Chapter 3
Matrices
Exercise 3.1

Question 1

In the matrix A

$$\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

Write
(i) The order of the matrix,
(ii) The number of elements,
(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}.

Solution:

(i) In given matrix,
Number of rows = 3
Number of columns = 4
Therefore, Order of the matrix is 3 × 4.
(ii) The number of elements in the matrix A is 3 ×4 = 12.
(iii) a_{13} = element in first row and third column = 19
a_{21} = element in second row and first column = 35
a_{33} = element in third row and third column =
a_{24} = element in second row and fourth column = 12
a_{23} = element in second row and third column = 5/2

Question 2

If a matrix has 24 elements, what are the possible orders it can have? What, if it has13 elements?

Solution:

We know that, a matrix of order mxn having mn elements.
There are 8 possible matrices having 24 elements of orders are as follows:
1 ×24, 2 ×12, 3 ×8, 4 ×6, 24 ×1, 12 ×2, 8 ×3, 6 ×4.
Prime number 13 = 1 x 13 and 13 x 1
Again,1 ×13(Row matrix) and 13 ×1(Column matrix) are 2 possible matrices whose product is 13.

Question 3

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution:

We know that, a matrix of order mxn having mn elements.
There are 6 possible matrices having 18 elements of orders:
1 ×18, 2 ×9, 3 ×6, 18 ×1, 9 ×2, 6 ×3.
Again, the product of 1 and 5 or 5 and 1 is 5.
Therefore, 1 ×5 (Row matrix) and 5 ×1 (Column matrix) are 2 possible matrices.

Question 4

Construct a 2 × 2 matrix, A = [aij], whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Solution:

(i) Construct 2 ×2 matrix for

$a_{ij} = \frac{(i+j)^2}{2}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = \frac{4}{2} = 2$

For a_{12}, i = 1 and j = 2

$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$

For a_{21}, i = 2 and j = 1

$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$

For a_{22}, i = 2 and j = 2

$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$

Required matrix is:

$\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$

(ii) Construct 2×2 matrix for

$a_{ij} = \frac{i}{j}$

Elements for 2 ×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{1}{1} = 1$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{2}{1} = 2$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{2}{2} = 1$$

The required matrix is

$$\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$$

(iii) Construct 2 ×2 matrix for

$$a_{\forall j} = \frac{(i+2j)^2}{2}$$

Elements for 2 ×2 matrix are: a_{11}, a_{12}, a_{21}, a_{22}

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{(1+4)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{(2+4)^2}{2} = \frac{(6)^2}{2} = \frac{36}{2} = 18$$

The required matrix is:

$$\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

Question 5

Construct a 3 × 4 matrix, whose elements are given by:

(i) $\boldsymbol{a_{ij} = \frac{1}{2}|-3i+j|}$

(ii) $\boldsymbol{a_{ij} = 2i - j}$

Solution:

(i) Construct 3 x 4 matrix for

$$a_{ij} = \frac{1}{2}|-3i+j|$$

Elements for 3 × 4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{13}, i = 1 and j = 3

$$a_{13} = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = \frac{1}{2}(0) = 0$$

For a_{14}, i = 1 and j = 4

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{1}{2}(4) = 2$$

For a_{23}, i = 2 and j = 3

$$a_{23} = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{1}{2}(3) = \frac{3}{2}$$

For a_{24}, i = 2 and j = 4

$$a_{24} = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

For a_{31}, i = 3 and j = 1

$$a_{31} = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{1}{2}(8) = 4$$

For a_{32}, i = 3 and j = 2

$$a_{32} = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{1}{2}(7) = \frac{7}{2}$$

For a_{33}, i = 3 and j = 3

$$a_{33} = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{1}{2}(6) = 3$$

For a_{34}, i = 3 and j = 4

$$a_{34} = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$$

The required matrix is

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii) Construct 3 ×4 matrix for

$a_{ij} = 2i - j$

Elements for 3 ×4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a_{11}, i = 1 and j = 1

$a_{11} = 2 - 1 = 1$
For a_{12}, i = 1 and j = 2
$a_{12} = 2 - 2 = 0$
For a_{13}, i = 1 and j = 3
$a_{13} = 2 - 3 = -1$
For a_{14}, i = 1 and j = 4
$a_{14} = 2 - 4 = -2$
For a_{21}, i = 2 and j = 1
$a_{21} = 4 - 3 = 3$
For a_{22}, i = 2 and j = 2
$a_{22} = 4 - 2 = 2$
For a_{23}, i = 2 and j = 3
$a_{23} = 4 - 3 = 1$
For a_{24}, i = 2 and j = 4
$a_{24} = 4 - 4 = 0$
For a_{31}, i = 3 and j = 1
$a_{31} = 6 - 1 = 5$
For a_{32}, i = 3 and j = 2
$a_{32} = 6 - 2 = 4$
For a_{33}, i = 3 and j = 3
$a_{33} = 6 - 3 = 3$
For a_{34}, i = 3 and j = 4
$a_{34} = 6 - 4 = 2$
The required matrix is

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Question 6

Find the values of x, y and z from the following equations:

(i)

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} \boldsymbol{x+y} & \boldsymbol{2} \\ \boldsymbol{5+z} & \boldsymbol{xy} \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} \boldsymbol{x+y+z} \\ \boldsymbol{x+z} \\ \boldsymbol{y+z} \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Solution:

(i)

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.
4 = y
3 = z
x = 1
(ii) Since both the matrices are equal, so their corresponding elements are also equal.
Find the value of unknowns by equating the corresponding elements

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

xy = 6 ...(1)
5 + z = 5 => z = 0
xy = 8 ....(2)
From equation (1), x = 6 – y
Substitute the value of x in equation (2)
(6 – y)y = 8
6y – y 2 = 8
or y^2 – 6y + 8 = 0
(y – 4)(y – 2) = 0
y = 4 or y = 2
Put values of y in equation (1), x+y = 6, we have x = 2 and x = 4
Therefore, x = 2, y = 4 and z = 0.
(iii) Since both the matrices are equal, so their corresponding elements are also equal.
Find the value of unknowns by equating the corresponding elements.

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

x + y + z = 9 ... (1)
x + z = 5 ... (2)
y + z = 7 ... (3)
equation (1) – equation (2), we get
y = 4
Equation (3): 4 + z = 7 => z = 3
Equation (2): x + 3 = 5 => x = 2
Answer: x = 2, y = 4 and z = 3

Question 7

Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} \boldsymbol{a-b} & \boldsymbol{2a+c} \\ \boldsymbol{2a-b} & \boldsymbol{3c+d} \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution:

Equate the corresponding elements of the matrices:
a - b = -1 ... (1)
2a + c = 5 ... (2)
2a – b = 0 ... (3)
3c + d = 13 ... (4)
Equation (1) - Equation (3)

-a = -1 => a = 1
Equation (1) => 1 - b = -1 => b = 2
Equation (2) => 2(1) + c = 5 = > c = 3
Equation (4) => 3(3) + d = 13 => d = 4
Therefore, a = 1, b = 2, c = 3 and d = 4

Question 8

A = [aij]m × n\ is a square matrix, if

(A) m < n **(B) m > n**

(C) m = n **(D) None of these**

Solution:

Option (C) is correct.
According to square matrix definition: Number of rows = number of columns (m = n)

Question 9

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$ **(B) Not Possible Find**

(C) $y = 7, x = \frac{-2}{3}$ **(D)** $x = \frac{-1}{3}, y = \frac{-2}{3}$

Solution:

Option (B) is correct
Explanation:
By equating all corresponding elements, we get
3x + 7 =0 = > x = -7/3
y – 2 = 5 = > y = 7
y + 1 = 8 => y = 7
2 – 3x = 4 => x =-2/3

Question 10

The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 **(B) 18**

(C) 81 **(D) 512**

Solution:

Option (D) is correct.
The number of elements of 3x3 matrix is 9

First element, a_11 is 2, can be 0 or 1, similarly the number of choices for each other element is 2.
Total possible arrangements = 2^9 = 512

Exercise 3.2

Question 1

Let

$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{-2} & \mathbf{5} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}$

Find each of the following:

(i) A + B

(ii) A – B

(iii) 3A – C

(iv) AB

(v) BA

Solution:

(i) A + B

$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii) A – B

$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii) 3A – C

$$3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv) AB

$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(v) BA

$$\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{vmatrix} 1(2)+3(3) & 1(4)+3(2) \\ (-2)2+5(3) & (-2)4+5(2) \end{vmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

Question 2

Compute the following

(i) $\begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{-b} & \boldsymbol{a} \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} \boldsymbol{a^2+b^2} & \boldsymbol{b^2+c^2} \\ \boldsymbol{a^2+c^2} & \boldsymbol{a^2+b^2} \end{bmatrix} + \begin{bmatrix} 2\boldsymbol{ab} & 2\boldsymbol{bc} \\ -2\boldsymbol{ac} & -2\boldsymbol{ab} \end{bmatrix}$

(iii) $\begin{bmatrix} \mathbf{-1} & \mathbf{4} & \mathbf{-6} \\ \mathbf{8} & \mathbf{5} & \mathbf{16} \\ \mathbf{2} & \mathbf{8} & \mathbf{5} \end{bmatrix} + \begin{bmatrix} \mathbf{12} & \mathbf{7} & \mathbf{6} \\ \mathbf{8} & \mathbf{0} & \mathbf{5} \\ \mathbf{3} & \mathbf{2} & \mathbf{4} \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 \boldsymbol{x} & \sin^2 \boldsymbol{x} \\ \sin^2 \boldsymbol{x} & \cos^2 \boldsymbol{x} \end{bmatrix} + \begin{bmatrix} \sin^2 \boldsymbol{x} & \cos^2 \boldsymbol{x} \\ \cos^2 \boldsymbol{x} & \sin^2 \boldsymbol{x} \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

(ii)

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Question 3

Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a(a)+b(b) & a(-b)+b(-a) \\ -b(a)+a(b) & (-b)(-b)+a(-a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -2ab \\ 0 & b^2-a^2 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1(1)+(-2)2 & 1(2)+(-2)3 & 1(3)+(-2)1 \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi)

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Question 4

If A = $\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$**, B =** $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ **and C =** $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ **then compute (A + B) and (B – C). Also, verify that A + (B – C) = (A + B) – C.**

Solution:

Find A + B:

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

Find B – C:

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0+2 & 3-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Verify that A + (B – C) = (A + B) – C

L.H.S. = A + (B – C)

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-2 & -3+0 \\ 5+4 & 0-1 & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

R.H.S. = (A + B) – C

$$\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1+2 & 4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

Question 5

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ **and** $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ **then compute 3 A – 5B.**

Solution:

Find 3 A – 5B:

$$3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 6

Simplify
$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

Solution:

Simplify first matrix

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

Simplify second matrix:

$$\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

Add results obtained from both the matrices, we get,

$$\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

We know that, $\sin 2x + \cos 2x = 1$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

This implies,

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Question 7

Find X and Y, if

(i) $\mathbf{X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}}$ **and** $\mathbf{X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}}$

(ii) $\mathbf{2x+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}}$ **and** $\mathbf{3X+2Y=\begin{bmatrix}-2 & -2\\ -1 & 5\end{bmatrix}}$

Solution:

(i) Add both the expressions:

$$(X+Y)+(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

Subtract both the expressions:

$$(X+Y)-(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$2Y=\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}$$

$$Y=\frac{1}{2}\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii) $2X + 3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$.... (1) and

$3X + 2Y = \begin{bmatrix}-2 & -2\\ -1 & 5\end{bmatrix}$... (2)

Multiply equation (1) by 2,

$4X+6Y=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}$.... (3)

Multiply equation (2) by 3

$9X+6Y=3\begin{bmatrix}-2 & 2\\ 1 & -5\end{bmatrix}=\begin{bmatrix}-6 & 6\\ 3 & -15\end{bmatrix}$.......... (4)

Subtract equation (3) from (4)

$$5X = \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ -5 & -15 \end{bmatrix}$$

$$X = \frac{1}{5}\begin{bmatrix} -10 & 0 \\ -5 & -15 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Question 8

Find X, if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X + Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution:

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question 9

Find x and y if

$$\mathbf{2}\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Solution:

Solving left hand side expression, we get

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Find x and y:

Equate corresponding elements of the matrices:

2 + y = 5 => y = 3

2x + 2 = 8 => x = 3

Question 10

Solve the equation for x, y, z and t and if

$$\mathbf{2}\begin{bmatrix} x & z \\ y & t \end{bmatrix} + \mathbf{3}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \mathbf{3}\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Solution:

$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

Find x, y, z and t by equating corresponding entries:

2x + 3 = 9 => x = 3

2y = 12 => y = 6

2z – 3 = 15 => z = 9

2t + 6 = 18 => t = 6

Question 11

If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ find the values of x and y.

Solution:

$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Find x and y by equating corresponding entries

2x – y = 10 ...(i)

3x + y = 5 ...(ii)

Add both the equation,

5x = 15 => x = 3

Put value of x in equation (ii),

9 + y = 5 => y =-4

Question 12

Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ find the values of x, y, z and w.

Solution:

$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$

Find x, y, z and w by equating corresponding entries

3x = x + 4 => x = 2

3z = -1 + z + w …… (1)

3y = 6 + x + y …… (2)

3w = 2w + 3 => w = 3

Put value of w in equation (1)

3z = -1 + z + 3 => 2z = 2 => z = 1
Put value of x in equation (2)
3y = 6 + 2 + y => y = 4.

Question 13

If $\mathbf{f}(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, show that F(x) F(y) = F(x + y).**

Solution:

Change x to y
$$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y + 0 & -\cos x\sin y - \sin x\cos y + 0 & 0 - 0 + 0 \\ \sin x\cos y + \cos x\sin y + 0 & -\sin x\sin y + \cos x\cos y + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$
$$= \begin{bmatrix} \cos(x + y) & -\sin(x + y) & 0 \\ \sin(x + y) & \cos(x + y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
= F (X + Y)
Hence proved.

Question 14

Show that

(I) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Solution:

(i) L.H.S.: $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$
Multiply both the matrices
$$\begin{bmatrix} 5(2) + (-1)3 & 5(1) + (-1)4 \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$$
$$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

R.H.S.: $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Multiply both the matrices

$= \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$

$= \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

L.H.S. ≠ R.H.S.

(ii) L.H.S.: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Multiply both the matrices

$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

R.H.S.:

$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1(1)+1(0)+0(1) & (-1)2+1(1)+0(1) & (-1)3+1(0)+0(0) \\ 0(1)+(-1)0+1(1) & (0)2+1(-1)+1(1) & (0)3+0(-1)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

L.H.S. ≠ R.H.S

Question 15

Find $A^2 - 5A + 6I$, if A is

$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Solution:

Find $A^2 = A \times A$

$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

Find 5 A

$$5\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}=\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}$$

Find 6I

$$6\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Now,

$$A^2-5A+6I=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$$

$$=\begin{bmatrix}1&-1&-3\\-1&-1&-10\\-5&4&4\end{bmatrix}$$

Question16

If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$, prove that A^3 – 6A2 + 7A + 2I = 0

Solution:

$$A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$A^2=A\times A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}$$

$$=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

$$6A^2=6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}=\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}$$

$A^3 = A^2 \times A$

$$=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Now,

$$A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Which is a zero matrix.

Therefore, A3 – 6A2 + 7A + 2I = 0 (Proved)

Question 17

If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,find k so that $A^2 = kA - 2I$.

Solution:

$A^2 = kA - 2I$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$$

Equate all the corresponding values to find the value of k,

1 = 3k – 2 => k = 1

-2 = -2k => k = 1

4 = 4k => k = 1

-4 = -2k – 2 => k = 1

The value of k is 1.

Question 18

If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$and I is the identity matrix of order 2, show that

$$\mathbf{I + A = (I - A)}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Solution:

$$1 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$$1 - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Now, R.H.S. =

$$(1 - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} \cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & \dfrac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

L.H.S. Hence proved.

Question 19

A trust fund has Rs. 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs.30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:
(a) Rs. 1800 (b) Rs. 2000

Solution:

Let x be the investment in first bond, then the investment in the second bond will be Rs. (30,000 – x).
Interest paid by first bond is 5% per year and interest paid by second bond is 7% per year.
Matrix of investment [x 30000-x]
Matrix of annual interest per year is

$$\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}$$

To obtain an annual total interest of Rs. 1800, we have

[X – 30000 – X $\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}$ = 1800

$$\left[\frac{5x}{100} + \frac{7(30000 - x)}{100}\right] = 1800$$

$$\frac{210000 - 2x}{100} = 1800$$

210000- 2x = 180000
x = 15000
The investment in first bond is Rs. 15,000
And investment in second bond is Rs. (30000 – 15000) = Rs. 15,000
To obtain an annual total interest of Rs. 2000, we have

$$\begin{bmatrix} x & 30000 - x \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\frac{210000 - 2x}{100} = 2000$$

or x = Rs. 5000
The investment in first bond is Rs. 5,000
And investment in second bond is Rs. (30000 – 5000) = Rs. 25,000

Question 20

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. Assume X, Y, Z, W and P. are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively. Choose the correct answer in Exercises 21 and 22.

Solution:

Let the selling prices of each book as a 3 x 1 matrix

$$\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

Total amount received by selling all books

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

= 9600 + 5760 + 4800

= 20160

Total amount received by selling all the books is Rs. 20160.

<u>Question 21</u>

The restriction on n, k and p so that PY + WY will be defined are:

(A) k = 3, p = n **(B) k is arbitrary, p = 2**

(C) p is arbitrary, k = 3 **(D) k = 2, p = 3**

Solution:

Option (A) is correct.

PY + WY = P (order of matrix, p x k) x Y(order of matrix, 3 x k) + W(order of matrix, n x k) Y(order of matrix, 3 x k)

Here k = 3 and p = n

<u>Question 22</u>

If n = p, then the order of the matrix 7X – 5Z is:

(A) p × 2 (B) 2 × n (C) n × 3 (D) p × n

Solution:

Option (B) is correct.

The order of matrices X and Z are equal, since n = p

The order of 7X – 5Z is same as the order of X and Z.

The order of 7X – 5Z is either 2xn or 2xp. (Given n = p)

<u>Exercise 3.3</u>

Question 1

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Solution:

(i)

Let A = $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$. then A' = $\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii)

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$. Then A' = $\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(iii)

Let A = $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$. Then A' = $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question 2

If A = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ = then verify that:

(i) $(A + B)^1 = A^1 + B^1$ (ii) $A – B)^1 = A^1 = B^1$

Solution:

(i) A + B = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

Now, $(A + B)^1 = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

Again,

$A^1 + B^1 = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

Proved.

(ii) $A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 9 & 2 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$(A - B)^1 = \begin{bmatrix} -1 & 4 & -3 \\ 2 & 5 & -2 \\ -2 & 9 & 0 \end{bmatrix}$

$A^1 - B^1 = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

Hence proved.

Question 3

If $A^1 = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then verify that:

(i) $(A + B)^1 = A^1 + B^1$

(ii) $(A - B)^1 = A^1 = B^1$

Solution:

$A^1 = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

As we know that $(A^1)^1 = A$, we have

$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

LHS:

$(A + B)^1 = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

RHS:

$A^1 + B^1 = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

LHS = RHS

(ii)

$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$

LHS

$(A - B)^1 = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

RHS:

$A^1 - B^1 = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

LHS = RHS

Question 4

If $A^1 = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$ then find $(A + 2B)'$.

Solution:

$(A')' = A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

Find A + 2 B

$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

And

$(A + 2B)' = \begin{bmatrix} -4 & 1 \\ 1 & 6 \end{bmatrix}$

Question 5

For the matrices A and B, verify that $(AB)^1 = B^1 A^1$, where:

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$. $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Solution:

LHS

$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$(AB)^1 = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

RHS:

$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

LHS = RHS

(ii)

$$AB = \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}1 & 5 & 7\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\1 & 5 & 7\\2 & 10 & 14\end{bmatrix}$$

LHS

$$(AB)' = \begin{bmatrix}0 & 1 & 2\\0 & 5 & 10\\0 & 7 & 14\end{bmatrix}$$

RHS:

$$B'A' = \begin{bmatrix}1\\5\\7\end{bmatrix}\begin{bmatrix}0 & 1 & 2\end{bmatrix} = \begin{bmatrix}0 & 1 & 2\\0 & 5 & 10\\0 & 7 & 14\end{bmatrix}$$

LHS = RHS

Question 6

(i) IF A = then verify that A'A = I.
(ii) IF A = then verify that A' A = I.

Solution:

$$L.H.S = AA' = \begin{bmatrix}\cos a & \sin a\\-\sin a & \cos a\end{bmatrix}\begin{bmatrix}\cos a & \sin a\\-\sin a & \cos a\end{bmatrix}$$

$$= \begin{bmatrix}\cos a & -\sin a\\\sin a & \cos a\end{bmatrix}\begin{bmatrix}\cos a & \sin a\\-\sin a & \cos a\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2 a + \sin^2 a & \cos a \sin a - \sin a \cos a\\\sin a \cos a - \cos a \sin a & \sin^2 a + \cos^2 a\end{bmatrix}$$

$$(i) = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I = R.H.S.$$

(ii)

$$L.H.S. = A'A = \begin{bmatrix}\sin a & \cos a\\-\cos a & \sin a\end{bmatrix}\begin{bmatrix}\sin a & \cos a\\-\cos a & \sin a\end{bmatrix}$$

$$= \begin{bmatrix}\sin^2 a + \sin^2 a & \sin a \cos a - \sin a \cos a\\\cos a \sin a - \sin a \cos a & \cos^2 a + \sin^2 a\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I = R.H.S.$$

Question 7

(i) Show that the matrix A = $\begin{bmatrix}1 & -1 & 5\\-1 & 2 & 1\\5 & 1 & 3\end{bmatrix}$ **is a symmetric matrix.**

(ii) Show that the matrix A = $\begin{bmatrix}0 & 1 & -1\\-1 & 0 & 1\\1 & -1 & 0\end{bmatrix}$ **is a skew symmetric matrix.**

Solution:

According to the symmetric matrix definition: A' = A

$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$

A is a symmetric matrix.

(ii) According to the shew symmetric matrix definition: A' = -A

$A^1 = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

= - A

A is a skew symmetric matrix.

Question 8

For a matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that.

(i) $(A + A^1)$ is a symmetric matrix.

(ii)$(A – A^1)$ is a skew symmetric matrix.

Solution:

(i)

$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 12 \end{bmatrix} = A + A^1$

(A + A') is a symmetric matrix.

(ii)

$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$A – A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A + A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} = -(A + A')$

(A - A') is a symmetric matrix.

Question 9

Find $1/2\ (A + A^1)$ and $1/2\ (A – A)^1$ when A is

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Solution:

A = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ then A' = $\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

Now, A + A' is

$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So, 1/2 (A + A') is

$\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Again,

A – A' = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$

1/2 (A – A') = 1/2 $\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Question 10

Express the following matrices as the sum of a symmetric and skew symmetric. Metrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & 1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Solution:

(i)

Let A = $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ then, A' = $\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$

$= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

And Skew symmetric matrix = 1/2(A – A')

$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$

$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Again,

Symmetric matrix + Skew symmetric matrix =

$$= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix.

(ii)

Let A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ then, A' = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Symmetric matrix = 1/2(A + A')

$$= \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Symmetric matrix = 1/2(A -A')

$$= \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix,

(iii)

Let A = $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ then, A' = $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Symmetric matrix = 1/2(A + A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

And Skew symmetric matrix = 1/2 (A – A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew Symmetric matrix.

(iv) Let A = $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ then, A' = $\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$$\frac{1}{2}\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

And Skew symmetric matrix = 1/2 (A – A')

$$\frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Which is A.

Given matrix is sum of symmetric matrix and Skew symmetric matrix.

Choose the correct answer in Exercises 11 and 12.

Question 11

If A and B are symmetric matrices of same order, AB – BA is a:
(A) Skew-symmetric matrix
(B) Symmetric matrix
(C) Zero matrix
(D) Identity matrix

Solution:

Option (A) is correct.

Question 12

IF A = $\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$. and A + A¹ = I, then the value of a is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

Solution:

Option (B) is correct.

$\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} + \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2\cos a & 0 \\ 0 & 2\cos a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By equating corresponding terms,

2 cos a = 1

Cos a = cos $\frac{\pi}{3}$

$a = \frac{\pi}{3}$

Exercise 3.4

Using elementary transformations, find the inverse of each of the matrices, if it exists in Exercises 1 to 17.

Question1

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

As we know A = I A

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$$[R_2 \rightarrow R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\left[R_2 \rightarrow \frac{1}{5}R_2\right]$$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$

$[R_1 \to R_1 + R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$

Therefore, $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

Question 2

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

As A = AI = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} A[R_2 \to R_2 - 2R_1]$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} A[R_1 \to (-1)R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} A[R_1 \to R_1 - R_2]$

$A^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

Question 3

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

As we know, A = AI

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A[R_1 \to R_1 - R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A[R_1 \to R_1 - 3R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} A[R_1 \to R_1 - R_2]$

$A^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

Question 4

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

As we know, A = AI

$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Again,

$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \ [R_1 \to R_2 - 2R_1]$

$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix} A \ [R_1 \leftrightarrow R_2]$

$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A \ [R_1 \leftrightarrow R_2 - 2R_1]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \ [R_1 \to R_1 - R_2]$

Therefore, the inverse of given matrix is:

$A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

Question 5

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

As we know, A =AI

$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Again,

$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A \ [R_1 \to R_2 - 3R_1]$

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -7 & 2 \end{bmatrix} A \ [R_1 \to R_1 - R_2]$

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -7 & 2 \end{bmatrix} A \ [R_2 \to R_2 - R_1]$

$A^{-1} = \begin{bmatrix} -4 & 1 \\ -7 & 2 \end{bmatrix}$

Question 6

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

As we know A = AI

$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Again,

$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A [R_1 \leftrightarrow R_2]$

$\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} A [R_1 \rightarrow R_2 - 2R_1]$

$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} A\ [R_1 \rightarrow (-1)R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad [R_1 \rightarrow R_1 - 3R_2]$

$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Question 7

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

As we know A = IA

$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} A\ \ [R_1 \rightarrow 2R_1]$

$\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} A\ [R_1 \rightarrow R_1 - R_2]$

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -10 & 6 \end{bmatrix} A\ [R_1 \rightarrow R_2 - 5R_1]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A\ \left[R_2 \rightarrow \frac{1}{2}R_2\right]$

$A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

Question8

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

As we know A = IA

$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \; [R_1 \to R_1 - R_2]$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \; [R_2 \to R_2 - 3R_1]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \; [R_1 \to R_1 - R_2]$

$A^{-1} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Question 9

$$\begin{bmatrix} \mathbf{3} & \mathbf{10} \\ \mathbf{2} & \mathbf{7} \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Let A = $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

As we know, A = IA

$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \; [R_1 \to R_1 - R_2]$

$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \; [R_1 \to R_1 - 2R_1]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \; [R_1 \to R_1 - 3R_2]$

$A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

Question 10

$$\begin{bmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{-4} & \mathbf{2} \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

As we know, A = IA

$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} -1 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \; [R_1 \to R_1 + R_2]$

$\begin{bmatrix} 1 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} A \; [R_1 \to (1)R_1]$

$\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} A\ [R_2 \to R_2 + 4R_1]$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A \left[R_2 \to \frac{-1}{2} R_2\right]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A\ [R_1 \to R_1 + R_2]$

$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$

Question 11

$$\begin{bmatrix} \mathbf{2} & \mathbf{-6} \\ \mathbf{1} & \mathbf{-2} \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

As we know that, A = IA

$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & -1 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A\ \ [R_1 \leftrightarrow R_2]$

$\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A\ \ [R_2 \to R_2 + 2R_1]$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} A \left[R_2 \to \frac{-1}{2} R_2\right]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A\ [R_2 \to R_2 + R_2]$

$A^{-1} = \begin{bmatrix} -1 & 3 \\ -1/_2 & 1 \end{bmatrix}$

Question 12

$$\begin{bmatrix} \mathbf{6} & \mathbf{-3} \\ \mathbf{-2} & \mathbf{1} \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

As we know, A = IA

$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \left[R_1 \to \frac{-1}{2} R_1\right]$

$\begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_2 \to R_2 + 2R_1]$

All entries in second row of left are zero, So A^{-1} does not exist.

Question 13

$$\begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that A = IA

$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying: $R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A.$

Applying: $R_2 \to R_2 + R_1$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A$

Applying: $R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

$A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Question 14

$$\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{4} & \mathbf{2} \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

We know that, A = IA

$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to \frac{1}{2} R_1$.

$\begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying: $R_2 \to R_2 - 4R_1$.

$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -2 & 1 \end{bmatrix} A$

All entries in second row of left side are zero, so inverse of the matrix does not exist.

Question 15

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that A =|A

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_3$

And $R_2 \to (-1)R_1$

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$

And $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

Applying: $R_2 \leftrightarrow R_3$

And $R_2 \to \left(-\frac{1}{5}\right) R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -\frac{3}{5} & 0 & \frac{2}{5} \\ 2 & 1 & -2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$

and $R_3 \to \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$$

Applying: $R_1 \to \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$$

Question 16

$$\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-2} \\ \mathbf{-3} & \mathbf{0} & \mathbf{-5} \\ \mathbf{2} & \mathbf{5} & \mathbf{0} \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that, A = |A

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$

and $R_3 \rightarrow R_3 - 2R_1$

and $R_2 \rightarrow \frac{1}{25} R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & -5 \\ 0 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_1 \rightarrow R_1 - 3R_2$

and $R_3 \rightarrow R_3 - 9R_2$

and $R_1 \rightarrow \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying: $R_1 \rightarrow R_1 - 10R_2$

And $R_2 \rightarrow R_2 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Question 17

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that A = IA

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$

And $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$

And $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$

and $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_3$

and $R_2 \to R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 18

Matrices A and B will be inverse of each other only if

AB = BA

AB = BA = 0

AB = 0, BA = I

AB = BA = I

Solution:

Option (A) is correct.

Miscellaneous Exercise

Question 1

Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ show that $(aI + bA)^n = a^nI + na^{n-1}\,bA$ where I is theidentity matrix of order 2and n ϵ N.

Solution:

Use Mathematical induction:
Step 1: Result is true for n = 1
$(aI + bA)^n = a^nI + na^{n-1}\,bA$
Step 2: Assume that result is true for n = k
That is
$(aI + bA)^k = a^kI + na^{k-1}\,bA$
Step 3: prove that. result is true for n = k + 1
That is
$(aI + bA)^{k+1} = a^{k+1}I + (K + 1)\,a^k\,bA$
LHS:
$(aI + bA)^{k+1} = (aI + bA)^k(aI + bA)$
$= (a^kI + ka^{k-1}bA)^k\,(aI + bA)$
$= a^{k+1}I \times I + ka^2bAI + a^kbAI + ka^{k-1}\,b^2A.A$
Here, A A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 0$
This implies
$= a^{k+1}I + (K + 1)\,a^k\,bA$
= R.H.S.
Thus, result is true.
Therefore, p(n) is true.

Question 2

If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Prove that A^n = $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, n ϵ N.

Solution:

Let us say, P(n) = A^n
Use Mathematic Induction:
Step 1: Result is true for n = 1
P(1) = A = $\begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Step 2: Assume that result is true for n = k

So,

P(1) = A = $\begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix}$ = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

P(k) = A^K = $\begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

Step 3: Prove that, result is true for n = k + 1

That is,

P(k + 1) = A^{k+1} = $\begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$

L.H.S.:

$A^{A+1} = A^k A$

$$\begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

= $\begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$

= R.H.S.

Thus, result is true.

Therefore, By Mathematical Induction p(n) is true for allnatural numbers.

Question 3

If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that A^n = $\begin{bmatrix} \frac{1+2n}{n} & \frac{-4n}{1-2n} \end{bmatrix}$ where is any positive integer.

Solution:

Use Mathematical Induction:

Step 1: Result is true for n = 1

A^1 = $\begin{bmatrix} \frac{1+2n}{n} & \frac{-4n}{1-2n} \end{bmatrix}$ = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Step 2: Assume that result is true for n = k

So,

A^1 = $\begin{bmatrix} \frac{1+2k}{k} & \frac{-4k}{1-2k} \end{bmatrix}$

Step 3: Prove that, result is true for n = k + 1

That is,

A^{k+1} = $\begin{bmatrix} \frac{1+2(k+1)}{(k+1)} & \frac{-4(k+1)}{1-2(k+1)} \end{bmatrix}$

L.H.S.

$A^{k+1} = A^k A$

= $\begin{bmatrix} \frac{1+2(k+1)}{(k+1)} & \frac{-4(k+1)}{1-2(k+1)} \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Using result from step 2.

= $\begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$

= $\begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$

= $\begin{bmatrix} \frac{1+2(k+1)}{(k+1)} & \frac{-4(k+1)}{1-2(k+1)} \end{bmatrix}$

= R.H.S.

Thus, result is true.

Therefore. By Mathematical induction result is true for all positive integers.

Question 4

If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix,

Solution:

Step 1: If A and B are symmetric matrices. Then A' = A and B' = B

Step 2: (AB – BA) = (AB)' – (BA)'

(AB – BA)' = B'A' – A'B' [using Reversal law]

(AB – BA)' = - (AB – BA)

Therefore, (AB –BA) is a skew symmetric.

Question 5

show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution:

We know that, (AB)' = B' A'

(B'AB)' = [B'(AB]' = (AB)' (B')'

This implies, (B'AB)' = B'A'B' ... say equation (1)

If A is a symmetric matrix, then A' = A

Using eq. (i) (B'AB') = B'AB

Therefore, B'AB is a symmetric matrix.

Again,

If A is a skew symmetric matrix then A'= — A

Using equation (i), (B'AB)' = B' (— A) B = — B' AB

So, B' AB is a skew symmetric matrix.

Question 6

Find the values of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the equation A'A = 1.

Solution:

Given matrix is A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

Transpose of A = A' = $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -y & z \end{bmatrix}$

Now. A'A = I (given)

$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 + x^2 + x^2 & 0 + xy - xy & 0 - xz + xz \\ 0 + xy - xy & 4y^2 + y^2 + y^2 & 2y^2 + yz + yz \\ 0 - zx + zx & 2yz - yz - yz & z^2 + z^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

This implies:

$\begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3y^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

To find the value of unknown, equate corresponding matrix entries;

We have,

$2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$6y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{6}}$ and

$3z^2 = 1 \Rightarrow z = \pm\frac{1}{\sqrt{3}}$

Question 7

For what value of x.

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

Solution:

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$\begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$\begin{bmatrix} 6 & 2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$[\,0 + 4 + 4x] = 0$

Therefore, $4 + 4x = 0 \Rightarrow x = -1$.

Question 8

IF A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. Show that $A^2 - 5A + 7I = 0$.

Solution:

$A^2 = A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7| = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

Now, $A^2 - 5A + 7I = 0$

L.H.S.

$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

R.H.S.

Hence Proved.

Question 9

Find x, if $[\boldsymbol{x} \quad \mathbf{-5} \quad \mathbf{-1}]\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \mathbf{4} \\ \mathbf{1} \end{bmatrix} = 0$

Solution:

$[x-0-2 \quad 0-10-0 \quad 2x-5-3]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$[x-2 \quad -10 \quad 2x-8]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$[(x-2) - 102x - 8)\ 1] = 0$

$[x^2 - 2x - 40x + 2x - 8] = 0$

$[x^2 - 48] = [0]$

Or $x^2 - 48 = 0 \Rightarrow x = \pm 4\sqrt{3}$

Question 10

A manufacturer product three products x. y. z which he sells in two markets. Annual sales are indicated below:

Market	Product	Product	Product
I	10.000	2.000	18.000
II	6.000	20.000	8.000

(a) If unit sales prices of x,y, and z are Rs 2.50, Rs 1.50 and Rs 1.00 respectively, find the total revenue in each market with the help of matrix algebra.
(b)If the unit costs of the above three commodities are Rs 2.00 Rs. 1.00 and 50 paise respectively. Find the grose profit.

Solution:

(a)
If unit sales prices of xy, and z are Rs 2.50 Rs 1.50 and Rs 1.00 respectively.
Total revenue n market I and II can be shown with help of matrix as. Basically, Revenue matrix

$$\begin{bmatrix} 10.000 & 2.000 & 18.000 \\ 6.000 & 20.000 & 8.000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

Solving above matrix, we have,

$= \begin{bmatrix} 25.000 + 3,000 + 18,000 \\ 15.000 + 30,000 + 8,000 \end{bmatrix}$

$= \begin{bmatrix} 46.000 \\ 53,000 \end{bmatrix}$

Therefore, the total revenue in market I = Rs 46.000 and in market II = Rs 53.000.
(b) If the unit costs of the above three commodities are Rs 2.00 Rs 1.00 and 50 paise respectively.
Total cost prices of all the products in market I and II can be shown with the help of matrix as:
Basically, Cost Matrix

$$\begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20.000 & 8,000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

Solving above matrix, we have,

$$\begin{bmatrix} 20,000 + 2,000 + 9,000 \\ 6.000 + 20,000 + 8,000 \end{bmatrix}$$

$= \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix}$

From (a) and (b),
The profit collected in two markets is given in matrix form as
Profit matrix = Revenue matrix – Cost matrix

$\begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} - \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} = \begin{bmatrix} 15,000 \\ 17,000 \end{bmatrix}$

Therefore, the gross profit in market I and II = Rs 15000 + Rs 17000 = Rs 32,000

Question 11

Find the matrix X so that X
$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} & \mathbf{6} \end{bmatrix} = \begin{bmatrix} \mathbf{-7} & \mathbf{-8} & \mathbf{-9} \\ \mathbf{2} & \mathbf{4} & \mathbf{6} \end{bmatrix}$

Solution:

Let X= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\begin{bmatrix} a + 4b & 2a + 5b & 3a + 6b \\ c + 4d & 2c + 5d & 3c + 6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Equate all the corresponding elements:
a + 4b = -7 … (1)
2a + 5b = -8 … (2)
3a + 6b = -9 …. (3)

$C + 4d = 2$ … (4)
$2c + 5d = 4$ ... (5)
$3c + 6d = 6$ …. (6)
Solving (1) and (2), we have $a = 1$ and $b = -2$
Solving (4) and (5), we have $c = 2$ and $d = 0$
So, $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Question 12

If A and B are square matrices of the same order such that Ab = BA, then Prove by Induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in N$.

Solution:

Use mathematical induction, to prove $AB^n = B^nA$
Step 1: Result is true for $n = 1$
$AB = BA$
Step 2: Assume that result is true for $n = k$
So,
$AB^k = B^kA$
Step 3: Prove that, result is true for $n = k + 1$
That is,
$AB^{k+1} = B^{k+1} A$
L.H.S.
$AB^{k+1} = AB^k B$
Using result of step 2, we have
$= B^kA B$
$= B^{k+1}A$
= R.H.S.
Thus, by Mathematical Induction the result s true.
Again, prove that $(AB)^n = A^nB^n$
Use Mathematical Induction:
Step 3: Result is true for $n = 1$
$(AB) = AB$
Step 2: Assume that result is true for $n = k$
So, $(AB)^k = A^kB^k$
Step 3: Prove that result is true for $n = k + 1$
That is, $(AB)^{k+1} = (AB)^K (AB)$
$= A^kB^k (AB)$ (using step 2 result)
$= A^K (B^K A) B$
$= A^K (A B^K) B$
$= (A^K A) (B^K B)$
$= A^{K+1} B^{K+1}$
= R.H.S
Thus, result is true for $n = k + 1$
Therefore, by Mathematical Induction we have $(AB)^n = A^n B^n$ for all $n \in N$.

Question 13

If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that A² = I, then:

(A) $1 + \alpha^2 + \beta\gamma = 0$
(B) $1 - \alpha^2 + \beta\gamma = 0$
(C) $1 - \alpha^2 - \beta\gamma = 0$
(D) $1 + \alpha^2 - \beta\gamma = 0$

Solution:

Option (c) is correct.
$A^2 = 1$
$\begin{bmatrix} a & b \\ y & -a \end{bmatrix}\begin{bmatrix} a & b \\ y & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\begin{bmatrix} a^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Or $a^2 + \beta\gamma = 1$
Or $1 - a^2 - \beta\gamma = 0$

Question 14

If the matrix A is both symmetric and skew symmetric, then:
(A) A is a diagonal matrix
(B) A is a zero matrix
(C) A is a square matrix
(D) None of these

Solution:

Option (B) is correct.

Question 15

If A is a square matrix such that A² = A, then (I + A)³ – 7A is equal to:
(A) A
(B) I = A
(C) I
(D) 3A

Solution:

Option (C) is correct.
Explanation:
As $A^2 = A$
Multiplying both sides by A.
$A^3 = A^2A = A = A^2 = A$
Again.
$(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$
Using $A^2 - A$ and $A^3 = A$, we have
$= I + 7A - 7A = 1$