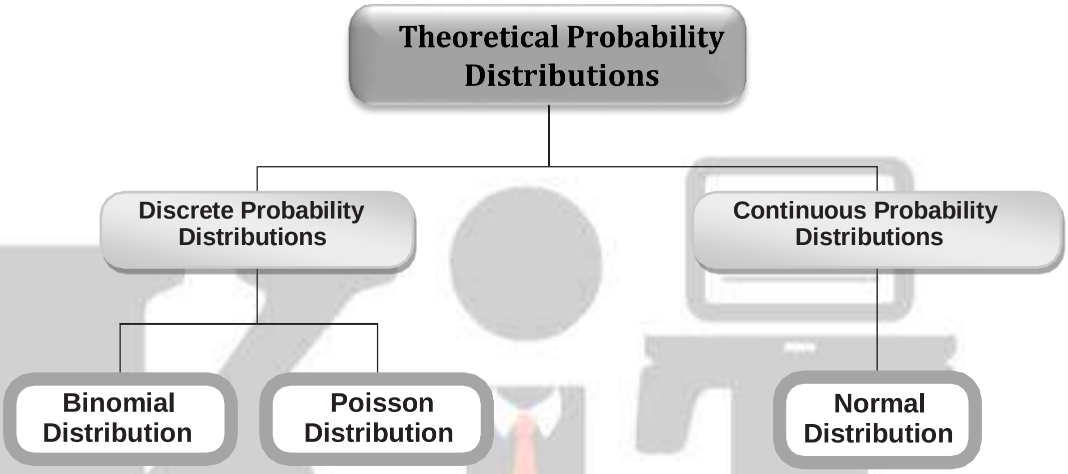
CHAPTER - 16
THEORETICAL DISTRIBUTIONS



THEORITICAL PROBABILITY	The total probability (i.e. one) is distributed to different mass points in case of a discrete random variable or to different class intervals In case of a continuous random variable
BINOMIAL DISTRIBUTION	One of the most important and frequently used discrete binomial distribution. The binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution such as the normal distribution. This is because the binomial distribution only counts two states, typically represented as 1 (for a success) or 0 (for a failure) given a number of trials in the data
Poisson Distribution	A random variable X is defined to follow Poisson distribution with parameter, to be denoted by X ~ P (m) if the probability mass function of x

Poisson Distribution Formula

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

x = 0, 1, 2, 3, …

λ = mean number of occurrences in the interval

e = Euler's constant ≈ 2.71828

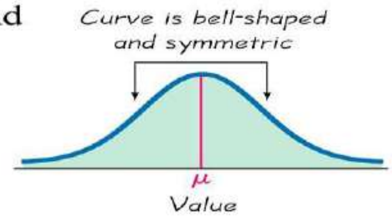
NORMAL DISTRIBUTIONS

NORMAL DISTRIBUTIONS

If a continuous random variable has a distribution with a graph that is symmetric and bell-shaped and can be described by the equation

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

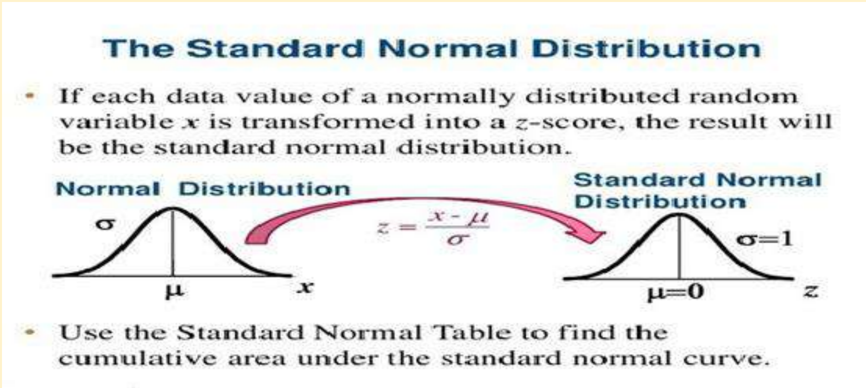
we say that it has a **normal distribution**.



Properties of the Normal Distribution

- The normal distribution curve is **bell-shaped**.
- The mean, median, and mode are **equal** and located at the center of the distribution.
- The normal distribution curve is **unimodal** (single mode).
- The curve is **symmetrical** about the mean.
- The curve is **continuous**.
- The curve **never touches the x-axis**.
- The total area under the normal distribution curve is **equal to 1 or 100%.**

STANDARD NORMAL DISTRIBUTION



POISSON DISTRIBUTION:

Question1

In a Poisson distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by

(a) m = n p (b) m = $(np)^2$

(c) m = n p (1-p) (d) m = p

Answer: a

Explanation:

For a discrete probability function, the mean value or the expected value is given by

Mean (μ) = $\sum_{x=0}^{n} xp\ (x)$

For Poisson Distribution P(x) = $\frac{e^{-m} m^x}{x!}$ substitute in above equation and solve to get μ = m = n p.

Question2

If 'm' is the mean of A Poisson Distribution, then variance is given by

(a) m^2 (b) $m^{\frac{1}{2}}$

(c) m (d) $m/2$

Answer: c

Explanation:

For a discrete probability function, the variance is given by

Variance (v) = $\sum_{x=0}^{n} x^2 p(x) - \mu^2$

Where μ is the mean, substitute P(x) = $\frac{e^{-m} m^x}{x!}$, in the above equation and put μ = m to obtain

V = m.

Question 3

The p.d.f of Poisson distribution is given by

(a) $\frac{e^{-m}m^x}{x!}$
(b) $\frac{e^{-m}x!}{m^x}$
(c) $\frac{x!}{m^x e^{-m}}$
(d) $\frac{e^{m}m^x}{x!}$

Answer: a

Explanation:

This is a standard formula for Poisson distribution, is needs no explanation. Even though if you are interested to know the derivations in detail, you can refer to any of the books or source on internet that speaks of this matter.

Question 4

If 'm' is the mean of a Poisson distribution, the standard deviation is given by

(a) $m^{1/2}$
(b) m^2
(c) m
(d) $m/2$

Answer: a

Explanation:

The variance of a Poisson distribution with mean 'm' is given by V = m, hence standard Deviation = $(\text{Variance})^{1/2}$ = $m^{1/2}$

Question 5

In a Poisson distribution the mean and variance are equal

(a) True
(b) False
(c) Can't say
(d) Not justifiable

Answer: a

Explanation:

Mean = m

Variance = m

∴ Mean = Variance.

Question 6

In a Poisson distribution, if mean (m) = e, then P(x) is given by

(a) $\frac{e^{-m}m^x}{x!}$
(b) $\frac{e^{-m}x!}{m^x}$
(c) $\frac{x!}{m^x e^{-m}}$
(d) $\frac{e^{m}m^x}{x!}$

Answer: a

Explanation:

Put m = e.

P(x) = $\frac{e^m m^x}{x!}$

Question 7

Poisson distribution is applied for

(a) Continuous Random variable

(b) Discrete Random variable

(c) Irregular Random variable

(d) Uncertain Random Variable

Answer: b

Explanation:

Poisson distribution along with Binomial Distribution is applied for discrete Random variable. Speaking more precisely, Poisson Distribution is an extension of Binomial Distribution for larger values 'n'. since Binomial Distribution is of discrete nature, so is its extension Poisson Distribution.

Question 8

If 'm' is the mean of Poisons Distribution, the P(0) is given by

(a) e^{-m}

(b) e^m

(c) e

(d) m^{-e}

Answer: a

Explanation:

P(x) = $\frac{e^{-m} m^x}{x!}$

Put x = 0, to obtain e^{-m}.

Question 9

In a Poisson distribution, the mean and standard deviation are equal

(a) True

(b) False

(c) Can't say

(d) Not justified

Answer: b

Explanation:

In a Poisson distribution,

Mean = m

Standard deviation = $m^{1/2}$

∴ Mean and Standard deviation are not equal.

Question 10

For a Poisson distribution, if mean (m) = 1, then P(1) is

(a) $\frac{1}{e}$

(b) e

(c) $\frac{e}{2}$

(d) Indeterminate

Answer: a

Explanation:

P (x) = $\frac{e^{-m}m^{x}}{x!}$

Put m = x = 1, (given) to obtain $^{1}/_{e}$.

Question 11

The recurrence relation between P(x) and P(x+1) in a Poisson distribution is given by

(a) P(x+1)- m P(x) = 0

(b) m P(x+1)- P(x) = 0

(c) (x+1) P(x+1) –m P(x) = 0

(d) (x+1) P(x) – x P(x+1) = 0

Answer: c

Explanation:

P(x) = $\frac{e^{-m}m^{x}}{x!}$

P (x+1) = e$^{-m}\frac{m^{x+1}}{(x+1)!}$

Divide P(x + 1) by P(x) and rearrange to obtain (x+1) P(x+1) – m P(x) = 0.

Question 12

The mean value for an event X to occur is 2 in a day. Find the probability of event X to occur thrice in a day.

(a) 0.1804

(b) 0.1804465

(c) 0.18

(d) None

Answer: b

Explanation:

Mean, m=2 $x = 3$

Probability of the event to occur thrice, P (3; 2) = $e^{-2}\frac{2^3}{3!}$ = 0.1804465

Question 13

A man was able to complete 3 files a day on an average. Find the probability that he can complete 5 files the next day.

(a) 0.108

(b) 0.1008

(c) 0.008

(d) None

Answer: b

Explanation:

Here we know this is a Poisson experiment with following values given:

μ = 3, average number of files completed a day

X = 5, the number of files required to be completed next day
And e = 2.71828 being a constant
On substituting the values in the Poisson distribution formula mentioned above we get the Poisson probability in this case.
We get
P(x, μ)= $\frac{(e^{-\mu})(\mu^{x})}{x!}$
→ P (5, 3) = $\frac{(2.71828)^{-3}(3^{5})}{5!}$
= 0.1008 approximately.
Hence the probability for the person to complete 5 files the next day is 0.1008 approximately.

Question 14

The number of calls coming per minute into a hotels reservation center is Poisson random variable with mean 3. Find the probability that no calls come in a given 1-minute period

(a) e^{-3}
(b) e^{3}
(c) e
(d) m^{-e}

Answer: a

Explanation:

Let x denote the number of calls coming in that given 1 minute period. X ~ Poisson (3)
P(x = 0) = $\frac{e^{-3}3^{0}}{0!}$
= e^{-3}

Question 15

If the random variable X follows a Poisson distribution with mean 3,4, find P(x=6)

(a) 0.071604409
(b) 0.00125948
(c) 0.0023698
(d) 0.015792

Answer: a

Explanation:

This can be written more quickly as: if X = Po (3, 4)
Find (x = 6)
Now
P(x = 6) = $\frac{e^{-\lambda}\lambda^{6}}{6!}$
= $\frac{e^{-3.4}(3.4)^{6}}{6!}$ (mean, λ = 3.4)

= 0.071604409 or 0.072(to 3 p.d.f)

BINOMIAL DISTRIBUTION:

Question 1

In a binomial Distribution, 'if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by

(a) np
(b) n
(c) p
(d) np(1 – p)

Answer: a

Explanation:

For a discrete probability function, the mean value or the expected value is given by

Mean (μ) $\sum_{x=0}^{n} xp(x)$

For Binomial Distribution P(x) = $^{x}C_{x}$ p^{x} $q^{(n-x)}$, substitute in the above equation and solve to get

μ = np.

Question 2

In the Binomial Distribution, If p, q and n are probability of success, failure and number of trials respectively then variance is given by

(a) np
(b) npq
(c) np^2q
(d) npq^2

Answer: b

Explanation:

For a discrete probability function, the variance is given by

Variance (V) = $\sum_{x=0}^{n} x^2\, p(x) - \mu^2$

Where μ is the mean, substitute P(x) = P(x) = $^{x}C_{x}$ p^{x} $q^{(n-x)}$, in the above equation and put μ = np to obtain

V = npq.

Question 3

If 'x' is a random variable, taking values 'x' probability of success and failure being 'p' and 'q' respectively and 'n' trials being conducted, then what is the probability that 'x' takes values 'x'? Use Binomial Distribution

(a) P(X = x) = $^{n}C_{x}\, p^{x}q^{x}$
(b) P(X = x) = $^{n}C_{x}$ p^{x} $q^{(n-x)}$
(c) P(X = x) = $^{n}C_{x}$ p^{x} $q^{(n-x)}$
(d) P (x =x) = $^{x}C_{n}p^{x}q^{x}$

Answer: b

Explanation:
It is the formula for Binomial Distribution that is asked here which is given by $P(X = x) = {}^{n}C_{x}\ px\ q^{(n-x)}$

Question 4
If 'p', 'q' and 'n' are probability of success, failure and number of trials respectively in a Binomial Distribution, what is its standard Deviation?
(a) $(np)^{1/2}$ (b) $(pq)^{1/2}$
(c) $(np)^{2}$ (d) $(npq)^{1/2}$
Answer: d
Explanation:
The variance (V) for a Binomial Distribution is given by V = npq

Question 5
In a Binomial Distribution, the mean and variance are equal
(a) True (b) False
(c) can't say (d) Not justifiable
Answer: b
Explanation:
Mean = np
Variance = npq
∴ Mean and Variance are not equal.

Question 6
It is suitable to use Binomial Distribution only for
(a) Large value of 'n' (b) Fractional values of 'n'
(c) Small values of 'n' (d) Any values 'n'
Answer: c
Explanation:
As the value of 'n' increase, it becomes difficult and tedious to calculate value of ${}^{n}C_{x}$.

Question 7
For larger values of 'n' Binomial Distribution
(a) Loses its discreteness (b) Tends to Poisson Distribution
(c) Stays as it is (d) Gives oscillatory values
Answer: b
Explanation:

Where m = np is the mean of Poisson distribution.

Question 8

In a Binomial Distribution, if p = q, then P(X = x) is given by

(a) ${}^nC_x\,(0.5)^n$

(b) ${}^xC_n(0.5)^n$

(c) ${}^nC_x\,p^{(n-x)}$

(d) ${}^xC_n p^{(n-x)}$

Answer: a

Explanation:

If p = q then p = 0.5

Substituting in $P(x) = {}^nC_x\,p^x\,q^{(n-x)}$ we get ${}^nC_x\,(0.5)^n$.

Question 9

Binomial Distribution is a

(a) Continuous distribution

(b) Discrete distribution

(c) Irregular distribution

(d) Not a Probability distribution

Answer: b

Explanation:

It is applied to a discrete Random variable, hence it is discrete distribution

Question 10

15 dates are selected at random, what is the probability of getting two Sundays?

(a) 0.29

(b) 34

(c) 56

(d) 78

Answer: a

Explanation:

If X denotes the number at Sundays. Then it is obvious that X follows binomial distribution with parameter n = 15 and p = probability of a Sunday in a week = $\frac{1}{7}$ and q = 1 – p = $\frac{6}{7}$

Then $f(x) = 15_{c_x}\left(\frac{1}{7}\right)^x.\left(\frac{6}{7}\right)^{15-x}$

For x = 0, 1, 2 .... 15.

Hence the probability of getting two Sundays

= f (2)

$= 15_{c_2}\left(\frac{1}{7}\right)^2,\left(\frac{6}{7}\right)^{15-2}$

$= \frac{10^5 \times 6^{13}}{7^{15}}$

= 0.29

Question 11

The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that out of 5 workmen, 3 or more will contract the disease?

(a) 890 (b) .0086

(c) .00086 (d) None

Answer: c

Explanation;

Let x denote the number of workmen in the sample. X follows binomial with parameters n = 5 and p = probability that a workman suffers from the occupational disease = 0.1

Hence q = 1 – 0.1 = 0.9

Thus f (x) = 5_{c_x} . $(0.1)^x$. $(0.9)^{5-x}$

For x = 0, 1, 2 ……5.

The probability that 3 or more workmen will contract the disease

= P (x ≥ 3)

= f (3) + f (4) + f (5)

= $5_{c_3}(0.1)^3(0.9)^{5-3}$ + $5_{c_4}(0.1)^4 . (0.9)^{5-4}$ + $5_{c_5}(0.1)^5$

= 10 × 0.001 × 0.814 + 5 × 0.0001 × 0.9 + 1 × 0.00001

= 0.0081 + 0.00045 + 0.00001

= 0.0086.

Question 12

Find the probability of a success for the binomial distribution satisfying the following relation 4 P (x = 4) = P (x = 2) and having the parameter n as six.

(a) P ≠ 1 (b) P ≠ -1

(c) P = 1 (d) P = 0

Answer: b

Explanation:

We are given that n = 6. The probability mass function of x is given by

F (x) = $n_{c_x}p^x q^{n-x} = 6_{c_x p^x q^{n-x}}$

For x = 0, 1 ….. , 6,

Thus P (x = 4) = f (4);

= $6_{c_4}p^4q^{6-4} = 15\, p^4q^2$

And P (x = 2) = f (2)

= $6_{c_4}p^2q^{6-2} = 15\, p^2q^4$

Hence 4 P (x = 4) = P (x = 2)
= 60 p^4q^2 = 15 p^2q^4
= 15 $p^2q^2(4p^2 - q^2)$ = 0
= $4p^2 - q^2 = 0$ (as p - 0, q -0)
= $4p^2$ – $(1-p)^2$ = 0 (as q = 1 – p)
= (2p + 1 – p) = 0 or (2p – 1 + p) = 0
= p = -1 or p = $\frac{1}{3}$ thus p = $\frac{1}{3}$ (as p ≠ -1)

NORMAL DISTRIBUTION:

Question 1

Normal distribution is applied for

(a) Continuous Random Distribution
(b) Discrete Random Variable
(c) Irregular Random Variable
(d) Uncertain Random Variable

Answer: a

Explanation:

Normal Distribution is applied for Continuous Random Distribution. A discrete probability distribution is a probability distribution characterized by a probability mass function. Thus, the distribution of a random variable x is discrete, and x is called a discrete random variable, if, as u runs through the set of all possible values of x.

Question 2

The shape of the Normal curve is

(a) Bell shaped
(b) Flat
(c) Circular
(d) Spiked

Answer: a

Explanation:

Due to the nature of the probability Mass function, a bell shaped curve is obtained.

Question 3

Normal Distribution is symmetric is about

(a) Variance
(b) Mean
(c) Standard deviation
(d) Covariance

Answer: b

Explanation:

Due to the very nature of p.m.f of Normal Distribution, the graph appears such that it is symmetric about its mean.

Question 4

For a standard normal variate, the value of mean is

(a) ∞ (b) 1

(c) 0 (d) Not defined

Answer: c

Explanation:

For a normal variate, if its mean = 0 and standard deviation = 1, then its called as standard Normal variate. Here, the converse is asked.

Question 5

The area under a standard normal curve is

(a) 0 (b) 1

(c) ∞ (d) Not defined

Answer: b

Explanation:

For any probability distribution, the sum of all probabilities is 1. Area under normal curve refers to sum of all probabilities.

Question 6

The standard normal curve is symmetric about the value.

(a) ∞ (b) 0

(c) 0.5 (d) 1

Answer: b

Explanation:

Normal curve is always symmetric about mean, for standard normal curve or variate mean = 0.

Question 7

For a standard normal variate. The value of standard deviation is

(a) 3 (b) 1

(c) ∞ (d) Not defined

Answer: b

Explanation:

If the mean and standard deviation of a normal variate are 0 and 1 respectively, it is called as standard normal variate.

Question 8

Normal Distribution is also known as

(a) Cauchy's Distribution
(b) Laplacian Distribution
(c) Gaussian Distribution
(d) Lagrangian Distribution

Answer; c

Explanation:

Named after the one who proposed it. For further details, refer to books or internet.

Question 9

Skewers of Normal distribution is

(a) Negative
(b) Positive
(c) 0
(d) Undefined

Answer: c

Explanation:

Since the normal curve is symmetric about its mean, its skewness is zero. This is a theoretical explanation for mathematical proofs, you can refer to books or website that

Speak on the same in detail.

Question 10

For a normal distribution its mean, median, mode are equal

(a) True
(b) False
(c) Not defined
(d) Can't say

Answer: a

Explanation:

It has theoretical evidence that requires some serious background on several topics for more details you can refer to any book or website that speaks on the same.

Question 11

In Normal distribution, the highest value of ordinate occurs at

(a) Mean
(b) Variance
(c) Extremes
(d) Same value occurs at all points

Answer: a

Explanation:

This is due the behavior of the p.d.f of Normal distribution.

Question 12

The shape of the normal curve depends on its

(a) Mean deviation (b) Standard deviation

(c) Quartile deviation (d) None

Answer: b

Explanation:

This can be seen in the p.d.f on the normal distribution where standard deviation is a variable.

Question 13

The value of constant 'e' appearing in normal distribution is

(a) 2.5185 (b) 2.7836

(c) 2.1783 (d) None

Answer: c

Explanation:

This is a standard constant.

Question 14

In standard normal distribution, the value of median is

(a) 1 (b) 0

(c) 2 (d) Not fixed

Answer: b

Explanation:

In a standard normal distribution the value of mean is 0 and in normal distribution mean, median and mode coincide.

Question 15

In a certain book, the frequency distribution of the number of words per page may be taken as approximately normal with mean 800 and standard deviation 50. If three pages are chosen at random, what is the probability that none of them has between 830 and 845 words each?

(a) 0.7536 (b) .7654

(c) .9084 (d) .8733

Answer: a

Explanation:

Let X is a normal variate which denotes the number of words per page. It is given that X – N (800, 50).

The probability that a page, select at random, does not have number of words between 830 and 845, is given by

1-P (830< X < 845) 1 – P $\left(\frac{830-800}{50} \leq = < \frac{845-800}{50}\right)$

= 1 –P (0.6 < = < 0.9) = 1 – P (0< = < 0.9) + P (0< = < 0.6)

= 1 – 0.3159 + 0.2257 = 0.9098 = 0.91

Thus, the probability that none of the three pages, selected at random, have number of words lying between 830 and 845 = (0.91)3 = 0.7536.

Question 16

The distribution of 1,000 examines according to marks percentage is given below:

% Marks	less than 40	40-75	75 or more	Total
No. of examines	430	420	150	1000

Assuming the marks percentage to follow a normal distribution, calculate the mean and standard deviation of marks. If not more than 300 examines are to fail, what should be the passing marks?

(a) 30%

(b) 40%

(c) 50%

(d) None

Answer: a

Explanation:

Let X denotes the percentage of marks and its mean and S.D. be m and s respectively. From the given table, we can write

P (x < 40) = 0.43 and $P(X \geq 75) = 0.15, \textit{which can also be written as}$

P $\left(= < \frac{40-\mu}{\sigma}\right)$ = 0.43 and P $\left(= \geq \frac{75-\mu}{\sigma}\right)$ = 0.15

The above equations respectively imply that

$\frac{40-\mu}{\sigma}$ = -0.175 or 40 – μ = - 0.175σ ...... (1)

And $\frac{75-\mu}{\sigma}$ = 1.04 or 75 – μ = 1.04σ ...... (2)

Solving the above equations simultaneously, we get μ = 45.04 and σ = 28.81

Let x, be the percentage or marks required to pass the examination.

Then we have P (x < x_1) = 0.3 or P (= <$\frac{x_1-45.04}{28.81}$) = 0.3

$\therefore \frac{x_1-45.04}{28.81}$ = - 0.525 → x_1 - 29.91 or 30% (approx)

Question 17

At a petrol station, the mean quantity of petrol sold to a vehicle is 20 litres per day with a standard deviation of 10 liters. If on a particular day, 100 vehicles took 25 or more litres of petrol, estimate the total

number of vehicles who took petrol from the station on the day. Assume that the quantity of petrol taken from the station by a vehicle is a normal variate.

(a) 333 (b) 343

(c) 324 (d) 567

Answer: c

Examination:

Let X denotes the quantity of petrol taken by a vehicle. It is given that X – N (20, 10).

$\therefore$ P (X ≥ 25) = P $\left(=\geq \frac{25-20}{10}\right)$= P (= ≥ 0.5)

= 0.5000-P (0 ≤= ≤ 0.5) = 0.5000 – 0.1915 = 0.3085

Let N be the total number of vehicles taking petrol on that day.

$\therefore$ 0.3085 × N = 100 or N = $\frac{100}{0.3085}$ = 324 (approx.)

Question 18

Using the table areas under the standard normal curve, find the following probabilities:

(i) P $(0 \leq z \leq 1.3)$

(ii) P $(-1 \leq z \leq 0)$

(iii) P $(-1 \leq z \leq 12)$

(a) 0 0.4032, 0.3413,0.8185 (b) 0.4072, 0.4413,0.8185

(c) 0.40456, 0.3456,0.8155 (d) None

Answer: a

Explanation:

The required probability, in each question, is indicated by the shaded are of the corresponding figure.

(a) From the table.

(b) (i) we can write $P(0 \leq z \leq 1.3) = 0.4032$.

(c) (ii) we can write $P(-1 \leq z \leq 1)$, because the distribution is symmetrical.

Question 19

Determine the value or values of z in the following situations:

(i) Area between 0 and z is 0.4495.

(ii) Area between - ∞ to z is 0.1401.

(a) -1.64, -1.08 (b) -1.08, -1.64

(c) 1.64, 1.08 (d) -1.64, 1.08

Answer: a

Explanation:

(i) On locating the value of z corresponding to an entry of area 0.4495 in the table of areas under the normal curve, we have z = 1.64 we note that same situations may correspond to a negative value of z. Thus, z can be 1.64 or – 1.64.
(ii) Since the area between -∞ to z<0.5, z will be negative. Further, the area between z and 0 = 0.5000 – 0.1401 = 0.3599. On locating the value of z corresponding to this entry in the table, we get z = -1.08

Question 20

In ____________ distribution, mean = variance:

(a) Binomial (b) Poisson
(c) Normal (d) None of these

Answer: b

Explanation:

Poisson; np=npq
Np = mean
Npq = variance

Question 21

In a Binomial Distribution, if p = q, then P(X = x) is given by

(a) $n_{C_x}(0.5)^n$ (b) $^nC_n\ (0.5)^n$
(c) $^nC_x\ p^{(n-x)}$ (d) $^nC_n\ p^{(n-x)}$

Answer: a

Explanation:

If p = q, then p = 0.5
Substituting in $P(x) = {}^nC_x\, p^x q^{(n-x)}$ we get $^nC_n\ (0.5)^n$.

Question 22

If Y ≥ x then mathematical expectation is

(a) E(X) > E(Y) (b) $E(X) \leq E(Y)$
(c) E (x) = E (Y) (d) E (X). E(Y) = 1

Answer: b

Explanation:

$E(X) \leq E(Y)$

Past Examination Questions

MAY - 2018

Question 1

The variance of a binomial distribution with the parameters n and p is:

(a) $np^2(1-p)$
(b) nq(1-q)
(c) $\sqrt{np-(1-p)}$
(d) $n^2p^2(1-p)^2$

Answer: b

Explanation:

= npq

= nqp

= nq(1-q)

Question 2

X is a passion variate satisfying the following condition 9 P(X = 4) + 90 (X = 6) = P (X = 2). What is the value of P (X ≤ 1)?

(a) 0.5655
(b) 0.5655
(c) 0.7358
(d) 0.8835

Answer: c

Explanation:

Given X ~ P (m)

P (x = 2) = 9 P(x = 4) + 90 P(x = 6)

$$\frac{e^{-m}.m^2}{2!} = + \frac{9.e^{-m}.m^4}{4\,!} + \frac{90.e^{-m}.m^e}{2!}$$

$$\frac{90.e^{-m}.m^e}{2\,!} + \frac{9.e^{-m}.m^4}{4\,!} - \frac{e^{-m}.m^2}{2\,!} = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{6!} + \frac{9m^2}{4!} - \frac{1}{2!}\right] = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{6!} + \frac{9m^2}{4!} - \frac{1}{2}\right] = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{6!} + \frac{9m^2}{4!} - \frac{1}{2}\right] = 0$$

$e^{-m}.m^2\left[\frac{m^4}{8}+\frac{3m^2}{8}-\frac{1}{2}\right]$ = 0

$\frac{e^{-m}}{2}\left[\frac{m^4+3m^2-4}{4}\right]$ = 0

$\frac{e^{m}.m^2}{8}(m^4+3m^2-4)=0$

$m^4+4m^2-m^2-4=0$

m^2 (m^2+4) - 1 (m² + 4) = 0

(m² + 4) (m² – 1) = 0

If m² + 4 = 0 if m² – 1 = 0

m^2= -4 if m² = + 1

m^2 = ≠ $\sqrt{1}$

m = (∵ m > 0)

$P(x \leq 1) = P(x=0) + P(x=1)$

$= \frac{e^{-1}.1^0}{0!} + \frac{e^{-1}.1!}{1!} = \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$

$\frac{2}{2.7182}$ = 0.7358

Question 3

What is the first quartile of x having the following probability of function?

f (x) $\frac{1}{\sqrt{72x}} e^{-(x-10)^{\frac{2}{72}}}$ for -∞<x<∞

(a)4 (b) 5

(c) 5.95 (d) 6.75

Answer: c

Explanation:

Given: f (x) $\frac{1}{\sqrt{72x}} e^{-(x-10)^{\frac{2}{72}}}$ for -∞<x<∞

f(x) $\frac{1}{\sqrt[6]{2x}} e^{-(x-10)^{\frac{2}{72}}}$

on company

f (x) $\frac{1}{\sqrt[6]{2x}} e^{\frac{-(x-\mu)^2}{2(o')^2}}$

we get

Ơ = 6, μ = 10

First quartile Q_1 = μ – 0.675Ơ

= 10-0.675×6

= 10-4.05

= 5.95

Question 4

An example of bi-parametric discrete probability distribution is

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Both a and b

Answer: d

Explanation:

Binomial distribution is an example of a bi- parametric discrete probability distribution.

Question 5

Probability distribution may be

(a) Discrete (b) Continuous

(c) Infinite (d) a or b

Answer: d

Explanation:

Probability distribution may be discrete or continuous.

Question 6

If the area of standard normal curve between z = 0 to z = 1 is 0.3413, then the value of ø (1) is.

(a) 0.5000 (b) 0.8413

(c) -0.5000 (d) 1

Answer: b

Explanation:

The area of standard of normal curve between z = 0 to z = 1 is 0.3413 then

Ø (1) = 0.3413 + 0.5

0.8413

NOV - 2018

Question 1

For a poisson variate X, P(X = 2) = 3P (X = 4), then the standard deviation of X is

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) 3

Answer: c

Explanation:

For Poisson Variate X,

$\frac{e^{-m}m^2}{2!} = \frac{3e^{-m}m^4}{4!}$

$\frac{m^2}{2} = \frac{3m^4}{4!}$

$6m^4 = 24\ m^2$

$m^2 = \frac{24}{6}$

$m^2 = 4$

m =2

S.D. = $\sqrt{m}$ = $\sqrt{2}$

Question 2

The mean of the Binomial distribution B $\left(4, \frac{1}{3}\right)$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{8}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Answer: d

Explanation:

X ₄ B (n, P) = B $\left(4, \frac{1}{3}\right)$

We get n = 4, P = $\frac{1}{3}$

Mean = np

= 4 × $\frac{1}{3}$ = $\frac{4}{3}$

Question 3

If for a normal distribution Q_1 = 54.52 and Q_3 = 78.86, then the median of the distribution is

(a) 12.17 (b) 12.17

(c) 66.369 (d) None

Answer: c

Explanation:

Q_1 = 54.52 and Q_3 = 78.86

We know that

$Q_1 = \mu$-0.675 = 54.52 _______ (1)

$Q_3 = \mu$-0.675 = 78.86 _______ (2)

On adding

2μ = 133.38

$\mu = \frac{133.28}{2}$

μ = 66.69

In normal distribution Mean, Median and mode are equal.

So, Median = Mean = 66.369

Question 4

What is the mean of X having the following density function?

F(X) = $\frac{1}{4\sqrt{2X}}$ e $\left(\frac{x-10}{32}\right)^e$ for -∞<x<∞

(a) 10 (b) 4

(c) 40 (d) None

Answer: a

Explanation:

Given Normal distribution

F(x) = $\frac{1}{4\sqrt{2X}}$ e $\left(\frac{x-10}{32}\right)^e$ for -∞<x<∞

On comparing from

f (x)= $\frac{1}{4\sqrt{2x}}$ e$\left(\frac{x-10}{32}\right)^e$ for -∞<x<∞

on comparing from

f (X) = $\frac{1}{0\sqrt{2x}} e^{\frac{x-\mu}{2(0\prime)^2}}$

we get

Mean (μ) = 10

Question 5

The probability that a student is not a Swimmer is $\frac{1}{5}$, then the probability that out of five student four are swimmer is

(a) $\left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)$ (b) $5_{C_1} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)$

(c) $5_{c_4} \left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^4$ (d) None

Answer: c

Explanation:

Given:

Probability that a student is not a swimmer (q) = $\frac{1}{5}$

Probability that a student is a swimmer (P) = 1-q = 1- $\frac{1}{5}$ = $\frac{4}{5}$

Total No. of student (n) = 5

P (Exactly 4 student are swimmer)

= P (x=4)

$5_{c_4}\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^4$ {∴ P (x = n) = $n_{c_{n.p^x.q^{n-x}}}$}

MAY - 2019

Question 1

If mean and variance are 5 and 3 respectively then relation between p & q is

(a) p > q
(b) p < q
(c) p = q
(d) p is symmetric

Answer: b

Explanation:

If mean and variance are 5 and 3 respectively then relation between p & q is p < q

Question 2

4 coins were tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time?

(a) $1600e^{-100}$
(b) $1000e^{-100}$
(c) $100e^{-1600}$
(d) e^{-100}

Answer: d

Probability of Head = 1/2

Probability of not head = 1 - 1/2 = 1/2

probability that all 4 coins do not turn head upward at a time

= 1 - Probability that 4 coins turn head upward at a time

= 1 - $^4C_4(1/2)^4(1/2)^0$

= 1 - 1/16

= 15/16

15/16 is the probability that all 4 coins do not turn head upward at a time

1600 * 15/16 = 1500

1500 times all 4 coins do not turn head upward at a time

Question 3

In a poisson distribution if p (x = 4) = p (x = 5) then the parameter of poisson distribution is:

(a) $\frac{4}{5}$
(b) $\frac{5}{4}$
(c) 4
(d) 5

Answer: (d)

Explanation:

In poisson distribution

P (x = 4) = p (x = 5)

$\frac{e^{-m}.m^4}{4!} = \frac{e^{-m}.m^5}{5!}$

$\frac{1}{4!} = \frac{m}{5!}$

$\frac{1}{24} = \frac{m}{120}$

24 m = 120

M = 5

Question 4

Area between = 1.96 to + in a normal distribution is:

(a) 95.45% (b) 95%

(c) 96% (d) 99%

Answer: (b)

Explanation:

Area between – 1.96 to + 1.96 in a normal distribution is 95%

Question 5

If the points of inflexion of a normal curve are 40 and 60 respectively then its mean deviation is:

(a) 8 (b) 45

(c) 50 (d) 60

Answer: (a)

Explanation:

If the point of inflexion of a normal Distribution Are 40 and 60.

Then

$\mu - \sigma = 40$ ______________ (1)

$\mu - \sigma = 60$ ______________ (2)

Solving eq. (1) and (2) we get

$\mu = 50$

Then M.O = $\frac{4}{5}$ S.D.

$= \frac{4}{5} \times 10$

$= 8$

NOV - 2019

Question 1

Area under U = 30'

(a) 99.73% (b) 99%

(c) 100% (d) 99.37%

Answer: a

Explanation:

(a) We know that 99.37 percent of the values of a normal variable lies between (u - 3Ơ) and (u + 3Ơ).

Thus probability that a value of x lies. Outside the limit is as low as

(100 – 99.73) = 0.27%

Question 2

For a Poisson distribution:

(a) mean and SD are equal (b) mean and variance are equal

(c) SD and Variance (d) Both a and b

Answer: b

Explanation:

(b) Poisson distribution is theoretical discrete probability distribution which can describe many processes

Mean is given by m i.e. U = m

Variance is also given by m i.e. $o^2 = m$

So in pass on distribution mean and variance are equal.

Question 3

Find mode when n = 15 and p = $\frac{1}{4}$ in binomial distribution?

(a) 4 (b) 4 and 3

(c) 4.2 (d) 3.7

Answer: b

Explanation:

(b) In binomial distribution,

m = (n + 1) p

m = (15 + 1) × $\frac{1}{4}$

m = 4

Since 4 is a integer so there will 2 modes

4 and (4 – 1)

Mode = 4 and 3

Question 4

In Poisson distribution, if P (x = 2) = $\frac{1}{2}$ p (x = 3) find m?

(a) 3

(b) $\frac{1}{6}$

(c) 6

(d) $\frac{1}{3}$

Answer: c

Explanation:

(c) In Poisson distribution P(x = x) = $\frac{e^{-m}.m^2}{x!}$

Here P (x = 2) = $\frac{1}{2}$ P(x = 3)

$\frac{e^{-m}.m^2}{2!} = \frac{1}{2} \times \frac{e^{-m}.m^3}{3!}$

$\frac{e^{-m}.m^2}{2!} = \frac{1}{2} \times \frac{e^{-m}.m^3}{3!}$

$\frac{m^2}{2} = \frac{1}{2} \times \frac{m^3}{6}$

$m^2 = \frac{2}{12} = \frac{1}{6} m^3$

$m^{-1} \frac{1}{6}$

$\frac{1}{m} = \frac{1}{6} = m = 6$

Question 5

In a binomial distribution B(n, p)

n = 4 P(x = 2) = 3 × P (x = 3) find P

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{6}{4}$

(d) $\frac{4}{3}$

Answer: a

Explanation:

We know P(x = 1) = $^{n}C_{r}$ $(p)^{r}$ $(q)^{n-r}$

Here p(x = 2) = 3 P(x = 3)

$4_{c_2}(p)^2(q)^{4-2}$ = 3 × $^{4}c_{3}$ $(p)^3(q)^1$

$\frac{4!}{(4-2)1\times 2!}(p)^2$ (1 – p² = 3 × $\frac{4!}{(4-3)1\times 3!} \times (p)^3$ (1 – p)

Since $^{n}C_{r} = \frac{n!}{(n-r)!1\times r!}$

6 × (1 – p) =3 × 4 p

6 – 6p = 12 p

18 p = 6

P = $\frac{1}{3}$

q= 1 - $\frac{1}{3}$ = $\frac{2}{3}$

What is the SD and mean

X if f(x) = $\frac{\sqrt{2}}{\sqrt{\Pi}} . e^{\frac{x-\ \mu}{2\ 0\prime^2}}$ ____________ (1)

Here, $\sqrt{\frac{2}{\Pi}} . e^{-2}(x-3)^2$

= $\sqrt{\frac{2}{\Pi}}$. e – $\left(\frac{1-3}{\frac{1}{2}}\right)^2$

On comparing with equation ----------- (1)

2 0² = $\frac{1}{2}$ u = 3

0² = $\frac{1}{4}$

0 = $\frac{1}{2}$

So SD = $\frac{1}{2}$, mean = 3

Question 6

Which is the SD and mean

x if (x) = $\frac{\sqrt{2}}{\sqrt{\pi}} e^{-2(x-3)^2}$, - ∞ < x < ∞

(a) 3,$\frac{1}{2}$ (b) 3,$\frac{1}{4}$

(c) 2,$\frac{1}{2}$ (d) 2,$\frac{1}{2}$

Answer: a

Explanation:

The standard from of probability density function is

F(x) = $\frac{1}{\sqrt{2\pi}}$ --------- (1)

Here, $\sqrt{\frac{2}{\pi}}. e^{-2(x-3)^2}$

= $\sqrt{\frac{2}{\pi}}\, e^{\left(\frac{13}{1/2}\right)^2}$

On comparing with equation -------------- (1)

$2\sigma^2$ = $\frac{1}{2}$ u = 3

σ^2 = $\frac{1}{4}$

$\sigma = \frac{1}{2}$

So SD = $\frac{1}{2}$, mean = 3

DEC - 2020

Question1

Which of the following is uni-parametric distribution?

(a) Normal
(b) Poisson
(c) Binomial
(d) Hyper geometric

Answer: b

Explanation:

Poisson distribution is uniparametric distribution. the parameter is m which is mean=np

Question2

If the probability of success in a binomial distribution is less than one – half, then the binomial distribution _____

(a) Is skewed to left
(b) Is skewed to right
(c) Has two modes
(d) Has median at a point > mean + ½

Answer: b

Explanation:

Is skewed to right

Question3

If we change the parameter(s) of a _____ distribution the Sharpe of probability curve does not change.

(a) Binomial
(b) Normal
(c) Poisson
(d) Non – Gaussian

Answer: b

Explanation:

If we change the parameter(s) of abnormal distribution the Sharpe of probability curve does not change.

Question4

Which one of the following has Poisson distribution?

(a) The number of days to get a Complete cure
(b) The number of defects per meter on Long roll Of coated

Polythene sheet.

(c) The errors obtained in repeated Measuring of The Length of a rod.

(d) The number of claims rejected By an Insurance agency.

Answer: b

Explanation:

The number of defects per meter on long roll of coated polythene sheet.

Question5

For a Poisson distributed variable X, we have P(X = 7) = 8. P (X = 9), the mean of the distribution is

(a) 4
(b) 3
(c) 7
(d) 9

Answer: b

Explanation:

$$P(X = n) = \frac{\lambda^7 e^{-\lambda}}{7!} = \frac{8.\lambda^9 e^{-\lambda}}{9!} \frac{9!}{7! \times 8} \lambda^2$$

λ =3

Question6

The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is ____

(a) 54.24
(b) 23.20
(c) 0.275
(d) 2.70

Answer: d

Explanation:

In normal distribution, quartile deviation is related to standard deviation as

Q.D. = 0.675 σ

Q.D. = 0.675 × 4

Q.D. = 2.70

Therefore, quartile deviation is 2.70.

Question7

If the parameter of poison distribution is m and (mean + S.D.= 25 6 then find m.

(a) $\frac{3}{25}$
(b) $\frac{1}{25}$
(c) $\frac{4}{25}$
(d) $\frac{3}{5}$

Answer: b

Explanation:

Let, Mean of the Poisson distribute =μ

For a Poisson distribution,

Standard Deviation (SD)=$\sqrt{\text{mean}}$

⇒ SD=$\sqrt{\mu}$

Mean+SD=$\frac{6}{25}$ (Given)

$\mu+\sqrt{\mu}=\frac{6}{25}$

$\Rightarrow\sqrt{\mu}=\frac{6}{25}-\mu$

On squaring both sides,

$(\sqrt{\mu})^2\left(\frac{6}{25}-\mu\right)^2$

$\mu=\mu^2-\frac{12}{25}\mu+\frac{36}{625}$

$\Rightarrow 0=\mu^2-\frac{37}{25}\mu+\frac{36}{625}$

$\Rightarrow 0=\left(\mu-\frac{1}{25}\right)\left(\mu-\frac{36}{25}\right)$

$\Rightarrow \mu=\frac{1}{25},\frac{36}{25}$

Maximum likelihood estimate of a sample from Poisson Distribution is the sample mean which is equal to parameter of Poisson's Distribution.

$\Rightarrow \mu=m=\frac{1}{25}$

∴ The correct option is B $\frac{1}{25}$

JAN - 2021

Question1

If X is a poisson variable, and P (X = 1 = P(X = 2), then P(X = 4) is

(a) $\frac{2}{3}e^2$ (b) $\frac{2}{3}e^4$

(c) $\frac{3}{2}e^2$ (d) $\frac{3}{2}e^4$

Answer: a

Explanation:

$P(x:\mu)=\frac{e^{-u}\mu^x}{x!}$

$P(X=1)=P(X=2)$

$\frac{e^{-u}\mu^1}{1!}=\frac{e^{-u}\mu^2}{2!}$

$\mu = 2$

$$P(X = 4) = \frac{e^{-u}\mu^x}{4!} = \frac{2}{3}e^2$$

Question2

Which one of the following is an uniparametric distribution?

(a) Poisson (b) Normal
(c) Binomial (d) Hyper geometric

Answer: a

Explanation:

Poisson distribution is uniparametric distribution. The parameter is m which is mean=np. Bcz it has λ as a parameter.

Question3

For a normal distribution, the value of third moment about mean is

(a) 0 (b) 1
(c) 2 (d) 3

Answer: a

Explanation:

E[(X−μ)3]=0 since X−μ is normally distributed with mean zero, then expand out the cube. If the distribution of a random variable X is symmetric about 0, meaning Pr(X>x)=Pr(X<−x) for every x>0, then its third moment, if it exists at all, must be 0, as must all of its odd-numbered moments.

Question4

A coin with probability for head as $\frac{1}{5}$ is tossed 100 times. The standard deviation of the number of head 5 turned up is.

(a) 3 (b) 2
(c) 4 (d) 6

Answer: a

Explanation:

Here n = 100

Probability of success (p) = $\frac{1}{5}$

Probability of failure (q) = 1-p

$= 1 - \frac{1}{5}$

$= \frac{1}{4}$

S.D. = $\sqrt{npq}$

$= \sqrt{100 \times \frac{1}{5} \times \frac{4}{5}}$

$= \sqrt{4 \times 4}$

= 4

JULY - 2021

Question 1

If is a Position variate such that I (x = 1) = 0.7, P (x =2) = 0.3, then P (x =0) =

(a) $e^{6/7}$ (b) $e^{-6/7}$

(c) $e^{-2/3}$ (d) $e^{-1/3}$

Answer: Options (b)

In a poisson variate

P(x = 1) = 0.7 and p (X = 2) 0.3

$\frac{e^{-m}.m^1}{1!} = 0.7$ $\frac{e^{-m}.m^2}{2!} = 0.3$

e^{-m}.m = 0.7 _______ (1) $e^{-m}\, m^2 = 0.3 \times 2!$

$e^{-m}\, m^2 = 0.3 \times 2$

$e^{-m}\, m^2 = 06$ _______ (2)

Eq (1) / eq (2)

$\frac{e^{-m}.m}{e^{-m}\, m^2} = \frac{0.7}{0.6}$

m = 6/7

Now p (x = 0)= $\frac{e^{-m}.m^0}{0!}$

$= \frac{e^{-\frac{6}{7}}.1}{1}$

$= e^{-6/7}$

Question 2

If X is a binomial variate with p = 1/3 for, the experiment of 90 trials, then the standard deviation is equal to

(a) $-\sqrt{5}$ (b) $\sqrt{5}$

(c) $2\sqrt{5}$ (d) $\sqrt{15}$

Answer: Options (c)

Explanation:

P if x ~ B (n, p)

Here n = 90, p = 1/3 q = 1 – p

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

S.D. = $\sqrt{npq}$

= $\sqrt{90 \times \frac{1}{3} \times \frac{2}{3}}$

= $\sqrt{20}$

S.D. = $2\sqrt{5}$

Question 3

For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of these mobiles and want to know the probability the Length of time will be between 50 and 70 hours is (Given $\Phi(1.33) = 0.9082$, $\Phi(0) = 0.5$)

(a) -0.4082 (b) 0.5

(c) 0.4082l (d) -0.5

Answer: Options (c)

Explanation:

Given,

μ=50 (mean)

σ=15 (standard/deviation)

find the probability for 50<x<70

Converting the problem in standard form

$Z = \frac{(x-\mu)}{\sigma}$

for x=50,

Z=0

For x=70,

Z= (70−50)/15=1.33

For finding the probability for 50<x<70

In the standard form 0<z<1.33

using Z-table, the area is equal to 0.4082

Question 4

In normal distribution mean, media and mode are:

(a) Zero (b) Not equal

(c) Equal (d) Null

Answer: Option (c)

Explanation:

In normal Distribution, Mean, Median and mode are equal.

Question 5

Which of the following diagram is the most appropriate to represents various heads in total cost?

(a) Pie chart
(b) Bar graph
(c) Multiple line chart
(d) Scatter plot

Answer: Option (c)

Explanation:

Pie chart is the most appropriate to represents various heads in total cost.