Chapter 2:
Relations and Functions

Exercise 2.1

Question 1

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$ the values of x and y.

Answer:

Given, $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$
As the ordered pairs are equal, the corresponding elements should also be equal.
Thus,
x/3 + 1 = 5/3 and y – 2/3 = 1/3
Solving, we get
x + 3 = 5 and 3y – 2 = 1 [Taking L.C.M and adding]
x = 2 and 3y = 3
Therefore,
x = 2 and y = 1

Question 2

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Answer:

Given, set A has 3 elements and the elements of set B are {3, 4, and 5}.
So, the number of elements in set B = 3
Then, the number of elements in (A × B) = (Number of elements in A) × (Number of elements in B)
= 3 × 3 = 9
Therefore, the number of elements in (A × B) will be 9.

Question 3

If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Answer:

Given, G = {7, 8} and H = {5, 4, 2}

We know that,
The Cartesian product of two non-empty sets P and Q is given as
P × Q = {(p, q): p ∈ P, q ∈ Q}
So,
G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Question 4

State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

Answer:

(i) The statement is False. The correct statement is:
If P = {m, n} and Q = {n, m}, then
P × Q = {(m, m), (m, n), (n, m), (n, n)}
(ii) True
(iii) True

Question 5

If A = {–1, 1}, find A × A × A.

Answer:

The A × A × A for a non-empty set A is given by
A × A × A = {(a, b, c): a, b, c ∈ A}
Here, given A = {–1, 1}
So,
A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

Question 6

If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Answer:

Given,
A × B = {(a, x), (a, y), (b, x), (b, y)}

We know that the Cartesian product of two non-empty sets P and Q is given by:
P × Q = {(p, q): p ∈ P, q ∈ Q}
Hence, A is the set of all first elements and B is the set of all second elements.
Therefore, A = {a, b} and B = {x, y}

Question 7

Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D

Answer:

Given,
A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)
Now, B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ
Thus,
L.H.S. = A × (B ∩ C) = A × Φ = Φ
Next,
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
Thus,
R.H.S. = (A × B) ∩ (A × C) = Φ
Therefore, L.H.S. = R.H.S
- Hence verified
(ii) To verify: A × C is a subset of B × D
First,
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
And,
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
Now, it's clearly seen that all the elements of set A × C are the elements of set B × D.
Thus, A × C is a subset of B × D.
- Hence verified

Question 8

Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Answer:

Given,
A = {1, 2} and B = {3, 4}
So,

A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
Number of elements in A × B is n(A × B) = 4
We know that,
If C is a set with n(C) = m, then n[P(C)] = 2m.
Thus, the set A × B has 24 = 16 subsets.
And, these subsets are as below:
Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Question 9

Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

Answer:

Given,
n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.
We know that,
A = Set of first elements of the ordered pair elements of A × B
B = Set of second elements of the ordered pair elements of A × B.
So, clearly x, y, and z are the elements of A; and
1 and 2 are the elements of B.
As n(A) = 3 and n(B) = 2, it is clear that set A = {x, y, z} and set B = {1, 2}.

Question 10

The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Answer:

We know that,
If n(A) = p and n(B) = q, then n(A × B) = pq.
Also, n(A × A) = n(A) × n(A)
Given,
n(A × A) = 9
So, n(A) × n(A) = 9
Thus, n(A) = 3
Also given that, the ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.
And, we know in A × A = {(a, a): a ∈ A}.
Thus, –1, 0, and 1 has to be the elements of A.
As n(A) = 3, clearly A = {–1, 0, 1}.
Hence, the remaining elements of set A × A are as follows:

(−1, −1), (−1, 1), (0, −1), (0, 0), (1, −1), (1, 0), and (1, 1)

Exercise 2.2

Question 1

Let A = {1, 2, 3, ... , 14}. Define a relation R from A to A by R = {(x, y): 3x – y = 0, where x, y ∈ A}. Write Down its domain, co domain and range.

Answer:

The relation R from A to A is given as:
R = {(x, y): 3x – y = 0, where x, y ∈ A}
= {(x, y): 3x = y, where x, y ∈ A}
So,
R = {(1, 3), (2, 6), (3, 9), (4, 12)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3, 4}
The whole set A is the co domain of the relation R.
Hence, Codomain of R = A = {1, 2, 3... 14}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {3, 6, 9, and 12}

Question 2

Define a relation R on the set N of natural numbers by R = {(x, y): y = x + 5, x is a natural number less than 4; x, y ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

Answer:

The relation R is given by:
R = {(x, y): y = x + 5, x is a natural number less than 4, x, y ∈ N}
The natural numbers less than 4 are 1, 2, and 3.
So,
R = {(1, 6), (2, 7), (3, 8)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {6, 7, 8}

Question 3

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference Between x and y is odd; x ∈ A, y ∈ B}. Write R in roster form.

Answer:

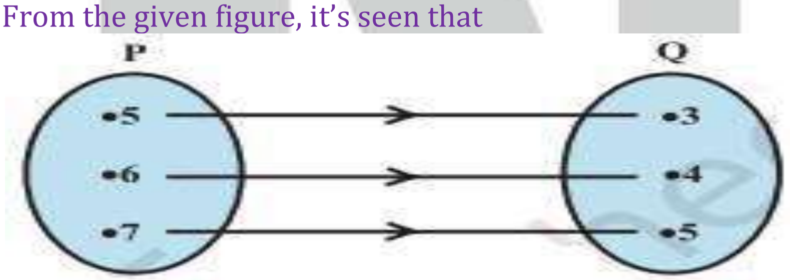
Given,
A = {1, 2, 3, 5} and B = {4, 6, 9}
The relation from A to B is given as:
R = {(x, y): the difference between x and y is odd; x ∈ A, y ∈ B}
Thus,
R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Question 4

The figure shows a relationship between the sets P and Q. write this relation
(i) in set-builder form (ii) in roster form.
What is its domain and range?

Answer:

From the given figure, it's seen that



P = {5, 6, 7}, Q = {3, 4, 5}
The relation between P and Q:
Set-builder form
(i) R = {(x, y): y = x – 2; x ∈ P} or R = {(x, y): y = x – 2 for x = 5, 6, 7}
Roster form
(ii) R = {(5, 3), (6, 4), (7, 5)}
Domain of R = {5, 6, 7}
Range of R = {3, 4, 5}

Question 5

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(a, b): a, b A, b is exactly divisible by a}.
(i) Write R in roster form

(ii) Find the domain of R
(iii) Find the range of R.

Answer:

Given,
A = {1, 2, 3, 4, 6} and relation R = {(a, b): a, b ∈ A, b is exactly divisible by a}
Hence,
(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
(ii) Domain of R = {1, 2, 3, 4, 6}
(iii) Range of R = {1, 2, 3, 4, 6}

Question 6

Determine the domain and range of the relation R defined by R = {(x, x + 5): x {0, 1, 2, 3, 4, 5}}.

Answer:

Given,
Relation R = {(x, x + 5): x ∈ {0, 1, 2, 3, 4, 5}}
Thus,
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
So,
Domain of R = {0, 1, 2, 3, 4, 5} and,
Range of R = {5, 6, 7, 8, 9, 10}

Question 7

Write the relation R = {(x, x3): x is a prime number less than 10} in roster form.

Answer:

Given,
Relation R = {(x, x^3): x is a prime number less than 10}
The prime numbers less than 10 are 2, 3, 5, and 7.
Therefore,
R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Question 8

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer:

Given, A = {x, y, z} and B = {1, 2}.

Now,
A × B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}
As n(A × B) = 6, the number of subsets of A × B will be 26.
Thus, the number of relations from A to B is 26.

Question 9

Let R be the relation on Z defined by R = {(a, b): a, b ∈ Z, a – b is an integer}. Find the domain and range of R.

Answer:

Given,
Relation R = {(a, b): a, b ∈ Z, a – b is an integer}
We know that the difference between any two integers is always an integer.
Therefore,
Domain of R = Z and Range of R = Z

Exercise 2.3

Question 1

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.
(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(1, 3), (1, 5), (2, 5)}

Answer:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
As 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation can be called as a function.
Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
As 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation can be called as a function.
Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}
(iii) {(1, 3), (1, 5), (2, 5)}
It's seen that the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation cannot be called as a function.

Question 2

Find the domain and range of the following real function:
(i) f(x) = –|x| (ii) f(x) = √(9 – x^2)

Answer:

(i) Given,
f(x) = –|x|, x ∈ R
We know that,
$|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$
$\therefore f(x) - |x| = \begin{cases} -x, x \geq 0 \\ x, x < 0 \end{cases}$
As f(x) is defined for x ∈ R, the domain of f is R.
It is also seen that the range of f(x) = –|x| is all real numbers except positive real numbers.
Therefore, the range of f is given by (–∞, 0].
(ii) f(x) = √(9 – x2)
As √(9 – x2) is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, for 9 – x2 ≥ 0.
So, the domain of f(x) is {x: –3 ≤ x ≤ 3} or [–3, 3].
Now,
For any value of x in the range [–3, 3], the value of f(x) will lie between 0 and 3.
Therefore, the range of f(x) is {x: 0 ≤ x ≤ 3} or [0, 3].

Question 3

A function f is defined by f(x) = 2x – 5. Write down the values of
(i) f(0), (ii) f(7), (iii) f(–3)

Answer:

Given,
Function, *f*(x) = 2x – 5.
Therefore,
(i) *f*(0) = 2 × 0 – 5 = 0 – 5 = –5
(ii) *f*(7) = 2 × 7 – 5 = 14 – 5 = 9
(iii) *f*(–3) = 2 × (–3) – 5 = – 6 – 5 = –11

Question 4

The function't' which maps temperature in degree Celsius into temperature
$t(C) = \frac{9C}{5} + 32$
Find (i) t (0) (ii) t (28) (iii) t (–10) (iv) The value of C, when t(C) = 212

Answer:

Given function $t(C) = \frac{9C}{5} + 32$

So,

I. $t(0) = \frac{9\times0}{5} + 32 = 0 + 32 = 32$

II. $t(28) = \frac{9\times28}{5} + 32 = \frac{252+160}{5} = \frac{412}{5}$

III. $t(0) = \frac{9\times(-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

IV. Given that $t(C) = 212$

$\therefore 212 = \frac{9C}{5} + 32$

➔ $\frac{9C}{5} = 212 - 32$

➔ $\frac{9C}{5} = 180$

➔ $9C = 180 \times 5$

➔ $c = \frac{180 \times 5}{9} = 100$

Therefore the value of t when $t(C) = 212$ is 100

Question 5

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x, x \in R, x > 0$.

(ii) $f(x) = x^2 + 2$, x is a real number.

(iii) $f(x) = x$, x is a real number.

Answer:

(i) Given,

$f(x) = 2 - 3x, x \in R, x > 0$.

We have,

$x > 0$

So,

$3x > 0$

$-3x < 0$ [Multiplying by -1 both the sides, the inequality sign changes]

$2 - 3x < 2$

Therefore, the value of 2 – 3x is less than 2.

Hence, Range = $(-\infty, 2)$

(ii) Given,

f(x) = x2 + 2, x is a real number

We know that,

$x^2 \geq 0$

So,

$x^2 + 2 \geq 2$ [Adding 2 both the sides]

Therefore, the value of x2 + 2 is always greater or equal to 2 for x is a real number.

Hence, Range = $[2, \infty)$

(iii) Given,

$f(x) = x$, x is a real number
Clearly, the range of f is the set of all real numbers.
Thus,
Range of $f = R$

Miscellaneous Exercise

Question 1

The relation f is defined by $f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x\ 3 \le x \le 10 \end{cases}$

$$g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x\ 2 \le x \le 10 \end{cases}$$

The relation g is defined by
Show that f is a function and g is not a function.

Answer:

The given relation f is defined as:

$$f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x\ 2 \le x \le 10 \end{cases}$$

It is seen that, for $0 \le x < 3$,
$f(x) = x^2$ and for $3 < x \le 10$,
$f(x) = 3x$
Also, at $x = 3$
$f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$
i.e., at $x = 3$, $f(x) = 9$ [Single image]
Hence, for $0 \le x \le 10$, the images of $f(x)$ are unique.
Therefore, the given relation is a function.
Now,
In the given relation g is defined as

$$g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x\ 2 \le x \le 10 \end{cases}$$

It is seen that, for $x = 2$
$g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$
Thus, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.
Therefore, this relation is not a function.

Question 2

If $f(x) = x2$, find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$

Answer:

Given,
$f(x) = x^2$
Hence,
$\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$

Question 3

Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Answer:

Given function,

$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x-6)(x-2)}$

It clearly seen that, the function f is defined for all real numbers except at $x = 6$ and $x = 2$ as the denominator becomes zero otherwise.
Therefore, the domain of f is R – {2, 6}.

Question 4

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x - 1)}$.

Answer:

Given real function,
$f(x) = \sqrt{(x - 1)}$
Clearly, $\sqrt{(x - 1)}$ is defined for $(x - 1) \geq 0$.
So, the function $f(x) = \sqrt{(x - 1)}$ is defined for $x \geq 1$.
Thus, the domain of f is the set of all real numbers greater than or equal to 1.
Domain of $f = [1, \infty)$.
Now,
As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{(x - 1)} \geq 0$
Thus, the range of f is the set of all real numbers greater than or equal to 0.
Range of $f = [0, \infty)$.

Question 5

Find the domain and the range of the real function *f* defined by *f* (*x*) = |*x* – 1|.

Answer:

Given real function,
$f(x) = |x - 1|$
Clearly, the function $|x - 1|$ is defined for all real numbers.
Hence,
Domain of *f* = R
Also, for $x \in R$, $|x - 1|$ assumes all real numbers.
Therefore, the range of *f* is the set of all non-negative real numbers.

Question 6

Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of *f*.

Answer:

Given

Given function,

$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$

Substituting values and determining the images, we have

$= \left\{(0,0)\left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{14}\right), \ldots\ldots\right\}$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[As the denominator is greater than the numerator.]

Or,
We know that, for x ∈ R,
$x^2 \geq 0$
Then,
$x^2 + 1 \geq x^2$
1 ≥ x2 / (x2 + 1)

Therefore, the range of *f* = [0, 1)

Question 7

Let f, g: R → R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x – 3$. Find $f + g$, $f – g$ and f/g.

Answer:

Given, the functions f, g: R → R is defined as
$f(x) = x + 1$, $g(x) = 2x – 3$
Now,
$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x – 3) = 3x – 2$
Thus, $(f + g)(x) = 3x – 2$
$(f – g)(x) = f(x) – g(x) = (x + 1) – (2x – 3) = x + 1 – 2x + 3 = – x + 4$
Thus, $(f – g)(x) = –x + 4$
$f/g(x) = f(x)/g(x)$, $g(x) \neq 0$, $x \in R$
$f/g(x) = x + 1/ 2x – 3$, $2x – 3 \neq 0$
Thus, $f/g(x) = x + 1/ 2x – 3$, $x \neq 3/2$

Question 8

Let $f = \{(1, 1), (2, 3), (0, –1), (–1, –3)\}$ be a function from Z to Z defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.

Answer:

Given, $f = \{(1, 1), (2, 3), (0, –1), (–1, –3)\}$
And the function defined as, $f(x) = ax + b$
For $(1, 1) \in f$
We have, $f(1) = 1$
So, $a \times 1 + b = 1$
$a + b = 1$ …. (i)
And for $(0, –1) \in f$
We have $f(0) = –1$
$a \times 0 + b = –1$
$b = –1$
On substituting $b = –1$ in (i), we get
$a + (–1) = 1 \Rightarrow a = 1 + 1 = 2$.

Therefore, the values of a and b are 2 and –1 respectively.

Question 9

Let R be a relation from N to N defined by R = $\{(a, b): a, b \in N \text{ and } a = b^2\}$. Are the following true?
(i) $(a, a) \in R$, for all $a \in N$
(ii) $(a, b) \in R$, implies $(b, a) \in R$
(iii) $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Answer:

Given relation R = {(a, b): a, b ∈ N and a = b2}
(i) It can be seen that 2 ∈ N; however, 2 ≠ 22 = 4.
Thus, the statement "(a, a) ∈ R, for all a ∈ N" is not true.
(ii) Its clearly seen that (9, 3) ∈ N because 9, 3 ∈ N and 9 = 32.
Now, 3 ≠ 92 = 81; therefore, (3, 9) ∉ N
Thus, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.
(iii) Its clearly seen that (16, 4) ∈ R, (4, 2) ∈ R because 16, 4, 2 ∈ N and 16 = 42 and 4 = 22.
Now, 16 ≠ 22 = 4; therefore, (16, 2) ∉ N
Thus, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true

Question 10

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the Following true?
(i) *f* is a relation from A to B (ii) *f* is a function from A to B.
Justify your answer in each case.

Answer:

Given,
A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}
So,
A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}
Also given that,
f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}
(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.
It's clearly seen that *f* is a subset of A × B.
Therefore, *f* is a relation from A to B.
(ii) As the same first element i.e., 2 corresponds to two different images (9 and 11), relation *f* is not a function.

Question 11

Let *f* be the subset of Z × Z defined by *f* = {(ab, a + b): a, b ∈ Z}. Is *f* a function from Z to Z: justify your answer.

Answer:

Given relation *f* is defined as
f = {(ab, a + b): a, b ∈ Z}

We know that a relation *f* from a set A to a set B is said to be a function if every element of set A has unique images in set B.
As 2, 6, −2, −6 ∈ Z, (2 × 6, 2 + 6), (−2 × −6, −2 + (−6)) ∈ *f*
i.e., (12, 8), (12, −8) ∈ *f*
It's clearly seen that, the same first element, 12 corresponds to two different images (8 and −8).
Therefore, the relation *f* is not a function.

Question 12

Let A = {9, 10, 11, 12, 13} and let *f*: A → N be defined by *f*(*n*) = the highest prime factor of *n*. Find the range of *f*.

Answer:

Given,
A = {9, 10, 11, 12, 13}
Now, *f*: A → **N** is defined as
f(*n*) = The highest prime factor of *n*
So,
Prime factor of 9 = 3
Prime factors of 10 = 2, 5
Prime factor of 11 = 11
Prime factors of 12 = 2, 3
Prime factor of 13 = 13
Thus, it can be expressed as
f(9) = The highest prime factor of 9 = 3
f(10) = The highest prime factor of 10 = 5
f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3
f(13) = The highest prime factor of 13 = 13
The range of *f* is the set of all *f*(*n*), where *n* ∈ A.
Therefore,
Range of *f* = {3, 5, 11, 13}