limits after integration,

$$I = 4 \times \left[\frac{x^4}{4} \right]_1^2 - 5 \times \left[\frac{x^3}{3} \right]_1^2 + 6 \times \left[\frac{x^2}{2} \right]_1^2 + 9 \times [x]_1^2$$

$$= 2^4 - 1^4 - 5 \left[\frac{2^3}{3} - \frac{1^3}{3} \right] + 6 \left[\frac{2^2}{2} - \frac{1^2}{2} \right] + 9[2 - 1]$$

$$= 16 - 1 - 5 \left[\frac{7}{3} \right] + 3(3) + 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{90-35}{3} = \frac{64}{3}$$

Therefore, $\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx = 64/3$

4. $\int_0^{\pi/4} \sin 2x dx$

Solution:-

$$I = \int_0^{\pi/4} \sin 2x dx$$

Applying limits after integration, we have

$$I = \left[-\frac{\cos 2x}{2} \right]_0^{\pi/4}$$

$$[\int \sin x dx = -\cos x]$$

$$I = -(\cos 2 \times \pi/4 - \cos 0)/2$$

$$I = -(\cos 2 \pi/2 - \cos 0)/2 = -(0 - 1)/2$$

$$I = 1/2$$

$$\int_0^{\pi/4} \sin 2x dx$$

Therefore, $\int_0^{\pi/4} \sin 2x dx = 1/2$

5. $\int_0^{\pi/2} \cos 2x dx$

Solution:-

$$\text{Let } I = \int_0^{\pi/2} \cos 2x dx$$

$$I = \int_0^{\pi/2} \cos 2x dx$$

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Integrating $\cos 2x$ and applying limits, we have

$$I = \left[\frac{\sin 2x}{2} \right]_0^{\pi/2}$$

$$[\int \cos x \, dx = \sin x + c]$$

$$I = \frac{1}{2} (\sin 2 \times \frac{\pi}{2} - \sin 2 \times 0)$$

$$I = \frac{1}{2} (\sin \pi - \sin 0)$$

$$I = \frac{1}{2} \times (0 - 0) = 0$$

$$\text{Therefore, } \int_0^{\pi/2} \cos 2x \, dx = 0,$$

$$6. \int_4^5 e^x \, dx$$

Solution:-

$$\text{Let } I = \int_4^5 e^x \, dx$$

$$I = \int_4^5 e^x \, dx$$

Applying the limits after integration, we get

$$I = [e^x]_4^5 = e^5 - e^4 \quad [\int e^x \, dx = e^x + c]$$

$$I = e^4 (e - 1)$$

$$\text{Therefore, } \int_4^5 e^x \, dx = e^4 (e - 1)$$

$$7. \int_0^{\pi/4} \tan x \, dx$$

Solution:-

$$\text{Let } I = \int_0^{\pi/4} \tan x \, dx$$

$$I = \int_0^{\pi/4} \tan x \, dx \quad [\text{Using } \int \tan x \, dx = -\log|\cos x| + C]$$

$$I = [-\log|\cos x|]_0^{\pi/4}$$

Applying limits after integrating, we have

$$I = -(\log|\cos \frac{\pi}{4}| - \log|\cos 0|)$$

$$\text{Therefore, } I = (\log|\frac{1}{\sqrt{2}}| - \log|1|) = -\log(2)^{\frac{1}{2}} + 0$$

$$8. \int_{\pi/6}^{\pi/4} \operatorname{cosec} x \, dx$$

Solution:-

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

Performing integration, we have

$$I = [\log|\operatorname{cosec} x - \cot x|]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

[using $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$]

Applying limits after integration, we get

$$I = \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right|$$

$$I = \log |\sqrt{2} - 1| - \log |2 - \sqrt{3}|$$

$$I = \log \left| \frac{\sqrt{2}-1}{2-\sqrt{3}} \right|$$

Therefore,
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x = \log \left(\frac{\sqrt{2}-1}{2-\sqrt{3}} \right)$$

9.
$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Solution:-

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Performing integration,

$$I = \int_0^1 \frac{dx}{\sqrt{1-x^2}} \quad \left[\text{Using } \int \frac{dx}{a^2-x^2} = \sin^{-1} \frac{x}{a} + c \right]$$

Applying limits after integration, we have

$$I = [\sin^{-1} x]_0^1$$

$$I = \sin^{-1}(1) - \sin^{-1}(0) = \pi/2 - 0$$

$$I = \pi/2$$

Therefore,
$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \pi/2$$

10.
$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Solution:-

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

We know that,

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

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Hence, on integrating we get

$$I = [\tan^{-1}x]_0^1$$

Applying limits, we have

$$I = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0$$

$$I = \frac{\pi}{4}$$

$$\text{Therefore, } \int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \frac{\pi}{4}$$

$$11. \int_2^3 \frac{dx}{x^2-1}$$

Solution:-

$$\text{Let } I = \int_2^3 \frac{dx}{x^2-1}$$

On integrating, we have

$$I = \int_2^3 \frac{dx}{x^2-1} \quad \left[\text{w. k. t } \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c \right]$$

Applying limits after integration, we get

$$I = \left[\frac{1}{2} \log \left| \frac{x-1}{x+1} \right| \right]_2^3 = \frac{1}{2} \left(\log \left| \frac{3-1}{3+1} \right| - \log \left| \frac{2-1}{2+1} \right| \right)$$

$$I = \frac{1}{2} \left(\log \left| \frac{2}{4} \right| - \log \left| \frac{1}{3} \right| \right) = \frac{1}{2} \log \frac{1/2}{1/3}$$

$$I = \frac{1}{2} \log 3/2$$

$$\text{Therefore, } \int_2^3 \frac{dx}{x^2-1} = \frac{1}{2} \log 3/2$$

$$12. \int_0^{\pi/2} \cos^2 x \, dx$$

Solution:-

$$\text{Let } I = \int_0^{\pi/2} \cos^2 x \, dx$$

We know that,

$$\cos 2x = 2\cos^2 x - 1$$

$$\text{So, } \cos^2 x = \frac{1+\cos 2x}{2}$$

Putting the value $\cos^2 x$ in I and splitting the integrals, we have

$$I = \int_0^{\pi/2} \frac{1+\cos 2x}{2} dx = \frac{1}{2} \int_0^{\pi/2} dx + \frac{1}{2} \int_0^{\pi/2} \cos 2x \, dx$$

$$[\int \cos x dx = \sin x + c]$$

Applying limits after integration. We get

$$I = \frac{1}{2} [x]_0^{\pi/2} + \frac{1}{2} \left[\frac{\sin 2x}{2} \right]_0^{\pi/2} = \frac{1}{2} (\pi - 0) + \frac{1}{4} \left(\sin 2 \times \frac{\pi}{2} - \sin 2 \times 0 \right)$$

$$I = \frac{\pi}{4} + \frac{1}{4} (0 - 0)$$

$$= \pi/4$$

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Therefore, $\int_0^{\pi/2} \cos^2 x \, dx = \pi/4$

13. $\int_2^3 \frac{x \, dx}{x^2+1}$

Solution:-

Let $I = \int_2^3 \frac{x \, dx}{x^2+1}$

Let's assume $x^2 + 1 = t$

So,

$$d(x^2 + 1) = dt$$

$$2x \, dx = dt$$

$$x \, dx = dt/2$$

$$\text{When } x = 2; t = 2^2 + 1 = 5$$

$$\text{When } x = 3; t = 3^2 + 1 = 10$$

Substituting $(x^2 + 1)$ and $x \, dx$ in I , we have

$$I = \int_5^{10} \frac{dt}{2t} = \frac{1}{2} \int_5^{10} \frac{dt}{t} \quad \left[\text{w. k. t } \int \frac{1}{x} \, dx = \log x \right]$$

Applying limits after integration, we get

$$I = \frac{1}{2} [\log t]_5^{10} = \frac{1}{2} (\log 10 - \log 5) = \frac{1}{2} \log \frac{10}{5}$$

$$I = \frac{1}{2} \log 2$$

$$\text{Therefore, } \int_2^3 \frac{x \, dx}{x^2+1} = \frac{1}{2} \log 2$$

14. $\int_0^1 \frac{2x+3}{5x^2+1} \, dx$

Solution:-

Let $I = \int_0^1 \frac{2x+3}{5x^2+1} \, dx$

Multiplying by 5 in numerator and denominator:

$$I = \frac{1}{5} \int_0^1 \frac{(2x+3)}{5x^2+1} \, dx = \frac{1}{5} \int_0^1 \frac{10x+15}{5x^2+1} \, dx$$

Splitting the fraction into two fractions, we have

$$\frac{1}{5} \int_0^1 \frac{10x}{5x^2+1} \, dx + 3 \int_0^1 \frac{1}{5x^2+1} \, dx$$

Now, $I = I_1 + I_2$

Where, $I_1 = \frac{1}{5} \int_0^1 \frac{10x}{5x^2+1} \, dx$

Let us take $5x^2 + 1 = t$ (1)

$$d(5x^2 + 1) = dt$$

$$10x \, dx = dt \text{ (2)}$$

$$\text{When } x = 0; t = 5 \times 0^2 + 1 = 1$$

$$\text{When } x = 1; t = 5 \times 1^2 + 1 = 6$$

Substituting (1) and (2) in I_1 , we have

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$$I_1 = \frac{1}{5} \int_1^6 \frac{dt}{t} = \frac{1}{5} [\log|t|]_1^6 \quad \left[\text{w. k. t } \int \frac{1}{x} dx = \log x \right]$$

Applying limits to integrals, we get

$$I_1 = \frac{1}{5} (\log|6| - \log|1|) = \frac{1}{5} (\log 6 - 0)$$

$$I_1 = \frac{1}{5} \int_0^1 \frac{10x}{5x^2+1} dx = \frac{\log 6}{5}$$

Next,

$$I_2 = 3 \int_0^1 \frac{1}{5x^2+1} dx = \frac{3}{5} \int_0^1 \frac{1}{x^2+\frac{1}{5}} dx \quad \left[\text{w. k. t } \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$$

Applying limits to integrals, we get

$$I_2 = \frac{3}{5} (\log|6| - \log|1|) = \frac{3}{5} (\log 6 - 0)$$

$$I_2 = \frac{3}{5} \int_0^1 \frac{10x}{5x^2+1} dx = \frac{\log 6}{5}$$

Next,

$$I_2 = 3 \int_0^1 \frac{1}{5x^2+1} dx = \frac{3}{5} \int_0^1 \frac{1}{x^2+\frac{1}{5}} dx \quad \left[\text{w. k. t } \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$$

$$I_2 = \frac{3}{5} \times \frac{1}{\sqrt{5}} \left[\tan^{-1} \sqrt{5x} \right]_0^1 = \frac{3}{5} \times \sqrt{5} (\tan^{-1} \sqrt{5} - \tan^{-1} 0)$$

$$I_2 = \frac{3}{\sqrt{5}} \tan^{-1} 5$$

$$I_2 = \frac{3}{5} \times \frac{1}{\sqrt{5}} \left[\tan^{-1} \sqrt{5x} \right]_0^1 = \frac{3}{5} \times \sqrt{5} (\tan^{-1} \sqrt{5} - \tan^{-1} 0)$$

$$I_2 = \frac{3}{\sqrt{5}} \tan^{-1} 5$$

Hence, $I = I_1 + I_2$

$$I = \frac{1}{5} \log 6 + \frac{3\sqrt{5}}{5} \tan^{-1} 5$$

$$\text{Therefore, } \int_0^1 \frac{2x+3}{5x^2+1} dx = \frac{1}{5} \log 6 + \frac{3\sqrt{5}}{5} \tan^{-1} 5$$

15. $\int_0^1 x e^{x^2} dx$

Solution:-

$$\int_0^1 x e^{x^2} dx$$

Let $I = 0$

On taking $x^2 = t \rightarrow 2x dx = dt$

When $x = 0$; $t = 0$

When $x = 1$; $t = 1$

Substituting t and dt in I ,

$$I = \int_0^1 \frac{e^t dt}{2} = \frac{1}{2} \int_0^1 e^t dt$$

$$[\int e^x dx = e^x + c]$$

$$I = \frac{1}{2} [e^t]_0^1 = \frac{1}{2} (e - e^0) = \frac{1}{2} (e - 1)$$

$$\text{Therefore, } \int_0^1 x e^{x^2} dx = \frac{1}{2} (e - 1)$$

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$$16. \int_1^2 \frac{5x^2}{x^2+4x+3}$$

Solution:-

$$\text{Let } I = \int_1^2 \frac{5x^2}{x^2+4x+3}$$

On dividing $5x^2$ by $x^2 + 4x + 3$ we get 5 as quotient and $-(20x + 15)$ as remainder

$$\text{So, } I = \int_1^2 \left(5 - \frac{20x+15}{x^2+4x+3} \right) dx$$

Splitting the integrals, we have

$$I = \int_1^2 5dx - \int_1^2 \frac{20x+15}{x^2+4x+3} = 5[x]_1^2 - \int_1^2 \frac{20x+16}{x^2+4x+3}$$

$$I = 5(2-1) - \int_1^2 \frac{20x+16}{x^2+4x+3}$$

$$I = 5 - I_1$$

Now,

$$I_1 = \int_1^2 \frac{20x+16}{x^2+4x+3}$$

Adding and subtracting 25 in the numerator, we get

$$I_1 = \int_1^2 \frac{20x+15+25-25}{x^2+4x+3} dx = \int_1^2 \frac{20x+40}{x^2+4x+3} dx - \int_1^2 \frac{25}{x^2+4x+3} dx$$

$$I_1 = 10 \int_1^2 \frac{2x+4}{x^2+4x+3} dx - 25 \int_1^2 \frac{1}{x^2+4x+3} dx$$

Let us assume $x^2 + 4x + 3 = t$

Then, $(2x+4) dx = dt$

So,

$$I_1 = 10 \int \frac{dt}{t} - 25 \int \frac{1}{x^2+4x+3+1-1} dx = 10 \log t + 25 \int \frac{1}{x^2+4x+3+1-1} dx$$

$$I_1 = 10 \log t - 25 \int \frac{1}{(x+2)^2-1^2} dx \quad \left[\text{w. k. t. } \int \frac{1}{x} dx = \log x \right]$$

$$I_1 = 10 \log t - 25 \left[\frac{1}{2} \log \left(\frac{x+2-1}{x+2+1} \right) \right] \quad \left[\text{w. k. t. } \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c \right]$$

Applying limits after integration, we get

$$I_1 = 10[\log(x^2 + 4x + 3)]_1^2 - \frac{25}{2} \left[\log \left(\frac{x+1}{x+3} \right) \right]_1^2$$

$$I_1 = 10[\log(5 \times 3) - \log(4 \times 2)] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4]$$

$$I_1 = 10 \log 5 + 10 \log 3 - 10 \log 4 - 10 \log 2 - \frac{25}{2} \log 3 + \frac{25}{2} \log 5 + \frac{25}{2} \log 2 - \frac{25}{2} \log 4$$

$$I_1 = \left(10 + \frac{25}{2} \right) \log 5 - \left(10 + \frac{25}{2} \right) \log 4 + \left(10 + \frac{25}{2} \right) \log 3 + \left(-10 + \frac{25}{2} \right) \log 2$$

$$I_1 = \frac{45}{2} \log 5 - \frac{45}{2} \log 4 - \frac{5}{2} \log 3 + \frac{5}{2} \log 2 = \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2}$$

As $I = 5 - I_1$

On substituting I_1 in I we get,

$$I = 5 - \frac{45}{2} \log \frac{5}{4} + \frac{5}{2} \log \frac{3}{2}$$

$$\text{Therefore, } \int_1^2 \frac{5x^2}{x^2+4x+3} = 5 - \frac{45}{2} \log \frac{5}{4} + \frac{5}{2} \log \frac{3}{2}$$

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$$17. \int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx$$

Solution:-

$$\text{Let } I = \int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx$$

Splitting the Given integral, we have

$$I = \int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx = 2 \int_0^{\frac{\pi}{4}} \sec^2 x dx + \int_0^{\frac{\pi}{4}} dx$$

Now, integration separately and applying limits, we get

$$I = 2 \left[\tan x \right]_0^{\frac{\pi}{4}} + \left[\frac{x^4}{4} \right]_0^{\frac{\pi}{4}} + 2 \left[x \right]_0^{\frac{\pi}{4}} \quad [\text{w. k. t } \int \sec^2 x dx = \tan x + c]$$

$$I = 2 \left(\tan \frac{\pi}{4} - \tan 0 \right) + \frac{1}{4} (\pi/4)^4 - 0 + 2 \left(\frac{\pi}{4} - 0 \right)$$

$$2 \times 1 + \frac{1}{4} \times \left(\frac{\pi}{4} \right)^4 + 2 \times \frac{\pi}{4}$$

Expanding the exponents, we have

$$I = 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

$$\text{Therefore, } \int_0^{\pi/4} (2\sec^2 x + x^3 + 2) dx = 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

$$18. \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

Solution:-

$$\text{Let } I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

We know that,

$$\cos x = \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}$$

So, substituting $\cos x = \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}$ in I we have

Applying the limits after integration, we get

$$I = \int_0^{\pi} \cos x dx = [\sin x]_0^{\pi}$$

$$[\text{W.k.t } \int \cos x dx = \sin x + c]$$

$$I = \sin \pi - \sin 0 = 0 - 0 = 0$$

$$\text{Therefore, } \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx = 0$$

$$19. \int_0^2 \frac{6x+3}{x^2+4} dx$$

Solution:-

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$$\text{Let } I = \int_0^2 \frac{6x+3}{x^3+4} dx$$

$$I = 3 \int_0^2 \frac{2x+1}{x^2+4} = 3 \int_0^2 \frac{2x}{x^2+4} dx + 3 \int_0^2 \frac{1}{x^2+4} dx$$

Now, we have $I = I_1 + I_2$

$$3 \int_0^2 \frac{2x}{x^2+4} dx$$

When $I_1 =$

$$\text{Let } x^2+4=t$$

$$2x dx = dt$$

$$\text{When } x = 0; t = 4$$

$$\text{When } x = 2; t = 2^2 + 4 = 8$$

Substituting t and dt in I_1

$$I_1 = 3 \int_4^8 \frac{dt}{t} = 3 [\log|t|]_4^8 \quad \left[\text{w. k. t } \int \frac{1}{x} dx = \log x \right]$$

$$I_1 = 3 [\log|8| - \log|4|] = 3 \log 8/4$$

$$I_1 = 3 \log 1/2 = -3 \log 2$$

$$\text{And, } I_2 = 3 \int_0^2 \frac{1}{x^2+4} dx = 3 \int_0^2 \frac{1}{x^2+2^2} dx \quad \left[\text{w. k. t } \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$$

$$I_2 = 3 \times \frac{1}{2} \left[\tan^{-1} \frac{x}{2} \right]_0^2 = \frac{3}{2} \left[\tan^{-1} \frac{2}{2} - \tan^{-1} \frac{0}{2} \right] = \frac{3}{2} [\tan^{-1} 1 - \tan^{-1} 0]$$

$$I_2 = \frac{3}{2} \times \frac{\pi}{4} = 3\pi/8$$

$$\text{Now } I = I_1 + I_2$$

$$I = 3 \log 1/2 + \frac{3\pi}{8}$$

$$\text{Therefore, } \int_0^2 \frac{6x+3}{x^2+4} dx = 3 \log 1/2 + 3\pi/8$$

$$20. \int_0^1 (x e^x + \sin \frac{\pi x}{4}) dx$$

Solution:-

$$\text{Let } I = \int_0^1 (x e^x + \sin \frac{\pi x}{4}) dx$$

Splitting the integrals, we have

$$I = \int_0^1 x e^x dx + \int_0^1 \sin \frac{\pi x}{4} dx$$

$$I = 0$$

$$\text{Now, } I = I_1 + I_2$$

$$I_1 = \int_0^1 x e^x dx \quad \left[\text{Using } u - v \text{ integral form: } u = x \text{ and } v = e^x \right]$$

$$x \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int e^x dx \right\} dx$$

$$I_1 = x e^x - \int e^x dx \quad \left[\text{w. k. t } \int e^x dx = e^x + c \right]$$

Now integration the reduced form and applying the limits, we get

$$I_1 = [x e^x - e^x]_0^1 = [(1 \times e^1 - e^1) - (0 \times e^0 - e^0)]$$

$$I_1 = e - e - 0 + 1$$

$$I_1 = 1$$

Next, taking I_2

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$$I_2 = \int_0^1 \sin \frac{\pi x}{4} dx \quad [\text{w. k. t } \int \sin x dx = -\cos x]$$

Applying the limits after integration, we get

$$I_2 = \left[-\frac{\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1 = -\frac{4}{\pi} \left[\cos \frac{\pi}{4} \times 1 - \cos \frac{\pi}{4} \times 0 \right] = -\frac{4}{\pi} \left[\cos \frac{\pi}{4} - \cos 0 \right]$$

$$I_2 = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{2}} \right) = \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

As, $I = I_1 + I_2$

$$\text{Hence, } I = 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

$$\text{Therefore, } \int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx = 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

21. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

- a) $\frac{\pi}{3}$
- b) $\frac{2\pi}{3}$
- c) $\frac{\pi}{6}$
- d) $\frac{\pi}{12}$

Solution:-

$$\text{Let } I = \int_1^{\sqrt{3}} \frac{dx}{x^2+1}$$

On integrating using standard form and applying limits, we get

$$I = [\tan^{-1}x]_1^{\sqrt{3}} = [\tan^{-1}\sqrt{3} - \tan^{-1}1] = \frac{\pi}{3} - \frac{\pi}{4}$$

$$[\text{w. k. t } \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c]$$

$$I = \frac{4\pi-3\pi}{12} = \frac{\pi}{12}$$

$$\text{Therefore } \int_1^{\sqrt{3}} \frac{dx}{x^2+1} = \pi/12$$

Hence, option (D) is correct

22. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

- a) $\frac{\pi}{6}$
- b) $\frac{\pi}{12}$
- c) $\frac{\pi}{24}$
- d) $\frac{\pi}{4}$

Solution:-

$$\text{Let } I = \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$$

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$$I = \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$$

Now, taking 9 common from Denominator in I, we have

$$I = \frac{1}{9} \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{9} \int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2} \quad \left[w. k. t \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$$

Using the standard form for integrating and applying the limits, we get

$$I = \frac{1}{9} \times \frac{3}{2} \left[\tan^{-1} \frac{x}{\frac{2}{3}} \right]_0^{\frac{2}{3}} = \frac{1}{9} \times \frac{3}{2} \left[\tan^{-1} \frac{3x}{2} \right]_0^{\frac{2}{3}}$$

$$I = \frac{1}{6} \left[\tan^{-1} \frac{3}{2} \times \frac{2}{3} - \tan^{-1} 0 \right] = \frac{1}{6} [\tan^{-1} 1 - \tan^{-1} 0]$$

$$I = \frac{1}{6} \times \left(\frac{\pi}{4} - 0 \right) = \frac{\pi}{24}$$

Hence, option (c) is correct.

Exercise 7.10

Evaluate the integrals in Exercise 1 to 8 by substitutions.

1. $\int_0^1 \frac{x}{x^2+1} dx$

Solution:-

Given integral: $\int_0^1 \frac{x}{x^2+1} dx$

Let's take $x^2+1 = t$

Then $2x dx = dt$

$x dx = \frac{1}{2} dt$

When $x = 0, t = 1$ and when $x = 1, t = 2$

Now,

$$\begin{aligned} \int_0^1 \frac{x}{x^2+1} dx &= \int_1^2 \frac{dt}{2t} \\ &= \frac{1}{2} \int_1^2 \frac{dt}{t} \\ &= \frac{1}{2} [\log|t|]_1^2 \\ &= \frac{1}{2} [\log 2 - \log 1] \\ &= \frac{1}{2} \log 2 \end{aligned}$$

2. $\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$

Solution:-

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Given integrals $\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$

Let's consider $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^4 \phi d\phi$

$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} (\cos^2 \phi)^2 \cos \phi d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} (1 - \sin^2 \phi)^2 \cos \phi d\phi$

Also, let $\sin \phi = t \rightarrow \cos \phi d\phi = dt$

So when, $\phi = 0, t = 0$ and when $\phi = \frac{\pi}{2}, t = 1$

Hence,

$$I = \int_0^1 \sqrt{t}(1 - t^2)^2 dt$$

Expanding and splitting the integrals, we have

$$= \int_0^1 t^{\frac{1}{2}} (1 + t^4 - 2t^2) dt$$

$$= \int_0^1 \left(t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right) dt$$

Integrating the terms individually by standard form, we get

$$= \left[\frac{3}{2} t^{\frac{3}{2}} + \frac{11}{2} t^{\frac{11}{2}} + \frac{7}{2} t^{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154+24-132}{231} = \frac{64}{231}$$

Therefore, $\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi = 64/231$

3. $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$

Solution:-

Given integral: $\int_0^1 \sin^{-1} \left(\frac{2x}{x^2+1} \right) dx$

Let us take $x = \tan \theta \rightarrow dx = \sec^2 \theta d\theta$

So, when $x = 0, \theta = 0$ and when $x = 1, \theta = \pi/4$

Let $I = \int_0^1 \sin^{-1} \left(\frac{2x}{x^2+1} \right) dx$

Now, by substitution I becomes

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1} \left(\frac{2 \tan \theta}{\tan^2 \theta + 1} \right) \sec^2 \theta d\theta$$

Transforming the trigonometric ratio into its simple form, we have

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1} (\sin 2\theta) \sec^2 \theta d\theta$$

Applying the inverse trigonometric ratio, we get

$$I = \int_0^{\frac{\pi}{4}} 2\theta \sec^2 \theta d\theta$$

$$I = 2 \int_0^{\frac{\pi}{4}} \theta \sec^2 \theta d\theta$$

Now, by applying product rule as:

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$$\int u \cdot v dx = u \cdot \int v dx - \int \frac{du}{dx} \cdot \{ \int v dx \} dx$$

$$\begin{aligned} I &= 2 \left[\theta \int \sec^{\theta} \theta d\theta - \int \frac{d}{d\theta} \theta \cdot \{ \int \sec^2 \theta d\theta \} \right]_0^{\frac{\pi}{4}} \\ &= 2 \left[\theta \tan \theta - \int 1 \cdot \tan \theta d\theta \right]_0^{\frac{\pi}{4}} \\ &= 2 \left[\theta \tan \theta - \log | \sec \theta | \right]_0^{\frac{\pi}{4}} \\ &= 2 \left[\frac{\pi}{4} \tan \frac{\pi}{4} - \log | \sec \frac{\pi}{4} | - 0 + \log \sec 0 \right] \\ &= 2 \left[\frac{\pi}{4} - \log(\sqrt{2}) + \log 1 \right] \\ &= 2 \left[\frac{\pi}{4} - \frac{1}{2} \log(2) \right] \\ &= \frac{\pi}{2} + \log(2) \end{aligned}$$

Therefore, $\int_0^1 \sin^{-1} \left(\frac{2x}{x^2+1} \right) dx = \frac{\pi}{2} + \log(2)$

4. $\int_0^2 x\sqrt{x+2}$ (Put $x+2 = t^2$)

Solution:-

Given integral: $\int_0^2 x + \sqrt{x+2} dx$

Let's take $x+2 = t^2 \rightarrow dx = 2t dt$

And, $x = t^2 - 2$

So when, $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

Hence, after substitution the given integral can be written as:

$$\int_0^2 x + \sqrt{x+2} dx = \int_{\sqrt{2}}^2 (t^2 - 2) \sqrt{t^2} 2t dt$$

Taking the square root we have,

$$= 2 \int_{\sqrt{2}}^2 (t^2 - 2)t \cdot t dt$$

$$= 2 \int_{\sqrt{2}}^2 (t^2 - 2) t^2 dt$$

$$= 2 \int_{\sqrt{2}}^2 (t^4 - 2t^2) dt$$

On integrating the terms separately, we get

$$= 2 \left[\frac{t^5}{5} - \frac{2t^3}{3} \right]_{\sqrt{2}}^2$$

Applying the limits after integration, we have

$$= 2 \left[\frac{(2)^5}{5} - \frac{2(2)^3}{3} - \frac{(2)^5}{5} + \frac{2(\sqrt{2})^3}{3} \right]_{\sqrt{2}}$$

$$= 2 \left[\frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{5}}{5} + \frac{4\sqrt{2}}{3} \right]$$

$$= 2 \left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15} \right]$$

[Taking L. C. M FOR addition]

$$= 2 \left[\frac{16+8\sqrt{2}}{15} \right]$$

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$$= \left[\frac{16(2+\sqrt{2})}{15} \right] \quad [\text{After taking common terms}]$$

$$= \frac{16\sqrt{2}(\sqrt{2}+1)}{15}$$

$$\text{Therefore, } \int_0^2 x\sqrt{x+2} dx = \frac{16\sqrt{2}(\sqrt{2}+1)}{15}$$

$$5. \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Solution:-

$$\text{Given integral: } \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

$$\text{Let } \cos x = t$$

On differentiating

$$-\sin x dx = dt$$

$$\sin x dx = -dt$$

So, where $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}$, $t = 0$

Hence, the given integration upon substitution will change as

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = - \int_1^0 \frac{\sin x}{1+\cos^2 x} dx$$

Given integral:

$$\text{Let } \cos x = t$$

On differentiating,

$$-\sin x dx = dt$$

$$\sin x dx = -dt$$

So, when $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}$, $t = 0$

Hence the given integration upon substitution will change as

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = - \int_1^0 \frac{dt}{1+t^2}$$

On integration, we have

$$- \int_1^0 \frac{dt}{1+t^2} = - \left[\frac{1}{1} \cdot \tan^{-1} t \right]_1^0 \quad \left[\text{As w. k. t } \int \frac{dt}{x^2+a^2} = \frac{1}{a} \cdot \tan^{-1} \frac{x}{a} + c \right]$$

$$= - [\tan^{-1} 0 - \tan^{-1} 1]$$

$$= - \left[0 - \frac{\pi}{4} \right]$$

$$= - \left[-\frac{\pi}{4} \right]$$

$$= \frac{\pi}{4}$$

$$\text{Therefore, } \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = \frac{\pi}{4}$$

$$6. \int_0^2 \frac{dx}{x+4-x^2}$$

Solution:-

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Given integral: $\int_0^2 \frac{dx}{x+4-x^2}$

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-(x^2-x-4)}$$

The given integral can be written as,

$$\int_0^2 \frac{dx}{-(x^2-x+\frac{1}{4}-\frac{1}{4}-4)}$$

[By completing its square method]

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2 - \frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{-\left[\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2\right]}$$

Now, taking suitable substitution

Let $x - \frac{1}{2} = t \Rightarrow dx = dt$

So when $x = 0$, $t = -\frac{1}{2}$ and when $x = 2$, $t = \frac{3}{2}$

After substitution, the integral changes as:

$$\int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - (t)^2\right]}$$

[As w. k. t, $\int \frac{dx}{[(a)^2 - (x)^2]} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$]

On integrating, we have

$$\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - (t)^2} = \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\left(\frac{\sqrt{17}}{2} + t\right)}{\sqrt{\frac{17}{2} - t}} \right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

Applying limits,

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\left(\frac{\sqrt{17}}{2} + \frac{3}{2}\right)}{\frac{\sqrt{17}-3}{2}} - \log \frac{\left(\frac{\sqrt{17}}{2} - \frac{1}{2}\right)}{\frac{\sqrt{17}-1}{2}} \right]$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{(\sqrt{17}+3)}{\sqrt{17}-3} - \log \frac{(\sqrt{17}-1)}{\sqrt{17}+1} \right]$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{(\sqrt{17}+3)}{\sqrt{17}-3} \times \log \frac{(\sqrt{17}+1)}{\sqrt{17}+1} \right]$$

[Using logarithmic properties]

$$= \frac{1}{\sqrt{17}} \left[\log \left\{ \frac{(\sqrt{17}+3)(\sqrt{17}+1)}{(\sqrt{17}-3)(\sqrt{17}-1)} \right\} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{5+\sqrt{17}}{5-\sqrt{17}} \right]$$

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$$= \frac{1}{\sqrt{17}} \log \left[\frac{(5+\sqrt{17})(5+\sqrt{17})}{(5-\sqrt{17})(5+\sqrt{17})} \right]$$

[Rationalising the surd]

$$= \frac{1}{\sqrt{17}} \log \left[\frac{(25+17+10\sqrt{17})}{25-17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{(42+10\sqrt{17})}{8} \right] = \frac{1}{\sqrt{17}} \log \left[\frac{(21+5\sqrt{17})}{4} \right]$$

7. $\int_{-1}^1 \frac{dx}{x^2+2x+5}$

Solution:-

Given integral: $\int_{-1}^1 \frac{dx}{x^2+2x+5}$

$$= \int_{-1}^1 \frac{dx}{(x^2+2x+1)+4}$$

$$= \int_{-1}^1 \frac{dx}{(x+1)^2+(2)^2}$$

[By completing the square]

Taking substitutions, $x + 1 = t$

So, $dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

Hence, the given integral is now changed as

$$\int_{-1}^1 \frac{dx}{(x+1)^2+(2)^2} = \int_0^2 \frac{dt}{(t)^2+(2)^2}$$

[As w. k. t $\int \frac{dt}{x^2+a^2} = \frac{1}{a} \cdot \tan^{-1} \frac{x}{a} + C$]

Therefore, $\int_{-1}^1 \frac{dx}{x^2+2x+5} = \frac{\pi}{8}$

8. $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$

Solution:-

Given integral: $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$

Taking substitute, $2x = t \rightarrow 2 dx = dt$

So, when $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

Hence, the given integral will change as:

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx = \int_2^4 \left(\frac{1}{\left(\frac{t}{2}\right)} - \frac{1}{2\left(\frac{t}{2}\right)^2} \right) e^t \left(\frac{dt}{2}\right)$$

$$= \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2} \right) e^t dt$$

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$$= \int_2^4 \frac{1}{2} \cdot (2) \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt$$

[Taking common and simplification]

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt$$

Further, Let $1/t = f(t)$

Then we have, $f(t) = -1/t^2$

Converting the integral into the required form,

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt = \int_2^4 (f(t) + f'(t)) e^t dt$$

[As, w. k. t $\int f(x) + F'(x) e^x dx = e^x f(x) + C$]

Up to Integration, we get

$$\begin{aligned} \int_2^4 (f(t) + f'(t)) e^t dt &= [e^t f(t)]_2^4 \\ &= \left[e^t \cdot \frac{1}{t} \right]_2^4 \\ &= \frac{e^4}{4} - \frac{e^2}{2} \\ &= \frac{e^4 - 2e^2}{4} = \frac{e^2(e^2 - 2)}{4} \end{aligned}$$

$$\text{Therefore, } \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx = \frac{e^2(e^2 - 2)}{4}$$

Choose the correct answer in Exercise 9 and 10.

9. The value of the integral $\int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$ is

- a) 6
- b) 0
- c) 3
- d) 4

Solution:-

$$\text{Given integral: } \int_{\frac{1}{3}}^1 \left(\frac{(x-x^3)^{\frac{1}{3}}}{x^4} \right) dx$$

$$\text{Let } I = \int_{\frac{1}{3}}^1 \left(\frac{(x-x^3)^{\frac{1}{3}}}{x^4} \right) dx$$

Now, taking $x = \sin \theta \rightarrow dx = \cos \theta d\theta$

So, when $x = \frac{1}{3}$, $\theta = \sin^{-1} \left(\frac{1}{3} \right)$ and when $x = 1$, $\theta = \pi/2$

Hence, after substituting the given integral will become:

$$I = \int_{\sin^{-1} \left(\frac{1}{3} \right)}^{\frac{\pi}{2}} \left(\frac{(\sin \theta - \sin^3 \theta)^{\frac{1}{3}}}{\sin^4 \theta} \right) \cos \theta d\theta$$

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$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left\{ \frac{(\sin \theta)^{\frac{1}{3}}(1-\sin^2\theta)^{\frac{1}{3}}}{\sin^4\theta} \right\} \cos\theta d\theta$$

[Taking common]

$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left\{ \frac{(\sin \theta)^{\frac{1}{3}}(\cos^2\theta)^{\frac{1}{3}}}{\sin^4\theta} \right\} \cos\theta d\theta$$

[Simplifying by using trigonometric identify]

$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left\{ \frac{(\sin \theta)^{\frac{1}{3}}(\cos \theta)^{\frac{2}{3}}}{\sin^2\theta \cdot \sin^2\theta} \right\} \cos\theta d\theta$$

$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left(\frac{(\cos \theta)^{\frac{2}{3}+1}}{(\sin \theta)^{2-\frac{1}{3}}} \right) \cdot \frac{1}{\sin^2\theta} d\theta$$

[Simplifying by using exponents properties]

$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left(\frac{(\cos \theta)^{\frac{5}{3}}}{(\sin \theta)^{\frac{5}{3}}} \right) \cdot \operatorname{cosec}^2\theta d\theta$$

$$= \int_{\sin^{-1}(\frac{1}{3})}^{\frac{\pi}{2}} \left((\cot \theta)^{\frac{5}{3}} \right) \cdot \operatorname{cosec}^2\theta d\theta \quad \dots\dots\dots (i)$$

Now, let $\cot \theta = t \Rightarrow -\operatorname{cosec}^2\theta d\theta$

So, when, $\theta = \sin^{-1}(\frac{1}{3})$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

After substitution, (i) becomes:

$$= \int_{2\sqrt{2}}^0 -(t)^{\frac{5}{3}} \cdot dt$$

On integrating and applying limits, we have

$$= \left[\frac{(t)^{\frac{5}{3}+1}}{\frac{5}{3}+1} \right]_{2\sqrt{2}}^0$$

$$= \left[\frac{(t)^{\frac{8}{3}}}{\frac{8}{3}} \right]_{2\sqrt{2}}^0$$

$$= -\frac{3}{8}[16]$$

$$= 6$$

Therefore, the correct option is (A).

10. If $f(x) = \int_0^x t \sin t dt$, then $f'(x)$ is

- a) $\cos x + x \sin x$
- c) $x \cos x$

- b) $x \sin x$
- d) $\sin x + x \cos x$

Solution:-

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Given integral function $f(x) = \int_0^x t \sin t dt$

Applying product rule, we have

$$\int u \cdot v dx = u \cdot \int v dx - \int \frac{du}{dx} \cdot \{v dx\} dx$$

So,

$$F(x) = t \int_0^x \sin t dt - \int_0^x \left\{ \left(\frac{d}{dt} t \right) \cdot \int \sin t dt \right\} dt dt = [t(-\cos t)]_0^x - \int_0^x (-\cos t) dt$$

Applying the limits, we get

$$= [-t(\cos t) + \sin t]_0^x$$

$$= -x \cos x + \sin x - 0$$

Thus, $f(x) = -x \cos x + \sin x$

On differentiating, we have

$$F(x) = - \left[x \cdot \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} x + \frac{d}{dx} \sin x \right]$$

$$F(x) = - [x(-\sin x)] + \cos x + \cos x$$

$$= x \sin x - \cos x + \cos x$$

$$= x \sin x$$

Therefore, the correct option is (B).

Exercise 7.11

By using the properties of definite integrals, evaluate the integrals in Exercise 1 to 19.

1. $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Solution:-

Given $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Let, $I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \dots (1)$

We know that, $\left\{ \int_0^a f(x) dx = \int_0^a (a-x) dx \right\}$

By using above formula, the given question can be written as

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2 \left(\frac{\pi}{2} - x \right) dx$$

From the standard integration formulae we have

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2(x) dx \dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} [\sin^2(x) + \cos^2(x)] dx$$

By using standard identities the above equation can be written as

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [1] dx$$

Now by applying the limits we get

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$$\begin{aligned} \Rightarrow 2I &= (X)_0^{\frac{\pi}{2}} \\ \Rightarrow 2I &= \frac{\pi}{2} - 0 \\ \Rightarrow 2I &= \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$

Solution:-

Given: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$

Let, $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \dots\dots\dots (1)$

As we know that, $\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\}$

By using the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x) + \sqrt{\cos(\frac{\pi}{2}-x)}}} dx$$

By substituting the standard identities we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x + \sqrt{\cos x}}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$$

$$\begin{aligned} \Rightarrow 2I &= [x]_0^{\frac{\pi}{2}} \\ \Rightarrow 2I &= \frac{\pi}{2} - 0 \\ \Rightarrow 2I &= \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

3. $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

Solution:-

Given $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

Let, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \dots\dots (1)$

As we know that

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$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By substituting the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)} dx$$

Again by substituting the standard identities we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx \quad (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

The above equation can be written as

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [1] dx$$

Integrating and applying the limit we get

$$\begin{aligned} \Rightarrow 2I &= [x]_0^{\frac{\pi}{2}} \\ \Rightarrow 2I &= \frac{\pi}{2} - 0 \\ \Rightarrow 2I &= \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

Solution:-

Given: $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

Let, $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x} dx \dots\dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By substituting the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)} dx$$

The above equation can be written as

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx \dots\dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx$$

The above equation becomes

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [1] dx$$

Now by integrating and applying the limits we get

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2} - 0$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$5. \int_{-5}^5 |x + 2| dx$$

Solution:-

$$\text{Given:- } \int_{-5}^5 |x + 2| dx$$

As we can see that $(x+2) \leq 0$ on $[-5, -2]$ and $(x+2) \geq 0$ on $[-2, 5]$

As we know that

$$\left\{ \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right\}$$

Now by substituting the formula we get

$$\Rightarrow I = \int_{-5}^{-2} -(x+2) dx + \int_{-2}^5 (x+2) dx$$

Integrating and applying the limits we get

$$\Rightarrow I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^5$$

On simplifying

$$\Rightarrow I = -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$\Rightarrow I = -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$

On Computing we get

$$\Rightarrow I = -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$\Rightarrow I = 29$$

$$6. \int_2^8 |x - 5| dx$$

Solution:

$$\text{Given } \int_2^8 |x - 5| dx$$

As we can see that $(x - 5) \leq 0$ on $[2, 5]$ and $(x - 5) \geq 0$ on $[5, 8]$

As we know that

$$\left\{ \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right\}$$

By applying the above formula we get

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$$\Rightarrow I = \int_2^5 -(x-5)dx + \int_5^8 (x-5)dx$$

Now by integrating the above equation

$$\Rightarrow I = - \left[\frac{x^2}{2} - 5x \right]_1^5 + \left[\frac{x^2}{2} - 5x \right]_5^8$$

Now by applying the limits we get

$$\Rightarrow I = - \left[\frac{(5)^2}{2} - 5(5) - \frac{(2)^2}{2} + 5(2) \right] + \left[\frac{(8)^2}{2} - 5(8) - \frac{(5)^2}{2} + 5(5) \right]$$

On computing

$$\Rightarrow I = - \left[\frac{25}{2} - 25 - 2 + 10 \right] + \left[\frac{64}{2} - 40 - \frac{25}{2} + 25 \right]$$

$$\Rightarrow I = \frac{25}{2} + 17 + 32 - 15 - \frac{25}{2}$$

On simplifying we get

$$\Rightarrow I = 34 - 25$$

$$\Rightarrow I = 9$$

$$7. \int_0^1 x(1-x)^n dx$$

Solution:-

$$\text{Given: } \int_0^1 x(1-x)^n dx$$

$$\text{Let, } \int_0^1 x(1-x)^n dx$$

As we know that

$$\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^1 (1-x)(1-(1-x))^n dx$$

The above equation can be written as

$$\Rightarrow I = \int_0^1 (1-x)(x)^n dx$$

By multiplying we get

$$\Rightarrow I = \int_0^1 (x)^n - (x)^{n+1} dx$$

On integrating

$$\Rightarrow I = \left[\frac{(x)^{n+1}}{n+1} - \frac{(x)^{n+2}}{n+2} \right]_0^1$$

Now by applying the limits we get

$$\Rightarrow I = \left[\frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$\Rightarrow I = \left[\frac{(n+2)-(n+1)}{(n+1)(n+2)} \right]$$

On simplification

$$\Rightarrow I = \left[\frac{1}{(n+1)(n+2)} \right]$$

$$8. \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

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Solution:-

Given: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$

Let, $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \dots\dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx$$

Again we know the standard formula

$$\left\{ \tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)} \right\}$$

By substituting the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left[1 + \frac{\tan\left(\frac{\pi}{4}\right) - \tan(x)}{1 + \tan\left(\frac{\pi}{4}\right)\tan(x)} \right] dx$$

Applying the values we get

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left[1 + \frac{1 - \tan(x)}{1 + \tan(x)} \right] dx$$

On simplification the above equation can be written as

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left[\frac{2}{1 + \tan(x)} \right] dx$$

Now by applying log formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left[\frac{2}{1 + \tan(x)} \right] dx$$

Now by applying log formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log[2] dx - \int_0^{\frac{\pi}{4}} \log[1 + \tan(x)] dx$$

From equation (1) we can write as

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log[2] dx - 1$$

On integration

$$\Rightarrow 2I = [x \log 2]_0^{\frac{\pi}{4}}$$

Now by applying the limits we get

$$\Rightarrow 2I = \frac{\pi}{4} \log 2 - 0$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

9. $\int_0^2 x\sqrt{\sqrt{2-x}} dx$

Solution:-

Given: $\int_0^2 x\sqrt{\sqrt{2-x}} dx$

$$\text{Let, } I = \int_0^2 x\sqrt{\sqrt{2-x}} dx \dots\dots (1)$$

As we know that

$$\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^2 (2-x)\sqrt{2-(2-x)}dx$$

On simplification the above equation can be written as

$$\Rightarrow I = \int_0^2 (2-x)\sqrt{x} dx$$

On multiplication we get

$$\Rightarrow I = \int_0^2 \left(2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx$$

On integration

$$\Rightarrow I = \left[2 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$$

$$\Rightarrow I = \left[\frac{4}{3} \left(x^{\frac{3}{2}} \right) - \frac{2}{5} \left(x^{\frac{5}{2}} \right) \right]_0^2$$

Now by applying the limits the above equation can be written as

$$\Rightarrow I = \left[\frac{4}{3} \left((2)^{\frac{3}{2}} \right) - \frac{2}{5} \left((2)^{\frac{5}{2}} \right) \right]$$

By Computing

$$\Rightarrow I = \frac{4}{3} \times 2\sqrt{2} - \frac{2}{5} \times 4\sqrt{2}$$

$$\Rightarrow I = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

On simplification

$$\Rightarrow I = \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$\Rightarrow I = \frac{16\sqrt{2}}{15}$$

$$10. \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

Solution:-

$$\text{Let } I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

Now by applying sin 2x formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{ 2 \log \sin x - \log (2 \sin x \cos x) \} dx$$

Applying log formula we can write above equation as

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log(2) - \log(\sin x) - \log(\cos x)) dx$$

On simplification

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{ \log \sin x - \log 2 - \log \cos x \} dx \dots\dots (1)$$

As we know that

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$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By using the formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{ \log \sin \left(\frac{\pi}{2} - x \right) - \log 2 - \log \cos \left(\frac{\pi}{2} - x \right) \right\}$$

Using allied angles formulae, the above equation becomes

$$\Rightarrow I = I = \int_0^{\frac{\pi}{2}} \{ \log \cos x - \log 2 - \log \sin x \} dx \dots\dots (1)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2) dx$$

By taking common

$$2I = -2 \log 2 \int_0^{\frac{\pi}{2}} (1) dx$$

On integration we get

$$\Rightarrow 2I = -2 \log 2 \left[x \right]_0^{\frac{\pi}{2}}$$

Now by applying the limits

$$\Rightarrow 2I = -2 \log 2 \left[\frac{\pi}{2} - 0 \right]$$

$$\Rightarrow 2I = -2 \log 2 \left(\frac{\pi}{2} \right)$$

On simplification we get

$$\Rightarrow I = \frac{\pi}{2} (-\log 2)$$

$$\Rightarrow I = \frac{\pi}{2} \left(\log \frac{1}{2} \right)$$

11. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$

Solution:-

As we can see $f(x) = \sin^2 x$ and $f(-x) = \sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$.

That is $f(x) = f(-x)$

So, $\sin^2 x$ is an even function.

It is also known that if $f(x)$ is an even function then, we have

$$\left\{ \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \right\}$$

Now by using this formula the given question can be written as

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\pi}{2}} (\sin^2 x) dx$$

Now by substituting $\sin^2 x$ formula we get

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx$$

On integrating we get

$$\Rightarrow I = \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}}$$

Now by applying the limits

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$$\Rightarrow I = \frac{\pi}{2} - \sin \pi - 0 + \sin 0$$

$$\Rightarrow I = \frac{\pi}{2}$$

12. $\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$

Solution:-

Given: $\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$

Let, $I = \int_0^{\pi} \frac{x \, dx}{1 + \sin x}$ (1)

As we know that

$$\left\{ \int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx \right\}$$

By using above formula we get

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} \, dx$$

Now by multiplying and simplifying the equation we get

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} \, dx \text{ (2)}$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi} \frac{(\pi - x) + x}{1 + \sin x} \, dx$$

$$2I = \int_0^{\pi} \frac{\pi}{1 + \sin x} \, dx$$

Now by multiplying and dividing the above equation by $(1 - \sin x)$ we get

$$2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} \, dx$$

On simplification we get

$$2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{\cos^2 x} \, dx$$

By splitting the numerator we get

$$2I = \pi \int_0^{\pi} \left\{ \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right\} \, dx$$

The above equation can be written as

$$2I = \pi \int_0^{\pi} \{ \sec^2 x - \tan x \sec x \} \, dx$$

$$\Rightarrow 2I = \pi (\tan x - \sec x) \Big|_0^{\pi}$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$

13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$

Solution:-

Given: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx$

Let, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx$

As we can see $f(x) = \sin^7 x$ and $f(-x) = \sin^7(-x) = (\sin(-x))^7 = (\sin(-x))^7 = -\sin^7 x$.

That is $f(x) = -f(-x)$

So, $\sin^2 x$ is an odd function.

It is also known that if $f(x)$ is an odd function then,

$$\left\{ \int_{-a}^a f(x) dx = 0 \right\}$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx = 0$$

14. $\int_0^{2\pi} \cos^5 x dx$

Solution:-

Let, $I = \int_0^{2\pi} \cos^5 x dx$

As we see, $f(x) = \cos^5 x$ and $f(2\pi - x) = \cos^5(2\pi - x) = \cos^5 x = f(x)$

Because, $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$, if $f(2a - x) = f(x)$

and $\int_0^{2a} f(x) dx = 0$, if $f(2a - x) = -f(x)$

$$\Rightarrow I = 2 \int_0^{\pi} (\cos^5 x) dx$$

Now $\{\cos^5(\pi - x) = -\cos^5 x\}$

$$\Rightarrow I = 0$$

15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

Solution:-

Given: $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

Let, $I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \dots\dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a - x) dx \right\}$$

By using the above formula in given equation it can be written as

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx$$

Now by applying allied angle formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \dots\dots\dots (2)$$

Adding (1) and (2), we get

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$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x + \cos x - \sin x}{1 + \sin x \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 - \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

16. $\int_0^{\pi} \log(1 + \cos x) dx$

Solution:-

Given:- $\int_0^{\pi} \log(1 + \cos x) dx$

Let, $I = \int_0^{\pi} \log(1 + \cos x) dx \dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

Now by using the above formula we get

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx$$

Here by allied angle formula we get

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi} \{ \log(1 + \cos x) + \log(1 - \cos x) \} dx$$

The above equation can be written as

$$2I = \int_0^{\pi} \log(1 - \cos^2 x) dx$$

By using trigonometric identities we get

$$2I = \int_0^{\pi} \log(\sin^2 x) dx$$

$$2I = \int_0^{\pi} 2 \cdot \log(\sin x) dx$$

$$2I = 2 \cdot \int_0^{\pi} \log(\sin x) dx$$

$$I = \int_0^{\pi} \log(\sin x) dx \dots (3)$$

Because, $\int_0^{2a} f(x) dx = 2 \cdot \int_0^a f(x) dx$, if $f(2a - x) = f(x)$

Here, if $f(x) = \log(\sin x)$ and $f(\pi - x) = \log(\sin(\pi - x)) = \log(\sin x) = f(x)$

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\pi}{2}} \log \sin x dx \dots (4)$$

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - x \right) dx$$

By using trigonometric equation we get

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\pi}{2}} \log \cos x dx \dots (5)$$

Adding (1) and (2), we get

$$\Rightarrow 2I = 2 \cdot \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

Now by adding and subtracting $\log 2$ we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

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The above equation can be written as

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log(2 \sin x \cos x) - \log 2) dx$$

Now by splitting the integral we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log(\sin 2x) dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

Let $2x = t \rightarrow 2dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \pi$

$$\Rightarrow I = \left[\frac{1}{2} \int_0^{\pi} (\log(\sin t) dt) \right] - \left(\frac{\pi}{2} \log 2 \right)$$

$$\Rightarrow I = \left[\frac{1}{2} \right] - \left(\frac{\pi}{2} \log 2 \right)$$

$$\Rightarrow I = - \left(\frac{\pi}{2} \log 2 \right)$$

$$\Rightarrow I = - (\pi \log 2)$$

17. $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

Solution:-

Given: $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

Let, $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \dots\dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \dots\dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

The Above equation becomes,

$$\Rightarrow 2I = \int_0^a [1] dx$$

On integrating we get

$$\Rightarrow 2I = [x]_0^a$$

Now by applying the limits

$$\Rightarrow 2I = a - 0$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

18. $\int_0^4 |x - 1| dx$

Solution:-

Given: $\int_0^4 |x - 1| dx$

As we can see that $(x - 1) \leq 0$ when $0 \leq x \leq 1$ and $(x - 1) \geq 0$ when $1 \leq x \leq 4$

As we know that

$$\left\{ \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right\}$$

By substituting the above formula we get

$$\Rightarrow I = \int_0^1 -(x - 1) dx + \int_1^4 (x - 1) dx$$

On integration

$$\Rightarrow I = - \left[\frac{x^2}{2} - x \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4$$

Now by applying the limit we get

$$\Rightarrow I = - \left[\frac{(1)^2}{2} - 1 - \frac{(0)^2}{2} + 0 \right] + \left[\frac{(4)^2}{2} - 4 - \frac{(1)^2}{2} + 1 \right]$$

$$\Rightarrow I = - \left[\frac{1}{2} - 1 \right] + \left[8 - 4 - \frac{1}{2} + 1 \right]$$

$$\Rightarrow I = \frac{1}{2} + 5 - \frac{1}{2}$$

$$\Rightarrow I = 5$$

19. Show that $\int_0^a f(x)g(x) dx = 2 \int_0^{\frac{a}{2}} f(x) dx$, if f and g are defined as $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$

Solution:-

Given:- $\int_0^a f(x)g(x) dx$

Let, $I = \int_0^a f(x)g(x) dx \dots \dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a - x) dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^a f(a - x)g(a - x) dx$$

$$\Rightarrow \int_0^a f(x)g(a - x) dx \dots \dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^a \{f(x)g(x) + f(x)g(a - x)\} dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a - x)\} dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{4\} dx \text{ as, } \{g(x) + g(a - x) = 4\}$$

$$\Rightarrow I = \frac{1}{2} \int_0^a f(x) \times 4 dx$$

$$\Rightarrow I = 2 \cdot \int_0^a f(x) dx$$

Choose the correct answer in Exercise 20 and 21.

20. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + \cos x + \tan^5 x + 1) dx$ is

- a) 0
- b) 2
- c) π
- d) 1

Solution:-

(c) π

Explanation:-

Given:- $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + \cos x + \tan^5 x + 1) dx$ is

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + \cos x + \tan^5 x + 1) dx$ is

Now by splitting the integrals we get

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3) dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x \cos x) dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\tan^2 x) dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1) dx$$

It is also known that if $f(x)$ is an even function then,

$$\left\{ \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \right\}$$

It is also known that if $f(x)$ is an odd function then,

$$\Rightarrow I = 0 + 0 + 0 + 2 \cdot \int_0^{\frac{\pi}{2}} (1) dx \left\{ \int_{-a}^a f(x) dx = 0 \right\}$$

$$\Rightarrow I = 2 \cdot [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I = 2 \cdot \frac{\pi}{2}$$

$$\Rightarrow I = \pi$$

Correct answer is C

21. The value of $\int_0^{\frac{\pi}{2}} \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) dx$ is

- a) 2
- b) $\frac{3}{4}$
- c) 0
- d) -2

Solution:-

(c) 0

Explanation:-

Given: $\int_0^{\frac{\pi}{2}} \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) dx$

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Let, $I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) dx \dots\dots (1)$

As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By using the above formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3 \sin \left(\frac{\pi}{2}-x \right)}{4+3 \cos \left(\frac{\pi}{2}-x \right)} \right) dx$$

By applying allied angles formulae we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3 \cos x}{4+3 \sin x} \right) dx \dots\dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \left\{ \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) + \left(\frac{4+3 \cos x}{4+3 \sin x} \right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

Substituting $\log 1 = 0$ we get

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0. dx$$

$$\Rightarrow I = 0$$

Correct answer is (c)

MISCELLANEOUS EXERCISE

Integrate the functions in Exercises 1 to 24.

1. $\frac{1}{x-x^3}$

Solution:-

Given:- $\frac{1}{x-x^3}$

Let $I = \frac{1}{x-x^3} = \frac{1}{x(1-x^2)} = \frac{1}{x(1+x)(1-x)}$

Using partial differentiation

Let $\frac{1}{x(1+x)(1-x)} = \frac{A}{x} + \frac{B}{1+x} + \frac{C}{1-x} \dots (1)$

By taking LCM we get

$$\Rightarrow \frac{1}{x(1+x)(1-x)} = \frac{A(1+x)(1-x) + B(x)(1-x) + C(x)(1+X)}{X(1+X)(1-X)}$$

$$\Rightarrow \frac{1}{x(1+x)(1-x)} = \frac{A(1-x^2) + Bx(1-x) + cx(1+x)}{x(1+x)(1-x)}$$

$$\Rightarrow 1 = A - Ax^2 + Bx - Bx^2 + Cx + Cx^2$$

$$\Rightarrow 1 = A + (B + C)x + (-A - B + C)x^2$$

Equating the coefficient of x, x^2 and constant value. We get;

(a) $A = 1$

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$$(b) B + C = 0 \rightarrow B = -C$$

$$(c) -A - B + C = 0$$

$$\rightarrow -1 - (-c) + c = 0$$

$$\rightarrow 2C = 1 \rightarrow C = \frac{1}{2}$$

$$\text{So, } B = -1/2$$

Put these values in equation (1)

$$\Rightarrow \frac{1}{x(1+x)(1-x)} = \frac{1}{x} + \frac{-\left(\frac{1}{2}\right)}{1+x} + \frac{\left(\frac{1}{2}\right)}{1-x}$$

$$\Rightarrow \int \frac{1}{x(1+x)(1-x)} dx = \int \frac{1}{x} dx - \frac{1}{2} \int \frac{1}{1+x} dx + \frac{1}{2} \int \frac{1}{1-x} dx$$

On integrating we get

$$= \log |x| - \frac{1}{2} \log |1+x| + \frac{1}{2} \log |1-x|$$

By using logarithmic formula the above equation can be written as

$$= \log |x| - \log \left| (1+x)^{\frac{1}{2}} \right| + \log \left| (1-x)^{\frac{1}{2}} \right|$$

$$= \log \left| \frac{x}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{1}{2}}} \right| + C$$

On simplification we get

$$= \log \left| \frac{(x^2)^{\frac{1}{2}}}{(1+x)(1-x)^{\frac{1}{2}}} \right| + C$$

$$= \log \left| \frac{(x^2)^{\frac{1}{2}}}{(1-x^2)^{\frac{1}{2}}} \right| + C$$

$$= \log \left| \left(\frac{x^2}{1-x^2} \right)^{\frac{1}{2}} \right| + C$$

$$\Rightarrow I = \frac{1}{2} \log \left| \frac{x^2}{1-x^2} \right| + C$$

$$2. \frac{1}{\sqrt{X+A} + \sqrt{X+B}}$$

Solution:-

$$\text{Given: } \frac{1}{\sqrt{X+A} + \sqrt{X+B}}$$

$$\text{Let } I = \frac{1}{\sqrt{X+A} + \sqrt{X+B}}$$

Multiply and divide by, $\sqrt{x+a} - \sqrt{x+b}$

$$\Rightarrow I = \frac{1}{\sqrt{X+A} + \sqrt{X+B}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$= \frac{\sqrt{x+a} - \sqrt{x+b}}{(\sqrt{x+a})^2 - (\sqrt{x+b})^2}$$

On simplification we get

$$= \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)}$$

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$$= \frac{\sqrt{x+a} - \sqrt{x+b}}{a-b}$$

Applying integration

$$\begin{aligned} \Rightarrow \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} dx &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{a-b} dx \\ &= \frac{1}{a-b} \int (\sqrt{x+a} - \sqrt{x+b}) dx \\ &= \frac{1}{a-b} \int \left((x+a)^{\frac{1}{2}} - (x+b)^{\frac{1}{2}} \right) dx \end{aligned}$$

On integrating we get

$$\begin{aligned} &= \frac{1}{a-b} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}} \right] \\ \Rightarrow I &= \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + C \end{aligned}$$

3. $\frac{1}{x\sqrt{ax-x^2}}$ [Hint: Put $x = \frac{a}{t}$]

Solution:-

Given: $\frac{1}{x\sqrt{ax-x^2}}$

Let $I = \frac{1}{x\sqrt{ax-x^2}}$

Put $x = \frac{a}{t} \rightarrow dx = -\frac{a}{t^2} dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}} dx = \int \frac{1}{\frac{a}{t} \sqrt{\frac{a \cdot a}{t} - \left(\frac{a}{t}\right)^2}} \cdot \frac{a}{t^2} dt$$

By taking a common we get

$$= \int \frac{-1}{at} \cdot \frac{1}{\sqrt{\frac{1}{t} - \left(\frac{1}{t}\right)^2}} dt$$

Now by multiplying t we get

$$= -\frac{1}{a} \int \frac{1}{\sqrt{\frac{t^2}{t} - \left(\frac{t}{t}\right)^2}} dt$$

The above equation becomes

$$= -\frac{1}{a} \int \frac{1}{\sqrt{t-1}} dt$$

$$= -\frac{1}{a} \int (t-1)^{\frac{1}{2}} dt$$

On integrating we get

$$= -\frac{1}{a} \left[\frac{\sqrt{(t-1)}}{\frac{1}{2}} \right] + C$$

$$= -\frac{2}{a} \left[\sqrt{\left(\frac{a}{x} - 1\right)} \right] + c \text{ because, } t = \frac{a}{x}$$

$$\Rightarrow I = -\frac{2}{a} \left[\sqrt{\left(\frac{a-x}{x}\right)} \right] + c$$

$$4. \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Solution:-

$$\text{Given:- } \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

$$\text{Let } I = \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Multiply and divide by x^3 , we get

$$\frac{x^{-3}}{x^2 \cdot x^{-3} (x^4+1)^{\frac{3}{4}}} = \frac{x^{-3} \cdot (x^4+1)^{-\frac{3}{4}}}{x^2 \cdot x^{-3}}$$

$$= \frac{(x^4+1)^{\frac{3}{4}}}{x^5 \cdot x^{-3} x^4}$$

On simplification the above equation can be written as

$$= \frac{1}{x^5} \cdot \left(1 + \frac{1}{x^4}\right)^{\frac{3}{4}}$$

$$\text{Let, } \frac{1}{x^4} = t = (x)^{-4} \rightarrow \frac{-4}{x^5} dx = dt \rightarrow \frac{1}{x^5} dx = -\frac{dt}{4}$$

$$\Rightarrow \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} \cdot dx = \int \frac{1}{x^5} \cdot \left(1 + \frac{1}{x^4}\right)^{\frac{3}{4}} \cdot dx$$

Substituting the above values we get

$$= \int (1+t)^{\frac{3}{4}} \cdot \left(-\frac{dt}{4}\right)$$

$$= -\frac{1}{4} \int (1+t)^{\frac{3}{4}} \cdot dt$$

On integrating

$$= -\frac{1}{4} \left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}} \right] + C$$

Now by substituting the value of t we get

$$= -\frac{1}{4} \left[\frac{\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}} \right] + C$$

$$= -\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + C$$

$$5. \frac{1}{x^2+x^3}$$

$$\left[\text{Hint: } \frac{1}{x^2+x^3} = \frac{1}{x^3\left(1+\frac{1}{x}\right)}, \text{ put } x = t^6 \right]$$

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Solution:-

Given $\frac{1}{x^2+x^3}$

Given question can be written as,

$$\frac{1}{x^2+x^3} = \frac{1}{x^2(1+x^6)}$$

Let $x = t^6 \rightarrow dx = 6t^5 dt$

$$\Rightarrow \int \frac{1}{x^3(1+x^6)} \cdot dx = \int \frac{6t^5}{t^2(1+t)} \cdot dt$$

On computing we get

$$= 6 \cdot \int \frac{t^3}{(1+t)} \cdot dt$$

After division we get,

$$\frac{1}{x^2+x^3} = 6 \cdot \int [t^2 - t + 1] - \frac{1}{(1+t)} \cdot dt$$

Now by splitting the integrals and computing

$$= 6 \cdot \left\{ \int t^2 \cdot dt - \int t \cdot dt + \int 1 \cdot dt - \int \left[\frac{1}{(1+t)} \right] \cdot dt \right\}$$

On integrating

$$= 6 \left[\left(\frac{t^3}{3} \right) - \left(\frac{t^2}{2} \right) + t - \log(1+t) \right]$$

Now by substituting the value of t we get

$$= 6 \left[\left(\frac{\left(\frac{1}{x^6} \right)^3}{3} \right) - \left(\frac{\left(\frac{1}{x^6} \right)^2}{2} \right) + \left(\frac{1}{x^6} \right) - \log \left(1 + \left(\frac{1}{x^6} \right) \right) \right] + C$$

$$= \left[\left(2x^{\frac{1}{2}} \right) - \left(3 \times \frac{1}{3} \right) + 6 \cdot x^{\frac{1}{6}} - 6 \cdot \log \left(1 + x^{\frac{1}{6}} \right) \right] + c$$

$$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log \left(1 + x^{\frac{1}{6}} \right) + C$$

6. $\frac{5x}{(x+1)(x^2+9)}$

Solution:-

Given: $\frac{5x}{(x+1)(x^2+9)}$

Let $I = \frac{5x}{(x+1)(x^2+9)}$

Using partial fraction

Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+9)} \dots (1)$

$$\Rightarrow \frac{5x}{(x+1)(x^2+9)} = \frac{A(x^2+9) + (Bx+C)(x+1)}{(x+1)(x^2+9)}$$

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$$\begin{aligned}\Rightarrow 5x &= A(x^2 + 9) + (Bx + c)(x + 1) \\ \Rightarrow 5x &= Ax^2 + 9A + Bx^2 + Bx + Cx + C \\ \Rightarrow 5x &= 9A + C + (B + C)x + (A+B)x^2\end{aligned}$$

Equating the coefficient of x , x^2 and constant value, we get

$$(a) 9A + C = 0 \Rightarrow C = -9A$$

$$(b) B + C = 5 \Rightarrow B = 5 - C \Rightarrow B = 5 - (-9A) \Rightarrow B = 5 + 9A$$

$$(c) A + B = 0 \Rightarrow A = -B \Rightarrow A = -(5 + 9A) \Rightarrow 10A = -5 \Rightarrow A = -1/2$$

And $C = 9/2$ and $B = 1/2$

Put these values in equation (1) we get

$$\begin{aligned}\Rightarrow \frac{5x}{(x+1)(x^2+9)} &= \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+9)} \\ \Rightarrow \frac{5x}{(x+1)(x^2+9)} &= \frac{-\frac{1}{2}}{(x+1)} + \frac{\left(\frac{1}{2}\right)x + \frac{9}{2}}{(x^2+9)}\end{aligned}$$

The above equation can be written as

$$\Rightarrow \frac{5x}{(x+1)(x^2+9)} = -\frac{1}{2} \cdot \frac{1}{(x+1)} dx + \frac{1}{2} \cdot \left(\frac{x+9}{x^2+9}\right)$$

Now by applying integrals on both sides we get

$$\Rightarrow \int \frac{5x}{(x+1)(x^2+9)} dx = -\frac{1}{2} \cdot \int \frac{1}{(x+1)} dx + \frac{1}{2} \cdot \int \frac{x}{(x^2+9)} dx + \frac{9}{2} \int \frac{1}{(x^2+9)} dx$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2+9)} dx = -\frac{1}{2} \cdot \int \frac{1}{(x+1)} dx + I_1 + \frac{9}{2} \int \frac{1}{x^2+(3^2)} dx$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2+9)} dx = -\frac{1}{2} \cdot \text{Log}|x+1| + I_1 + \frac{9}{2} \cdot \left(\frac{1}{3} \tan^{-1} \frac{x}{3}\right) \dots\dots (2)$$

Now solving for I_2 we get

$$I_1 = \frac{1}{2} \cdot \int \frac{x}{(x^2+9)} dx$$

Put $x^2 = t \Rightarrow 2x dx = dt$

$$\Rightarrow I_1 = \frac{1}{2} \cdot \int \frac{1}{(t+9)} \cdot \frac{dt}{2}$$

$$\Rightarrow I_1 = \frac{1}{4} \log|t+9|$$

$$\Rightarrow I_1 = \frac{1}{4} \log|x^2+9|$$

Put the value in equation (2)

$$\Rightarrow \int \frac{5x}{(x+1)(x^2+9)} dx = -\frac{1}{2} \cdot \log|x+1| + \frac{1}{4} \log|x^2+9| + \frac{3}{2} \cdot \left(\tan^{-1} \frac{x}{3}\right) + C$$

7. $\frac{\sin x}{\sin(x-a)}$

Solution:-

Given:- $\frac{\sin x}{\sin(x-a)}$

Let $I = \frac{\sin x}{\sin(x-a)}$

Let $x-a = t \Rightarrow x = t+a \Rightarrow dx = dt$

$$\Rightarrow \int \frac{\sin x}{\sin(x-a)} dx = \int \frac{\sin(t+a)}{\sin(t)} dt$$

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As we know that, $\{\sin (A+B) = \sin A \cos B + \cos A \sin B\}$

$$\Rightarrow \int \frac{\sin x}{\sin (x-a)} dx = \int \frac{\sin t \cos a + \cos t \sin a}{\sin (t)} dt$$

The above equation becomes

$$= \int \frac{\sin t \cos a}{\sin t} + \frac{\cos t \sin a}{\sin t} dt$$

On simplification

$$= \int (\cos a + \cot t \sin a) dt$$

Now by splitting the integrals we get

$$= \int (\cos a) dt + \int (\cot t \sin a) dt$$

$$= (\cos a) \int 1. dt + \sin a. \int \cot t) dt$$

On integrating we get

$$= (\cos a). t + \sin a. \log |\sin t| + C$$

Now by substituting the value of t we get

$$= (\cos a). (x - a) + \sin a. \log |\sin(x - a)| + C$$

$$= \sin a. \log |\sin(x - a)| + x. \cos a - a. \cos a + C$$

$$= \sin a. \log |\sin(x - a)| + x. \cos a + C_2$$

8. $\frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}}$

Solution:-

Given $\frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}}$

Let, $I = \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}}$

Now by taking common and above equation can be written as

$$\Rightarrow \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} = \frac{e^{4 \log x} (e^{\log x} - 1)}{e^{2 \log x} (e^{\log x} - 1)}$$

On simplification

$$= e^{2 \log x}$$

$$= e^{\log x^2}$$

$$= x^2$$

Applying integrals

$$= \int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx = \int x^2 dx$$

$$= \frac{x^3}{3} + C$$

9. $\frac{\cos x}{\sqrt{4 - \sin^2 x}}$

Solution:-

Given; $\frac{\cos x}{\sqrt{4 - \sin^2 x}}$

Let $I = \frac{\cos x}{\sqrt{4-\sin^2 x}}$

Put $\sin x = t \rightarrow \cos x dx = dt$

The given equation can be written as

$$\begin{aligned} \Rightarrow \int \frac{\cos x}{\sqrt{4-\sin^2 x}} dx &= \int \frac{1}{\sqrt{4-t^2}} dt \\ &= \int \frac{1}{\sqrt{(2^2-t^2)}} dt \end{aligned}$$

On integrating we get

$$\begin{aligned} &= \sin^{-1} \left(\frac{t}{2} \right) + C \\ \Rightarrow I &= \sin^{-1} \left(\frac{\sin x}{2} \right) + C \end{aligned}$$

10. $\frac{\sin^8 - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

Solution:-

Given: $\frac{\sin^8 - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

Let, $I = \frac{\sin^8 - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

As we know that $a^2 - b^2 = (a + b)(a - b)$

Now by using this formula we get

$$\begin{aligned} \Rightarrow \frac{\sin^8 - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} &= \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{(\sin^4 x + \cos^4 x)(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)} \\ \Rightarrow \frac{(\sin^4 x - \cos^4 x)}{(\sin^2 x - \sin^2 x \cos^2 x) + (\cos^2 x - \sin^2 x \cos^2 x)} \end{aligned}$$

We know that $\cos^2 + \sin^2 x = 1$, using this in above equation

$$\begin{aligned} &= \frac{(\sin^4 x + \cos^4 x)(\sin^2 x - \cos^2 x) \cdot (1)}{(\sin^2 x(1 - \cos^2 x) + \cos^2 x(1 - \sin^2 x))} \\ &= \frac{-(\sin^4 x + \cos^4 x)(\cos^2 x - \sin^2 x)}{\sin^2 x(\sin^2 x) + \cos^2 x(\cos^2 x)} \end{aligned}$$

On simplification we get

$$= \frac{-(\sin^4 x + \cos^4 x)(\cos^2 x - \sin^2 x)}{(\sin^4 x + \cos^4 x)}$$

$$= (\sin^2 x - \cos^2 x)$$

$$= -\cos 2x$$

$$\Rightarrow \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx = \int -\cos 2x dx$$

On integrating

$$\Rightarrow I = -\frac{\sin 2x}{2} + C$$

11. $\frac{1}{\cos(x+A)\cos(x+B)}$

Solution:-

Given:- $\frac{1}{\cos(x+A)\cos(x+B)}$

Let $I = \frac{1}{\cos(x+A)\cos(x+B)}$

Multiply and divide by $\sin(a-b)$, we get

$$I = \frac{1}{\sin(a-b)} \cdot \left(\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)} \right)$$

Now by adding and subtracting x from the numerator

$$= \frac{1}{\sin(a-b)} \cdot \left(\frac{\sin(a-b+x-x)}{\cos(x+a)\cos(x+b)} \right)$$

By grouping we get

$$= \frac{1}{\sin(a-b)} \cdot \left(\frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)} \right)$$

As we know that $\{\sin(A-B) = \sin A \cos B - \cos A \sin B\}$

By using this formula we get

$$\Rightarrow I = \frac{1}{\sin(a-b)} \cdot \left(\frac{\sin(x+a)}{\cos(x+a)} - \frac{\sin(x+b)}{\cos(x+b)} \right)$$

$$\Rightarrow \frac{1}{\sin(a-b)} \cdot [\tan(x+a) - \tan(x+b)]$$

Taking integrals on both sides we get

$$\Rightarrow \int \frac{1}{\cos(x+a)\cos(x+b)} dx = \int \frac{1}{\sin(a-b)} \cdot [\tan(x+a) - \tan(x+b)] dx$$

$$= \frac{1}{\sin(a-b)} \left\{ \int \tan(x+a) dx - \int \tan(x+b) dx \right\}$$

On integrating we get

$$= \frac{1}{\sin(a-b)} [-\log|\cos(x+a)| - (-\log|\cos(x+b)|)]$$

$$= \frac{1}{\sin(a-b)} [-\log|\cos(x+a)| + \log|\cos(x+b)|]$$

$$\Rightarrow I = \frac{1}{\sin(a-b)} \cdot \log \left| \frac{\cos(x+b)}{\cos(x+a)} \right| + C$$

12. $\frac{x^3}{\sqrt{1-x^8}}$

Solution:-

Let $I = \frac{x^3}{\sqrt{1-x^8}}$

Now, let $x^4 = t \Rightarrow 4x^3 dx = dt$

And $x^3 dx = \frac{dt}{4}$

Substituting these values in given question we get

$$\Rightarrow \int \frac{x^2}{1-x^8} dx = \int \frac{1}{\sqrt{1-t^2}} \left(\frac{dt}{4} \right)$$

$$= \frac{1}{4} \int \frac{1}{\sqrt{1-t^2}} \cdot dt$$

On integrating we get

$$= \frac{1}{4} \sin^{-1}t + c$$

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Now by substituting t value we get

$$\Rightarrow I = \frac{1}{4} \sin^{-1}(x^4) + C$$

13. $\frac{e^x}{(1+e^x)(2+e^x)}$

Solution:-

Given: $\frac{e^x}{(1+e^x)(2+e^x)}$

Let, $I = \frac{e^x}{(1+e^x)(2+e^x)}$

Let $e^x = t \rightarrow e^x dx = dt$

Now substituting these values in given question we get

$$\begin{aligned} \Rightarrow \int \frac{e^x}{(1+e^x)(2+e^x)} dx &= \int \frac{1}{(1+t)(2+t)} dt \\ &= \int \left[\frac{1}{1+t} - \frac{1}{2+t} \right] dt \end{aligned}$$

Now by splitting the integrals we get

$$= \int \left[\frac{1}{1+t} \right] dt - \int \left[\frac{1}{2+t} \right] dt$$

On integrating we get

$$= \log|(1+t)| - \log|2+t| + C$$

$$= \log \left| \frac{1+t}{2+t} \right| + C$$

$$\Rightarrow I = \log \left| \frac{1+e^x}{2+e^x} \right| + C$$

14. $\frac{1}{(x^2+1)(x^2+4)}$

Solution:-

Given: $\frac{1}{(x^2+1)(x^2+4)}$

Let $I = \frac{1}{(x^2+1)(x^2+4)}$

Using partial fraction method, we get

Let $\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$ (1)

$$\Rightarrow \frac{1}{(x^2+1)(x^2+4)} = \frac{(Ax+B)(x^2+4) + (Cx+D)(x^2+1)}{(x^2+1)(x^2+4)}$$

$$\Rightarrow 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$$

$$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$$

$$\Rightarrow 1 = (A+C)x^3 + (B+D)x^2 + (4A+C)x + (4B+D)$$

Equating the coefficient of x , x^2 , x^3 and constant value. We get:

(a) $A + C = 0 \rightarrow C = -A$

(b) $B + D = 0 \rightarrow B = -D$

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$$(c) 4A + C = 0 \rightarrow 4A = -C \rightarrow 4A = A \rightarrow 3A = 0 \rightarrow A = 0 \rightarrow C = 0$$

$$(d) 4B + D = 1 \rightarrow 4B - B = 1 \rightarrow B = 1/3 \rightarrow D = -1/3$$

Put these values in equation (1)

$$\Rightarrow \frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$$

$$\Rightarrow \frac{1}{(x^2+1)(x^2+4)} = \frac{(0)x + \frac{1}{3}}{(x^2+1)} + \frac{(0)x + (-\frac{1}{3})}{(x^2+4)}$$

$$\Rightarrow \frac{1}{(x^2+1)(x^2+4)} = \frac{\frac{1}{3}}{(x^2+1)} + \frac{(-\frac{1}{3})}{(x^2+4)}$$

Now by taking integrals on both sides we get

$$\Rightarrow \int \frac{1}{(x^2+1)(x^2+4)} dx = \frac{1}{3} \cdot \int \frac{1}{(x^2+1)} dx - \frac{1}{3} \cdot \int \frac{1}{(x^2+4)} dx$$

$$\Rightarrow \int \frac{1}{(x^2+1)(x^2+4)} dx = \frac{1}{3} \cdot \int \frac{1}{(x^2+1^2)} dx - \frac{1}{3} \cdot \int \frac{1}{(x^2+2^2)} dx$$

On integrating we get

$$= \frac{1}{3} \cdot \tan^{-1}x - \frac{1}{3} \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

$$\Rightarrow I = \frac{1}{3} \cdot \tan^{-1}x - \frac{1}{6} \tan^{-1} \frac{x}{2} + C$$

15. $\cos^3 x e^{\log \sin x}$

Solution:-

Given: $\cos^3 x e^{\log \sin x}$

Let $I = \cos^3 x e^{\log \sin x}$

Logarithmic and exponential functions cancels each other in above equation then we get
 $= \cos^3 x \cdot \sin x$

Let $\cos x = t \rightarrow -\sin x dx = dt \rightarrow \sin x dx = -dt$

Substituting these values in given question we get

$$\Rightarrow \int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \cdot \sin x dx$$

$$= \int t^3 \cdot (-dt)$$

$$= - \int t^3 \cdot dt$$

On integrating

$$= - \frac{t^4}{4} + C$$

Now by substituting the value of t we get

$$= - \frac{\cos^4 x}{4} + C$$

16. $e^{3 \log x} (x^4 + 1)^{-1}$

Solution:-

Given:- $e^{3 \log x} (x^4 + 1)^{-1}$

Let $I = e^{3 \log x} (x^4 + 1)^{-1}$

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$$= e^{\log^3 (x^4 + 1)^{-1}}$$

Logarithmic and exponential functions cancels each other in above equation then we get

$$= \frac{x^3}{x^4+1}$$

$$\text{Let } x^4 = t \rightarrow 4x^3 dx = dt \rightarrow x^3 dx = dt / 4$$

Now by substituting these values in given question we get

$$\Rightarrow \int e^{3 \log x} (x^4 + 1)^{-1} = \int \frac{x^3}{x^4+1} dx$$

$$\Rightarrow \int \frac{1}{t+1} \cdot \frac{dt}{4}$$

$$\Rightarrow \frac{1}{4} \log (t + 1) + c$$

Now by substituting the values of t we get

$$\Rightarrow I = \frac{1}{4} \log(x^4 + 1) + C$$

17. $f'(ax + b) [f(ax + b)]^n$

Solution:-

$$\text{Given: } f'(ax + b) [f(ax + b)]^n$$

$$\text{Let } f(ax + b) = t \rightarrow a \cdot f(ax + b) dx = dt$$

Now by substituting these values in given question we get

$$\Rightarrow \int f'(ax + b) [f(ax + b)]^n = \int t^n \left(\frac{dt}{a}\right)$$

$$= \frac{1}{a} \int \frac{t^{n+1}}{n+1} + C$$

$$= \frac{1}{a(n+1)} \cdot (f(ax + b))^{n+1} + C$$

18. $\frac{1}{\sqrt{\sin^3 x \cos (x + \alpha)}}$

Solution:-

$$\text{Given: } \frac{1}{\sqrt{\sin^3 x \cos (x + \alpha)}}$$

$$\text{Let } I = \frac{1}{\sqrt{\sin^3 x \cos (x + \alpha)}}$$

As we know that, $\{ \sin (A + B) = \sin A \cos B + \cos A \sin B \}$

Using this formula we get

$$\Rightarrow I = \frac{1}{\sqrt{\sin^3 x (\sin x \cos \alpha + \cos x \sin \alpha)}}$$

Multiplying and dividing by $\sin x$ to denominator we get

$$\Rightarrow I = \frac{1}{\sqrt{\sin^2 x (\sin x \cos \alpha + \cos x \frac{\sin x}{\sin x} \sin \alpha)}}$$

On rearranging we get

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$$= \frac{1}{\sqrt{\sin^2 x (\sin x \cos \alpha + \sin x \frac{\cos x}{\sin x} \sin \alpha)}}$$

Simplifying we get

$$= \frac{1}{\sqrt{\sin^4 x (\cos \alpha + \cot x \sin \alpha)}}$$

$$= \frac{1}{\operatorname{cosec}^2 x \sqrt{(\cos \alpha + \cot x \sin \alpha)}}$$

Now, let $(\cos \alpha + \cot x \sin \alpha) = t \rightarrow -\operatorname{cosec}^2 x \cdot \sin \alpha dx = dt$

Now by substituting these values in given question we get

$$\Rightarrow \int \frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} dx = \int \frac{\operatorname{cosec}^2 x}{(\cos \alpha + \cot x \sin \alpha)} dx$$

$$\Rightarrow \int \frac{1}{\sqrt{t}} \cdot -\frac{dt}{\sin \alpha}$$

$$= -\frac{1}{\sin \alpha} \int \frac{1}{\sqrt{t}} \cdot dt$$

$$= -\frac{1}{\sin \alpha} \int t^{-\frac{1}{2}} \cdot dt$$

On integrating we get

$$= -\frac{1}{\sin \alpha} \left[\frac{t^{-\frac{1}{2} + 1}}{-\frac{1}{2} + 1} \right] + C$$

$$= -\frac{2}{\sin \alpha} [\sqrt{t}] + C$$

Now by substituting the value of t

$$= -\frac{2}{\sin \alpha} \left[\sqrt{\sqrt{(\cos \alpha + \cot x \sin \alpha)}} \right] + C$$

Computing and simplifying

$$= -\frac{2}{\sin \alpha} \left[\sqrt{\frac{\cos \alpha \sin x + \cos x \sin \alpha}{\sin x}} \right] + C$$

$$\Rightarrow I = -\frac{2}{\sin \alpha} \left[\sqrt{\frac{\sin(x + \alpha)}{\sin x}} \right] + C$$

19. $\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}, x \in [0, 1]$

Solution:-

Given:- $\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}$

Let $I = \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}} \dots (1)$

As we know, $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$

Now using this identify we get

$$\Rightarrow I = \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}} dx = \int \frac{\left(\frac{\pi}{2}-\cos^{-1}\sqrt{x}\right)-\cos^{-1}\sqrt{x}}{\left(\frac{\pi}{2}\right)} dx$$

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$$\Rightarrow \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx = \int \frac{\left(\frac{\pi}{2} - \cos^{-1}\sqrt{x}\right) - \cos^{-1}\sqrt{x}}{\left(\frac{\pi}{2}\right)} dx$$

$$= \left(\frac{2}{\pi}\right) \int \left(\frac{\pi}{2} - 2\cos^{-1}\sqrt{x}\right) dx$$

Now by splitting the integral we get

$$= \left(\frac{2}{\pi}\right) \int \left(\frac{\pi}{2} \cdot dx\right) - \left(\frac{2}{\pi}\right) \int 2 \cdot (\cos^{-1}\sqrt{x} \cdot dx)$$

$$= \int (1 \cdot dx) - \left(\frac{4}{\pi}\right) \int (\cos^{-1}\sqrt{x} \cdot dx)$$

On integration we get

$$\Rightarrow I = x - \left(\frac{4}{\pi}\right) I_1 \dots (2)$$

Now, first solve for I_2 :

$$\text{As, } I_1 = \int (\cos^{-1}\sqrt{x} \cdot dx)$$

$$\text{Let } \sqrt{x} = t \Rightarrow \frac{1}{2} x^{\frac{1}{2}} dx = dt \Rightarrow \frac{dx}{\sqrt{x}} = 2 \cdot dt \Rightarrow dx = 2 \cdot t dt$$

$$\Rightarrow I_1 = \int (\cos^{-1} t \cdot 2t \cdot dt)$$

$$\Rightarrow 2 \int t \cdot \cos^{-1} t dt$$

$$\text{Because, } \int u \cdot v dx = u \cdot \int v dx - \int \frac{du}{dx} \cdot \{ \int v dx \} dx$$

$$\Rightarrow 2 \int t \cdot \cos^{-1} t dt = 2 \cdot \left[\cos^{-1} t \int t dt - \int \frac{d(\cos^{-1} t)}{dt} \cdot \{ \int t dt \} dt \right]$$

$$= 2 \cdot \cos^{-1} t \cdot \frac{t^2}{2} - 2 \cdot \int \left(-\frac{1}{\sqrt{1-t^2}} \right) \cdot \left\{ \frac{t^2}{2} \right\} dt$$

$$= t^2 \cdot \cos^{-1} t - \int \left(\frac{-t^2}{\sqrt{1-t^2}} \right) dt$$

Now by adding and subtracting 1 to numerator we get

$$= t^2 \cdot \cos^{-1} t - \int \left(\frac{-1+1-t^2}{\sqrt{1-t^2}} \right) dt$$

Splitting the denominator

$$= t^2 \cdot \cos^{-1} t + \int \left(\frac{-1}{\sqrt{1-t^2}} + \frac{1-t^2}{\sqrt{1-t^2}} \right) dt$$

Splitting the integral we get

$$= t^2 \cdot \cos^{-1} t + \int \left(\frac{1}{\sqrt{1-t^2}} dt \right) - \int (\sqrt{1-t^2}) \cdot dt$$

$$= t^2 \cdot \cos^{-1} t + \int \left(\frac{1}{\sqrt{1-t^2}} dt \right) - \frac{t}{2} \cdot \sqrt{1-t^2}$$

$$\text{As, } \int (\sqrt{a^2 - x^2}) \cdot dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{1} \sin^{-1} \left(\frac{x}{a} \right)$$

$$\Rightarrow I_1 = t^2 \cdot \cos^{-1} t + \sin^{-1} t - \frac{t}{2} \sqrt{1-t^2} - \frac{1}{2} \sin^{-1}(t)$$

$$\Rightarrow I_1 = t^2 \cdot \cos^{-1} t - \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t$$

Put it in equation. (2)

$$\Rightarrow I = x - \left(\frac{4}{\pi}\right) \left[t^2 \cdot \cos^{-1} t - \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t \right] \dots (2)$$

Now substitute the value of t we get

$$\Rightarrow I = x - \left(\frac{4}{\pi}\right) \left[(\sqrt{x})^2 \cdot \cos^{-1} \sqrt{x} - \frac{\sqrt{x}}{2} \sqrt{1 - (\sqrt{x})^2} + \frac{1}{2} \sin^{-1} \sqrt{x} \right]$$

Computing and simplifying we get

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$$\begin{aligned}
&= x - \left(\frac{4}{\pi}\right) \left[x \cdot \cos^{-1}\sqrt{x} - \frac{\sqrt{x}}{2} \sqrt{1-x} + \frac{1}{2} \sin^{-1}\sqrt{x} \right] \\
&= x - \left(\frac{4}{\pi}\right) \left[x \cdot \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right) - \frac{(\sqrt{x-x^2})}{2} + \frac{1}{2} \sin^{-1}\sqrt{x} \right] \\
&= x - 2x + \frac{4x}{\pi} \sin^{-1}\sqrt{x} + \frac{2}{\pi} \sqrt{x-x^2} - \frac{2}{\pi} \sin^{-1}\sqrt{x} \\
&= -x + \frac{2}{\pi} \left[(2x-1) \sin^{-1}\sqrt{x} \right] + \frac{2}{\pi} \sqrt{x-x^2} + C \\
\Rightarrow I &= \frac{2(2x-1)}{\pi} \sin^{-1}\sqrt{x} + \frac{2}{\pi} \sqrt{x-x^2} - x + C
\end{aligned}$$

20. $\int \frac{\sqrt{1-\sqrt{x}}}{1+\sqrt{x}}$

Solution:-

Given: $\int \frac{\sqrt{1-\sqrt{x}}}{1+\sqrt{x}}$

Let $I = \int \frac{\sqrt{1-\sqrt{x}}}{1+\sqrt{x}}$

Let $x = \cos^2\theta \rightarrow dx = -2\sin\theta \cos\theta d\theta$

$\Rightarrow \sqrt{x} = \cos\theta$ or $\theta = \cos^{-1}\sqrt{x}$

Substituting these values in given question we get

$\Rightarrow I = \int \frac{\sqrt{1-\cos^2\theta}}{1+\cos^2\theta} (-2\sin\theta \cos\theta) d\theta$

$\Rightarrow \int \frac{\sqrt{1-\cos^2\theta}}{1+\cos^2\theta} (-2\sin\theta \cos\theta) d\theta$

Substituting the standard formulae we get

$= \int - \frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)} (2\sin\theta \cos\theta) d\theta$

Multiplying and dividing by 2 we get

$= \int - \frac{\sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)} \left(2\sin 2\frac{\theta}{2} \cos 2\frac{\theta}{2}\right) d\theta$

Using standard identities the above equation can be written as

$= \int - \frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} \cdot (2) \cdot \left(2\sin\frac{\theta}{2} \cos\frac{\theta}{2}\right) \cdot \left(2\cos^2\left(\frac{\theta}{2}\right) - 1\right) d\theta$

$\Rightarrow \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx = \int -4 \cdot \left[\sin^2\left(\frac{\theta}{2}\right)\right] \left(2\cos^2\left(\frac{\theta}{2}\right) - 1\right) d\theta$

$\Rightarrow \int -4 \cdot \left\{ \left[2 \cdot \sin^2\left(\frac{\theta}{2}\right) \cos^2\left(\frac{\theta}{2}\right)\right] - \sin^2\left(\frac{\theta}{2}\right) \right\} d\theta$

Splitting the integrals we get

$= \int -2 \cdot \left(2\sin\frac{\theta}{2} \cos\frac{\theta}{2}\right)^2 d\theta + 4 \int \sin^2\left(\frac{\theta}{2}\right) d\theta$

Again by using standard identifies above equation can be written as

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$$\begin{aligned}
 &= -2 \int \sin^2 \theta d\theta + 4 \int \sin^2 \left(\frac{\theta}{2}\right) d\theta \\
 &= -2 \int \frac{1+\cos 2\theta}{2} d\theta + 4 \int \frac{1-\cos \theta}{2} d\theta \\
 &\text{On integrating we get} \\
 &= -2 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right] + 4 \left[\frac{\theta}{2} - \frac{\sin \theta}{2} \right] + C \\
 &= -\theta + \frac{\sin 2\theta}{2} + 2\theta - 2 \sin \theta + C
 \end{aligned}$$

Computing and simplifying

$$\begin{aligned}
 &= \theta + \frac{2 \sin \theta \cos \theta}{2} - 2 \sin \theta + C \\
 &= \theta + \frac{2\sqrt{1-\cos^2 \theta} \cos \theta}{2} - 2\sqrt{1-\cos^2 \theta} + C
 \end{aligned}$$

Substituting the values we get

$$\begin{aligned}
 &= \cos^{-1} \sqrt{x} + \sqrt{1-x} \cdot \sqrt{x} - 2\sqrt{1-x} + C \\
 &= \cos^{-1} \sqrt{x} + \sqrt{x(1-x)} - 2\sqrt{1-x} + C \\
 I &= \cos^{-1} \sqrt{x} + \sqrt{x(1-x)} - 2\sqrt{1-x} + C
 \end{aligned}$$

21. $\frac{2+\sin 2x}{1+\cos 2x} e^x$

Solution:-

Let $I = \frac{2+\sin 2x}{1+\cos 2x} e^x$

Substituting the $\sin 2x = 2 \sin x \cos x$ formula we get

$$= \left(\frac{2+2 \sin x \cos x}{2 \cos^2 x} \right) e^x$$

Now by taking 2 common

$$= 2 \cdot \left(\frac{1+\sin x \cos x}{2 \cos^2 x} \right) e^x$$

On simplification

$$= \left(\frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x} \right) e^x$$

$$= (\sec^2 x + \tan x) e^x$$

Substituting integrals both the sides we get

$$\Rightarrow \int \frac{2+\sin 2x}{1+\cos 2x} e^x dx = \int (\sec^2 x + \tan x) e^x dx$$

Now let $\tan x = f(x)$

$$\Rightarrow F(x) = \sec^2 x dx$$

$$\Rightarrow \int \frac{2+\sin 2x}{1+\cos 2x} e^x dx = \int f(x) + f'(x) e^x dx$$

On integrating we get

$$= e^x f(x) + C$$

$$= I = e^x \tan x + c$$

22. $\frac{x^2+x+1}{(x+1)^2(x+2)}$

Solution:-

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Given: $\frac{x^2+x+1}{(x+1)^2(x+2)}$

Let $I = \frac{x^2+x+1}{(x+1)^2(x+2)}$

Using partial fraction we get

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{b}{(x+1)^2} + \frac{c}{(x+2)}$ (1)

$$\Rightarrow \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A(x+1)(x+2) + B(x+2) + C(x+1)^2}{(x+1)^2(x+2)}$$

$$\Rightarrow \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A(x^2+3x+2) + B(x+2) + C(x^2+2x+1)}{(x+1)^2(x+2)}$$

$$\Rightarrow x^2 + x + 1 = Ax^2 + 3Ax + 2A + Bx + 2B + Cx^2 + 2Cx + C$$

$$\Rightarrow x^2 + x + 1 = (2A + 2B + C) + (3A + B + 2C)x + (A + C)x^2$$

Equating the coefficient of x , x^2 and constant value. We get:

(a) $A + C = 1$

(b) $3A + B + 2c = 1$

(c) $2A + 2B + C = 1$

After solving the above equation we get

$A = -2, B = 1$ and $C = 3$

Substituting the values of A, B and C we get

$$\Rightarrow \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)}$$

Taking integrals on both sides

$$\Rightarrow \int \frac{x^2+x+1}{(x+1)^2(x+2)} dx = \int \left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)} \right) dx$$

Splitting the integrals we get

$$= -2 \cdot \int \left(\frac{1}{x+1} \right) dx + \int \left(\frac{1}{(x+1)^2} \right) dx + 3 \cdot \int \left(\frac{1}{(x+2)} \right) dx$$

$$= -2 \cdot \int \left(\frac{1}{x+1} \right) dx + \int ((x+1)^{-2}) dx + 3 \cdot \int \left(\frac{1}{(x+2)} \right) dx$$

On integrating we get

$$= -2 \log|x+1| + \left(\frac{(x+1)^{-1}}{(-1)} \right) + 3 \log|x+2| + c$$

$$= -2 \log|x+1| - \frac{1}{(x+1)} + 3 \log|x+2| + C$$

23. $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$

Solution:-

Given:- $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$

Let $I = \tan^{-1} \sqrt{\frac{1-x}{1+x}}$

Let $x = \cos \theta \rightarrow dx = -\sin \theta d\theta$

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$$\Rightarrow \theta = \cos^{-1}x$$

Now by substituting these values in given question we get

$$\Rightarrow I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx = \int \tan^{-1} \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} (-\sin \theta) d\theta$$

Using standard identifies the above equation can be written as

$$= - \int \tan^{-1} \sqrt{\frac{2\sin^2(\frac{\theta}{2})}{2\cos^2(\frac{\theta}{2})}} (\sin \theta) d\theta$$

$$= - \int \tan^{-1} \sqrt{\tan^2(\frac{\theta}{2})} (\sin \theta) d\theta$$

On simplification we get

$$= - \int \tan^{-1} \tan^{\frac{\theta}{2}} (\sin \theta) d\theta$$

$$= - \frac{1}{2} \int \theta (\sin \theta) d\theta$$

Now by using product rule

$$\int u \cdot v \cdot dx - u \cdot \int v dx - \int \frac{du}{dx} \cdot \{ \int v dx \} dx$$

$$= - \frac{1}{2} \int \theta (\sin \theta) d\theta = - \frac{1}{2} \left[\sin \theta d\theta - \int \frac{d\theta}{d\theta} \cdot \{ \int \sin v d\theta \} d\theta \right]$$

Computing and integrating we get

$$= - \frac{1}{2} [\theta \cdot (-\cos \theta) - \int 1 \cdot (-\cos \theta) d\theta]$$

$$= - \frac{1}{2} [-\theta \cdot \cos \theta + \sin \theta]$$

Substituting the values we get

$$= \frac{1}{2} \theta \cdot \cos \theta - \frac{1}{2} \sqrt{1 - \cos^2 \theta}$$

$$= \frac{1}{2} \cos^{-1}x \cdot x - \frac{1}{2} \sqrt{1 - x^2} + C$$

$$= \frac{1}{2} (x \cdot \cos^{-1} x - \sqrt{1 - x^2}) + C$$

$$24 \sqrt{\frac{x^2+1 [\log(x^2+1)-2 \log x]}{x^4}}$$

Solution:-

Given:- $\frac{\sqrt{x^2+1} [\log(x^2+1)-2 \log x]}{x^4}$

Let I = $\frac{\sqrt{x^2+1} [\log(x^2+1)-2 \log x]}{x^4}$

$$= \frac{\sqrt{x^2+1}}{x^2} [\log(x^2 + 1) - \log x^2]$$

Using logarithmic identities we get

$$= \frac{1}{x^3} \sqrt{\frac{x^2+1}{x^2}} \left[\log \left(\frac{x^2+1}{x^2} \right) \right]$$

On Computing

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$$= \frac{1}{x^2} \sqrt{1 + \frac{1}{x^2}} \left[\log \left(1 + \frac{1}{x^2} \right) \right]$$

Now let $1 + \frac{1}{x^2} = t \Rightarrow -\frac{2}{x^2} dx = dt$

Substituting these values in given question we get

$$\begin{aligned} \Rightarrow \int \frac{\sqrt{x^2+1} [\log(x^2+1)-2 \log x]}{x^4} dx &= \int \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} \left[\log \left(1 + \frac{1}{x^2} \right) \right] dx \\ &= \int -\frac{1}{2} \cdot \sqrt{t}(\log(t))dt \end{aligned}$$

By using product rule

$$\begin{aligned} \int u \cdot v dx &= u \cdot \int v dx - \int \frac{du}{dx} \cdot \{ \int v dx \} dx \\ &= \int -\frac{1}{2} \cdot \sqrt{t}(\log(t))dt = -\frac{1}{2} \left[\log t \cdot \int \sqrt{t} dt - \int \frac{d}{dt} \log t \cdot \{ \int \sqrt{t} dt \} dt \right] \end{aligned}$$

Computing and simplifying we get

$$\begin{aligned} &= -\frac{1}{2} \left[\log t \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{1}{t} \cdot \left\{ \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right\} dt \right] \\ &= -\frac{1}{2} \left[\frac{2}{3} t^{\frac{3}{2}} \log t - \int \left\{ \frac{t^{\frac{3}{2}-1}}{\frac{3}{2}} \right\} dt \right] \\ &= -\frac{1}{2} \left[\frac{2}{3} t^{\frac{3}{2}} \log t - \frac{2}{3} \int t^{\frac{1}{2}} dt \right] \end{aligned}$$

On integration we get

$$\begin{aligned} &= -\frac{1}{2} \left[\frac{2}{3} t^{\frac{3}{2}} \log t - \frac{2}{3} \int t^{\frac{1}{2}} dt \right] \\ &= \left[-\frac{1}{2} \cdot \frac{2}{3} t^{\frac{3}{2}} \log t + \frac{2}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right] \\ &= \left[-\frac{1}{2} \cdot \frac{2}{3} t^{\frac{3}{2}} \log t + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot t^{\frac{3}{2}} \right] \\ &= -\frac{1}{3} t^{\frac{3}{2}} \left[\log t - \frac{2}{3} \right] \end{aligned}$$

Substituting the value of t we get

$$\Rightarrow I = -\frac{1}{3} \left(1 + \frac{1}{x^2} \right)^{\frac{3}{2}} \left[\log \left(1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + C$$

Evaluate the definite integrals in Exercise 25 to 33.

25. $\int_{-\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

Solution:-

Given: $\int_{-\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

Let, $I = \int_{-\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

Substituting the standard identities for $1 - \sin x$ and $1 - \cos x$ we get

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$$= \int_{-\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \left(\frac{x}{2} \right)} \right) dx$$

Now splitting the denominator

$$= \int_{-\frac{\pi}{2}}^{\pi} e^x \left(\frac{2}{2 \sin^2 \left(\frac{x}{2} \right)} - \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \left(\frac{x}{2} \right)} \right) dx$$

$$= \int_{-\frac{\pi}{2}}^{\pi} \left(e^x \left(\frac{1}{2} \operatorname{cosec}^2 \left(\frac{x}{2} \right) - \cot \frac{x}{2} \right) \right) dx$$

Now let $f(x) = -\cot \frac{x}{2}$

Substituting these values we get

$$\Rightarrow F'(x) = - \left(-\frac{1}{2} \operatorname{cosec}^2 \left(\frac{x}{2} \right) \right) = \frac{1}{2} \operatorname{cosec}^2 \left(\frac{x}{2} \right)$$

$$\Rightarrow \int_{-\frac{\pi}{2}}^{\pi} \left(\frac{1}{2} \operatorname{cosec}^2 \left(\frac{x}{2} \right) - \cot \frac{x}{2} \right) dx = \int_{-\frac{\pi}{2}}^{\pi} (f(x) + f'(x)) e^x dx$$

On integration we get

$$= [e^x f(x)]_{-\frac{\pi}{2}}^{\pi}$$

$$= [e^x (-\cot \frac{x}{2})]_{-\frac{\pi}{2}}^{\pi}$$

Now by applying the limits we get

$$= - \left[e^{\pi} \left(\cot \frac{\pi}{2} \right) - e^{\frac{\pi}{2}} \left(\cot \frac{\pi}{4} \right) \right]$$

$$= - \left[e^{\pi} \left(\cot \frac{\pi}{2} \right) - e^{\frac{\pi}{2}} \left(\cot \frac{\pi}{4} \right) \right]$$

$$= - \left[e^{\pi} (0) - e^{\frac{\pi}{2}} (1) \right]$$

$$= - \left[0 - e^{\frac{\pi}{2}} \right]$$

On simplification we get

$$\Rightarrow I = e^{\frac{\pi}{2}}$$

26. $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Solution:-

Given:- $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Let, $I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Taking $\cos^4 x$ as common we get

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x \left(1 + \frac{\sin^4 x}{\cos^4 x} \right)} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{(1 + \tan^4 x)} dx$$

Now let $\tan^2 x = t \rightarrow 2 \tan x \sec^2 x dx = dt$

Ans when $x=0$ then $t=0$ and when $x=\pi/4$ then $t=1$

Now by substituting these values in above equation we get

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$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{(1 + \tan^4 x)} dx = \int_0^1 \frac{1}{(1+t^2)} \left(\frac{dt}{2}\right)$$

On integration

$$\Rightarrow I = \frac{1}{2} [\tan^{-1}t]_0^1$$

Now by applying the limits we get

$$= \frac{1}{2} [\tan^{-1}1 - \tan^{-1}0]$$

$$\Rightarrow I = \frac{1}{2} \cdot \frac{\pi}{4}$$

$$\Rightarrow I = \frac{\pi}{8}$$

27. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4 \sin^2 x}$

Solution:-

Given: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4 \sin^2 x}$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4 \sin^2 x}$ (1)

Substituting $\sin^2 x$ formula we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1 - \cos^2 x)} dx$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1 - 4\cos^2 x)} dx$$

On Computing we get

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{4 - 3\cos^2 x} dx$$

Now multiplying and dividing by 3 to the numerator we get

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{1}{3} \cdot 3\cos^2 x}{4 - 3\cos^2 x} dx$$

Again by adding and subtracting 4 to the numerator we get

$$= -\frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{3\cos^2 x + 4 - 4}{4 - 3\cos^2 x} dx$$

The above equation can be written as

$$= -\frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{4 - 3\cos^2 x - 4}{4 - 3\cos^2 x} dx$$

Now splitting the integrals we get

$$= \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} (1) dx + \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{4}{4 - 3\cos^2 x} dx$$

$$= \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} (1) dx + \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{4}{4 - 3\left(\frac{1}{\sec^2 x}\right)} dx$$

Applying the limits we get

$$= \frac{1}{3} \cdot [x]_0^{\frac{\pi}{2}} + \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{4\sec^2 x}{4\sec^2 x - 3} dx$$

$$= -\frac{1}{3} \cdot \left[\frac{\pi}{2}\right] + \frac{1}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{4\sec^2 x}{4\sec^2 x - 3} dx$$

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$$= I = -\frac{\pi}{6} + \frac{2}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x} dx$$

$$\Rightarrow I = -\frac{\pi}{6} + I_2 \dots\dots (2)$$

First solve for I_1 :

$$I_1 = \frac{2}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x} dx$$

Let $2 \tan x = t \rightarrow 2 \sec^2 x dx = dt$

When $x = 0$ then $t = 0$ and when $x = \frac{\pi}{2}$ then $t = \infty$

Substituting these values for above equation we get

$$\Rightarrow \frac{2}{3} \cdot \int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x} dx = \frac{2}{3} \cdot \int_0^{\infty} \frac{1}{1+t^2} dt$$

Integrating and applying the limits we get

$$\Rightarrow I_1 = \frac{2}{3} [\tan^{-1} t]_0^{\infty}$$

$$\Rightarrow \frac{2}{3} [\tan^{-1} \infty - \tan^{-1} 0]$$

$$\Rightarrow I_1 = \frac{2}{3} \cdot \frac{\pi}{2}$$

$$\Rightarrow I_1 = \frac{\pi}{6}$$

28. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Solution:-

Given: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Let, $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

On rearranging we get

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{-\sqrt{\sin 2x}} dx$$

Now by substituting the $\sin 2x$ formula we get

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1+1-2 \sin x \cos x)}} dx$$

1 can be written as $\sin^2 x + \cos^2 x$

Substituting this in above equation we get

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{1-(\sin^2 x + \cos^2 x - 2 \sin x \cos x)}} dx$$

As we know $(a + b)^2 = a^2 + b^2$ using this in above equation we get

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1+1-2 \sin x \cos x)}} dx$$

Now let $\sin x - \cos x = t \rightarrow (\cos x + \sin x) dx = dt$

When $x = \frac{\pi}{6} \rightarrow t = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$ and when $x = \frac{\pi}{3} \rightarrow t = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

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Substituting these values in above equation we get

$$\Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{1 - (t)^2}} dt$$

$$\Rightarrow = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{1 - (t)^2}} dt$$

Let $f(x) = \frac{1}{\sqrt{1 - (t)^2}}$ and $f(-x) = \frac{1}{\sqrt{1 - (-t)^2}} = \frac{1}{\sqrt{1 - (t)^2}} = f(x)$

That is $f(x) = f(-x)$

So, $f(x)$ is an even function.

It is also known that, if $f(x)$ is even function then, we have

$$\left\{ \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \right\}$$

By using the above formula we get

$$\Rightarrow I = 2 \cdot \int_0^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{1 - (t)^2}} dt$$

On integrating

$$\Rightarrow I = [2 \cdot \sin^{-1} t]_0^{\frac{\sqrt{3}-1}{2}}$$

Now by applying the limits

$$\Rightarrow I = 2 \cdot \sin^{-1} \left(\frac{\sqrt{3}-1}{2} \right)$$

29. $\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$

Solution:-

Given: $\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$

Let, $I = \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$

Now multiply and divide $\sqrt{1+x} + \sqrt{x}$ to the above equation we get

$$= \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}} \times \frac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} + \sqrt{x}}$$

$$= \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1+x-x} dx$$

On simplification

$$= \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1} dx$$

Now by splitting the integrals we get

$$= \int_0^1 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx$$

$$= \int_0^1 ((1+x)^{\frac{1}{2}}) dx + \int_0^1 (x)^{\frac{1}{2}} dx$$

On integrating we get

$$\Rightarrow I = \left[\frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 + \left[\frac{(x)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1$$

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Now by applying the limits we get

$$= \frac{2}{3} \cdot [(1+1)^3 - (1+0)^3] + \frac{2}{3} \cdot [(1)^3]$$

Computing and simplifying we get

$$= \frac{2}{3} \cdot [(2)^3 - (1)^3] + \frac{2}{3} \cdot [(1)^3]$$

$$= \frac{2}{3} \cdot [(2)^3] + \frac{2}{3} \cdot (1)^3 \cdot [1]$$

$$= \frac{2}{3} \cdot [2\sqrt{2}]$$

$$\Rightarrow I = \frac{4\sqrt{2}}{3}$$

$$30. \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Solution:-

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Also, let $\sin x - \cos x = t$

Differentiating both sides, we get,

$$(\cos x \sin x) dx = dt$$

When $x = 0$, $t = -1$

And when $x = \frac{\pi}{4}$, $t = 0$

$$\text{Now, } (\sin x - \cos x)^2 = t^2$$

$$1 - 2 \sin x \cos x = t^2$$

$$\sin 2x = 1 - t^2$$

Putting all the values, we get the integral,

$$I = \int_{-1}^0 \frac{dt}{9 + 16(1 - t^2)}$$

$$I = \int_{-1}^0 \frac{dt}{25 - 16t^2}$$

The above equation can be written as

$$I = \int_{-1}^0 \frac{dt}{(5)^2 - (4t)^2}$$

On integrating we get

$$I = \frac{1}{4} \left[\frac{1}{2(5)} \log \left| \frac{5+4t}{5-4t} \right| \right]_{-1}^0$$

Now by applying the limits we get

$$I = \frac{1}{40} \left[\log 1 - \log \frac{1}{9} \right]$$

$$I = \frac{1}{40} \log 9$$

$$31. \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

Solution:-

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Given:- $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

Let, $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

$= \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \cdot \tan^{-1}(\sin x) dx$

Let $\sin x = t \rightarrow \cos x dx = dt$

When $x=0$ then $t = 0$ and when $x = \frac{\pi}{2}$ then $t = 1$

Now by substituting these values in above equation we get

$\Rightarrow \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \cdot \tan^{-1}(\sin x) dx = \int_0^1 2t \cdot \tan^{-1}(t) dt$

Using product rule

$\int u \cdot v dx = u \cdot \int v dx - \int \frac{du}{dx} \cdot \{ \int v dx \} dx$

$\Rightarrow 2 \int_0^1 t \cdot \tan^{-1}(t) dt = 2 \left[\tan^{-1}(t) \cdot \int t dt - \left(\frac{d}{dt} \right) \cdot \{ \int \tan^{-1}(t) \} dt \right]$

Computing using product rule we get

$= 2 \left[\tan^{-1}(t) \cdot \frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt \right]$

$= 2 \left[\tan^{-1}(t) \cdot \frac{t^2}{2} - \frac{1}{2} \cdot \int \frac{-1+1+t^2}{1+t^2} dt \right]$

Splitting the integrals we get

$= 2 \left[\tan^{-1}(t) \cdot \frac{t^2}{2} - \frac{1}{2} \cdot \left\{ \int -\frac{1}{1+t^2} dt + \int \frac{1+t^2}{1+t^2} dt \right\} \right]$

On simplification we get

$= 2 \left[\tan^{-1}(t) \cdot \frac{t^2}{2} - \frac{1}{2} \cdot \left\{ \int -\frac{1}{1+t^2} dt + \int 1 dt \right\} \right]$

$= 2 \left[\tan^{-1}(t) \cdot \frac{t^2}{2} - \frac{1}{2} \cdot \{ -\tan^{-1}(t) + t \} \right]$

$= [t^2 \cdot \tan^{-1}(t) - \{ -\tan^{-1}(t) + t \}]$

Computing we get

$\Rightarrow 2 \int_0^1 t \cdot \tan^{-1}(t) dt = [t^2 \cdot \tan^{-1}(t) - \{ -\tan^{-1}(t) + t \}]_0^1$

Now by applying the limits

$= [1^2 \cdot \tan^{-1}(1) - \{ -\tan^{-1}(1) + 1 \}] - [0^2 \cdot \tan^{-1}(0) - \{ -\tan^{-1}(0) + 0 \}]$

$= \left[1 \cdot \frac{\pi}{4} - \left\{ -\frac{\pi}{4} + 1 \right\} \right]$

$= \left[\frac{\pi}{4} + \frac{\pi}{4} - 1 \right]$

$\Rightarrow I = \left[\frac{\pi}{2} - 1 \right]$

32. $\int_0^x \frac{x \tan x}{\sec x + \tan x} dx$

Solution:-

Given: $\int_0^x \frac{x \tan x}{\sec x + \tan x} dx$

Let $I = \int_0^x \frac{x \tan x}{\sec x + \tan x} dx \dots (1)$

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As we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

Using this in above equation we get

$$\Rightarrow I = \int_0^\pi \frac{(\pi-x)(-\tan(x))}{\sec(\pi-x)+\tan(\pi-x)} dx$$

Using standard allied angles the above equation can be written as

$$\begin{aligned} &= \int_0^\pi \frac{(\pi-x)(-\tan(x))}{(-\sec x) + (-\tan x)} dx \\ &= \int_0^\pi \frac{-(\pi-x)(\tan(x))}{- [(-\sec x) + (-\tan x)]} dx \\ &= \int_0^\pi \frac{(\pi-x)(-\tan(x))}{\sec x + \tan x} dx \dots\dots\dots (2) \end{aligned}$$

Adding (1) and (2) we get

$$2I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} + \int_0^\pi \frac{(\pi-x)(-\tan(x))}{\sec x + \tan x} dx$$

Now by adding we get

$$2I = \int_0^\pi \frac{\pi \tan x}{\sec x + \tan x} dx$$

Tan x can be written as

$$= \int_0^\pi \frac{\pi \cdot \frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$2I = \pi \cdot \int_0^\pi \frac{\pi \tan x}{(1 + \sin x)} dx$$

$$= \pi \cdot \int_0^\pi \frac{(-1 + 1 \sin x)}{(1 + \sin x)} dx$$

Now by splitting the integrals we get

$$= \pi \cdot \int_0^\pi \frac{(-1)}{(1 + \sin x)} dx + \pi \int_0^\pi \frac{(1 + \sin x)}{(1 + \sin x)} dx$$

Again by multiplying and dividing above equation by $1 - \sin x$ we get

$$= \pi \cdot \int_0^\pi \frac{(-1)}{(1 + \sin x)} \times \frac{(1 - \sin x)}{(1 - \sin x)} dx + \pi \cdot \int_0^\pi 1 \cdot dx$$

Splitting the integrals

$$= -\pi \int_0^\pi \frac{(1 - \sin x)}{(1 - \sin^2 x)} dx + \pi \cdot \int_0^\pi 1 \cdot dx$$

$$2I = -\pi \int_0^\pi \frac{(1 - \sin x)}{\cos^2 x} dx + \pi \cdot \int_0^\pi 1 \cdot dx$$

$$2I = -\pi \int_0^\pi \left\{ \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right\} dx + \pi \cdot \int_0^\pi 1 \cdot dx$$

$$2I = -\pi \int_0^\pi \{ \sec^2 x - \tan x \sec x \} dx + \pi \cdot \int_0^\pi 1 \cdot dx$$

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On integrating we get

$$21 = -\pi [\tan x - \sec x]_0^\pi + [x]_0^\pi$$

Now by applying the limits we get

$$\Rightarrow 21 = -\pi[\tan \pi - \sec \pi - \tan 0 + \sec 0] + \pi \cdot [\pi - 0]$$

$$\Rightarrow 21 = -\pi[0 - 1 - 0 + 1] + \pi \cdot [\pi]$$

$$\Rightarrow 21 = \pi[-2 + \pi]$$

$$\Rightarrow 1 = \frac{\pi}{2} \cdot [\pi - 2]$$

$$33. \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$$

Solution:-

$$\text{Given } \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$$

Let,

$$\Rightarrow I = \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$$

Now by splitting the integrals we get

$$\Rightarrow I = \int_1^4 [|x - 1|] dx + \int_1^4 [|x - 2|] dx + \int_1^4 [|x - 3|] dx +$$

$$\text{Let } I = I_1 + I_2 + I_3$$

First solve for I_1 :

$$I_1 = \int_1^4 [|x - 1|] dx$$

As we can see that $(x - 1) \geq 0$ when $1 \leq x \leq 4$

$$\Rightarrow I_1 = \int_1^4 (x - 1) dx$$

On integrating we get

$$\Rightarrow \left[\frac{x^2}{2} - x \right]_1^4$$

Now by applying the limits we get

$$\Rightarrow I_1 = \left[\frac{(4)^2}{2} - 4 - \frac{(1)^2}{2} + 1 \right]$$

$$\Rightarrow I_1 = \left[8 - 4 - \frac{1}{2} + 1 \right]$$

$$\Rightarrow I_1 = \left[5 - \frac{1}{2} \right]$$

$$\Rightarrow I_1 = \frac{9}{2}$$

Now solve for I_2 :

$$I_2 = \int_1^4 [|x - 2|] dx$$

As we can see that $(x - 2) \leq 0$ when $1 \leq x \leq 2$ and $(x - 2) \geq 0$ when $2 \leq x \leq 4$

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As we know that

$$\left\{ \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \right\}$$

By using this we get

$$\Rightarrow I_2 = \int_1^2 -(x-2)dx + \int_2^4 (x-2)dx$$

On integrating

$$\Rightarrow I_2 = - \left[\frac{x^2}{2} - 2x \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^4$$

Now by applying the limits we get

$$\Rightarrow I_2 = - \left[\frac{(2)^2}{2} - 2(2) - \frac{(1)^2}{2} + 2(1) \right] + \left[\frac{(4)^2}{2} - 2(4) - \frac{(2)^2}{2} + 2(2) \right]$$

$$\Rightarrow I_2 = - \left[2 - 4 - \frac{1}{2} + 2 \right] + [8 - 8 - 2 + 4]$$

$$\Rightarrow I_2 = \left[\frac{1}{2} + 2 \right]$$

$$\Rightarrow I_2 = \frac{5}{2}$$

Now solve for I_3

$$I_3 = \int_1^4 [|x-3|] dx$$

As we can see that $(x-3) \leq 0$ when $1 \leq x \leq 3$ and $(x-3) \geq 0$ when $3 \leq x \leq 4$

As we know that

$$\left\{ \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \right\}$$

By using this we get

$$\Rightarrow I_3 = \int_1^3 -(x-3)dx + \int_3^4 (x-3)dx$$

On integrating

$$\Rightarrow I_3 = - \left[\frac{x^2}{2} - 2x \right]_1^3 + \left[\frac{x^2}{2} - 2x \right]_3^4$$

Now by applying the limits

$$\Rightarrow I_3 = - \left[\frac{(3)^2}{2} - 3(3) - \frac{(1)^2}{2} + 2(1) \right] + \left[\frac{(4)^2}{2} - 3(4) - \frac{(3)^2}{2} + 3(3) \right]$$

$$\Rightarrow I_3 = - \left[\frac{9}{2} - 9 - \frac{1}{2} + 2 \right] + \left[8 - 12 - \frac{9}{2} + 9 \right]$$

$$\Rightarrow I_3 = \left[2 + \frac{1}{2} \right]$$

$$\Rightarrow I_3 = \frac{5}{2}$$

as $I = I_1 + I_2 + I_3$

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Substituting the above all values we get

$$\Rightarrow I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2}$$

$$\Rightarrow I = \frac{19}{2}$$

Prove the following (Exercises 34 to 39)

$$34. \int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$$

Solution:-

$$\text{Given } \int_1^3 \frac{dx}{(x^2)(x+1)}$$

$$\text{To prove: } \int_1^3 \frac{dx}{(x^2)(x+1)} = \frac{2}{3} + \log \frac{2}{3}$$

$$\text{Let } I = \frac{dx}{(x^2)(x+1)}$$

Using partial fraction

$$\text{Let } \frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{c}{x+1} \dots \dots (1)$$

$$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{A(x)(x+1) + B(x+1) + C(x^2)}{(x+1)(x^2)}$$

$$\Rightarrow 1 = A(x^2 + x) + (Bx + B) + Cx^2$$

$$\Rightarrow 1 = Ax^2 + Ax + B + Bx + Cx^2$$

$$\Rightarrow 1 = B + (A + B)x + (A + C)x^2$$

Equating the coefficients of x , x^2 and constant value we get

$$a) B = 1$$

$$b) A + B = 0 \Rightarrow C = -A \Rightarrow C = 1$$

$$c) A + C = 0 \Rightarrow C = -A \Rightarrow C = 1$$

Putting this values in equation (1)

$$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{c}{x+1}$$

$$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$$

Taking integrals on both side we get

$$\Rightarrow \int \frac{1}{(x^2)(x+1)} dx = \int -\frac{1}{x} dx + \int \frac{1}{(x^2)} dx + \int \frac{1}{(x+1)} dx$$

$$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = [-\log|x - x^{-1} + \log|x + 1|]_1^2$$

$$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = \left[-\frac{1}{x} + \log \left| \frac{x+1}{x} \right| \right]_1^3$$

Now by applying the limits we get

$$= \left[-\frac{1}{3} + \log \left| \frac{3+1}{3} \right| - \left(-\frac{1}{1} + \log \left| \frac{1+1}{1} \right| \right) \right]$$

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$$= \left[-\frac{1}{3} + \log \left| \frac{4}{3} \right| + \left(1 - \log \left| \frac{2}{1} \right| \right) \right]$$

Computing and simplifying we get

$$= \left[-\frac{1}{3} + 1 + \log \left| \frac{4}{3} \times \frac{1}{2} \right| \right]$$

$$\Rightarrow I = \left[\frac{2}{3} + \log \left| \frac{2}{3} \right| \right]$$

\Rightarrow L. H. S = R. H. S.

Hence proved

$$35. \int_0^1 x e^x dx = 1$$

Solution:-

Given: $\int_0^1 x e^x dx$

To prove: $\int_0^1 x e^x dx = 1$

Let $I = \int_0^1 x e^x dx$

Using product rule we get

$$\int u \cdot v dx = \int v dx \int \frac{du}{dx} \cdot \left\{ \int v dx \right\} dx$$

$$\Rightarrow \int_0^1 x e^x dx = x \cdot \int_0^1 e^x dx - \int_0^1 \frac{dx}{dx} \left\{ \int v dx \right\} dx$$

On integrating

$$\Rightarrow \int_0^1 x e^x dx = [x e^x]_0^1 - \int_0^1 1 \cdot e^x dx$$

Now by applying the limits we get

$$\Rightarrow \int_0^1 x e^x dx = [x e^x]_0^1 - [e^x]_0^1$$

$$\Rightarrow \int_0^1 x e^x dx = [1 \cdot e^1 - 0 e^0] - [e^1 - e^0]$$

$$\Rightarrow \int_0^1 x e^x dx = e - 0 - e + 1$$

$$\Rightarrow \int_0^1 x e^x dx = 1$$

Therefore L.H.S = R.H.S.

Hence proved

$$36. \int_{-1}^1 x^{17} \cos^4 x dx = 0$$

Solution:-

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Given: $\int_{-1}^1 x^{17} \cos^4 x dx$

To prove $\int_{-1}^1 x^{17} \cos^4 x dx = 0$

Let $I = \int_{-1}^1 x^{17} \cos^4 x dx$

As we can see $f(x) = x^{17} \cdot \cos^4 x$ and $f(-x) = (-x)^{17} \cdot \cos^4(-x) = -x^{17} \cos^4 x$

That is $f(x) = -f(-x)$

So, it is an odd function

It is also known that if $f(x)$ is an odd function then we have

$$\left\{ \int_{-a}^a f(x) dx = 0 \right\}$$

$$\Rightarrow I = \int_{-1}^1 x^{17} \cdot \cos^4 x dx = 0$$

Hence proved

37. $\int_0^{\pi/2} \sin^3 x dx = \frac{2}{3}$

Solution:-

Given: $\int_0^{\pi/2} \sin^3 x dx$

To prove $\int_0^{\pi/2} \sin^3 x dx = \frac{2}{3}$

Let $I = \int_0^{\pi/2} \sin^3 x dx \dots \dots \dots (1)$

Above equation can be written as

$$= \int_0^{\pi/2} \sin x \cdot \sin^2 x dx$$

$$= \int_0^{\pi/2} \sin x \cdot (1 - \cos^2 x) dx$$

Now by splitting the integrals

$$\Rightarrow I = \int_0^{\pi/2} \sin x dx - \int_0^{\pi/2} x \cdot \cos^2 x dx$$

$$\Rightarrow I = [-\cos x]_0^{\pi/2} - I_1 \dots \dots \dots (2)$$

First solve for I_1

$$\Rightarrow I_1 = \int_0^{\pi/2} \sin x \cdot \cos^2 x dx$$

Let $\cos x = t \Rightarrow -\sin x dx = dt \Rightarrow \sin x dx = -dt$

When $x = 0$ then $t = 1$ and when $x = \pi/2$ then $t = 0$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{2}} t^2(-dt)$$

$$= - \int_1^0 t^2(dt)$$

On integrating we get

$$= \left[\frac{t^3}{3} \right]_1^0$$

Now by applying the limits we get

$$= - \left\{ -\frac{1}{3} \right\}$$

$$\Rightarrow I = -\frac{1}{3}$$

$$\Rightarrow I = \frac{2}{3}$$

L.H.S = R.H.S.

38. $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

Solution:-

Given: $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx$

To prove $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

Let $I = \int_0^{\frac{\pi}{4}} 2 \tan^3 x dx \dots \dots \dots (1)$

The above equation can be written as

$$= \int_0^{\frac{\pi}{4}} 2 \tan x \cdot \tan^2 x dx$$

Substituting $\tan^2 x$ formula we get

$$= 2 \int_0^{\frac{\pi}{4}} \tan x (\sec^2 x - 1) dx$$

Now by splitting the integral we get

$$\Rightarrow I = 2 \left\{ - \int_0^{\frac{\pi}{4}} \tan x dx + \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx \right\}$$

$$\Rightarrow I = - [2 \log \sec x]_0^{\pi/4} + 2 \cdot I_1 \dots \dots \dots (2)$$

First solve for I_1 :

$$\Rightarrow I_1 \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

When $x=0$ then $t=0$ and when $x = \pi/2$ then $t = 1$

$$\Rightarrow I_1 = \int_0^1 t \cdot dt$$

On integrating we get

$$= \left[\frac{t^2}{2} \right]_0^1$$

Applying the limits we get

$$\Rightarrow I_1 = \frac{1}{2}$$

Substitute in equation (2)

$$\Rightarrow I = [2 \log \cos x]_0^{\pi/4} + 2 \cdot \frac{1}{2}$$

On simplification we get

$$\Rightarrow I = 2 \left\{ \log \cos \frac{\pi}{4} - \log \cos 0 \right\} + 1$$

Substituting the values of $\cos 0 = 1$ we get

$$\Rightarrow I = 2 \left\{ \log \frac{1}{\sqrt{2}} - \log 1 \right\} + 1$$

$$\Rightarrow I = \left\{ \log \left(\frac{1}{\sqrt{2}} \right)^2 - \log(1)^2 \right\} + 1$$

$$\Rightarrow I = 1 - \log 2 + \log 1$$

$$\Rightarrow I = 1 - \log 2$$

L.H.S = R.H.S

Hence the proof.

$$39. \int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$$

Solution:-

Given: $\int_0^1 \sin^{-1} x dx$

To prove $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$

Let $I = \int_0^1 \sin^{-1} x \cdot 1 dx$

Using product rule we get

$$\int u \cdot v dx = u \cdot \int v dx - \int \frac{du}{dx} \cdot \left\{ \int v dx \right\} dx$$

$$\Rightarrow \int_0^1 x e^x dx = \sin^{-1} x \cdot \int_0^1 1 \cdot dx - \int_0^1 \frac{d}{dx} \sin^{-1} x \cdot \left\{ \int_0^1 1 \cdot dx \right\} \cdot dx$$

On integrating we get|

$$\Rightarrow \int_0^1 x e^x dx = [\sin^{-1} x \cdot x]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x dx$$

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$$\Rightarrow I_1 [\sin^{-1} x \cdot x]_0^1 - I_1 \dots \dots \dots (2)$$

First solve for I_1 :

$$\Rightarrow I_1 = \int_0^1 \frac{1}{\sqrt{x-x^2}} \cdot x dx$$

$$\text{Let } 1-x^2 = t \Rightarrow -2x dx = dt$$

When $x = 0$ then $t = 1$ and when $x = 1$ then $t = 0$

$$\Rightarrow I_1 = \int_1^0 \frac{1}{\sqrt{t}} \cdot \frac{-dt}{2}$$

$$= -\frac{1}{2} \left[\frac{t^{1/2}}{1/2} \right]_1^0$$

$$\Rightarrow I_1 = \sqrt{1}$$

$$\Rightarrow I_1 = 1$$

Substitute in equation (2)

$$\Rightarrow I = [\sin^{-1} x \cdot x]_0^1 - 1$$

$$\Rightarrow I = \sin^{-1}(1) - 0 - 1$$

$$\Rightarrow I = \frac{\pi}{2} - 1$$

L.H.S = R.H.S

Hence proved

40. Evaluate $\int_0^1 e^{2-3x} dx$ as a limit of a sum.

Solution:-

Given: $\int_0^1 e^{2-3x} dx$

Let $I = \int_0^1 e^{2-3x} dx$

Because, $\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots \dots \dots + f(a+(n-1)h)]$

Where, $h = \frac{b-a}{n}$

Here, $a = 0, b = 1$, and $f(x) = e^{2-3x}$ and h

$$= \lim_{n \rightarrow \infty} \frac{1}{n} [e^2 + e^2 \cdot e^{3h} + e^2 \cdot e^{6h} + \dots \dots \dots + e^2 e^{-3(n-1)h}]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} [e^2 \{1 + e^{3h} + e^{6h} + \dots \dots \dots + e^{-3(n-1)h}\}]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[e^2 \left\{ \frac{1 - (e^{-3h})^n}{1 - e^{-3h}} \right\} \right]$$

$$= \lim_{h \rightarrow \infty} \frac{1}{h} \left[e^2 \left\{ \frac{1 - \left(e - \frac{3}{n}\right)^n}{1 - \left(e - \frac{3}{n}\right)} \right\} \right] \text{ as, } h = \frac{1}{h}$$

$$= \lim_{h \rightarrow \infty} \frac{1}{h} \left[e^2 \left\{ \frac{(e^{-3})^1 - 1}{\left(e - \frac{3}{n}\right) - 1} \right\} \right]$$

$$= e^2 \cdot (e^{-3} - 1) \lim_{h \rightarrow \infty} \frac{1}{h} \cdot \left(-\frac{n}{3}\right) \left[\left\{ \frac{-\frac{3}{n}}{\left(e - \frac{3}{n}\right) - 1} \right\} \right]$$

On simplification we get|

$$= - \frac{(e^2 \cdot (e^{-3} - 1))}{3} \lim_{h \rightarrow \infty} \left[\left\{ \frac{-\frac{3}{n}}{\left(e - \frac{3}{n}\right) - 1} \right\} \right]$$

We know that

$$\lim_{x \rightarrow \infty} \left[\frac{x}{(e^x) - 1} \right] = 1$$

Substituting this in above equation we get

$$= \frac{-e^{-1} + e^2}{3} (1)$$

$$\Rightarrow I = \frac{1}{3} \left(e^2 - \frac{1}{e} \right)$$

Choose the correct answer in Exercise 41 to 44

41. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(A) $\tan^{-1}(e^x) + C$

(B) $\tan^{-1}(e^{-x}) + C$

(C) $\log(e^x - e^{-x}) + C$

(D) $\log(e^x + e^{-x}) + C$

Solution:-

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(A) $\tan^{-1}(e^x) + C$

Explanation:

Given: $\int \frac{dx}{e^x + e^{-x}}$

Let $I = \int \frac{dx}{e^x + e^{-x}}$

The above equation can be written as

$$= \int \frac{dx}{e^{-x}(e^{2x} + 1)}$$

$$= \int \frac{e^x dx}{(e^{2x} + 1)}$$

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Put $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \frac{e^x dx}{(e^{2x} + 1)} = \int \frac{dt}{(t^2 + 1)}$$

$$= \tan^{-1} t + C$$

$$= \tan^{-1} (e^x) + C$$

Hence, correct option is (A).

42. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

(A) $\frac{-1}{\sin x + \cos x} + C$

(C) $\log |\sin x - \cos x| + C$

(B) $\log |\sin x + \cos x| + C$

(D) $\frac{1}{(\sin x \cos x)^2}$

Solution:-

(B) $\log |\sin x + \cos x| + C$

Explanation:

Given: $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Let $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Substituting $\cos 2x$ formula we get

$$= \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$$

By using $a^2 - b^2 = (a + b)(a - b)$ we get

$$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$$

On simplification

$$= \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

Put $\sin x + \cos x = t \Rightarrow \cos x - \sin x = dt$

$$\Rightarrow \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx = \int \frac{dt}{t}$$

$$= \log |t| + C$$

$$= \log |\sin x + \cos x| + C$$

Hence, correct option is (B).

43. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x) dx$ is equal to

(A) $\frac{a+b}{2} \int_a^b f(b-x) dx$

(B) $\frac{a+b}{2} \int_a^b f(b+x) dx$

(C) $\frac{b-a}{2} \int_a^b f(b-x) dx$

(D) $\frac{a+b}{2} \int_a^b (x) dx$

Solution:-

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(D) $\frac{a+b}{2} \int_a^b (x)dx$

Explanation:

Given: $\int_a^b x f(x)dx$

Let, $I = \int_a^b x f(x) dx$

As we know that
 $\{f(x) = f(a + b - x)\}$

Using this we get

$$\Rightarrow I = \int_a^b (a + b)f(x)dx - \int_a^b (x)f(x)dx$$

$$\Rightarrow I = \int_a^b (a + b)f(x)dx - I$$

$$\Rightarrow 2I = \int_a^b (a + b)f(x)dx$$

$$\Rightarrow I = \frac{(a + b)}{2} \int_a^b f(x)dx$$

Hence, correct option is (D)

44. The value of $\int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$ is

- (A) 1
- (C) -1

- (B) 0
- (D) π

Solution:-

(B) 0

Explanation:

Given: $\int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$

Let $I = \int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$

The above equation can be written as

$$= \int_0^1 \tan^{-1} \left(\frac{x + x - 1}{1 + x(1 - x)} \right) dx$$

$$= \int_0^1 \tan^{-1} \left(\frac{x - (1 - x)}{1 + x(1 - x)} \right) dx$$

As we know that

$$\tan^{-1} \left(\frac{A - B}{1 + AB} \right) = \tan^{-1} (A) - \tan^{-1} (B)$$

By using the formula we get

$$= \int_0^1 [\tan^{-1} (x) - \tan^{-1} (1 - x)] dx \dots\dots\dots (1)$$

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Again as we know that

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

By using this we can write as

$$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-(1-x))] dx$$

$$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)] dx \dots (2)$$

Adding (1) and (2) we get

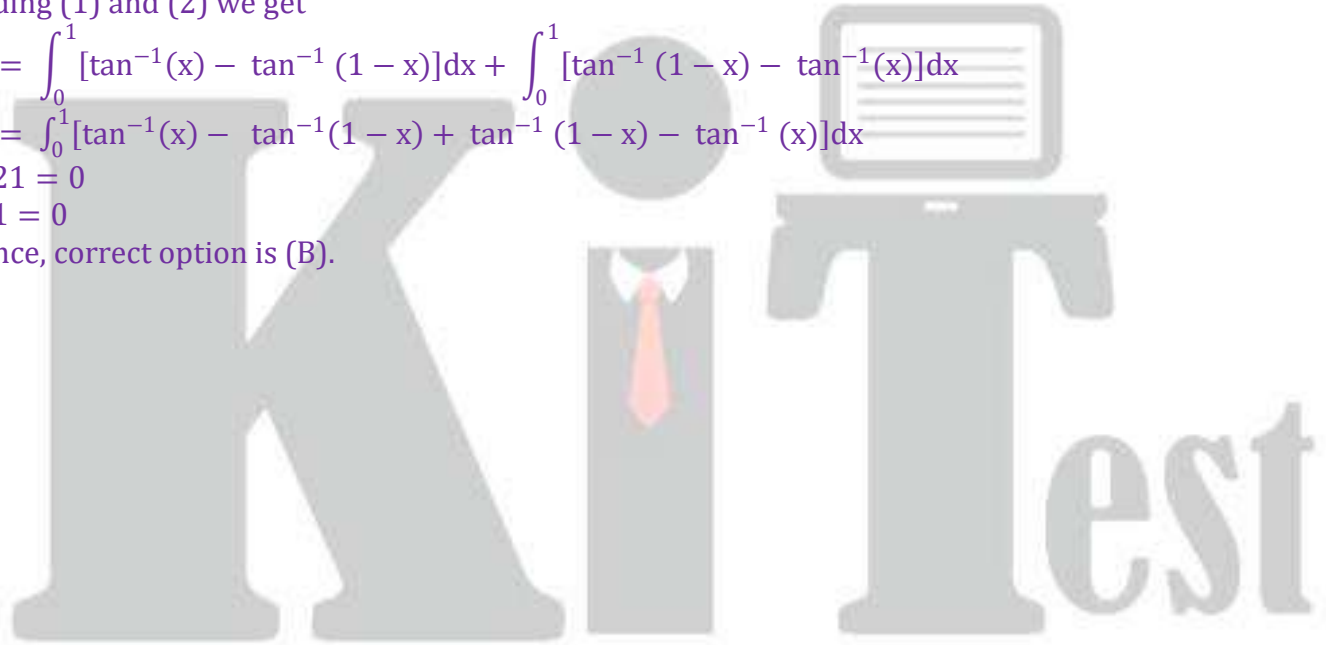
$$2I = \int_0^1 [\tan^{-1}(x) - \tan^{-1}(1-x)] dx + \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)] dx$$

$$2I = \int_0^1 [\tan^{-1}(x) - \tan^{-1}(1-x) + \tan^{-1}(1-x) - \tan^{-1}(x)] dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

Hence, correct option is (B).



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