Chapter 15
Statistics

Exercise 15.1

Question 1

Find the mean deviation about the mean for the data in Exercises 1 and 2.
1. 4, 7, 8, 9, 10, 12, 13, 17

Solution:

First we have to find $(\bar{x})$of the given data

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{80}{8} = 10$$

So, the respective values of the deviations from mean,
i.e., $x_i - \bar{x}$ are, 10 – 4 = 6, 10 – 7 = 3, 10 – 8 = 2, 10 - 9 = 1,
10 – 10 = 0, 10 – 12 = - 2, 10 – 13 = - 3, 10 – 17 = - 7
6, 3, 2, 1, 0, -2, -3, -7
Now absolute values of the deviations,
6, 3, 2, 1, 0, 2, 3, 7

$$\therefore \sum_{i}^{8} |x_i - \bar{x}| = 24$$

MD = sum of deviations/ number of observations
= 24/8
= 3
So, the mean deviation for the given data is 3.

Question 2

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:

First we have to find $(\bar{x})$ of the given data

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = \frac{500}{10} = 50$$

So, the respective values of the deviations from mean,
i.e., xi – x are, 50̅ – 38 = -12, 50 -70 = -20, 50 – 48 = 2, 50 – 40 = 10, 50 – 42 = 8,
50 – 55 = - 5, 50 – 63 = - 13, 50 – 46 = 4, 50 – 54 = -4, 50 – 44 = 6
-12, 20, -2, -10, -8, 5, 13, -4, 4, -6
Now absolute values of the deviations,
12, 20, 2, 10, 8, 5, 13, 4, 4, 6

$\therefore \sum_{i=1}^{10} |x_i - \bar{x}| = 84$

MD = sum of deviations/ number of observations

= 84/10

= 8.4

So, the mean deviation for the given data is 8.4.

Question 3

Find the mean deviation about the median for the data in Exercises 3 and 4.
3. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:

First we have to arrange the given observations into ascending order,
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.
The number of observations is 12
Then,
Median = $((12/2)^{th}$ observation + $((12/2) + 1)^{th}$ observation)/2
$(12/2)^{th}$ observation = 6^{th} = 13
$(12/2) + 1)^{th}$ observation = 6 + 1
= 7^{th} = 14
Median = (13 + 14)/2
= 27/2
= 13.5
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are 3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

$\therefore \sum_{i=1}^{12} |X_i - M| = 28$

Mean Division,

M.D. (M) = $\frac{1}{12} \sum_{i=1}^{12} |X_i - M|$

= (1/12) × 28

= 2.33

So, the mean deviation about the median for the given data is 2.33.

Question 4

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:

First we have to arrange the given observations into ascending order,
36, 42, 45, 46, 46, 49, 51, 53, 60, 72.
The number of observations is 10
Then,
Median = $((10/2)^{th}$ observation + $((10/2) + 1)^{th}$ observation)/2
$(10/2)^{th}$ observation = 5^{th} = 46
$(10/2)+ 1)^{th}$ observation = 5 + 1

= 6 th = 49
Median = (46 + 49)/2
= 95
= 47.5
So, the absolute values of the respective deviations from the median, i.e., |xi – M| are 11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

$$\therefore \sum_{i=1}^{10} |X_i - M| = 70$$

Mean Deviation,

M.D. (M) = $\therefore \frac{1}{10}\sum_{i=1}^{10}|X_i - M|$
= (1/10) × 70
= 7

So, the mean deviation about the median for the given data is 7.

Question 5

Find the mean deviation about the mean for the data in Exercises 5 and 6.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution:

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		150

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 25 \sum_{i=1}^{5} f_i x_i = 350$$

Now we have to find ($\bar{x}$) by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i|x_i - \bar{x}| = 158$

Therefore M.D. $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i|x_i - \bar{x}|$
= (1/25) × 158
= 6.32

So, the mean deviation about the mean for the given data is 6.32.

Question 6

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution:

Let us make the table of the given data and append other columns after calculations.

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$x_i\lvert x_i - \bar{x}\rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 80 \sum_{i=1}^{5} f_ix_i = 4000$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_ix_i = \frac{1}{80} \times 4000 = 50$$

The absolute values of the deviations from the mean, i.e,$|x_i - \bar{x}|$, as shown in the table

From the table, $\sum_{i=1}^{5} f_i|x_i - \bar{x}| = 1280$

Therefore M.D $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i|x_i - \bar{x}|$

$= (1/80) \times 1280$

$= 16$

So, the mean deviation about the mean for the given data is 16.

Question 7

Find the mean deviation about the median for the data in Exercises 7 and 8.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution:

Let us make the table of the given data and append other columns after calculations.

x_i	f_i	c.f.	$\lvert x_i - M\rvert$	$f_i\lvert x_i - M\rvert$
5	8	8	2	16
7	6	14	0	0

9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48

Now, N = 26, which is even.
Median is the mean of the 13th and 14th observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.
Then,
Median = (13th observation + 14th observation)/2
= (7 + 7)/2
= 14/2
= 7
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.
Therefore $\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i |x_i - M| = 84$
And M.D. (M) $= \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$
= (1/26) × 84
= 3.23
Hence, the mean deviation about the median for the given data is 3.23.

Question 8

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution:

x_i	f_i	c.f.	$\lvert x_i - \bar{x}\rvert$	$x_i\lvert x_i - \bar{x}\rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40

Now, N = 29, which is odd
So 29/2 = 14.5
The cumulative frequency for greater than 14.5 is 21, for which the corresponding observation is 30.
Then,
Median = (15th observation + 16th observation)/2
= (30 + 30)/2
= 60/2
= 30
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{5} f_i = 29$ and $\sum_{i=1}^{5} f_i |x_i - M| = 148$

And M.D. (M) $= \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$

= (1/29) × 148

= 5.1

Hence, the mean deviation about the median for the given data is 5.1.

Question 9

Find the mean deviation about the mean for the data in Exercises 9 and 10.

Income per day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of person	4	8	9	10	7	5	4	3

Solution:

Let us make the table of the given data and append other columns after calculations.

Income per day in ₹	Number of person f_i	Mid- point x_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	5	550	2750	192	960
600-700	4	650	2600	292	1160
700-800	3	750	2250	392	1176
	50		17900		7896

The sum of calculated data,

$$N = \sum_{i=1}^{8} f_i = 50, \sum_{i=1}^{8} f_i x_i = 17900$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

The absolute values of the deviations from the mean, i.e, $|x_i - \bar{x}|$, as shown in the table

So, $\sum_{i=1}^{8} f_i|x_i - \bar{x}| = 7896$

And M.D $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i|x_i - \bar{x}|$

= (1/50) × 7896

= 157.92

Hence, the mean deviation about the mean for the given data is 157.92.

Question 10

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Solution:

Let us make the table of the given data and append other columns after calculations.

Height in cms	Number of boy f_i	Mid- point x_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247
	100		12530		1128.8

The sum of calculated data,

$$N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

The absolute values of the deviations from the mean, i.e,$\lvert x_i - \bar{x}\rvert$, as shown in the table

So, $\sum_{i=1}^{6} f_i\lvert x_i - \bar{x}\rvert = 1128.8$

And M.D $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i\lvert x_i - \bar{x}\rvert$

$= (1/100) \times 1128.8$

$= 11.28$

Hence, the mean deviation about the mean for the given data is 11.28.

Question 11

Find the mean deviation about median for the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of girls	6	8	14	16	4	2

Solution:

Let us make the table of the given data and append other columns after calculations.

Marks	Number of girls f_i	Cumulative frequency (c.f)	Mid – point x_i	$\lvert x_i - Med \rvert$	$f_i \lvert x_i - Med \rvert$
0-10	6	6	5	22.85	137.1
10-20	8	14	15	12.85	102.8
20-30	14	28	25	2.85	39.9
30-40	16	44	35	7.15	114.4
40-50	4	48	45	17.15	68.6
50-60	2	50	55	27.15	54.3
	50				517.1

The class interval containing $N^{th}/2$ or 25^{th} item is 20-30
So, 20-30 is the median class.
Then,
Median = l + (((N/2) - c)/f) × h
Where, l = 20, c = 14, f = 14, h = 10 and n = 50
Median = 20 + (((25 - 14))/14) × 10
= 20 + 7.85
= 27.85
The absolute values of the deviations from the median, i.e,$\lvert x_i - Med \rvert$, as shown in the table
So, $\sum_{i=1}^{6} f_i \lvert x_i - Med. \rvert = 517.1$
And $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i \lvert x_i - Med. \rvert$
$= (1/50) \times 517.1$
$= 10.34$
Hence, the mean deviation about the median for the given data is 10.34.

Question 12

Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

[Hint Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval]

Solution:

The given data is converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding the 0.5 to the upper limit of each class intervals and append other columns after calculations

Age	Number f_i	Cumulative frequency (c.f)	Mid – point x_i	$\lvert x_i - Med\rvert$	$f_i\lvert x_i - Med\rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	100				735

The class interval containing $N^{th}/2$ or 50^{th} item is 35.5 – 40.5
So, 35.5 – 40.5 is the median class.
Then,
Median = $l + (((N/2) - c)/f) \times h$
Where, l = 35.5, c = 37, f = 26, h = 5 and N = 100
Median = $35.5 + (((50 - 37))/26) \times 5$
$= 35.5 + 2.5$
$= 38$
The absolute values of the deviations from the median, i.e,$\lvert x_i - Med\rvert$, as shown in the table
So, $\sum_{i=1}^{8} f_i\lvert x_i - Med.\rvert = 735$
And M.D (M) $= \frac{1}{N}\sum_{i=1}^{6} f_i\lvert x_i - Med.\rvert$
$= (1/00) \times 735$
$= 7.35$
Hence, the mean deviation about the median for the given data is 7.35.

Exercise 15.2

Question 1

Find the mean and variance for each of the data in Exercise 1 to 5.
6, 7, 10, 12, 13, 4, 8, 12

Solution:

We have,
Mean $\bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$
Where, n = number of observation
$\sum_{i=1}^{a} x_i$ = sum of total observation
So, $\bar{x} = (6+7+10+12+13+4+8+12)/8$
$= 72/8$
$= 9$
Let us make the table of the given data and append other columns after calculations.

X_i	Deviations from mean$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	6-9=-3	9
7	7-9=-2	4
10	10-9=1	1
12	12-9=3	9
13	13-9=4	16
4	4-9=-5	25
8	8-9=-1	1
12	12-9=3	9
		74

We know that Variance,

$$\sigma^2 = \frac{1}{n}\sum_i^a (x_i - \bar{x})^2$$

$\sigma^2 = (1/8) \times 74$

$= 9.2$

∴Mean = 9 and Variance = 9.25

Question 2

First n natural numbers

Solution:

We know that Mean = Sum of all observations/Number of observations

∴Mean, $\bar{x}$ = ((n (n + 1)) 2)/n

= (n + 1)/2

and also WKT Variance,

$$\sigma^2 = \frac{1}{n}\sum_i^a (x_i - \bar{x})^2$$

By substitute that value of $\bar{x}$ we get,

$$= \frac{1}{n}\sum_{i=1}^{n} x_i \left(x_i - \frac{n+1}{2}\right)^2$$

We know that $(a-b)^2 = a^2-2ab+b^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \frac{1}{n}\sum_{i=1}^{n} 2x_i\left(\frac{n+1}{2}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

Substituting the summation values

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{n+1}{n}\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

Multiplying and Computing

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

By taking LCM and simplifying, we get

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

By taking (n+1) common from each term, we get

$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$

$= \frac{(n+1)(n-1)}{12}$

WKT $(a+b)(a-b) = a^2 - b^2$

$\sigma^2 = (n^2-1)/12$

∴Mean = $(n+1)/2$ and Variance = $(n^2-1)/12$

Question 3

First 10 multiples of 3

Solution:

First we have to write the first 10 multiples of 3, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

We have,

Mean = $\bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\bar{x}$ = (3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30)/10

= 165/10

= 16.5

Let us make the table of the data and append other columns after calculations.

X_i	Deviation from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	3-16.5=-13.5	182.25
6	6-16.5=-7.5	110.25
9	9-16.5=-7.5	56.25
12	12-16.5=-4.5	20.25
15	15-16.5=-1.5	2.25
18	18-16.5=1.5	2.25
21	21-16.5=4.5	20.25
24	24-16.5=7.5	56.25
27	27-16.5=10.5	110.25
30	30-16.5=13.5	182.25
		742.5

Then, Variance

$\sigma^2 = \frac{1}{n}\sum_{i}^{a}(x_i - \bar{x})^2$

= (1/10)× 742.5

= 74.25

∴Mean = 16.5 and Variance = 74.25

Question 4

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution:

Let us make the table of the given data and append other columns after calculations.

x_i	f_i	f_ix_i	Deviation from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	6-9=13	169	338
10	4	40	10-19=-9	81	324
14	7	98	14-19=-5	25	175
18	12	216	18-19=-1	1	12
24	8	192	24-19=5	25	200
28	4	112	28-19=9	81	324
30	3	90	30-19=11	121	363
	N=40	760			1736

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 760/70

= 19

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_i^a f_i(x_i - \bar{x})^2$

= (1/40)× 1736

= 43.4

∴Mean = 19 and Variance = 43.4

Question 5

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:

x_i	f_i	f_ix_i	Deviation from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	92-100=-8	64	192
93	2	186	93-100=-7	49	98
97	3	291	97-100=-3	9	27
98	2	196	98-100=-2	4	8

102	6	612	102-100=2	4	24
104	3	312	104-100=4	16	48
109	3	327	109-100=9	81	243
	N=22	2200			640

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x} = 2200/22$

= 100

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i}^{a}(x_i - \bar{x})^2$

= (1/22)× 640

= 29.09

∴Mean = 100 and Variance = 29.09

Question 6

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution:

Let the assumed mean A = 64. Here h = 1

We obtain the following table from the given data.

x_i	Frequency f_i	$Y_i=(x_i - A)/h$	Y_i^2	f_iy_i	f_iy_i
60	2	-4	16	-8	38
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	39
68	5	4	16	20	80
				0	286

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where A = 64, h = 1

So, $\bar{x}$ = 64 + ((0/100) × 1)

= 64 + 0

= 64

Then, variance,

$\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i {y_i}^2 - (\sum f_i y_i)^2\right]$

$\sigma^2 = (1^2/100^2)[100(286) - 0^2]$

= (1/10000)[28600 – 0]

= 2.86

Hence, standard deviation = σ = √2.886

= 1.691

∴ Mean = 64 and Standard Deviation = 1.691

Question 7

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution:

Let us make the table of the given data and append other columns after calculations.

Classes	Frequency f_i	Mid – point x_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0-30	2	15	30	-92	8464	16928
30-60	3	45	135	-62	3844	11532
60-90	5	75	375	-32	1024	5120
90-120	10	105	1050	-2	4	40
120-150	3	135	405	28	784	2352
150-180	5	165	825	58	3364	16820
180-210	2	195	390	88	7744	15488
	N =30		3210			68280

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x} = 3210/30$

= 107

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_i^a f_i (x_i - \bar{x})^2$

= (1/30)× 68280

= 2276

∴Mean = 107 and Variance = 2276

Question 8

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:

Let us make the table of the given data and append other columns after calculations.

Classes	Frequency f_i	Mid – point x_i	f_ix_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0-10	5	5	25	-22	484	2420
10-20	8	15	120	-12	144	1152
20-30	15	25	375	-2	4	60
30-40	16	35	560	8	64	1024
40-50	6	45	270	18	324	1944
	N = 50		1350			6600

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 1350/50

= 27

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_i^a f_i(x_i - \bar{x})^2$

= (1/50)× 6600

= 132

∴Mean = 27 and Variance = 132

Question 9

Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	150-110	110-115
Frequencies	3	4	7	7	15	9	6	6	3

Solution:

Let the assumed mean, A = 92.5 and h = 5

Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children Frequency f_i	Midpoint X_i	$Y_i=(x_i - A)/h$	Y_i^2	Y_i^2	$f_iy_i^2$
70-75	2	72.5	-4	16	-12	48
75-80	1	77.5	-3	9	-12	36
80-85	12	82.5	-2	4	-14	28
85-90	29	87.5	-1	1	-7	7
90- 95	25	92.5	0	0	0	0
95-100	12	97.5	1	1	9	9
100-105	10	102.5	2	4	12	24

105-110	4	107.5	3	9	18	54
110-115	5	112.5	4	6	12	48
	N = 60				6	254

, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where, A = 92.5, h = 5

So, $\bar{x}$= 92.5 + ((6/60) × 5)

= 92.5 + ½

= 92.5 + 0.5

= 93

Then, Variance,

$\sigma^2 = \frac{h^2}{N^2}[N \sum f_i {y_i}^2 - (\sum f_i {y_i}^2]$

σ 2 = (5^2 /60^2) [60(254) – 6^2]

= (1/144) [15240 - 36]

= 15204/144

= 1267/12

= 105.583

Hence, standard deviation = σ = √105.583

= 10.275

∴ Mean = 93, variance = 105.583 and Standard Deviation = 10.275

Question 10

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.
[Hint first make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Solution:

Let the assumed mean, A = 42.5 and h = 4

Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children (Frequency f_i)	Midpoint x_i	$Y_i=(x_i – A)/h$	Y_i^2	Y_i^2	$f_i y_i^2$
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	48.5	-1	1	-17	17
40.5-44.5	21	42.5	-0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100

	N = 100				25	199

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where, A = 42.5, h = 4

So, $\bar{x}$= 42.5 + (25/100) × 4

= 42.5 + 1

= 43 .5

Then, Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N \sum f_i {y_i}^2 - (\sum f_i {y_i}^2]$$

σ 2 = (4^2 /100^2) [100(199) – 25^2]

= (1/625) [19900 - 625]

= 19275/625

= 771/25

= 30.84

Hence, standard deviation = σ = √30.84

= 5.553

∴ Mean = 43.5, variance = 30.84 and Standard Deviation = 5.553.

Exercise 15.3

Question 1

From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution:-

For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater C.V. is said to be more variable than the other.

The series having lesser C.V. is said to be more consistent than the other.

Co-efficient of variation (C.V.) = (σ/ $\bar{x}$)× 100

Where, σ = standard deviation, $\bar{x}$= mean

For Group A

Marks	Group A f_i	Midpoint X_i	$Y_i=(x_i – A)/h$	Y_i^2	Y_i^2	$f_i y_i^2$
10-20	9	15	((15-45)/10=-3	$(-3)^2$= 9	-27	81
20-30	17	25	((25-45)/10))=-2	$(-2)^2$= 4	-34	68
30-40	32	35	((35-45)/10))=-1	$(1)^2$= 1	-32	32

40-50	33	45	((45-45)/10))=0	0^2=	0	0
50-60	40	55	((55-45)/10))=1	$1^2 = 1$	40	40
60-70	10	65	((65-45)/10))=2	$2^2 = 4$	20	40
70-80	9	75	((75-45)/10))=3	$3^2 = 9$	27	81
Total	150				-6	342

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A =45,

And $y_i = (x_i - A)/h$

Here h = class size = 20 – 10

h = 10

So, $\bar{x}$ = 45 + ((-6/150) × 10)

= 45-0.4

= 44.6

Then, Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\sum f_i {y_i}^2 - (\sum f_i {y_i}^2]$$

σ 2 = $(10^2 /150^2)$ $[150(342) – (-6)^2]$

= (100/22500) [51,300 -36]

= (100/22500)× 51264

= 227.84

Hence, standard deviation = $\sigma = \sqrt{227.84}$

= 15.09

∴C.V for group A = $(\sigma/ \bar{x})$× 100

= (15.09/44.6) × 100

= 33.83

Now, for group B.

Marks	Group B f_i	Midpoint X_i	$Y_i=(x_i – A)/h$	Y_i^2	Y_i^2	$f_i y_i^2$
10-20	10	15	((15-45)/10=-3	$(-3)^2$= 9	-30	90
20-30	20	25	((25-45)/10))=-2	$(-2)^2$= 4	-40	80
30-40	30	35	((35-45)/10))=-1	$(1)^2$= 1	-30	30
40-50	25	45	((45-45)/10))=0	0^2=	0	0
50-60	43	55	((55-45)/10))=1	$1^2 = 1$	43	43
60-70	15	65	((65-45)/10))=2	$2^2 = 4$	30	60

70-80	7	75	((75-45)/10))=3	$3^2 = 9$	21	63
Total	150				-6	366

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$
Where A =45,
h = 10
So, $\bar{x}$ = 45 + ((-6/150) × 10)
= 45-0.4
= 44.6
Then, Variance,
$\sigma^2 = \frac{h^2}{N^2}[N \sum f_i {y_i}^2 - (\sum f_i {y_i}^2]$
σ 2 = $(10^2 / 150^2)$ [150(366) – $(-6)^2$]
= (100/22500) [54,900 -36]
= (100/22500)× 54,864
= 243.84
Hence, standard deviation = σ = √243.84
= 15.61
∴ C.V for group A = (σ/ $\bar{x}$)× 100
= (15.61/44.6) × 100
= 35
By comparing C.V. of group A and group B.
C.V of Group B > C.V. of Group A
So, Group B is more variable.

Question 2

From the prices of shares X and Y below, find out which is more stable in value:

X	35	54	52	53	56	58	52	50	51	49
y	108	107	105	105	106	107	104	103	104	101

Solution:

From the given data,
Let us make the table of the given data and append other columns after calculations.

X(x_i)	Y(y_i)	X_i^2	Y_i^2
35	108	1125	11664
54	107	2916	11449
52	105	2704	11025
53	105	2809	11025
56	106	8136	11236
58	107	3364	11449
52	104	2704	10816
50	103	2500	10609

51	104	2601	10816
49	101	2401	10201
Total = 510	1050	26360	110290

We have to calculate Mean for x,
Mean $\bar{x} = \sum x_i/n$
Where, n = number of terms
= 510/10
= 51
Then, Variance for x $= \frac{1}{n^2}[N\sum x_i^2 - (\sum x_i)^2]$
$= (1/10^2)[(10 \times 26360) - 510^2]$
= (1/100) (263600 - 260100)
= 3500/100
= 35
WKT Standard deviation = √variance
= √35
= 5.91
So, co-efficient of variation = (σ/ x̄) × 100
= (5.91/51) × 100
= 11.58
Now, we have to calculate Mean for y,
Mean $\bar{y} = \sum y_i/n$
Where, n = number of terms
= 1050/10
= 105
Then, Variance for y $= \frac{1}{n^2}[N\sum y_i^2 - (\sum y_1)^2]$
$= (1/10^2)[(10 \times 110290) - 1050^2]$
= (1/100) (1102900 - 1102500)
= 400/100
= 4
WKT Standard deviation = √variance
= √4
= 2
So, co-efficient of variation = (σ/ x̄) × 100
= (2/105) × 100
= 1.904
By comparing C.V. of X and Y.
C.V of X > C.V. of
So, Y is more stable than X.

Question 3

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wages earners	586	648

Mean of monthly wages	Rs 5253	Rs 5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?
(ii) Which firm, A or B, shows greater variability in individual wages?

Solution:

From the given table,
Mean monthly wages of firm A = Rs 5253
and Number of wage earners = 586
Then,
Total amount paid = 586 × 5253
= Rs 3078258
Mean monthly wages of firm B = Rs 5253
Number of wage earners = 648
Then,
Total amount paid = 648 × 5253
= Rs 34, 03,944
So, firm B pays larger amount as monthly wages.
(ii) Variance of firm A = 100
We know that, standard deviation (σ) = $\sqrt{100}$
=10
Variance of firm B = 121
Then,
Standard deviation (σ) =$\sqrt{(121)}$
=11
Hence the standard deviation is more in case of Firm B that means in firm B there is greater variability in individual wages.

Question 4

The following is the record of goals scored by team A in a football session:

No. of goals scored	**0**	**7**	**2**	**3**	**4**
No. of matches	**1**	**9**	**7**	**5**	**3**

For the team B, mean number of goals scored per match was 2 with standard deviation 1.25 goals. Find which team may be considered more consistent?

Solution:

From the given data,
Let us make the table of the given data and append other columns after calculations.

Number of goals scored x_i	Number of matches f_i	f_ix_i	X_i^2	$f_ix_i^2$

0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
Total	25	50		130

First we have to calculate Mean for team A,

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{50}{25} = 2$

Then,

Variance $= \frac{1}{N^2}[N \sum f_i {y_i}^2 - (\sum f_i y_i)^2]$

$$= \frac{1}{25^2}[25 \times 130 - 2500] = \frac{750}{625} = 1.2$$

We know that, Standard deviation σ = √variance =√1.2 = 1.09

Hence co-efficient of variation of team A,

C.V.$_A \frac{\sigma}{\bar{x}} \times 100 = \frac{1.09}{2} \times 100 = 54.5$

For team B

Given, $\bar{x} = 2$

Standard deviation σ = 1.25

So, co-efficient id variation of team B,

$\Rightarrow$ C. V.$_B = \frac{1.25}{2} \times 100 = 62.5$

Since C.V. of firm B is greater

∴ Team A is more consistent.

Question 5

The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} {x_i}^2 = 902.8, \sum_{i=1}^{50} y_i = 261, \sum_{i=1}^{50} {y_i}^2 = 1457.6$$

Which is more varying, the length or weight?

Solution:

First we have to calculate Mean for Length x

Mean = $\bar{x} = \frac{\sum x_i}{n} = \frac{212}{50} = 4.24$

Then,

Variance = $\frac{1}{N^2}[N \sum f_i {x_i}^2 - (\sum f_i {x_i}^2]$

= (1/50²)[(50×902.8)- 212²]

= (1/2500)(45140-44944)

= 196/2500

=0.0784

We know that, Standard deviation σ =√variance

$=\sqrt{0.0784}$

$= 0.28$

Hence co-efficient of variation of team A,

C.V.$_x = \frac{\sigma}{\bar{x}} \times 100 = \frac{0.28}{4.24} \times 100 = 6.603$

Now we have to calculated mean of Weight y

$\bar{y} = \sum y_i/n$

$=261/50$

$=5.22$

Then,

Variance $= \left(\frac{1}{N^2}\right) [(N\sum f_i y_i^2) - (\sum f_i y_i)^2$

$= (1/50^2)[(50\times1457.6)- 261^2]$

$= (1/2500)(72880-68121)$

$= 4759/2500$

$=1.9036$

We know that, Standard deviation $\sigma = \sqrt{\text{variance}}$

$=\sqrt{1.9036}$

$= 1.37$

So, co-efficient of variation of team B,

C.V.$_y = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.37}{5.22} \times 100 = 26.24$

Since C.V of firm weight y is greater

∴ Weight is more varying.

Miscellaneous Exercise

Question 1

The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

From the question is given that,

Variance of eight observations 9 and 9.25.

There are six observations given 6, 7, 10, 12, 12, and 13

Let us assume the remaining two observation to be x and y respectively such that,

Observations 6, 7, 10, 12, 12, 13, x, y.

We have calculated the mean of given observations,

∴Mean, $\bar{x} = \frac{6+7+10+12+12+13+x+y}{8}$

$$9 = \frac{6 + 7 + 10 + 12 + 12 + 13 + x + y}{8}$$

$60+x+y=72$

$x+y =12$ ... [we call it as as equation (i)]

Now, Variance $= \frac{1}{n}\sum_{i=1}^{8}(x_i - x)^2$

$9.25 = \frac{1}{8}[(-3)^2 + (-2)^2 + 1^2 + 3^2 + 4^2 + x^2 + y^2 - 18(x+y) + 2 \times 9^2]$

By using equation (i) substitute 12 instead of (x+y)

$9.25 = \frac{1}{8}[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18 \times 12 + 162]$

$9.25 = \frac{1}{8}[48 + x^2 + y^2 - 216 + 162]$

$9.25 = \frac{1}{8}[x^2 + y^2 - 6]$

$x^2 + y^2 = 80$ ... [we call it as equation (ii)]

So, from equation (i) we have:

$$x^2 + y^2 = 2xy = 144$$

Thus from (ii) and (iii), we have

2xy = 64 (iv)

Now by subtracting (iv) from (ii), we get:

$x^2 + y^2 - 2xy = 80 - 64$

$$x - y = \pm 4(v)$$

Hence, from equation (i) and (v) we have:

When x - y = 4

Then, x = 8 and y = 4

And, when x – y = -4

Then, x = 4 and y = 8

∴ The remaining observations are 4 and 8

Question 2

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

Solution:

From the question is given that,

Variance of seven observations 8 and 16.

There are 6 observations given 2, 4, 10, 12, and 14

Let us assume the remaining two observation to be x and y respectively such that,

Observations 2, 4, 10, 12, 14, x, y.

We have calculated the mean of given observations,

∴Mean, $\bar{x} = \frac{2+4+10+12+14+x+y}{7}$

x+y =14 .... [we call it as equation (i)]

In the question it is given that,

Variance = 16

We know that,

Variance = $\frac{1}{n}\sum_{i=1}^{n}(x_i - x)^2$

$16 = \frac{1}{7}[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2 \times 8(x+y) + 2 \times (8)^2]$

By using equation (i) substitute 14 instead of (x+y)

$16 = \frac{1}{7}[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)]$

$16 = \frac{1}{7}[12 + x^2 + y^2]$
$x^2 + y^2 = 112 - 12$
$x^2 + y^2 = 100$ ... [we call it as equation (ii)]
So, from equation (i) we have:
$x^2 + y^2 = 2xy = 196$ ... [we call it as equation (iii)]
Thus from equation (ii) and (iii), we have
$2xy = 196 – 100$
$2xy = 96$ (iv)
Now by subtracting (iv) from (ii), we get:
$x^2 + y^2 - 2xy = 100 - 96$
$(x - y)^2 = 4$

$$x - y = \pm 2 \text{ (v)}$$

Hence, from equation (i) and (v) we have:
When x - y = 2
Then, x = 8 and y = 6
And, when x – y = -2
Then, x = 2 and y = 8
∴The remaining observations are 6 and 8

Question 3

The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:

From the question it is given that,
Mean of six observations = 8
Standard deviation of six observations = 4
Let us assume the observations be $x_1, x_2, x_3, x_4, x_5, x_6$
So, mean of assumed observations,

$$\therefore \text{Mean } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

Then, as per the question if each observation is multiplied by 3 and the resulting observations are y_i then, we have:
$Y_i = 3x_i$
Hence, $x_i = \frac{1}{3} y_i$ (For i = 1 to 6)

$$\therefore \text{New mean}, \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$= 3 \times 8$
$= 24$
We know that,
Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \bar{x})^2}$
By squaring on the both sides

$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$

$\sum_{i=1}^{6}(x_i - \bar{x})^2 = 96 (ii)$

Hence, from (i) and (ii) we have:

$\bar{y} = 3\bar{x}$

$\bar{x} = \frac{1}{3}\bar{y}$

Now, by substituting the values of x_i and $\bar{x}$ in (ii) we have:

$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}\bar{y}\right)^2 = 96$

Thus, $\sum_{i=1}^{6}(y_i - \bar{y})^2 = 864$

So, the variance of new observation = $(1/6) \times 864$

= 144

Therefore, standard deviation of new observation = $\sqrt{144}$

= 12

Question 4

Given that $\bar{x}$ is the mean and σ2 is the variance of n observations $x_1, x_2, ..., x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, ..., ax_n$ are $a\bar{x}$ and $a^2 \sigma^2$, respectively, ($a \neq 0$).

Solution:

From the question it is given that, n observations are $x_1, x_2, ...x_n$

Mean of the n observation = $\bar{x}$

Variance of the n observation = σ^2

As we know that,

Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_i(x_i - \bar{x})^2$ ..... [equation (i)]

As per the condition given in the question, if each of the observation is being multiplied by 'a' and the new observation are y_i the, we have:

$$y_i = ax_i$$

Thus, $x_i = \frac{1}{a}y_i$

$\therefore \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} ax_i$

$\bar{y} = \frac{a}{n}\sum_{i=1}^{n} x_i$

$\bar{y} = a\bar{x}$

Therefore, mean of the observations ax_1, ax_2 ...... ax_n is $a\,\bar{x}$

Now, by substituting the values of x_i and $\bar{x}$in equation (i), we get:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_i - \frac{1}{a}\bar{y}\right)^2$$

$$a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

∴ the variance of the given observation $ax_i, ax_2, \ldots ax_n$ is $a^2\sigma^2$

Question 5

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect.
Calculate the correct mean and standard deviation in each of the following cases: (i) If wrong item is omitted. (ii) If it is replaced by 12

Solution:

(i) If wrong item is omitted,
From the question it is given that,
The number of observations i.e. n = 20
The incorrect mean = 20
The incorrect standard deviation = 2

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{20} X_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} X_i$$

$$\sum_{i=1}^{20} X_i = 200$$

By the calculation the incorrect sum of observation = 200
Hence, correct sum of observations = 200 -8
= 192
Therefore the correct mean = correct sum/19
= 192/19
= 10.1

We know that, Standard deviation (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2}$

$$2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - (\bar{x})^2}$$

$$4 = \frac{1}{20}\text{ Incorrect }\sum_{i=1}^{n} {x_i}^2 - 100$$

Incorrect $\sum_{i=1}^{n} {x_i}^2 = 2080$
Therefore, correct $\sum_{i=1}^{n} {x_i}^2$ = Incorrect $\sum_{i=1}^{n} {x_i}^2 - (8)^2$

= 2080-64
= 2016

Finally we came to calculate correct standard deviation,

Hence, correct standard deviation $= \sqrt{\frac{\text{Correct } \sum X_1{}^2}{n} - (\text{correct Mean})^2}$

$= \sqrt{\frac{2016}{19} - (10.1)^2}$

$= \sqrt{1061.1 - 102.1}$

= 2.02

(ii) If it is replaced by 12,
From the question it is given that,
The number of incorrect sum observations i.e. n = 200
The correct sum of observations n = 200 – 8 + 12
n = 204
Then, correct mean = correct sum/20
= 204/20
= 10.2

Standard deviation (σ) $= \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2}$

$$2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - (\bar{x})^2}$$

$$4 = \frac{1}{20} \text{ Incorrect} \sum_{i=1}^{n} x_i{}^2 - 100$$

Incorrect $\sum_{i=1}^{n} x_i{}^2 = 2080$

Thus, correct $\sum_{i=1}^{n} x_i{}^2$ = Incorrect $\sum_{i=1}^{n} x_i{}^2 - (8)^2 + (12)^2$
= 2080-64 + 144
= 2160

Hence, correct standard deviation $= \sqrt{\frac{\text{Correct } \sum X_1{}^2}{n} - (\text{correct Mean})^2}$

$= \sqrt{\frac{2160}{20} - (10.2)^2}$

$= \sqrt{108 - 104.04}$

$= \sqrt{3.96}$

= 1.98

Question 6

The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry

Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solution:

From the question it is given that,
Mean of Mathematics = 42
Standard deviation of Mathematics = 12
Mean of Physics = 32
Standard deviation of physics = 15
Mean of Chemistry = 40.9
Standard deviation of chemistry = 20
As we know that,
Coefficient of variation (C.V) $= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$
Then,
C.V. in Mathematics = (12/42) × 100
= 28.57
C.V. in Mathematics = (15/32) × 100
= 46.87
C.V. in Mathematics = (20/40.9) × 100
= 48.89
Hence, subject with highest variability in marks is chemistry as subject with the greater C.V is more variable than others

Question 7

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution:

From the question it is given that,
The total number of observations (n) = 100
Incorrect mean, $(\bar{x}) = 20$
And, Incorrect standard deviation (σ) = 3

$$\therefore 20 = \frac{1}{100}\sum_{i=1}^{100} x_1$$

By cross multiplied, we get

$$\sum_{i=1}^{100} x_1 = 20 \times 100$$

$$\sum_{i=1}^{100} x_1 = 2000$$

Hence, incorrect sum of observation is 2000

Now, correct sum of observation = 2000-21-21-18

= 2000-60

=1940

Therefore correct Mean = Correct sum/ (100-3)

=1940/97

=20

We know that, Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_1 - \frac{1}{n^2}(\sum_{i=1}^{n} x)^2}$

$$3 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_1}^2 - (\bar{x})^2}$$

$$3 = \sqrt{\frac{1}{100} \times \text{Incorrect} \sum {x_1}^2 - (20)^2}$$

Incorrect $\sum {x_1}^2 = 100(9 + 400)$

Incorrect $\sum {x_1}^2 = 40900$

Correct $\sum_{i=1}^{n} {x_1}^2 =$ Incorrect $\sum_{i=1}^{n} {x_1}^2 - (21)^2 - (21)^2 - (18)^2$

= 40900 – 441 – 441 – 324

= 40900 -1206

= 39694

Hence, correct standard deviation $= \sqrt{\frac{Correct \sum {X_1}^2}{n} - (\text{correct Mean})^2}$

$$= \sqrt{\frac{39694}{97} - (20)^2}$$

$$= \sqrt{409.216 - 400}$$

= 3.036