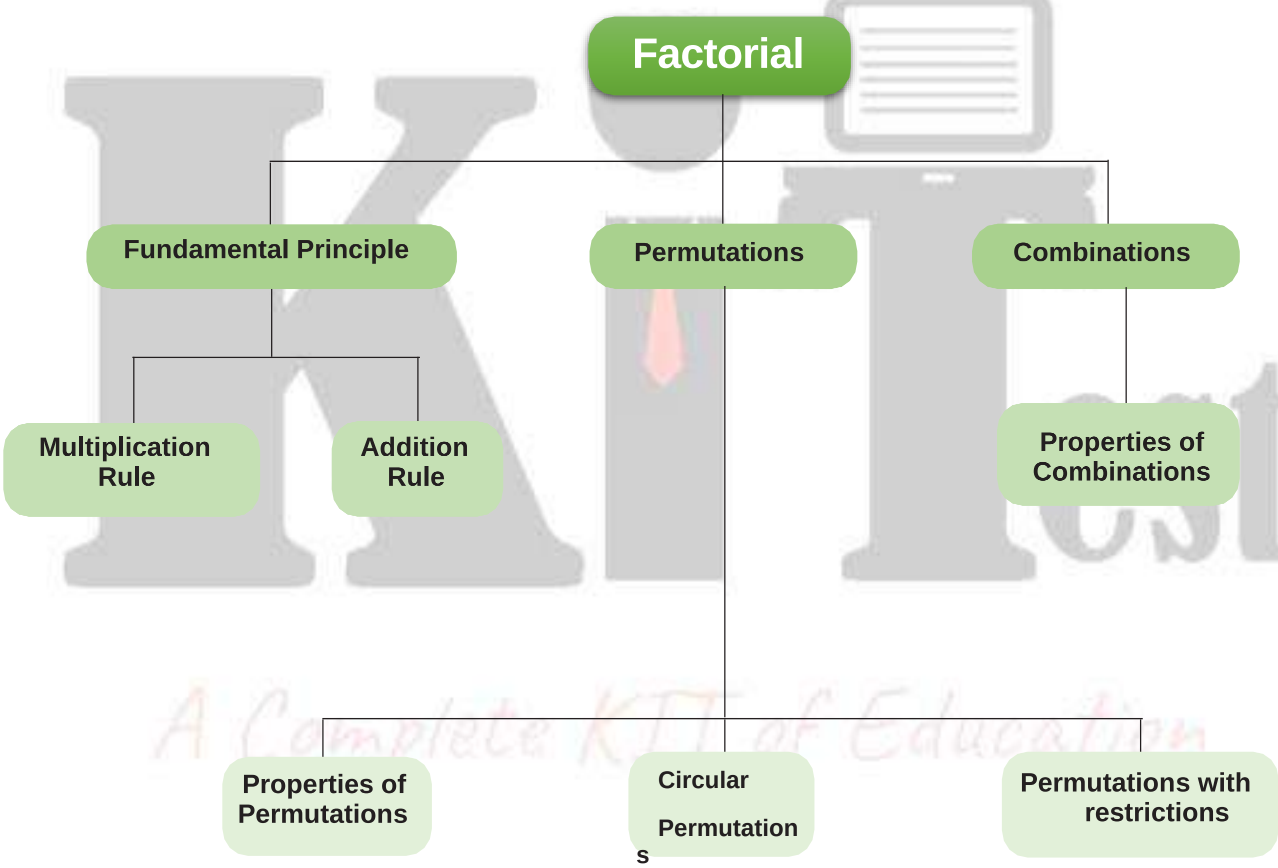
CHAPTER - 5
BASIC CONCEPTS OF PERMUTATIONS AND COMBINATIONS



Fundamental principles of counting	**Multiplication Rule**: If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways then total number of ways of doing both things simultaneously = m×n. **Addition Rule**: It there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in (m+n) ways.
Factorial	The factorial n, written as n! or n , represents the product of all integers from 1 ton both inclusive. To make the notation meaningful, when n=0, we define 0! or 0= 1. Thus, n!=n(n−1)(n−2)...3.2.1
Permutations	The ways of arranging or selecting smaller or equal number of persons or objects from a group of persons or collection of objects with due regard being paid to the order of arrangement or selection, are called permutations. The number of permutations of n things chosen r at a time is given by $^nP_r = n\ (n-1)\ (n-2)\ \ldots\ (n-r+1)$ Where the product has exactly r factors.

Circular Permutations	(a) n ordinary permutations equal one circular permutation. Hence there are ${}^{n}P_{n}/$ n ways in which all the *n* things can be arranged in a circle. This equals $(n-1)!$ (b) The number of necklaces formed with n beads of different colors • Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}p_r$. • Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement
Combinations	The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations. ${}^{n}C_{r}$ = n!/r! (n – r)! ${}^{n}C_{r} = {}^{n}C_{n-r}$ $\mathbf{{}^{n}C_{0}}$ = n!/{0! (n–0)!} = n! / n! =**1**. $\mathbf{{}^{n}C_{n}}$ = n!/{n! (n–n)!} = n! / n! . 0! = **1**. ${}^{n}C_{r}$ has a meaning only when r and n are integers 0 = r = n and ${}^{n}C_{n-r}$ has a meaning only when0= n–r = n. • ${}^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}$ • ${}^{n}P_{r} = {}^{n-1}P_{r} + r^{n-1}\ P_{r-}$
Permutations	Permutations when some of the things are alike, taken all at a time Permutations when each thing may be repeated once, twice, up tor times in any arrangement =n!. The total number of ways in which it is possible to form groups by taking some or all of n things $(2^{n}-1)$.

The total, number of ways in which it is possible to make groups by taking some or all out of n (=$n_1 + n_2 + n_3 + \ldots$) things, where n_1 things are alike of one kind and so on, is given by

$\{(n_1 + 1)\,(n_2 + 1)\,(n_3 + 1)\ldots\} - 1$

The combinations of selecting r_1 things from a set having n_1 objects and r_2 things from a set having n_2 objects where combinations of r1 things, r2 things are independent

Question 1

An examination paper consists of 12 questions divided into parts A and B Part A contains 7 questions and part B contains 5 questions. A candidate is required to attempt 8 questions selecting at least from each part. In how many maximum ways can the candidate select the question?

(a) 35
(b) 175
(c) 210
(d) 420

Answer: d

Explanation:

The candidate can select 8 questions by selecting at last '' three from each part in the following ways:

3 questions from part A and 5 questions from part B = $7_{C_3} \times 5_{C_5}$ = 35 ways

4 questions from part A and part B each

= $7_{C_4} \times 5_{C_4}$ = 175 Ways.

Questions from part A and 3 questions from part B = $7_{C_5} \times 5_{C_3}$ = 210 ways
Hence, the total number of ways in which the candidate can select the question will be = 35 + 175 + 210 = 420 ways

Question 2

Code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. How many such code words are there?

(a) 6,15,800
(b) 46,800
(c) 7,19,500
(d) 4,10,800

Answer: b

Explanation:

The number of ways filling the first two places with English alphabets = 26 × 25 = 650
The number of ways of filling the last two places with distinct numbers = 9 × 8 = 72
The numbers of code words that can be formed are = 650 × 72
= 46800

Question 3

A boy has 3 library tickets and 8 books of his interest in the library of these 8, he does not want to borrow Mathematics part – II unless Mathematics part – I is also borrowed? In how many ways can he choose the three books to be borrowed?

(a) 41
(b) 51
(c) 61
(d) 71

Answer: a

Explanation:

There are two cases possible
CASE 1: When Mathematics Part – II is borrowed (i.e. it means Mathematics Part – I has also been borrowed
Numbers of ways = 6_{C_1} = 6 ways
CASE 2: When Mathematics part – II is not borrowed (i.e. 3 books are to be selected out of 7)
Number of ways = 7_{C_3} = 35 Ways
Therefore, total number ways
35 + 6 = 41 ways

Question 4

Find 5!, 4! And 6!

(a) 720 (b) 120

(c) 380 (d) 620

Answer: a

Explanation:

5! = 5 × 4 × 3 × 2 × 1 = 120: 4! = 4 × 3 × 2 × 1 = 24; 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720

Question 5

Find $\frac{9!}{6!}; \frac{10!}{7!}$

(a) 630,504 (b) 504,720

(c) 920,630 (d) 121,720

Answer: b

Explanation:

$\frac{9!}{6!} = \frac{9\times8\times7\times6!}{6!} = \frac{9\times8\times7}{7!} = \frac{504}{7!}; \frac{10!}{7!} = \frac{10\times9\times8\times7!}{7!} = 720$

10 × 9 × 8 = 720

Question 6

Find x if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$

(a) 121 (b) 112

(c) 211 (d) 111

Answer: a

Explanation:

We have,

$\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$

- ➔ $\frac{1}{9!} + \frac{1}{10\times9!} = \frac{x}{11\times10\times9!}$
- ➔ $\frac{1}{9!}\left[1 + \frac{1}{10}\right] = \left(\frac{x}{11\times10}\right) \times \frac{1}{9!}$
- ➔ $1 + \frac{1}{10} = \frac{x}{11\times10}$
- ➔ $\frac{11}{10} = \frac{x}{11\times10}$
- ➔ X = 11 × 11 = 121

Question 7

Evaluate each of 5_{P_3}, 10_{P_2}, 11_{P_5}

(a) 540 (b) 55440
(c) 5440 (d) 5540

Answer: b

Explanation:

$5_{P_3} = 5 \times 4 \times (5\text{-}3\text{+}1) = 5 \times 4 \times 3 = 60.$

$10_{P_2} = 10 \times .... \times (10\text{-}2\text{+}1) = 10 \times 9 = 90$

$11_{P_5} = \frac{11!}{(11-5)} = 11 \times 10 \times 9 \times 8 \times 7 \times \frac{6!}{6!} = 11 \times 10 \times 9 \times 8 \times 7 = 55440$

Question 8

How many three letters words can be formed using the letters of the word SQUARE?

(a) 110 (b) 12
(c) 120 (d) 210

Answer: c

Explanation:

Since the word 'SQUARE' consists of 6 different letters, the number of permutations of choosing 3 letters out of six equals $6_{P_3} = 6 \times 5 \times 4 = 120$

Question 9

In how many different ways can five persons stand a line for a group photograph?

(a) 110 ways (b) 120 ways
(c) 130 ways (d) 20 ways

Answer: b

Explanation:

Here we know that the order is important, hence this is the number of permutation of n five things taken all at a time. Therefore, this equals $5_{P_5} = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

Question 10

How many three letters words can be formed using the letters the word HEXAGON?

(a) 110 (b) 12
(c) 120 (d) 210

Answer: d

Explanation:

Since the word 'HEXAGON' contains 7 different letters, the number of permutations is $7_{P_3} = 7 \times 6 \times 5 = 210$.

Question 11

First, second and third are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many dif? ferent

(a) 1110 ways (b) 1320 ways
(c) 1830 ways (d) 1716 ways

Answer: d

Explanation:

Here, order of selection is important and repetitions are not meaningful as no exhibit can receive more than one prize. Hence the answer is the number of permutations of 13 things taken three at a time. Therefore, we find 13_{P_3} $= \frac{13!}{10!} = 13 \times 12 \times 11 = 1{,}716$ ways

Question 12

In how many different ways can 3 students be associated with 4 chartered accountants, assuming that each chartered accountant can take at most one student?

(a) 10 (b) 12
(c) 20 (d) 24

Answer: d

Explanation:

This equals the number of permutations of choosing 3 persons out of 4, hence the answer is $4_{p_3} = 4 \times 3 \times 2 = 24$.

Question 13

Compute the sum of 4 digit numbers which can be formed with the four digits 1, 3, 5, 7, if each digit is used only once in each arrangement.

(a) 1,06,656 (b) 1,46,800
(c) 7,19, 500 (d) 4,10,800

Answer: a

Explanation:

The number of arrangement of 4 different digits taken 4 at a time is given by $4_{p_4} = 4! = 24$. All the four digits will occur equal number of times at each of the positions, namely ones, tens, hundreds, thousands.

Thus, each digit will occur $\frac{24}{4} = 6$ times in each of the positions. The sum of digits in one's position will be $6 \times (1+3+5+7) = 96$. Similar is the case in ten's, hundred's and thousand's places. Therefore, the sum will be 96 + 96 × 100 + 96 × 1000 = 106,656.

Question 14

In how many different ways can a club with 10 members select a President, Secretary and Treasurer, if no member can hold two offices and each member is eligible for any office?

(a) 720 (b) 780
(c) 960 (d) 630

Answer: a

Explanation:

The answer is the number of permutations of 10 persons chosen three at a time.

Therefore, it is 10_{P_3}= 10×9×8 = 720

Question 15

When jiana arrives in New York, she has eight shops to see, but he has time only to visit six of them. In how many different ways can he arrange her schedule in New York?

(a) 20,160 (b) 2016
(c) 26105 (d) 21560

Answer: a

Explanation:

She can arrange his schedule in 8_{P_6} = 8 × 7 × 6 × 5 × 4 × 3 = 20,160 ways

Question 16

When Dr. Ramanujan arrives in his dispensary, he finds 12 patients waiting to see him. If he can see only one patient at a time. Find the number of ways; he can schedule his patients if they all want their turn.

(a) 479001600 (b) 79833600
(c) 34879012 (d) 67800983

Answer: b

Explanation:

There are 12-3 = 9 patients. They can be seen 12_{P_9}= 79,833,600 ways.

Question 17

How many arrangements can be made out of the letters of the word 'DRAUGHT' the vowels never beings separated?

(a) 1440 (b) 720
(c) 740 (d) 750

Answer: a

Explanation:

The word 'DRAUGHT' consists of 7 letters of which 5 are consonants and two are vowels. In the arrangement we are to take all the 7 letters but the restriction is that the two vowels should not be separated.

We can view the two vowels as one letter. The two vowels A and U in this one letter can be arranged in 2! = 2 ways. (i) AU or (ii) UA. Further, we can arrange the six letters: 5 consonants and one letter consisting of two vowels. The total number of ways of arranging them is 6_{P_6} = 6! = 720 ways.

Hence, by the fundamental principle, the total number of arrangements of the letters of the word DRAUGHT, the vowels never being separated = 2 × 720 = 1440 ways.

Question 18

An examination paper with 10 questions consists of 6 questions in mathematics and 4 questions in statistic part. At least one question from each part is to be attempted in how many ways can this be done?

(a) 1024 (b) 945
(c) 1000 (d) 1022

Answer: b

Explanation:

Total question = 10

No. of Mathematics questions = 6 No. of statics questions = 4.

No. of ways at least one question of Mathematics

= (2^6 1) = (64 – 1) = 63

No. of ways at least one question of statics

= (2^4 1) = (16 – 1) = 15

Total no. of ways = 63 × 15 = 945

Questions 19

A student has three books on computer, three books on Economics and five books on Commerce. If these books are to be arranged subject wise, then these can be placed on a shelf in the number of ways:

(a) 25290 (b) 25092
(c) 4320 (d) 25920

Answer: d

Explanation:

No. of ways = 3! 3! 5! 3!
= 6 × 6 × 120 × 6
= 216 × 120
= 25, 920

Questions 20

A person has ten friends of whom six are relatives. If h invites five guests 'SUCH' that three are his relatives, then the total number of ways in which he can invite then are:

(a) 30 (b) 60
(c) 120 (d) 75

Answer: c

Explanation:

Total friend: 10
No. of Relative = 6
No. of friend = 4
No. of ways to invite five guests such that three of them are his relatives.
$= 6_{C_3} \times 4_{C_2}$
$= \frac{6!}{3! \times 3!} \times \frac{4!}{2! \times 2!}$
20 × 6 = 120

Questions 24

Six seats of articled clerks are vacant in a 'Chartered Accountant Firm'. How many different batches of candidates can be chosen out of ten candidates?

(a) 216 (b) 210
(c) 220 (d) 230

Answer: b

Explanation:

The number of ways in which 6 articled clerks can be selected out of 10 caildidats
$= 10_{C_6}$ = 210 ways.

Question 25

Six persons A, B, C, D, E and F are to be seated at a circular table. In how many ways can this be done, if a must always has either B or C on his right and B must always have either C or D on his right?

(a) 3 (b) 6

(c) 12 (d) 18

Answer: d

Explanation:

Using the given restrictions, we must have AB or AC and AB or BD

Therefore, we have the following alternatives

ABC, D, E, F, which gives (4 – 1)! Or 3! ways.

ABC, D, E, F which gives (4 – 1)! Or 3! ways.

AC, BD, E, F, which gives (4 – 1) or 3! ways.

Hence, the total number of ways are

= 3! + 3! +3!

= 6 + 6 + 6 = 18 ways

Question 26

A fundamental principle of counting is:

(a) m × n, m - n (b) m × n, m + n

(c) m + n, m ÷ n (d) m ÷ n, m – n

Answer: b

Explanation:

Fundamental principles of counting

a. Multiplications Rule: m × n
b. Addiction Rule: m + n

Question 27

If $n_{C_r} = n_{C_{r-1}}$| and n_{P_r} and$n_{P_{r+1}}$, then the value of n is 27.

(a) 3 (b) 4

(c) 2 (d)5

Answer: a

Explanation:

The conditions provided that n – r = r – 1 ρ r = $\frac{n+1}{2}$ so if

We put n = 3, then r =2 satisfies the conditions

Question 28

$n_{P_r} \div n_{C_r}$ =

(a) n! (b) (n - r)!

(c) 48 (d) r!

Answer: d

Explanation:

Question 30

The number of ordered triplets of positive integers which are solutions of the equation x+y+z = 100 is

(a) 6005 (b) 4851

(c) 5081 (d) none of these

Answer: b

Explanation:

The number of triplets of positive integers which re solutions of

X + y + z = 100 = coefficient of x^{100} in $(x + x^2 + x^3 + \ldots\ldots.)^{3}$

= coefficient of x^{100} in $x^3(1 - x)^{-3}$ = coefficient of x^{100} in

$X^3 \left(1 + 3x + 6x^2 + \ldots\ldots.. + \frac{(n+1)(n+2)}{2}x^n + \ldots\ldots.\right)$

$= \frac{(97+1)(97+2)}{2} = 49 \times 99 = 4851$

Question 32

The number of way to sit 3 men and 2 women in a bus such that total number of sited men and women on each side is 3

(a) 5! (b) $6_{C_5} \times 5!$

(c) $6! \times 6_{P_5}$ (d) $5! + 6_{C_5}$

Answer: b

Explanation:

3 men and 2 women equal to 5. A group of 5 members make 5! Permutations with each other. The number of ways to sit 5 members = 5! 6 places are filled by 5 members by 6_{C_5} ways. The total number of ways to sit 5 members on 6 seats of a bus = $6_{C_5} \times 5!$

Question 33

If P (n,r)=1680 and C (n,r) = 70, then 69n+r! =

(a) 128 (b) 576

(c) 256 (d) 625

Answer: b

Explanation:

P (n, r) = 1680 $\frac{n!}{(n-r)!}$ = 1680?... (i) C (n, r) = 70ρ

$\frac{n!}{r!(n-r)!}$ = 70? (ii) $\frac{1680}{r!}$ = 70. [From (i) and (ii)]

r! = $\frac{1680}{70}$ = 24ρ r = 4 ∵ P (n, 4) = 1680∵

n (n – 1)(n – 2)(n – 3) = 1680 ρ n = 8 ∵

8 × 7 × 6 × 5 = 1680 + r! = 69 × 8 + 4! = 552 + 24

= 576

Question 34

Number of divisors of n = 38808 (except 1 and n) is

(a) 70 (b) 68

(c) 72 (d) 74

Answer: a

Explanation:

Since 38808 = 8 × 4851

$8\times9\times539=8\times9\times7\times7\times11=2^3 \times 3^2 \times 7^2 \times 11$

Number of divisors = (3 + 1) (2 + 1) (2 + 1) (1 + 1) = 72. This includes two divisors 1 and 38808. Hence the required number off divisors = 72 – 2 = 70

Question 35

If eleven members of a committee sit at a round table so that the president and secretary always sit together, then the number of arrangement is

(a) 10! × 2 (b) 10!

(c) 9! × 2 (d) None of these

Answer: c

Explanation:

Required number of ways 9! × 2 (by fundamental property of Circular permutation).

Question 36

In how many ways can 5 keys be put in a ring?

(a) $\frac{1}{2}4!$ (b) $\frac{1}{2}5!$

(c) 4! (d) 5!

Answer: a

Explanation:

Mark the keys as 1, 2, 3, 4, 5

Assume the ring as a circle with 5 positions.

First position can be taken by any one of them.

The 2nd positions has 4 possibility, 3rd has 3, 4th has 2, 5th has 1

Totally 4×3×2×1 = 24.

Question 37

A question paper is divided into two parts A and B and each part contains 5 questions. The number of ways in which a candidate can answer 6 questions selecting at least two questions from each part is

(a) 80
(b) 810
(c) 200
(d) None of these

Answer: d

Explanation:

The number of ways that the candidate may select 2 questions from A and 4 from B=$5_{C_2} \times 5_{C_3}$ 4 questions from A and 2 from B = $5_{C_4} \times 5_{C_2}$. Hence total numbers of ways are 200.

Question 38

How many number lying between 10 and 1000 can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition is allowed)

(a) 1024
(b) 810
(c) 2346
(d) None of these

Answer: b

Explanation:

The total number between 10 and 1000 are 989 but we have to form the numbers by using numerals 1, 2,……9, i.e. 0 is not occurring so the numbers containing any?0? would be excluded i.e., Required number of ways

$$= 989 - \begin{Bmatrix} 20,\ 30, 40, \dots \dots \dots \dots 100 = 9 \\ 101, 102, \dots \dots \dots ..300 = 19 \\ 201, \dots \dots \dots \dots \dots ..300 = 19 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. \\ 901, \dots \dots \dots \dots \dots \dots ..990 = 18 \end{Bmatrix}$$

= 989-(9+18+19×8) = 810. Alter: Between 10 and 1000, the numbers are of 2 digits And 3 digits. Since repetition is allowed, so each digit can be filled in 9 ways. Therefore number of 2 digit numbers = 9×9 = 81 and number of 3 digit numbers 9 × 9 × 9 = 729. Hence total ways = 81 + 729 = 810

Question 39

The number of ways in which the letters of the word TRIANGLE can be arranged such that two vowels do not occur together is

(a) 1200
(b) 2400
(c) 14400
(d) 14400

Answer: c

Explanation:
·T·R·N·G·L Three vowels can be arrange at 6 places in 6_{P_3} = 120 ways. Hence the required number of arrangements = 120×5! = 14400

Question 40
There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls one in each box, could be such that a ball does not go to box of its own colour is
(a) 8 (b) 7
(c) 9 (d) None of these
Answer: c
Explanation:
Since the number of derangements in such a problems is given by
$n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\dots\dots\dots.(-1)^n\frac{1}{n!}\right\}$
$\therefore$ Number of derangements are = $4!\left\{\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right\}$
= 12-4+1 = 9

Question 41
If$56_{P_{r+6}}$:$54_{P_{r+3}}$ = 30800:1, then r =
(a) 31 (b) 41
(c) 51 (d) none of these
Answer: b
Explanation:
$$\frac{56!}{(50-r)!}\times\frac{(51-r)!}{54!}$$
$\frac{30800}{1}$ = 56×55× (51 – r) = 30800
r = 41

Question 42
The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player just one card, is
(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^2}$
(c) 52! (d) none
Answer: a
Explanation:

For the first set number of ways $52_{C_{17}}$. Now out of 35 cards left 17 cards can be put for second in $35_{C_{17}}$ways similarly for 3rd in $18_{C_{17}}$. One card for the last set can be put in only one way. Therefore the required number of ways for the proper distribution = $\frac{52!}{35!17!} \times \frac{35!}{18!17!} \times \frac{18!}{17!1!} \times 1! = \frac{52!}{(17!)^3}$

Question 43

m men and n women are to be seated in a row so that no two women sit together. If m>n, them then the number of ways in which can be seated is

(a) $\frac{m!(m+1)!}{(m-n+1)!}$ (b) $\frac{m!(m-1)!}{(m-n+1)!}$

(c) $\frac{(m-1)!(m+1)!}{(m-n+1)!}$ (d) none

Answer: a

Explanation:

First arrange m men, in arrow in m! Ways. Since n<m and no two women can sit together, in any one of the m! Arrangement, there are places in which n women can be arranged in $m + 1_{P_n} = \frac{m!(m+1)!}{[(m+1)-n)!} = \frac{m!(m+1)!}{(m-n+1)!}$

Question 44

The number of times the digit 3 will be written when listing the integers from 1 to 1000 is:

(a) 369 (b) 300

(c) 271 (d) 302

Answer: b

Explanation:

To find number of times 3 occurs in listing the integer from 1 to 999. (Since 3 does not occur in 1000). Any number between 1 to 999 is a 3 digit number xyz where the digit x, y, z are any digits from 0 to 9. Now, we first count the numbers in which 3 occurs once only. Since 3 can occur at one place in 3_{C_1} ways. There are 3_{C_1}. (9 × 9) + 3 × 1 = 300

Question 45

Ten persons, amongst whom are A, B, and c to speak at a function. The number of ways in which it can be done. If A wants to speak before B and B wants to speak before C is

(a) $\frac{10!}{6}$ (b) 3 ! 7 !

(c) 10_{P_3}.7! (d) None of these

Answer: a

Explanation:

For A, B, C, to speak in order of alphabets 3 places out of 10 may be chosen first in 1. 3_{C_2} = 3 ways. The remaining 7 persons can speak in 7! Ways.
Hence, the number of ways in which all the 10 person can speak is 10_{C_3}. 7! = $\frac{10!}{3!} = \frac{10!}{6}$

Question 46

How many words can be made out from the letters of the word INDEPENDENCE, in which vowels always come together?

(a) 16800 (b) 16630
(c) 1663200 (d) None of these

Answer: a

Explanation:

Required numbers of ways are $\frac{8!}{2!3!} \times \frac{5!}{4!}$ = 16800. {Since IEEEENDPNDNC = 8 letters}.

Question 47

The exponent of 3 in 100! Is

(a) 33 (b) 44
(c) 48 (d) 52

Answer: c

Explanation:

Let E (n) denote the exponent of 3 in n. the greatest integer less than 100 divisible by 3 is 99. We have
E (100!) = E (1 .2 .3. 4....99. 100)
= E (3 .6. 9....99)
= E [(3. 1)(3 . 2) (3 . 3)...... (3. 33)]
= 33 + E (1. 2. 3......33) Now
E (1. 2. 3.......33) =E (3. 6. 9....33)
= E [(3. 1) (3. 2) (3. 3)....... (3. 11)]
= 11+ E (1. 2. 3 .....11) And
E (1 .2. 3 ....11) = E [(3. 1) (3. 2) (3. 3)]
3 + E (1 .2. 3) = 3 + 1 = 4 Thus
E (100!) = 33 + 11 + 4 = 48

Question 48

A dictionary is printed consisting of 7 lettered words only that can be made with a letter of the word CRICKET. If the words are printed at the alphabetical order, as in an ordinary dictionary, then the number of word before the word CRICKET is

(a) 530 (b) 480
(c) 531 (d) 481

Answer: a

Explanation:

The number of words before the word CRICKET is 4×5! +2×4! +2! = 530

Question 49

The number of positive integral solutions of abc = 30 is

(a) 30 (b) 27
(c) 8 (d) none of these

Answer: b

Explanation:

We have, 30 = 2 × 3 × 5. So, 2 can be assigned to either a or b or c i.e. 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways. Thus the no. of solutions are 3 × 3 × 3 = 27.

Question 50

The number of different words that can be formed out of the letters of the word 'MORADABAD' taken four at a time is

(a) 500 (b) 600
(c) 620 (d) 626

Answer: d

Explanation:

In MORADABAD, we have 6 different types of letters 3A^S, 2D^S and rest four different. We have to form words of 4 letters. (i) All letters 6_{P_4} = 6 × 5 × 4 × 3 = 360. (II) Two different two a like $2_{C_1} \times 5_{C_2} \times \frac{4!}{2!}$ = 240 (iii) 3 alike 1 different $1_{C_1} \times 5_{C_1} \times \frac{4!}{2!}$ = 20 (iv) 2 alike of one type and 2 alike of other type $2_{C_2} \times \frac{4!}{3!}$ = 6
Therefore total number of words
= 360 + 240 + 20 + 6 = 626

PREPARE FOR WORST

Question 1

How many 3 letter words with or without meaning can be formed out of the letters of the word MONDAY when repetition of words is allowed?

(a) 125 (b) 216
(c) 120 (d) 320

Question 2

In how many ways the letters in the word TOOTH can be arranged?

(a) 120 (b) 40
(c) 20 (d) 30

Type – 2

Question 1

How many five letters words with or without meaning, can be formed from the word 'COMPLEXIFY', if repetition of letters is not allowed?

(a) 43200 (b) 30240
(c) 12032 (d) 36000

Question 2

In how many different ways can the letters of the word 'LOGARITHMS' be arranged so that the vowels always come together?

(a) 6720 (b) 241920
(c) 40320 (d) 360344

Question 3

How many three digit numbers can be formed from the digits 3, 4, 5, 7, 8, and 9. Also, the number formed should be divisible by 5 and no repetition is allowed?

(a) 20 (b) 24
(c) 25 (d)

Type 3

Question 1

An ice cream seller sells 5 different ice-creams. John wants to buy 15 ice creams for his friends. In how many ways can he buy the ice-cream?

(a) 1450 (b) 3768
(c) 3879 (d) 1540

Question 2

There are 5 types of soda flavor available in a shop. In how many ways

can 10 soda flavors be selected?

(a) 1454 (b) 1001

(c) 1211 (d)1540

Type - 4

Question 1

A wooden box contains 2 grey balls, 3 pink balls and 4 green balls. Fins out in how many ways 3 balls can be drawn from the wooden box. Make sure that at least one pink ball is included in the draw?

(a) 64 (b) 46

(c) 56 (d) 65

Question 2

There are 5 boys and 10 girls in a classroom. In how many ways teacher can select 2 boys and 3 girls to make a dance group?

(a) 720 (b) 1200

(c) 240 (d) 840

Question 3

There are 10 consonants and 5 vowels. Out of which how many words of 5 consonants and 2 vowels can be made?

(a) 2520 (b) 1200

(c) 210 (d) 720

Question 4

A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when at least 2 women are included?

(a) 196 (b) 186

(c) 190 (d) 200

Question 5

If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number:

(a) 601 (b) 600

(c) 603 (d) 602

Question 6

A college has 10 basketball players. A 5-member team and a captain will

be selected out of these 10 players. How many different selections can be made?

(a) 1260 (b) 1400
(c) 1250 (d) 1600

Question 7

When four fair dice are rolled simultaneously, in how many outcomes will at least one of the dice show 3?

(a) 620 (b) 671
(c) 625 (d) 567

Question 8

A letter lock consists of three rings each marked with six different letters. The number of distinct unsuccessful attempts to open the lock is at the most?

(a) 215 (b) 268
(c) 254 (d) 216

Question 9

In how many ways can the letters of the word EDUCATION be rearranged so that the relative position of the vowels and consonants remain the same as in the word EDUCATION?

(a) 4! x 4! (b) 5! x 5!
(c) 4! x 5! (d) 3! x 4!

Question 10

In a Plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no lines passes through both points A and B , and no two are parallel. Find the number of points of intersection of the straight lines.

(a) 525 (b) 535
(c) 545 (d) 555

Question 11

How many different four letter words can be formed (the words need not be meaningful using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R?

(a) 59 (b) 56
(c) 64 (d) 55

Question 12
In how many ways can 5 different toys be packed in 3 identical boxes such that no box is empty, if any of the boxes may hold all of the toys?

(a) 36 (b) 25
(c) 24 (d) 72

Question 13
In a box, there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen?

(a) 180 (b) 220
(c) 240 (d) 160

ANSWERS AVAILABLE ON:
- **TELEGRAM CHANNEL: t.me/KINSHUKInstitute**
- **WEBSITE : WWW.KITest.IN**
- **KITest APP**

Past Examination Questions

MAY - 2018

Question 1
The number of triangle that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is:

(a) 185 (b) 175
(c) 115 (d) 105

Answer: a

Explanation:

Here n = 12, k = 7

No. of triangle are formed from 'n' point

In which (k) points are collinear = n_{C_3}-k_{C_3}

= 12_{C_3} - 7_{C_3}

$= \frac{12\times11\times10}{3\times2\times1} - \frac{7\times6\times5}{3\times2\times1}$

=220-35

=185

Question 2

If $1000_{C_{98}}$- +$999_{C_{97}}$ + $x_{C_{901}}$, find x:

(a) 999 (b) 998

(c) 997 (d) 1000

Answer: a

Explanation:

If $1000_{C_{98}}$ - $999_{C_{97}}$ + $x_{C_{901}}$

$\therefore n_{C_r} + n_{C_{r-1}}$ = n + 1_{C_r}

Then x = $999\left[999_{C_{901}} + 999_{C_{98}}\right]$

Nov - 2018

Question 1

A bag contains 4 red, 3 black, and 2 white balls. In how many ways 3 balls can be drawn from his bag so that they include at least one black ball?

(a) 64 (b) 46

(c) 85 (d) None

Answer: a

Explanation:

No. of total balls = 4 Red+3 Black + 2 white = 9 balls

Total number of ways

= 3C3 + (3C2 x 6C1) + (3C1 x 6C2) [because 6 are nonblack]

=1+[3×6]+[3×(6×52×1)]=1+18+45=64

Question 2

The number of words from the letter word BHARAT, in which B and H will never come together, is

(a) 360 (b) 240

(c) 120 (d) None

Answer: b

Explanation:

Given word

'B H A R A T'

1 2 3 4 5 6

Total No. of ways arrange the letter word = $\frac{6!}{2!} = \frac{720}{2} = 360$

If Letter 'B' and 'H' are never taken together

= 360-120

=240

Question 3

The value of N in = $\frac{1}{7!} + \frac{1}{8!} + \frac{N}{9!}$ ***is***

(a) 81 (b) 78

(c) 89 (d) 64

Answer: a

Explanation:

If $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$

$$\frac{9 \times 8 \times 1}{9 \times 8 \times 7!} - \frac{9 \times 1}{9 \times 8!} = \frac{N}{9!}$$

$$\frac{72}{9!} + \frac{9}{9!} = \frac{N}{9!}$$

$$\frac{81}{9!} = \frac{N}{9!}$$

N = 81

Question 4

If n_{P_r}=720, n_{P_r} = 120, then r is

(a) 3 (b) 4

(c) 5 (d) 6

Answer: a

Explanation:

Given n_{P_r} = 720, n_{C_r} = 120

We know that

$$\frac{n_{C_r}}{n_{C_r}} = \frac{1}{r}$$

$$\frac{120}{720} = \frac{1}{r}$$

$$\frac{1}{6} = \frac{1}{r}$$

R = 3

MAY - 2019

Question 1

If 11_{C_r} = $11_{C_{2x-4}}$ and x≠4 then the value of 7_{C_x} =

(a) 20 (b) 21

(c) 22 (d) 23

Answer: b

Explanation:

Equate bases of LHS and RHS

So x=4

Therefore, LHS = RHS

11-x = 2x-4

x=5

$7_{C_3} = 7_{C_2} = 21$

Question 2

Which of the following is not a correct statement?

(a) n_{P_n}=$n_{P_{n-1}}$ (b) n_{P_n}=2.$n_{P_{n-2}}$

(c) n_{P_n}=3.$n_{P_{n-3}}$ (d) n_{P_n}= n. $n.(n-1)_{Pn-1}$

Answer: d

Explanation:

LHS ≠ RHS

In case of d option

Question 3

How many words can be formed with the letter of the world ''PARALLEL''. So that all L's do not come together?

(a) 2000 (b) 3000

(c) 4000 (d) None of these

Answer: b

Explanation:

There are 8! ways of arranging the eight letters of ''PARALLEL'', but since there are three ''L''s and two ''A''s, we must divide through by 3!×2! to get a total of $\frac{8!}{3!\times2!}$ permutations.

Okay, so how many of these have all three ''L''s together?

$\frac{8!}{3!\times2!}$ - 6×$\frac{5!}{2!}$ = 3000

Question 4

The Indian cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket eleven be selected if we

have to select 1 wicket keeper and atleast 4 bowlers?

(a) 1024 (b) 1900
(c) 2000 (d) 1092

Answer: d

Explanation:

We are to choose 11 players including 1 wicket keeper and 4 bowlers or, 1 wicket keeper and 5 bowlers.

Number of ways of selecting 1 wicket keeper, 4 bowlers and 6 other player's in

$2_{C_1} \times 5_{C_4} \times 9_{C_6} = 840$

Number of ways of selecting 1 wicket keeper, 5 bowlers and 5 other players in

$2_{C_1} \times 5_{C_4} \times 9_{C_5} = 252$

Total number of ways of selecting the term = 840 + 252 = 1092.

Nov - 2019

Question 1

Three girls and five boys are to be seated in a row so that no two girls sit together. Total no. of ways of this arrangement are:

(a) 14,400 (b) 120
(c) 5_{P_3} (d) 3! × 5!

Answer: a

Explanation:

(a) Required arrangement

X B_1 X B_2 X B_3 X B_4 X B_5 X

No. of ways of arranging 3 girls in 6 places

$= 5_{p_3}$

Total ways = $^6p_3 \times {}^5p_5$

$= \frac{6!}{(6-3)!} \times 5!$

$= \frac{6 \times 5 \times 4 \times 3!}{3!} \times 120$ = Rs. 14,400

Question 2

How many numbers can be formed with the help of 2, 3, 4, 5, 6, 1 which is not divisible by 5, given that it is a five-digit no. and not repeating?

(a) 600 (b) 400
(c) 1200 (d) 1400

Answer: a

Explanation:

(a) No's 2, 3, 4, 5, 6, 1.
A no. is divisible by 5 when it ends with 0 or 5
T TH TH H O
— — — — —
No. of ways of filling one's digit = 5 (all except 5)
No. of ways of filing ten's digit = 5
No. of ways of filling thousand place = 4
No. of ways of filling ten thousand place = 3
No. of ways of filling hundred's place = 2
Total ways = 5 × 5 × 4 × 3 × 2
= 600 ways

Question 3

How many different groups of 3 people can be formed from a group of 5 people?

(a) 5 (b) 6
(c) 10 (d) 9

Answer: c

Explanation:

(c) We know,
No. of ways to choose r objects out of n objects is ^nC_r
Using the formula,
Choosing 3 distinct objects (groups) from
$5 = {}^5C_3 = \frac{5!}{(5-3)!\times 3}$
$= \frac{5!}{2!\times 3!}$
$= \frac{5\times 4\times 3!}{2\times 3!}$
10 ways

Question 4

In how many ways can 4 people be selected at random from 6 boys and 4 girls if there are exactly 2 girls?

(a) 90 (b) 360
(c) 92 (d) 480

Answer: a

Explanation

(a) **Boys** (6)**Girls** (4)
2 **2**
No. of ways of selecting 2 boys out of 6 = $^6C_2$

No. of ways of selecting 2 girls out of 4 = $^4C_2$
Total ways = $^6C_2 \times {}^4C_2$

$$= \frac{6!}{(6-2)! \times 2} \times \frac{4!}{21 \times (4-2)!}$$

$= \frac{6\times5\times4!}{4!\times2} \times \frac{4\times3\times2!}{2!\times2}$

= 15 × 6 = 90 ways.

Question 5

$^np_3 : {}^np_2 = 2:1$

(a) 4 (b) 7/2
(c) 5 (d) 2/7

Answer: a

Explanation:

(a) $^np_r = \frac{n!}{(n-r)!}$

$^nP_r : {}^nP_2 = 2:1$

$$\frac{n!}{(n-3)!} : \frac{n!}{(n-2)!} = \frac{2}{1}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-2)(n-3)!}{n!} = \frac{2}{1}$$

(n – 2) = 2
N = 4

DEC - 2020

Question 15

If $^np_4 = 20\ {}^np_2$ = where P denotes the number of permutations n =__________

(a) 4 (b) 2
(c) 5 (d) 7

Answer: d

Explanation:

$$n_{P_r} = \frac{n!}{(n-r)!}$$

Here,

$$n_{P_4} = 20n_{P_2}$$

$$= \frac{n!}{(n-4)!} = 20 = \frac{n!}{(n-2)!}$$

$$(n-2)! = 20(n-4)!$$

$(n-2)(n-3)(n-4)! = 20(n-4)!$
$(n-2)(n-3) = 20$
$n^2 - 5n + 6 = 20$
$n^2 - 5n - 14 = 0$
$n^2 - 7n + 2n - 14 = 0$
$n(n-7) + 2(n-7) = 0$
$(n+2)(n-7) = 0$
If n+2 = 0 ⇒ n = -2 (Not possible)
If n-7 = 0 ⇒ n = 7
Thus, the value of n is 7.

Question 16
A fruit basket contains 7 apples, 6 bananas and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?
(a) 120 ways (b) 35 ways
(c) 168 ways (d) 70 ways
Answer: c
Explanation:
Given:
Number of Bananas = 6
Number of Apples = 7
Number of Mangoes = 4
To find: Number of ways can a person make a selection of fruits from the basket.
Number of ways to select zero or more bananas = 6 + 1 = 7 ways
Number of ways to select zero or more apples = 7 +1 = 8 ways
Number of ways to select zero or more mangoes in 4 +1 = 5 ways
So, Total number of ways = 5 x 8 x 7 = 280
We included a case of 0 Banana, 0 apple and 0 mangoes, so we have to subtract this from total number of ways,
⇒ Number of ways = 280 – 1 = 279 ways
Therefore, A person can make a selection of fruits from the basket is 279 ways.
∴3 fruits can be made so that all 3 are apples is 35

Question 17
Out of 7 boys and 4 girls a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is___
(a) 429 (b) 439
(c) 419 (d) 441
Answer: d

Explanation:
The Team Consist of 4 girls +1 boy
Number of selections $4_{C_3} \times 7_{C_1} = 1 \times 7 = 7$
Hence, the total number of teams that can be formed = 140+210+84+7= 441

Question 18
From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done?
(a) 201 (b) 168
(c) 202 (d) 220
Answer: a
Explanation:
4 Women, 1 Man
$7_{C_3} \times 6_{C_1} = 35 \times 6 = 210$

JAN - 2021

Question 1
Eight chairs are numbered from 1 to 8. Two women and three men are to be seated by allowing one chair for each. First, the women choose the chairs from the chairs numbered 1 to 4 and then men select the chairs from the remaining. The number of possible arrangements is:
(a) 120 (b) 288
(c) 32 (d) 1440
Answer: d
Explanation:
1440
Step-by-step explanation:
First women can take any of the chairs marked 1 to 4 in 4 different way.
Second women can take any of the remaining 3 chairs from those marked 1 to 4 in 3 different ways.
So, total no of ways in which women can take seat =4×3
⇒4P2
4P2=4!(4−2)!
=4×3×2×12×1
=12
After two women are seated 6 chairs remains

First man take seat in any of the 6 chairs in 6 different ways, second man can take seat in any of the remaining 5 chairs in 5 different ways
Third man can take seat in any of the remaining 4 chairs in 4 different ways.
So, total no of ways in which men can take seat =6×5×4
⇒6P3
6P3=6!(6−3)!
⇒6×5×4×3×2×13×2×1
⇒120
Hence total number of ways in which men and women can be seated =120×12
⇒1440

Question 2

'n' locks and 'n' corresponding keys are available but the actual combination is not known. The maximum number of trials that are needed to assigns the keys to the corresponding locks is.

(a) (n – 1) C_2 (b) (n + 1) C_2
(c) $\sum_{k-2}^{n}(k-1)$ (d) $\sum_{k-2}^{n} K$

Answer: d

Question 3

The harmonic mean of the roots of the equation
$(5+\sqrt{2})x2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$, is

(a) 2 (b) 4
(c) 6 (d) 8

Answer: b

Explanation:

let the 2 roots be α, β

$$\text{H.M.} = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2\times\frac{8+\sqrt[2]{5}}{5+\sqrt{2}}}{\frac{4+\sqrt{5}}{6+\sqrt{2}}} = \frac{2\times 2\,(4+\sqrt{5})}{(4+\sqrt{5})} = 4$$

Question 4

There are ten fights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different fight, is

(a) 90 (b) 95
(c) 80 (d) 78

Answer: a

Explanation:

To go from A to B = 10 Flight
& to go from B to A = 9 flights
(as cannot comping in some flight)
10×9 = 90 ways

Question 5

How many odd numbers of four digits can be formed with digits 0, 1, 2, 3, 4, 7 and 8?

(a) 150 (b) 180
(c) 120 (d) 210

NOTE: The correct Ans is: 300

Answer: b

Explanation:

5 × 5 × 4 × 3 = 300
(0 cannot be here & 1 used in last cannot be here)
(1,3,7) can be on last place as it should be odd

Question 6

In how many different ways, can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd numbered position?

(a) 32 (b) 36
(c) 48 (d) 60

Answer: b

Explanation:

Since detail has 6 letters, there are 3 odd positions, the 1st, 3rd, and 5th spots. Let's determine how many ways the word can be arranged when the vowels occupy the odd positions.

1st spot: 3 options (any of the 3 vowels)
2nd spot: 3 options (any of the 3 consonants)
3rd spot: 2 options (any of the 2 remaining vowels)
4th spot: 2 options (any of the 2 remaining consonants)
5th spot: 1 option (the last remaining vowel)
6th spot: 1 option (the last remaining consonant)
So, the word can be arranged in 3 x 3 x 2 x 2 x 1 x 1 = 36 ways.

Question 7

$^nC_p + 2\,^nC_{P-1} + ^nC_{p-2}$?

(a) $^{n+}C_P$ (b) $^{n+2}C_p$

(c) $^{n+1}C_{p+1}$ (d) $^{n+2}C_{p-1}$

Answer: d

Explanation:

Direct Formula

for refer another origin formula

$^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Question 8

A business house wishes to simultaneously elevate two of its six branch heads. In how many ways these elevations can take place?

(a) 12 (b) 3

(c) 6 (d) 15

Answer: d

Explanation:

$^{6}C_{2} = \frac{6\times5}{2} = 15$

JULY - 2021

Question 1

If $^{n}p_{6} = 20\ {}^{n}p_{4}$ then the value of n is given by

(a) n =5 (b) n =3

(c) n = 9 (d) n = 8

Answer: Options (c)

Explanation:

$^{n}p_{6} = 20\ {}^{n}p_{4}$

$$\frac{[n}{[n-6} = 20\frac{[n}{[n-4}$$

$$\frac{[n}{[n-6} \times \frac{[n-4}{[n} = 20$$

$$\frac{[n-4}{[n-6} = 20$$

$$\frac{(n-4)\,(n-5)\,[(n-6)}{[n-6}$$

(n- 4) (n – 5) = 20

(n – 4) (n – 5) = 5 × 4

So n – 4 = 5

n = 9

Question 2

How many number of seven digit numbers which can be formed for the digits 3,4,5,6,7,8,9 no digits being repeated are not divisible by 5?

(a) 4320 (b) 4690

(c) 3900 (d) 3890

Answer: Options (a)

If no should not ÷ 5 then 5 not on last plag (3,4,5,6,7,8,9)

$6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 6$

4320

Question 3

A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to "A" by any mode other than the one earlier. The number of different ways, the entire journey can be complete is_

(a) 110 (b) 10^{10}

(c) 9^5 (d) 10^9

Answer: Options (a)

If a person has 11 ways of going and cannot come from same place 10 ways of coming. $11 \times 10 = 110$

Question 4

The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent is___

(a) 2550 (b) 2880

(c) 625 (d) 2476

Answer: Options (b)

Explanation:

5 boys can sit around the circular table in (5–1)! = 4! Ways For boys and girls to occupy alternate positions, 5 girls have to sit in the gap between the 5 boys.

The girls can be arranged in these gaps in 5! ways

Therefore, total number of seating arrangements = 4! * 5! = 24 * 120 = 2880