Chapter 4
Determinants
Exercise 4.1

Evaluate the following determinants in Exercise 1 and 2.

Question 1

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Solution:

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ = 2(-1) $-$ 4 (-5)= 18

Question 2

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Solution:

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ = $\cos\theta$ x $\cos\theta$ $-$ ($\sin\theta$) x $\sin\theta$ =1

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ = (x^2 $-$ x +1)(x + 1) $-$ (x + 1)(x $-$ 1)= x^3 $-$x^2+2

Question 3

IF A = $\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$ then show that |2A|= 4|A|.

Solution:

A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

2A = $\begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

L.H.S. = |2A| = $\begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$ = 8 $-$32 = -24

R.H.S. = 4|A| = 4$\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$ = 4 (2 $-$ 8) = -24

LHS = RHS

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Question 4

IF A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that |3A| | = 27 |A|

Solution:

$3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

LHS:

$|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$

$= 3 \times 36 = 108$

RHS

$27\ |A| = 27 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$= 27(4) = 108$

LHS= RHS

Question 5

Evaluate the Determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Solution:

(i)

$\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 3\begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

= -15+3 −0 = -12

(ii)

$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

=3(7) + 4(5) + 5(1)

=46

(iii)

$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} -1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} +2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

=0 – 1(-6) + 2(-3-0)

= 0

(iv)

$$\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} -(-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} +(-2)\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$$

2(-5) + (0+5) – 2(0-6)

= 5

Question 6

If A $= \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$ find |A|.

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} -1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} +(-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

= 1 (-9 + 12) − (-18 + 15) − 2(8 - 5)

= 0

Question 7

Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Solution:

(i)

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

2 – 20 = 2x^2 −24

2x^2 = 3

Or x = $\pm\sqrt{3}$

(ii)

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

10 – 12 = 5x – 6x

X=2

Question 8

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then is equal to

(A) 6   (B) ± 6

(C) -6 (D) 0

Solution:

Option (B) is correct.
Explanation:
$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$
X^2 -36 = 36 – 32
X^2 = 36
X = ± 6

Exercise 4.2

Using the property of determinants and without expanding in Exercises 1 to 7, provethat.

Question 1

$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

Solution:

L.H.S.

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$$

Applying: $C_1 + C_2$

$$\begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix}$$

Elements of Column 1 and Column 2 are same. So determinant value is zero as per determinant properties.
=0
= RHS
Proved.

Question 2

$\begin{vmatrix} a-b & b-c & a-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Solution:

$$\begin{vmatrix} a-b & b-c & a-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Appling: $C_1 \rightarrow C_2 + C_2 + C_2$

$$\begin{vmatrix} 0 & b-c & a-b \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$$

= 0

All entries of first column are zero. (As per determinant properties)

Question 3

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Solution:

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

Applying: $C_3 \rightarrow C_3 - C_1$

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} = 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$

Elements of 2 columns are same, so determinant is zero.
=0
Proved.

Question 4

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Solution:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

Applying: $C_3 \rightarrow C_3 + C_2$

$$\begin{vmatrix} 1 & bc & ab+ab+ac \\ 1 & ca & ab+ab+ac \\ 1 & ab & ab+ab+ac \end{vmatrix}$$

(ab + ab + ac) is a common element in 3rd row.

$$= (ab + ab + ac)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

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4. 5

Two columns are identical, so determinant is zero.
=0

Question 5

Prove that
$$\begin{vmatrix} (b+c) & q+r & y+z \\ (c+a) & r+p & z+x \\ (a+b) & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Solution:

LHS:
Applying: $R_1 \rightarrow R_1 + R_2 + R_2$

$$\begin{vmatrix} b+c++c+a+a+b & q+r+r+p+p+q & y+z+z+x+x+y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} (a+b+c) & (p+q+r) & (x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

Applying:

$$R_1 \rightarrow R_1 - R_2$$

and $R_3 \rightarrow R_3 - R_1$

$$2\begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$$

Again, $R_2 \rightarrow R_2 - R_3$

$$2\begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$$

Interchanging rows, we have

$$2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

=RHS
Proved.

Question 6

Prove that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

Solution:

Let $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Taking (-1) common from all the 3 rows. Again, interchanging rows and columns, we have

$\Delta = - \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

$\Delta = - \Delta$

Which shows that, $2\Delta = 0$ or $\Delta = 0$. Proved.

Question 7

Prove that
$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

Solution:

LHS:

$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking a, b, c from row 1, row and row 3 respectively,

$= abc \begin{vmatrix} -a & a & a \\ d & -b & b \\ d & b & -c \end{vmatrix}$

$\qquad\qquad\qquad\qquad R_1 \to R_1 + R_2$

$= abc \begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

$= abc\,(2c) \begin{vmatrix} a & -b \\ a & b \end{vmatrix}$

$= 2abc^2\,(ab + ab)$

$= 4a^2b^2c^2$

=RHS
Proved.

By using properties of determinants, in Exercise 8 to 14, show that:

Question 8

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)\,(b-c)\,(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)\,(a+b+c)$

Solution:

(i)LHS:

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^{2-a^2} \\ 1 & c-a & c^{2-a^2} \end{vmatrix}$$

Expanding 1ˢᵗ column,

$$= 1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

Taking (b-a) common from first row,

$$= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$$

Simplifying above expression, we have

= (b-c)(c-a) (c-b)
= (a-b)(b-c)(c-a)
= RHS
Proved.

(ii) LHS

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-c & c-a \\ a^3 & b^3-a^3 & c^3 a^3 \end{vmatrix}$$

Expanding first row

$$= 1 \begin{vmatrix} b-c & c-a \\ (b-c)(b^2+a^2+ab) & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

$$= (b-c)(c-a) \begin{vmatrix} 1 & 1 \\ (b^2+a^2+ab) & (c^2+a^2+ac) \end{vmatrix}$$

$= (b-c)(c-a)(c^2+a^2+ac-b^2-a^2-ab)$
$= (b-a)(c-a)[(c-b)(c+b)+a(c-b)]$
$= (b-a)(b-c)(c-a)(a+b+c)$
= RHS
Proved,

Question 9

Prove that

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Solution:

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LHS

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$$

Multiplying R_1, R_2, R_3 by x, yz respectively

$$\begin{vmatrix} x & x^3 & xyz \\ y & y^3 & xyz \\ z & z^3 & xyz \end{vmatrix}$$

$$= \frac{xyz}{xyz} \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix} = \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1. \ R_2 \to R_3 - R_1$

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{vmatrix}$$

$$= 1 \begin{vmatrix} y^2 - x^2 & y^3 - x^3 \\ z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

$$= \begin{vmatrix} (y-x)(y+x) & (y-x)(y^2 + x^2 + yx) \\ (z-x)(z+x) & (z-x)(z^2 + x^2 + zx) \end{vmatrix}$$

$$= (y-x)\ (z-x) \begin{vmatrix} y+x & y^2 + x^2 + yx \\ z+x & z^2 + x^2 + zx \end{vmatrix}$$

$= (y-x)\ (z-x)\ [yz^2 + yx^2 + xyz + xz^2 + x^3 + x^2z - zy^2 - zx^2 - xyz - xy^2 - x^3x^2y\ ]$

$= (y-x)\ (z-x)\ [yz^2 - zy^2 + xz^2 - xy^2]$

$= (y-x)\ (z-x)[yz\ (z-y) + x\ (x^2 - y^2)]$

$= (y-x)\ (z-x)\ [yz\ (z-y) + x\ (z-y)(z+y)]$

$= (y-x)\ (z-x)\ (z-y)[yz + x\ (z+y)]$

$= (y-x)\ (z-x)(z-y)\ [yz + x(z+y)]$

$= (x-y)\ (y-z)\ (z-x)\ (xy + yz + zx)$

= RHS (Proved)

Question10

(i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)\ (4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = K^2(3y + k)$

Solution:

(i) LHS

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$[R_1 \rightarrow R_1 + R_2 + R_3]$$

$$= \begin{bmatrix} 5x + 4 & 5x + 4 & 5x + 4 \\ 2x & x + 4 & 2x \\ 2x & 2x & x + 4 \end{bmatrix}$$

$$= (5x+4) \begin{bmatrix} 1 & 1 & 1 \\ 2x & x + 4 & 2x \\ 2x & 2x & x + 4 \end{bmatrix}$$

$C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$= (5x+4) \begin{vmatrix} 1 & 0 & 0 \\ 2x & 4 - x & 0 \\ 2x & 0 & 4 - x \end{vmatrix}$$

$$= (5x+4).1 \begin{vmatrix} 4 - x & 0 \\ 0 & 4 - x \end{vmatrix}$$

$= (5x+4)(4-x)^2$

= RHS (Proved)

(ii)LHS

$$\begin{vmatrix} y + k & y & y \\ y & y + k & y \\ y & 2x & y + k \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 3y + k & y & y \\ 3y + k & y + k & y \\ 3y + k & y & y + k \end{vmatrix}$$

$$= (3y + k) \begin{vmatrix} 1 & y & y \\ 1 & y + k & y \\ 1 & y & y + k \end{vmatrix}$$

$C_2 \rightarrow C_2 - C_1$ and $C_2 \rightarrow C_3 \rightarrow C_3 - C_1$

$$= (3y+k) \begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

$$= (3y + k).1 \begin{vmatrix} l & 0 \\ 0 & k \end{vmatrix}$$

$= K^2(3y + k)$

= RHS (Proved)

Question 11

Prove that,

(i) $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a - b - c)^3$

(ii) $\begin{vmatrix} x + z + 2x & x & y \\ z & y + z + 2x & y \\ z & x & z + x + 2y \end{vmatrix} = 2(x + y + z)^2$

Solution:

LHS

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_2$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c)(1) \begin{vmatrix} -b-c-a & 0 \\ 0 & -c-a-b \end{vmatrix}$$

$= (a+b+c)[-(b+c+a)][-(c+a+b)]$

$= (a+b+c)^3$

$=$ RHS

(ii) LHS

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

Taking $2(x + y + z)$ common from first column. Then apply operations:

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)(1) \begin{vmatrix} x+y+z & 0 \\ 0 & x+y+z \end{vmatrix}$$

$= 2(x+y+z)[(x+y+z)^2 - 0]$

$= 2(x+y+z)\text{^}3$

$=$ RHS (Proved)

Question 12

Prove that.

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$$

Solution:

LHS

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (1+x+x^2) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2) \begin{vmatrix} 1-x^2 & x-x^2 \\ x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2) \begin{vmatrix} (1-x)(1+x) & x(1-x) \\ -x(1-x) & 1-x \end{vmatrix}$$

$$= (1+x+x^2)[(1-x)^2(1+x) + x^2(1-x)^2]$$

$= (1+x+x^2)^2(1-x)^2$

$= (1-x+x-x^2+x^2-x^3)^2$

$= (1-x^3)^2$

$= $ RHS

Proved.

Question 13

Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Solution:

LHS

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2b & 1-a^2-b^2 \end{vmatrix}$$

$C_1 \to C_1 - b\,C_3$ and $C_2 \to C_2 + aC_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2b^2 \end{vmatrix}$$

$R_3 \to R_3 - b\,R_1$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

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4. 12

$= (1+a^2 + b^2)^2 \begin{vmatrix} 1 & 2a \\ -a & 1-a^2+b^2 \end{vmatrix}$

$= (1+a^2+b^2)^2 (1-a^2+b^2+2a^2)$

$= (1+a^2+b^2)^3$

= RHS

Proved,

Question 14

Prove that.

$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

Solution:

LHS

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Multiply, C_1, C_2, C_3 by a, b, c respectively

Then divide the determinant by abc

$= \frac{1}{abc} \begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix}$

$= \frac{abc}{abc} \begin{vmatrix} a(a^{2+1}) & ab^2 & ac^2 \\ a^2b & b(b^{2+1}) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix}$

$= \frac{abc}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\frac{abc}{abc} \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$

$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$.

$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= (1+a^2+b^2+c^2)(1)(1-0)$

$= 1+a^2+b^2+c^2$

-RHS

(Proved)

Choose the correct answer in Exercise 15 and 16

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Question 15

Let A be a square matrix of order 3 x 3, then |KA| is equal to
(A) K | A | (B) K^2 |A| (C) K^3|A| (D) 3k |A|

Solution:

Option (C) is correct.

Question 16

Which of the following is correct?
(A) Determinant is a square matrix.
(B) Determinant is a number associated to a matrix.
(C) Determinant is a number associated to a square matrix.
(D) None of these

Solution:

Option (C) is correct.

Exercise 4.3

Question 1

Find area of the triangle with vertices at the point given in each of the following:
(i) (1,0), (6,0), (4,3)
(ii) (2,7), (1,1),(10, 8)
(iii) (-2, -3), (3,2), (-1, -8)

Solution:

Formula for Area of triangle:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(i)

$$\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$= \frac{1}{2}[1\,(0-3) - 0\,(6-4) + 1(18-0)]$
$= \frac{15}{2}$ sq. units
(ii)

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Area = $\frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)]$

$= \frac{47}{2}$ sq. units

(iii)

Area = $\frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$

$= \frac{1}{2}[-2(10) + 3(4) - 22]$

= 15 sq. units

Question 2

Show that point: A (a, b + c), B (b, c, + a), C (c, a + b) are collinear.

Solution:

Points are collinear if area of triangle is equal to zero.
i.e., Area of triangle = 0

Area of Triangle = $\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$= \frac{1}{2}[a(c+a-a-b) - (b+c)(b-c) + 1\{b(a+b) - c(c+a)\}]$

$\frac{1}{2}(ac - ab - b^2 + c^3 + ab + b^2 - c^2 - ac)$

=0

Therefore, points are collinear.

Question 3

Find values of K if area of triangle is 4 sq. units and vertices are
(i) $(K, 0), (4, 0), (0, 2)$
(ii) (-2, 0), (0, 4), (0, K)

Solution:

(i)
Area of triangle = ± 4 (Given)

$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$

$$\frac{1}{2}[k(0-2) - 0 + 1(8-0)] = 4$$
$$^1/_2(-2k + 4) = 4$$

-k + 4 = 4

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Now: -k + 4 = ± 4

-k +4 = 4 and –k + 4 =-4

K = 0 and k= 8

$$\frac{1}{2}\begin{vmatrix} x_1 & x_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$$

(ii)

$$\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$$

1/2 (-8+2K) = 4

Or $- k + 4 = \pm 4$

Now: -k + 4 and –k + 4 = -4

K = 0 and K =8

Question 4

(i) Find equation of line joining (1,2) and (3,6) using determinants.
(ii) Find equation of line joining (3,1) and (9,3) using determinants.

Solution:

Let A (x, y) be any vertex of a triangle.

All points are on one line if area of triangle is zero.

$$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$\frac{1}{2}[x(2-6) - y(1-3) + 1(6-6)] = 0$

-4x + 2y = 0

Y = 2x

Which is equation of line.

(ii) Let A(x, y) be any vertex of a triangle.

All points are on one line if area of triangle is zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$\frac{1}{2}[x(1-3) - y(3-9) + 1(9-9)] = 0$

-2x + 6y = 0

X – 3y = 0

Which is equation of line.

Question 5

If area of triangle is 35 sq. units with vertices (2,– 6), (5,4) and (k,4). Then K is
(A) 12   **(B) – 2**
(C)−12, −2   **(D) 12, −2**

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Solution:

Option (D) is correct.
Explanation:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 35$$

$\frac{1}{2}[2(4-4) - (-6)(5-k) + 1(20-4k)] = 35$

Solving above expression, we have

25 – 5k = 35 and 25 – 5k = -35

K =-2 and k = 12.

Exercise 4.4

Write Minors and Cofactors of the elements of following determinants:

Question 1

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Solution:

Find Minors of elements:
Say, M_{ij} is minor of element a_{ij}
M_{11} = Minor of element a_{11} = 3
M_{12} = Minor of element a_{12} = 0
M_{21} = Minor of element a_{21} = -4
M_{22} = Minor of element a_{22} = 2
Find cofactor of a_{ij}
Let cofactor of a_{ij} is A_{ij}, which is $(-1)^{i+j}M_{ij}$
A_{11} = $(-1)^{1+1}M_{11}$ = $(-1)^2$ (3) = 3
A_{12} = $(-1)^{1+2}M_{12}$ = $(-1)^3$ (0) = 0
A_{21} = $(-1)^{2+1}M_{21}$ = $(-1)^3$ (-4) = 4
A_{22} = $(-1)^{2+2}M_{22}$ = $(-1)^4$ (2) = 2

(ii)

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

Solution:

Find Minors of elements:
Say, M_{ij} is minor of element a_{ij}
M_{11} = Minor of element a_{11} = d
M_{12} = Minor of element a_{12} = b

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M_{21} = Minor of element a_{21} = c
M_{22} = Minor of element a_{22} = a
Find cofactor of a_{ij}
Let cofactor of a_{ij} is A_{ij}. Which is $(-1)^{i+j}M_{ij}$
$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 (d) = d$
$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 (b) = -b$
$A_{21} = (-1)^{2+1}M_{21} = (-1)^3 (c) = -c$
$A_{22} = (-1)^{2+2}M_{22} = (-1)^4 (a) = a$

Question 2

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

Solution:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Find Minor and cofactors of elements:
Say, M_{ij} is minor of element a_{ij} and A_{ij} is cofactor of a_{ij}

M_{11} = Minor of element a_{11} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{11} = 1

M_{12} = Minor of element a_{12} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{12} = 0

M_{13} = Minor of element a_{13} = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{13} = 0

M_{21} = Minor of element a_{21} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{21} = 0

M_{22} = Minor of element a_{22} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{22} = 0

M_{23} = Minor of element a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{23} = 0

M_{31} = Minor of element a_{31} = $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{31} = 0

M_{32} = Minor of element a_{32} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{32} = 0

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

M_{33} = Minor of element a_{33} =   and A_{33} = 1

(ii)
$$\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$$

Find Minors and cofactors of elements:

Say, M_{ij} is minor of element a_{ij} and A_{ij} is cofactor of a_{ij}

$$\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 - (-1) = 11$$

M_{11} = Minor of element a_{11} =   and A_{11} = 11

$$\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$$

M_{12} = Minor of element a_{12} =   and A_{12} = -6

$$\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

M_{13} = Minor of element a_{13} =   and A_{13} = 3

$$\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$$

M_{21} = Minor of element a_{21} =   and A_{21} = 4

$$\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

M_{22} = Minor of element a_{22} =   and A_{22} = 2

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

M_{23} = Minor of element a_{23} =   and A_{23} = -1

$$\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$$

M_{31} = Minor of element a_{31} =   and A_{12} = -20

$$\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 2 = -13$$

M_{32} = Minor of element a_{32} =   and A_{32} = 13

$$\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$$

M_{33} = Minor of element a_{33} =   and A_{33} = 5

Question 3

Using Cofactors of elements of second row, evaluate Δ.

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Solution:

Find Cofactors of elements of second row:

$$(-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix}$$

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A_{21} = Cofactor of element a_{21} = $-(-1)^2 (9-16) = 7$

A_{22} = Cofactor of element a_{22} = $(-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix}$ $-(-1)^4 (15-8) = 7$

A_{23} = Cofactor of element a_{23} = $(-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$ $-(-1)^5 (10-3) = 7$

Now, $\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 14 + 0 - 7 = 7$

Question 4

Using Cofactors of elements of third column, evaluate Δ

$$\Delta = \begin{bmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{bmatrix}$$

Solution:

Find Cofactors of elements of third column:

A_{13}= Cofactor of element a_{13} = $(-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix}$ = $(-1)^4 (z-y) = Z$

A_{23}= Cofactor of element a_{23} = $(-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix}$ $-(-1)^5 (z-x) = x-z$

A_{33} = Cofactor of element a_{33} = $(-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix}$ $-(-1)^6 (y-x) = y-x$

Now, $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz\ (z-y) + zx\ (x-z) + xy\ (y-x)$
$= (yz^2 - y^2z) + (xy^2 - xz^2) + (xz^2 - x^2z)$
$= (y-z)\,[-yz + x\,(y+z) - x^2]$
$= (y-z)\,[-y\,(z-x) + x\,(z-x)]$
$= (x-y)\,(y-x)\,(z-x)$

Question 5

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ **and** A_{ij} **is cofactor of** a_{ij} **then value of** Δ **is given by:**

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Solution:

Option (D) is correct.

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Exercise 4.5

Find adjoin of the matrices in Exercises in 1 and 2.

Question 1

1.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Solution:

1. Let A =

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Cofactors of the above matrix are
$A_{11} = 4$
$A_{12} = -3$
$A_{21} = -2$
$A_{22} = 1$
adj. $A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2.

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Cofactors of the above matrix are

$A_{11} = +\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$ $\quad A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = 1$ $\quad A_{31} = +\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -11$

$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -12$ $\quad A_{22} = +\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5$ $\quad A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -1$

$A_{13} = +\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = +$ $\quad A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 2$ $\quad A_{33} = +\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5$

Therefore,

Adj. $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

Verify A (adj A) = (adj A) A = |A| I in Exercises 3 and 4

Question 3

Let A $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ $\qquad$ 4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Solution:

3.

Let A = $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Adj. A = $\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

A. (adj. A) = $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Again, |A| = $\begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix}$ = −12+12 = 0

|A| I = (0) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

LHS =RHS
Verified.

4.

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Cofactors of A,

A_{11} =+ $\begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix}$= 0 A_{21} = − $\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix}$ = 3 A_{31} = + $\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix}$ = 2

A_{12} =− $\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix}$= −11 A_{22} = + $\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$ = 1 A_{32} = + $\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$ = 8

A_{13} =+ $\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}$= 0 A_{23} = − $\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$ = -1 A_{33} = + $\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix}$ = 3

Now,

Adj. A = = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

Now, Verify A (adj A) A = |A| I

A (adj. A) = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 0 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

|A| = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ = 1(0) − (-1)(11) + 2(0) = 11

(adj. A) A = $\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

|A|I = 11$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Verified.

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

Question 5

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$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

|A| = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ = 14 ≠ 0

Since determinant of the matrix is not zero, so inverse of this matrix is possible.

As we know, formula to find inverse is:

$A^{-1} = \frac{1}{|A|}$adj.A

Adj.A = $\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

This implies,

$A^{-1} = \frac{1}{14}\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Question 6

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

|A|= $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$=13 ≠ 0

Since determinant of the matrix is not zero, so inverse of this matrix is possible.

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

Adj. A = $\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

This implies,

$A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Question 7

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$|A| = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ = 1(10) - 2 (0) + 3 (0) = 10 $\neq$ 0

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = + \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10$ $A_{21} = - \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10$ $A_{31} = + \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2$

$A_{12} = - \begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = 0$ $A_{22} = + \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5$ $A_{32} = + \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4$

$A_{13} = + \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$ $A_{23} = - \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0$ $A_{33} = + \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2$

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

$A^{-1} = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

Question 8

$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Solution:

Let A $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$|A| = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ = 1(−3) − 0 + 0 = −3 $\neq$ 0

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = + \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3$ $A_{21} = - \begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -0$ $A_{31} = + \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{12} = - \begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = 3$ $A_{22} = + \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5$ $A_{32} = + \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{13} = + \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = -9$ $A_{23} = - \begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} = -2$ $A_{33} = + \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3$

Adj. A = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

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$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Question 9

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

$|A| = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

$= 2(-1)-1) + (4) + 3(1) = -3$

$\neq 0$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = +\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1$ $A_{21} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5$ $A_{31} = +\begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = 3$ $A_{22} = +\begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = 23$ $A_{32} = +\begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 12$

$A_{13} = +\begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = -1$ $A_{23} = -\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -11$ $A_{33} = +\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$

Adj, $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ -1 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

Question 10

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

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$$|A| = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$= 1(2) + 1(9) + 2(-6) = -1$$

$\neq 0$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = + \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 2$ $\qquad A_{21} = - \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = 0$ $\qquad A_{31} = + \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = -1$

$A_{12} = - \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -9$ $\qquad A_{22} = + \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2$ $\qquad A_{32} = - \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 3$

$A_{13} = + \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = -6$ $\qquad A_{23} = - \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -1$ $\qquad A_{33} = + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$

$$\text{Adj.A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Question 11

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,a & sin\,a \\ 0 & sin\,a & -cos\,a \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,a & sin\,a \\ 0 & sin\,a & -cos\,a \end{bmatrix}$

$|A| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,a & sin\,a \\ 0 & sin\,a & -cos\,a \end{bmatrix}$

$= (cos^2 a - sin^2 a) - 0 + 0$

$= -(cos^2 a + sin^2 a) = -1 \neq 0$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = + \begin{vmatrix} cos\,a & sin\,a \\ sin\,a & -cos\,a \end{vmatrix} = -1$ $\quad A_{21} = - \begin{vmatrix} 0 & 0 \\ sin\,a & -cos\,a \end{vmatrix} = 0$ $\quad A_{31} = + \begin{vmatrix} 0 & 0 \\ cos\,a & sin\,a \end{vmatrix} = 0$

$A_{12} = - \begin{vmatrix} 0 & sin\,a \\ 0 & -cos\,a \end{vmatrix} = 0$ $\quad A_{22} = + \begin{vmatrix} 1 & 0 \\ 0 & -cos\,a \end{vmatrix} = -cos\,a$ $\quad A_{32} = - \begin{vmatrix} 0 & 0 \\ 0 & sin\,a \end{vmatrix} = -sin\,a$

$A_{13} = + \begin{vmatrix} 0 & cos\,a \\ 0 & -sin\,a \end{vmatrix} = 0$ $\quad A_{23} = + \begin{vmatrix} 1 & 0 \\ 0 & sin\,a \end{vmatrix} = sin\,a$ $\quad A_{33} = + \begin{vmatrix} 1 & 0 \\ 0 & cos\,a \end{vmatrix} = cos\,a$

adj. A = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & -\cos a \end{bmatrix}$

As we know, formula to find matrix inverse is:

$A^{-1} = \frac{1}{|A|}$adj. A

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$

Question 12

Let A = $\begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Solution:

A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 1 \neq 0$

$A^{-1} = \frac{1}{|A|}$adj. A

$A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Again,

B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$|B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = -2 \neq 0$

$B^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

Now Multiply A and B,

AB $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

Find determinants of AB:

$|AB| = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix} = 4087 - 4089 = -2 \neq 0$

Now, verify $(AB)^{-1} = B^{-1}A^{-1}$

LHS:

$|AB| = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix} = 4087 - 4089 = -2 \neq 0$

And

$(AB)^{-1} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

RHS:

$B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

This implies, LHS = RHS (Verified)

Question 13

If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}

Solution:

$A^2 + AA$
$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$
LHS $= A^2 - 5A + 7I$
$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix}$
$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$
= RHS. (Proved)
To Find A^1
Multiply $A^2 - 5A + 7I$ A^1 , we have
(Consider I is 2x2 matrix)
$A^2 A^{-1} - 5A. A^{-1} + 7I A^{-1} = 0. A^{-1}$
$A - 5I + 7A^{-1} = 0$
$7A^{-1} = -A + 5I$
$\quad\quad = \begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
$\quad\quad = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$
$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Question 14

For the Matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Solution:

$A^2 = AA = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$
Since $A^2 + aA + bI = O$
$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} = a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$
$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
$\begin{bmatrix} 11 + 3a + b & 8 + 2a + 0 \\ 4 + a + 0 & 3 + a + b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Equate corresponding elements, we get
$11 + 3a + b = 0$ ... (1)
$8 + 2a = 0 \Rightarrow 4$
Substitution the value of a in equation (1),

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11 + 3 (-4) + b = 0
11 – 12 + b = 0
B = 1.

Question 15

For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Solution

$A^2 = AA = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+1+ & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1+6+9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$A^3 = A^2A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

Now, LHS = $A^3 - 6A^2 + 5A + 11\,|$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$= \begin{bmatrix} 8-24+5+11 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10+11 & -69+84-15 \\ 32-42+10 & -13+18\pm5 & 58-84+15+11 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= 0$

RHS (Proved)

Now, find $\mathbf{A^{-1}}$

Multiply $A^3 - 6A^2 + 11|$ by A^{-1}, we have

(Consider | is 3x3 matrix)

$$A^3 A^{-1} - 6A^2 A^{-1} + 5AA^{-1} + 11I\,A^{-1} = 0A^{-1}$$
$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$11\,A^{-1} = 6A - 5I - A^2$

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$11\,A^{-1} = 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} -5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} -\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$= \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

Therefore,

$A^{-1} = \dfrac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

Question 16

If A=$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4| = 0$ and hence find A^{-1}.

Solution:

$$A^2 = AA$$
$$\begin{bmatrix} 4+1+1 & -2-2-1 & 1+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+1 \end{bmatrix}$$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

Again, $A^3 = A^2 A$

$$\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

Now, $A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} -6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} +9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} -4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22+36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} +\begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$

$= \begin{bmatrix} -14+14 & 9-9 & -9+9 \\ 9-9 & -14+14 & 9-9 \\ -9+9 & 9-9 & -14+14 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

= 0 (RHS)

Multiply $A^3 - 6A^2 + 9A - 4l = 0$ by A^{-1}, (here I is 3x3 matrix)

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$A^3 A^{-1} - 6A^2 A^{-1} + 9A - 4LA^{-1} = 0 \ A^{-1}$

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$4A^{-1} = A^2 - 6a + 9I$$

Now Placing all the matrices,

$$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Inverse of the matrix is:

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

<u>Question 17</u>

Let A be a non-singular matrix of order 3 x 3. Then |adj. A| is equal to:
(A) |A| **(B) |A|^2**
(C) |A|^3   **(D) 03|A|**

Solution:

Option (B) is correct.
Explanation:
$|adj. \ A| = |A|^{n-1} = |A|^2$ (for n = 3)

<u>Question 18</u>

If A is an invertible matrix of order 2, then det (A^{-1}) is equal to:
(A) det A   **(B) 1/det A**
(C) 1   **(D) 0**

Solution:

Option (B) is correct.
Explanation:
$A \ A^{-1} = 1$
Det $(AA^{-1} = l)$
Det (A) det $(A^{-1}) = 1$
Det $(A^{-1}) = 1/$ det A

<u>Exercise 4.6</u>

Examine the consistency of the system of equations in Exercises 1 to 6.

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Question 1

x +2y = 2: and 2x + 3y = 3

Solution:

Given set equations is: x +2y = 2: and 2x + 3y = 3
This set of equation can be written in the form of matrix as AX = B, where
A = $\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$ and B = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and
X = $\begin{bmatrix} x \\ y \end{bmatrix}$
So, AX = B is
$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
|A| = $\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0$
Inverse of matrix exists. So, system of equation is consistent.
Question 2

2x – y = 5 and x + y = 4

Solution:

Given set of equations is: 2x – y = 5 and x + y = 4
This set of equation can be written in the form of matrix as AX = B, where
A = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$
So, AX = B is
$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$
|A| = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = 3 \neq 0$

Inverse of matrix exists. So, system of equation is consistent.

Question 3

x + 3y and 2x + 6y = 8

Solution:

Given set of equations is: x+ 3y and 2x + 6y = 8
This set of equation can be written in the form of matrix as AX = B
$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$
Where,
A = $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$

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$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$

Adj A $= \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

And

$(AdjA) B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$

The given educations are inconsistent.

Question 4

X + y z =1; 2x + 3y + 2z = 2 and ax + ay +2az =4

Solution:

Given set of educations is: x + y + z 1; 2x + 3y + 2z = 2 and ax +ay +2az = 4
The given education can be written in the form of matrix as AX = B

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix} = 1(6a-2a)-1(4a-2a)+1(2a-3a)$

$= 4a - 2a - a = a \neq 0$
System of equation is consistent.

Question 5

3x – y – 2z = 2; 2y – z -1 and 3x – 5y = 3

Solution:

Given set of equations is: 3x – y − 2z − 2; 2y – z = -1 and 3x – 5y = 3
This set of equation can be written in the form of matrix as AX = B

$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 3(-5) +(3) -2(-6)$
$= 15 – 15$
$= 0$
Now.

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(adj. A) = $\begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$

(adj. A) B = $\begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$

Question 6

Given set of equations is:
5x – y + 4z = 5
2x + 3y + 5z = 2
5x – 2y + 6z = −1

Solution:

Given set equation is: 5x – y + 4z = 5; 2x + 3y + 5z = 2; 5x – 2y + 6z = −1
This set of equations can be written in the form of matrix as AX = B

$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

A = $\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$

|A| = $\begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$

= 5(18 + 10) + 1(12-25) + 4(-4 – 15)
= 140 – 13 – 76
= 140-89
= 51
≠ 0
System of equations is consistent.

Solve system of linear equations, using matrix method, in Exercises 7 to 14.

Question 7

5x + 2y = 4 and 7x + 3y = 5

Solution:

Given set of equations is: 5x + 2y =4 and 7x + 3y = 5
This set of equation can be written in the form of matrix as AX = B

$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Where

A = $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

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And |A| = 1 ≠ 0
System is consistent.
Now,
X = $A^{-1}B = \frac{1}{|A|}$ (adj. A) B

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

=>X = 2 and y = -3

Question 8

2x – y = -2 and 3x + 4y = 3

Solution:

Given set of equations is: 2x – y = -2 and 3x + 4y = 3
This set of equation can be written in the form of matrix as AX = B

$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

|A| = 11 ≠ 0
System is consistent,
So,
X = $A^{-1}B = \frac{1}{|A|}$ (adj. A) B

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix}$

Therefore, x = -5/11 and y = 12/11

Question 9

4x – 3y = 3 and 3x – 5y = 7

Solution:

Given set of equations is: 4x – 3y = 3 and 3x – 5y = 7
This set of equation can be written in the form of matrix as AX = B

$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Where;

$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$

And |A| = -20 + 9 = -11 ≠ 0
System is consistent.
So,

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

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$$= \frac{1}{-11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 6 \\ 19 \end{bmatrix}$$

Therefore, x = 6/-11 and y = 19/-11

Question 10

5x + 2y = 3 and 3x + 2y = 5

Solution:

Given set of equations is: 5x + 2y = 3 and 3x + 2y = 5
This set of equation can be written in the form of matrix as AX = B
$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$
Where
$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$$
And |A| = 4 ≠ 0
System is consistent.
So,
$$X = A^{-1}B = \frac{1}{|A|} (adj.A)\, B$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$
Therefore, x = -1 and y 4

Question 11

$2x + y + z = 1$ and $x - 2y\, z = \dfrac{3}{2}$ and 3y – 5z = 9

Solution:

Given setoff equations is: 11. $2x + y + z = 1$ and $x - 2y\, z = \dfrac{3}{2}$ and 3y – 5z = 9
This set of equation can be written in the form of matrix as AX = B
$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$
Where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

And
$$|A| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$
= 34 ≠ 0
System is consistent.
So,

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$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A) \ B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

$= \frac{1}{34} \begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 15 \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$

Therefore' $x = 1$' $y = 1/2$ and z= $3/2$

Question 12

$x - y + z = 4 \ and \ 2x + y - 3z = 0$ and x + y + z =2

Solution:

Given set of equations is: x – y + z = 4 and $2x + y - 3z = 0 \ and \ x + y + z = 2$
This set of equation can be written in the form of matrix as AX = B

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Where

$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

And

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1 \ (1 + 3) + 1 \ (2 - 1) = \ 1 \ (4) \ + 1 \ (5) \ + 1 \ (1) \ 4 \ + 5 + 1$

$= 10 \neq 0$

System is consistent.
So,

$X = A^{-1}B = \begin{bmatrix} 1 \\ |A| \end{bmatrix} (\text{adj. } A) \ B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$= \frac{1}{10} \begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 - 0 + 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Therefore, x = 2, y = -1 and z = 1

Question 13

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

Solution:

Given set of equations is:
$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

And,

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2 (4 + 1) - 3 (-2 - 3) + 3 (-1 + 6) = 2 (5) + 3 (5) = 10 + 15 + 15$$

$= 40 \neq 0$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A) B$$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z = -1.

<u>Question 14</u>

X – y + 2z = 7
3x + 4y – 5z = – 5
2x – y +3z = 12

Solution:

Given set of equations is:
X – y + 2z – 7
3x + 4y – 5z = – 5
2x – y + 3z = 12

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$$

And

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$$|A| = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} = 1\,(12-5) + 1\,(9+10) + 2\,(-3-8) = 1\,(7) + 1\,(19) + 2\,(-11) = 7 + 19 - 22$$

$= 4 \neq 0$

System is consistent

So,

$X = A^{-1}B = \frac{1}{|A|}$ (adj A) B

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, X = 2, y = 1 and z = 3.

Question 15

If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, **find** A^{-1}.**Using** A^{-1} **solve the system of equation.**

2x – 3y + 5z = 11

3x + 2y – 4z = −5

X + y − 2z = −3

Solution:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = 2\,(-4+4) + 3\,(-6+4) + 5\,(3-2) = 2\,(0) + 3\,(-2) + 5\,(1) = -6 + 5$$

$= -1 \neq 0$: inverse of matrix exists,

Find the inverse of matrix:

Cofactors of matrix:

$$A_{11} = 0, A_{12} = 2, A_{13} = 1$$
$$A_{21} = -1, A_{22} = -9, A_{23} = -5$$
$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

Adj. A = $\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

So,

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Now, matrix of equation can be written as:

AX = B

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$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

And, $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z =3.

Question 16

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4 kg wheat and 2 kg rice are Rs. 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice are Rs. 70. Find cost of each item per kg by matrix method.

Solution:

Let x, y and z be the per kg. prices of onion, wheat and rice respectively.
According to given statement, we have following equations,
4x + 3y + 2z = 60
2x + 4y + 6z = 90
6x + 2y + 3z = 70
The above system of equations can be written in the form of matrix as, AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

And

$$|A| = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

= 4 (0) – 3(-30) + 2 (-20)
= 50 ≠ 0
System is consistent, and $X = A^{-1} B$
First find invers of A.
Cofactors of all the elements of A are:

$$A_{11} = 0, A_{12} = 30, A_{13} = -20$$
$$A_{21} = -5, A_{22} = 0, A_{23} = 10$$
$$A_{31} = 10, A_{32} = -20, A_{33} = 10$$

$$\text{Adj. A} = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Again,

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Therefore, x = 5, y = 8 and z = 8,

The cost of onion, wheat and rice per kg are Rs. 5, Rs. 8, respectively.

Miscellaneous Examples

Question 1

Prove that the determinant $\begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$ **is independent of θ.**

Solution:

Let $\Delta = \begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$

$\Delta = x \begin{bmatrix} -x & 1 \\ 1 & x \end{bmatrix} - \sin\theta \begin{bmatrix} \sin\theta & 1 \\ \cos\theta & x \end{bmatrix} + \cos\theta \begin{bmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{bmatrix}$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= -x^3$

Which is independent of θ (Proved)

Question 2

Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & B^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Solution:

Start with LHS:

$$\begin{vmatrix} a & a^2 & bc \\ b & B^2 & ca \\ c & c^2 & ab \end{vmatrix}$$

Multiplying R1 by a R2 by b and R3 by c, we have

$$\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$$

Taking out common elements

$$\frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Interchanging C_1 and C_2

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & a^3 & a^2 \\ 1 & b^3 & b^2 \\ 1 & c^3 & c^2 \end{vmatrix}$$

Interchanging C_2 and C_3

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

= RHS (proved)

Question 3

Evaluate

$$\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Solution:

$$\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

= $\cos\alpha\cos\beta(\cos\alpha\cos\beta - 0) - \cos\alpha\sin\beta(-\cos\alpha\sin\beta - 0) - \sin\alpha(-\sin\alpha\sin^2\beta - \sin\alpha\cos^2\beta)$

= $\cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha(\sin^2\beta + \cos^2\beta$

= $\cos^2\alpha + \sin^2\alpha = 1$

Question 4

If a, b and c are real numbers, and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either a + b + c = 0 or a = b = c

Solution:

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying: $R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+c & b+c & c+a \end{vmatrix}$$

Since Δ = 0

This implies,

Either 2(a + b + c) = 0 or

$$\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+c & b+c & c+a \end{vmatrix} = 0$$

Case 1: If 2(a + b + c) = 0

Then (a + b + c) = 0

Case 2:

$$\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+c & b+c & c+a \end{vmatrix} = 0$$

Applying: $C_2 \rightarrow C_2 - C_3 \rightarrow C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ c+a & a+b-c-a & b+c-c-a \\ a+b & b+c-a-b & c+a-a-b \end{vmatrix} = 0$$

$$\begin{vmatrix} b-c & b-a \\ c-a & c-b \end{vmatrix} = 0$$

Or

=>$(b-c)(c-b) - (b-a)(c-a) = 0$

=>$bc - b^2 - c^2 + bc - bc + ab + ac - a^2 = 0$

=>$- a^2 - b^2 - c^2 + ab + bc + ca = 0$

=>$a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca = 0$

=> $(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ca) = 0$

=>$((a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

Above expression only possible, if (a – b) = 0 and (c- a) = 0

That is a = b and b = c and c = a

Therefore, we have result, either a+b+c = 0 or a=b=c.

Question 5

Solve the equation

$$\Delta = \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$$

Solution:

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

Applying;

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$(3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+1 & x \\ x & x & x+a \end{vmatrix} = 0$$

Case 1: Either 3x + a = 0

Then x = -a/3

Case 2: or

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x+1 & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying:

$$C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

a^2 = 0

Or a = 0

Not possible, as we are given a ≠ 0.

So, x = -a/3 is only the solution.

Question 6

Prove that

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Solution:

LHS:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} a & c & (a+c) \\ (a+b) & b & a \\ b & (b+c) & c \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$= abc\begin{vmatrix} a-a-b-b & c-b-b-c & a+c-a-c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_1$$

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$$= abc \begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = abc \begin{vmatrix} -2b & 0 & 0 \\ a+b & -a & a \\ b & c & c \end{vmatrix}$$

$$= abc \, (-2b)(-ac-ac) = 4a^2b^2c^2$$

RHS (Proved)

Question 7

If $A^{-1} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$ find $(AB)^{-1}$

Solution:

As we know, $(AB)^{-1} = B^{-1}A^{-1}$ ... (1)

First find inverse of matrix B.

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$= 1(3-0) - 2(-1-0) + (-2)(2-0) = 1 \neq 0$ (Inverse of B is possible)

Find cofactors of B:

$B_{11} = 3, B_{12} = 1, B_{13} = 2$

$B_{21} = 2, B_{22} = 1, B_{23} = 2$ and

$b_{31} = 6, B_{32} = 2, B_{33} = 5$

So, adj. of B is

$$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$B^{-1} = \frac{1}{|B|}(\text{adj. } B) = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

From equation (1),

$$(AB)^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Question 8

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$, verify that

(i) $(\text{adj.} A)^{-1} = \text{adj. }(A^{-1})$

(ii) $(A^{-1})^{-1} = A$

Solution:

$|A| = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

= -13 ≠ 0 (Inverse of A exists)

Cofactors of A are:

$A_{11} = 14, A_{12} = 11, A_{13} = -5$

$A_{21} = 11, A_{22} = 4, A_{23} = -3$

$A_{31} = 5, A_{32} = -3, A_{33} = -1$

So, adjoint of A is

$$\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -5 \\ -5 & -3 & -1 \end{bmatrix}$$

Now, A^{-1}

$$\frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -5 \\ -5 & -3 & -1 \end{bmatrix}$$

Again,

$|B| = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -5 \\ -5 & -3 & -1 \end{bmatrix}$

= 169 ≠ 0 (inverse of A exists)

Cofactors of B are:

$B_{11} = -13, B_{12} = 26, B_{13} = -13$

$B_{21} = 26, B_{22} = -39, B_{23} = -13$

$B_{31} = -13, B_{32} = -13, B_{33} = -65$

Therefore, inverse of B is

$$\frac{-1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

Find: $adjA^{-1}$

$A^{-1} = \frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$|A^{-1}| = -1/13 \neq 0$ (After solving the determinant we get the value. Try at your own)

Inverse of A^{-1} exists.

Let say cofactors of A^{-1} are represented as C_{ij}, we have

$C_{11} = \frac{-1}{13}, C_{12} = \frac{2}{13}, C_{13} = \frac{-1}{13}$

$C_{21} = \frac{2}{13}, C_{22} = \frac{-3}{13}, C_{23} = \frac{-1}{13}$ and

$C_{31} = \frac{-1}{13}, C_{32} = \frac{-1}{13}, C_{33} = \frac{-5}{13}$

Therefore:

$(A^{-1})^{-1} = c^{-1} = \frac{1}{|C|}(adj.\ C)$

Which implies,

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

= A

Which is again given matrix A.

(i) $(adj. A)^{-1} = adj. (A^{-1})$

$$\frac{-1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \frac{-1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

(ii) $(A^{-1})^{-1} = A$

$$\frac{-1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \frac{-1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

Question 9

Evaluate

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Solution:

Consider,

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Operation: $[R_1 \rightarrow R_1 + R_2 + R_3]$

$$\begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Taking $2(x + y)$ common from first now

$$2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$[C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$

Operation:

$$2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x+y-y & x-y \\ x+y & x-x-y & y-x-y \end{vmatrix}$$

$$2(x+y).1\begin{vmatrix} x & x-y \\ -y & -x \end{vmatrix}$$

$2(x+y)\{-x^2 + (x-y)\}$

-2(x^3 + y^3)

Δ = -2(x^3 + y^3)

Question 10

Evaluate

$$\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

Solution:

Consider

$$\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

$[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$

Operation:

$$\begin{vmatrix} 1 & x & y \\ 0 & x+y & 0 \\ 0 & 0 & x+y-y \end{vmatrix}$$

$$\begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$$

=xy

$\Delta = xy$

Using properties of determinants in Exercises 11 to 15, that:

Question 11

$$\begin{vmatrix} a & a^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

Solution:

LHS

$$\begin{vmatrix} a & a^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$[C_3 \rightarrow C_3 + C_1]$

Operation:

$$\begin{vmatrix} a & a^2 & a+\beta+\gamma \\ \beta & \beta^2 & a+\beta+\gamma \\ \gamma & \gamma^2 & a+\beta+y \end{vmatrix}$$

$$(a+\beta+\gamma)\begin{vmatrix} a & a^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$$

$[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$

Operation:

$$(a+\beta+\gamma)\begin{vmatrix} a & a^2 & 1 \\ \beta & \beta^2-a^2 & 0 \\ \gamma & \gamma^2-a^2 & 0 \end{vmatrix}$$

$$= (a+\beta+\gamma)\begin{vmatrix} \beta-a & (\beta-a)(\beta+a) \\ \gamma-a & (y-a)(y+a) \end{vmatrix}$$

$$=(a + \beta + \gamma)(\beta - a)(\gamma - a)\begin{vmatrix} 1 & (\beta + a) \\ 1 & (y + a) \end{vmatrix}$$

$$= (a + \beta + \gamma)(\beta - a)(\gamma - a)(\gamma + a - \beta - a)$$

$$= (a + \beta + \gamma)[-(a - \beta)](\gamma - \alpha)[-(\beta - \gamma)]$$

$$= (a + \beta + y)(a - \beta)(\beta - y)(\gamma - a)$$

= RHS

Question 12

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

Solution:

LHS=

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix}$$

=

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix}$$

We have two determinants; say Δ 1 and Δ 2

= Δ 1 + Δ 2

$$\Delta_1 = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$$[R_2 \to R_2 - R_1 \, and \, R_3 \to R_3 - R_1]$$

Operation:

$$= \begin{vmatrix} x & x^2 & 1 \\ y - x & y^2 - x^2 & 0 \\ z - x & z^2 - x^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} y - x & (y - x)(y + x) \\ z - x & (z - x)(z + x) \end{vmatrix}$$

$$= (y - x)(z - x)\begin{vmatrix} 1 & y + x \\ 1 & z + x \end{vmatrix}$$

$$= (y - x)(z - x)(z + x - y - x)$$

$$= (x - y)(y - z)(z - x)$$

Again:

$$\Delta_2 = \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix}$$

$$pxyz\Delta_2 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= -pxyz \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$$

$$= pxyz \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$= pxyz\Delta_1$

Therefore: $\Delta_1 + \Delta_2$

LHS

$$= (y - x)(z - x)(z - y) + pxyz\,(y - x)\,(z - x)(z - y)$$
$$= (1 + pxyz)\,(y - x)\,(z - x)(z - y)$$

= RHS

(Proved)

13. Prove that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3\,(a+b+c)\,(ab + bc + ca)$$

Solution:

LHS

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$[C_1 \rightarrow C_1 + C_2 + C_3]$

Operation:

$$\begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

$[R_2 \rightarrow R_2 - R_1 \, and \, R_3 \rightarrow R_3 - R_1]$

Operation:

$$(a+b+c).1 \begin{vmatrix} 2b+a & a-b \\ a-c & 2c+a \end{vmatrix}$$
$$= (a+b+c)\,[(2b+a)(2c+a) - (a-b)(a-c)]$$
$$= (a+b+c)\,[4bc + 2ab + a^2 - a^2 + ac + ab - bc]$$
$$= 3(a+b+c)(ab + bc + ac)$$

= RHS

Hence Proved.

Question 14

Prove that

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Solution:

LHS

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$$

$[R_2 \rightarrow R_2 - 2R_1 \ and \ R_3 \rightarrow R_3 - 3R_1]$

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+q \\ 0 & 3 & 7+3p \end{vmatrix}$$

$1\begin{vmatrix} 1 & 2+p \\ 3 & 7+3p \end{vmatrix} = 0 + 0$

$= 7 + 3p - 6 - 3p$

$= 1$

= RHS

Hence Proved.

Question 15

Prove that

$$\begin{vmatrix} \sin a & \cos a & \cos(a+\delta) \\ \sin \beta & \cos \beta & \cos(\beta+\delta) \\ \sin \gamma & \cos \gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$$

Solution:

LHS

$$\begin{vmatrix} sina & cosa & cos(a+\delta) \\ sin\beta & cos\beta & cos(\beta+\delta) \\ sin\gamma & cos\gamma & cos(\gamma+\delta) \end{vmatrix}$$

$$= \begin{vmatrix} sina & cosa & cosacos\delta - sinasin\delta \\ sin\beta & cos\beta & cos\beta cos\delta - sin\beta sin\delta \\ sin\gamma & cos\gamma & cos\gamma cos\delta - sin\gamma sin\delta \end{vmatrix}$$

$[C_3 \rightarrow C_3 + (sin\delta)C_1]$

Operation:

$$= \begin{vmatrix} sin\alpha & cosa & cosacos\delta - sinasin\delta + sinasin\delta \\ sin\beta & cos\beta & cos\beta cos\delta - sin\beta sin\delta + sin\beta sin\delta \\ sin\gamma & cos\gamma & cos\gamma cos\delta - sin\gamma sin\delta + sin\gamma sin\delta \end{vmatrix}$$

$$= \begin{vmatrix} sina & cosa & cosacos\delta \\ sin\beta & cos\beta & cos\beta cos\delta \\ sin\gamma & cos\gamma & cos\gamma cos\delta \end{vmatrix}$$

$$= \cos\delta \begin{vmatrix} sina & cosa & \cos a \\ sin\beta & cos\beta & \cos \beta \\ sin\gamma & cos\gamma & \cos \gamma \end{vmatrix}$$

Column 2 and column 3 are identical, as per determinant property, value is zero.

= 0

= RHS

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Question 16

Solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$: $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

Solution:

Let

$$\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w.$$

We have

$$2u + 3y + 10w = 4:$$
$$4u - 6v + 5w = 1: and$$
$$6u + 9v - 20w = 2$$

Below is the matrix from the given equations: AX = B

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Let say

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$$

Then,

|A| = 1200 ≠ 0

A^{-1} exists.

Cofactors of A are:

$$A_{11} = 75, A_{12} = 110. A_{13} = 72$$
$$A_{21} = 150, A_{22} = -100, A_{23} = 0 \text{ and}$$
$$A_{31} = 75, A_{32} = 30, A_{33} = -24$$

adj. A = $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Inverse of A is

$A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Re substitute the values, to get answer in the form of x.y and z.

Since AX = B

X = A^{-1}B

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

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Which means:

U = ½, v = 1/3 and w = 1/5

This implies:

X = 1/u = 2

Y = 1/v = 3

Z = 1/w = 5

Answer!

Choose the correct answer in Exercise 17 to 19.

<u>Question 17</u>

If a, b, c is in A.P., then determinant is $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$:

(A) 0   (B) 1

(C) X   (D)2x

Solution:

Option (A) is correct.

Explanation:

Since a, b, c is in A.P.

So, b – a = c – b

Let

$$\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$[R_2 \to R_2 - R_1 \, and \, R_3 \to R_3 - R_1]$$

Operation:

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 1 & 1 & 2(c-b) \end{vmatrix}$$

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 1 & 1 & 2(b-a) \end{vmatrix}$$

Row 2 and row 3 are identical, so value is zero

= 0

<u>Question 18</u>

If x, y, z are non-zero real numbers, then Inverse of matrix A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **is:**

(A)

$$\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^1 \end{bmatrix}$$

(B)

X, y, z $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^1 \end{bmatrix}$

(C)

$$\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

(D)

$$\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

Option (A) is correct,
Explanation:

Let A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

|A| = x, y, z $\neq$ 0: (A^{-1} exists)
Now: Cofactors of A are:

$A_{11} = yz, A_{12} = 0, A_{13} = 0$
$A_{21} = 0, A_{22} = xz, A_{23} = 0$

and

$A_{31} = 0, A_{32} = 0, A_{33} = xy$

Therefore:

$A^{-1} = \frac{adj .A}{|A|} = \frac{1}{xyz} \begin{vmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{vmatrix}$

$= \begin{vmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{vmatrix}$

$= \begin{vmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{vmatrix}$

Question 19

Let A = $\begin{bmatrix} 1 & sin\theta & 1 \\ sin\theta & 1 & sin\theta \\ -1 & -sin\theta & 1 \end{bmatrix}$ **where** $0 \leq \theta \leq 2\pi$. **Then:**

(A) Det (A) $\in$ 0
(B) Det (A) $\in$ (2,∞)
(C) Det (A) $\in$ (2, 4)
(D) Det (A) $\in$ [2,4]

Solution:

Option (D) is correct.
Explanation:

Let A = $\begin{bmatrix} 1 & sin\theta & 1 \\ -sin\theta & 1 & sin\theta \\ -1 & -sin\theta & 1 \end{bmatrix}$

|A| = 2 + 2 sin^2 $\neq$ 0 : (A^{-1} exists)

Since:

$$-1 \leq sin\theta \leq 1$$
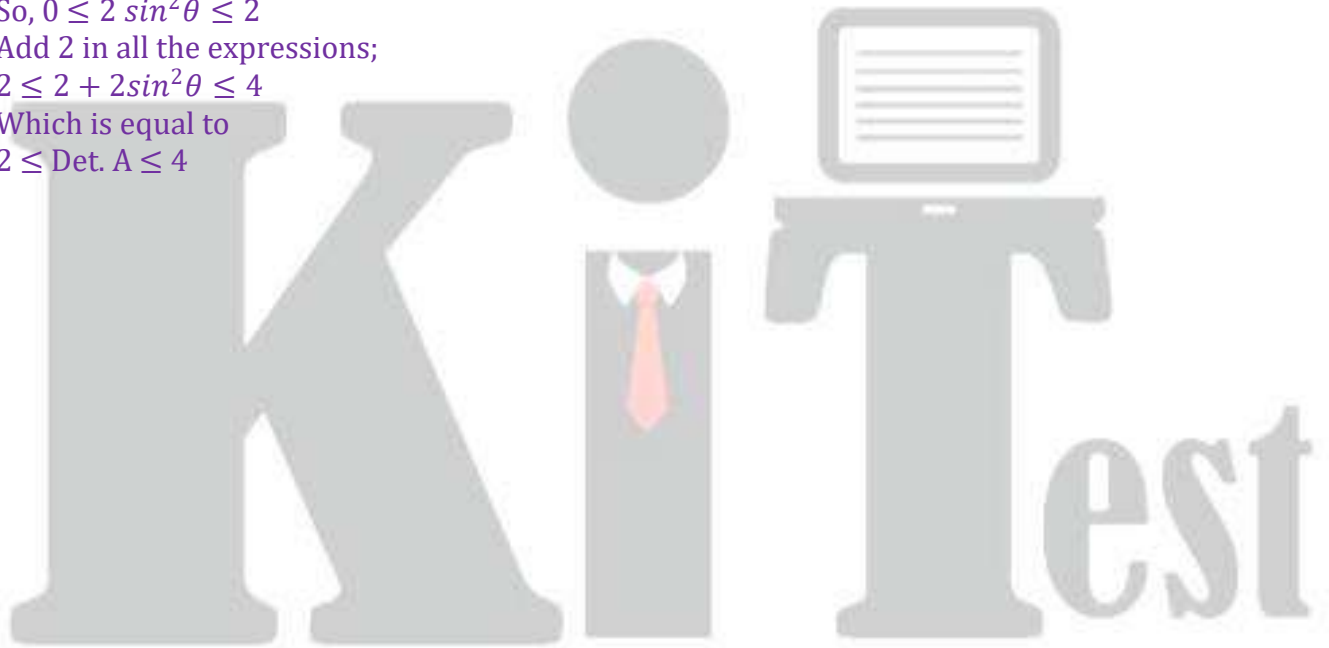$$0 \leq A^2\ \theta \leq 1$$

(The value of θ cannot be negative)
So, $0 \leq 2\ sin^2\theta \leq 2$
Add 2 in all the expressions;
$2 \leq 2 + 2sin^2\theta \leq 4$
Which is equal to
$2 \leq Det.\ A \leq 4$