

Chapter 14 Mathematical Reasoning

Exercise 14.1

Question 1

Which of the following sentences are statements? Give reasons for your answer.

- (i) There are 35 days in a month.
- (ii) Mathematics is difficult.
- (iii) The sum of 5 and 7 is greater than 10.
- (iv) The square of a number is an even number.
- (v) The sides of a quadrilateral have equal length.
- (vi) Answer this question.
- (vii) The product of (-1) and 8 is 8.
- (viii) The sum of all interior angles of a triangle is 180° .
- (ix) Today is a windy day. (x) All real numbers are complex numbers.

Solution:

- (i) The maximum number of days in a month is 31, so this sentence is incorrect. Therefore it is a statement
- (ii) This sentence is subjective. For some people, Mathematics can be easy and for some others it can be difficult. Therefore it is not a statement
- (iii) The sum of 5 and 7 is 12 and it is greater than 10. Therefore this sentence is always correct. Hence, it is a statement
- (iv) This sentence can be sometimes correct and sometimes incorrect. For example, the square of 2 is even number but the square of 3 is odd number. Hence, it is not a statement
- (v) This sentence can be sometimes correct and sometimes incorrect. For example, square and rhombus have sides of equal lengths whereas trapezium and rectangle have sides of unequal lengths. Therefore it is not a statement
- (vi) It is an order. Hence, it is not a statement
- (vii) The given sentence is incorrect because the product of (-1) and 8 is -8 . Hence, it is a statement
- (viii) The given sentence is correct and therefore, it is a statement
- (ix) The given sentence is not a statement because the day that is being referred to is not evident from the sentence. (x) The given sentence is always correct because all real numbers can be written as $a \times 1 + 0 \times i$. Hence, it is a statement.

Question 2

Give three examples of sentences which are not statements. Give reasons for the answers.

Solution:

The three examples of sentences, which are not statements are given below:

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(i) He is a doctor

In the given sentence, it is not evident as to whom 'he' is referred to. Hence, it is not a statement

(ii) Geometry is difficult

For some people, geometry can be easy and for some others, it can be difficult. Hence, this is not a statement

(iii) Where is she going?

In this question, it is not evident as to whom 'she' is referred to. Hence, it is not a statement

Exercise 14.2

Question 1

Write the negation of the following statements:

(i) Chennai is the capital of Tamil Nadu.

(ii) $\sqrt{2}$ is not a complex number.

(iii) All triangles are not equilateral triangle.

(iv) The number 2 is greater than 7.

(v) Every natural number is an integer

Solution

(i) Chennai is not the capital of Tamil Nadu

(ii) $\sqrt{2}$ is a complex number

(iii) All triangles are equilateral triangles

(iv) The number 2 is not greater than 7

(v) Every natural number is not an integer

Question 2

Are the following pairs of statements negations of each other?

(i) The number x is not a rational number.

The number x is not an irrational number.

(ii) The number x is a rational number.

The number x is an irrational number

Solution:

(i) The negation of the first statement is 'the number x is a rational number'.

This is same as the second statement because if a number is not an irrational number then the number is a rational number Hence; the given statements are negations of each other

(ii) The negation of the first statement is 'the number x is not a rational number.

This means that the number x is an irrational number which is same as the second statement. Hence, the given statements are negations of each other

Question 3

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Find the component statements of the following compound statements and check whether they are true or false.

- (i) Number 3 is prime or it is odd.**
- (ii) All integers are positive or negative.**
- (iii) 100 is divisible by 3, 11 and 5.**

Solution:

(i) The component statements are

(a) Number 3 is prime

(b) Number 3 is odd

Here, both the statements are true

(ii) The component statements are

(a) All integers are positive

(b) All integers are negative

Here, both the statements are false.

(iii) The component statements are

(a) 100 is divisible by 3

(b) 100 is divisible by 11

(c) 100 is divisible by 5

Here, the statements (a) and (b) are false and (c) is true.

Exercise 14.3

Question 1

For each of the following compound statements first identify the connecting words and then break it into component statements.

(i) All rational numbers are real and all real numbers are not complex.

(ii) Square of an integer is positive or negative.

(iii) The sand heats up quickly in the Sun and does not cool down fast at night.

(iv) $x = 2$ and $x = 3$ are the roots of the equation $3x^2 - x - 10 = 0$.

Solution:

(i) In this sentence 'and' is the connecting word

The component statements are as follows

(a) All rational numbers are real

(b) All real numbers are not complex

(ii) In this sentence 'or' is the connecting word

The component statements are as follows

(a) Square of an integer is positive

(b) Square of an integer is negative

(iii) In this sentence 'and' is the connecting word

The component statements are as follows

(a) The sand heats up quickly in the Sun

(b) The sand does not cool down fast at night

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(iv) In this sentence 'and' is the connecting word

The component statements are as follows

(a) $x = 2$ is the root of the equation $3x^2 - x - 10 = 0$

(b) $x = 3$ is the root of the equation $3x^2 - x - 10 = 0$

Question 2

Identify the quantifier in the following statements and write the negation of the statements.

(i) There exists a number which is equal to its square.

(ii) For every real number x , x is less than $x + 1$.

(iii) There exists a capital for every state in India.

Solution:

(i) Here, the quantifier is 'there exists'.

The negation of this statement is as follows

There does not exist a number which is equal to its square

(ii) Here, the quantifier is 'for every'.

The negation of this statement is as follows

There exist a real number x , such that x is not less than $x + 1$

(iii) Here, the quantifier is 'there exists'.

The negation of this statement is as follows

In India there exists a state, which does not have a capital.

Question 3

Check whether the following pair of statements is negation of each other. Give reasons for the answer.

(i) $x + y = y + x$ is true for every real numbers x and y .

(ii) There exists real number x and y for which $x + y = y + x$.

Solution:

The negation of (i) statement is given below

There exists real number x and y for which $x + y \neq y + x$

Now, this statement is not same as (ii) statement

Hence, the given statements are not the negation of each other

Question 4

State whether the "Or" used in the following statements is "exclusive or" inclusive. Give reasons for your answer.

(i) Sun rises or Moon sets.

(ii) To apply for a driving licence, you should have a ration card or a passport.

(iii) All integers are positive or negative.

Solution:

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- (i) It is not possible for the Sun to rise and the Moon to set together. Hence, the 'or' in the given statement is exclusive.
- (ii) Since a person can have both a ration card and a passport to apply for a driving license. Hence, the 'or' in the given statement is inclusive.
- (iii) Since all integers cannot be both positive and negative. Hence, the 'or' in the given statement is exclusive

Exercise 14.4

Question 1

Rewrite the following statement with "if-then" in five different ways conveying the same meaning.

If a natural number is odd, then its square is also odd.

Solution:

The five different ways of the given statement can be written as follows

- (i) A natural number is odd indicates that its square is odd.
 (ii) A natural number is odd only if its square is odd.
 (iii) For a natural number to be odd, it is necessary that its square is odd.
 (iv) It is sufficient that the number is odd, for the square of a natural number to be odd.
 (v) If the square of a natural number is not odd, then the natural number is not odd

Question 2

Write the contra positive and converse of the following statements.

- (i) If x is a prime number, then x is odd.
 (ii) If the two lines are parallel, then they do not intersect in the same plane.
 (iii) Something is cold implies that it has low temperature.
 (iv) You cannot comprehend geometry if you do not know how to reason deductively.
 (v) x is an even number implies that x is divisible by 4

Solution:

(i) The contra positive of the given statement is as follows

If a number x is not odd, then x is not a prime number.

The converse of the given statement is as follows

If a number x is odd, then it is a prime number

(ii) The contra positive of the given statement is as follows

If two lines intersect in the same plane, then the two lines are not parallel.

The converse of the given statement is as follows

If two lines do not intersect in the same plane, then they are parallel

(iii) The contra positive of the given statement is as follows

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If something does not have low temperature, then it is not cold.
 The converse of the given statement is as follows
 If something is at low temperature, then it is cold.

(iv) The contra positive of the given statement is as follows
 If you know how to reason deductively, then you can comprehend geometry.
 The converse of the given statement is as follows
 If you do not know how to reason deductively, then you cannot comprehend geometry.

(v) The given statement can be written as 'if x is an even number, then x is divisible by 4'.
 The contra positive of the given statement is as follows
 If x is not divisible by 4, then x is not an even number.
 The converse of the given statement is as follows
 If x is divisible by 4, then x is an even number.

Question 3

Write each of the following statement in the form "if-then".

- (i) You get a job implies that your credentials are good.**
- (ii) The Banana trees will bloom if it stays warm for a month.**
- (iii) A quadrilateral is a parallelogram if its diagonals bisect each other.**
- (iv) To get A+ in the class, it is necessary that you do the exercises of the book.**

Solution:

- i. If you get a job, then your credentials are good.
- ii. If the Banana trees stays warm for a month, then the trees will bloom.
- iii. If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
- iv. If you want to score an A+ in the class, then you do all the exercises of the book.

Question 4

Given statements in (a) and (b). Identify the statements given below as contra positive or converse of each other.

- (a) If you live in Delhi, then you have winter clothes.**
- (i) If you do not have winter clothes, then you do not live in Delhi.**
- (ii) If you have winter clothes, then you live in Delhi.**
- (b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.**
- (i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.**
- (ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.**

Solution:

- (a) If you live in Delhi, then you have winter clothes.
- (i) If you do not have winter clothes, then you do not live in Delhi [Contra positive of statement (a)]
- (ii) If you have winter clothes, then you live in Delhi [Converse of statement (a)]

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- (b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.
 (i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram [Contra positive of statement (b)]
 (ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram [Converse of statement (b)]

Exercise 14.5

Question 1

Show that the statement p:

“If x is a real number such that $x^3 + 4x = 0$, then x is 0” is true by

- (i) direct method**
(ii) Method of contradiction
(iii) Method of contra positive

Solution:

Let p: ‘If x is a real number such that $x^3 + 4x = 0$, then x is 0’

q: x is a real number such that $x^3 + 4x = 0$

r: x is 0

(i) We assume that q is true to show that statement p is true and then show that r is true

Therefore, let statement q be true

Hence, $x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

Since x is real, it is 0.

So, statement r is true.

Hence, the given statement is true.

(ii) By contradiction, to show statement p to be true, we assume that p is not true. Let x be a real number such that $x^3 + 4x = 0$ and let $x \neq 0$

Hence, $x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x = 0$ or $x + 4 = 0$

$x = 0$ or $x^2 = -4$

However x is real. Hence, $x = 0$, which is a contradiction since we have assumed that $x \neq 0$

Therefore, the given statement p is true.

(iii) By contrapositive method, to prove statement p to be true, we assume that r is false and prove that q must be false

$\sim r$: $x \neq 0$

Clearly, it can be seen that

$(x^2 + 4)$ will always be positive

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$x \neq 0$ implies that the product of any positive real number with x is not zero.

Now, consider the product of x with $(x^2 + 4)$

$$\therefore x(x^2 + 4) \neq 0$$

$$x^3 + 4x \neq 0$$

This shows that statement q is not true.

Hence, proved that

$$\sim r \Rightarrow \sim q$$

Hence, the given statement p is true

Question 2

Show that the statement “For any real numbers a and b , $a^2 = b^2$ implies that $a = b$ ” is not true by giving a counter-example.

Solution:

The given statement can be written in the form of ‘if then’ is given below

If a and b are real numbers such that $a^2 = b^2$, then $a = b$

Let p : a and b are real numbers such that $a^2 = b^2$

q : $a = b$

The given statement has to be proved false. To show this, two real numbers, a and b , with $a^2 = b^2$ are required such that $a \neq b$

Let us consider $a = 1$ and $b = -1$

$$a^2 = (1)^2$$

$$= 1 \text{ and}$$

$$b^2 = (-1)^2$$

$$= 1$$

$$\text{Hence, } a^2 = b^2$$

However, $a \neq b$

Therefore, it can be concluded that the given statement is false.

Question 3

Show that the following statement is true by the method of contra positive. p : If x is an integer and x^2 is even, then x is also even.

Solution:

Let p : If x is an integer and x^2 is even, then x is also even

Let q : x is an integer and x^2 is even

r : x is even

By contra positive method, to prove that p is true, we assume that r is false and prove that q is also false

Let x is not even

To prove that q is false, it has to be proved that x is not an integer or x^2 is not even x is not even indicates that x^2 is also not even.

Hence, statement q is false.

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Therefore, the given statement p is true

Question 4

By giving a counter example, show that the following statements are not true.

- (i) p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.**
(ii) q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Solution:

(i) Let q: All the angles of a triangle are equal r:

The triangle is an obtuse angled triangle

The given statement p has to be proved false.

To show this, required angles of a triangle should not be an obtuse angle

We know that, sum of all the angles of a triangle is 180° . Therefore, if all the three angles are equal then each angle measures 60° , which is not an obtuse.

In an equilateral triangle, all angles are equal. However, the triangle is not an obtuse angled triangle.

Hence, it can be concluded that the given statement p is false

(ii) The given statement is q:

The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

This statement has to be proved false

To show this, let us consider,

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

One root of the equation $x^2 - 1 = 0$, i.e. the root $x = 1$, lies between 0 and 2

Therefore, the given statement is false.

Question 5

Which of the following statements are true and which are false? In each case give a valid reason for saying so.

- (i) p: Each radius of a circle is a chord of the circle.**
(ii) q: The centre of a circle bisects each chord of the circle.
(iii) r: Circle is a particular case of an ellipse.
(iv) s: If x and y are integers such that $x > y$, then $-x < -y$.
(v) t: $\sqrt{11}$ is a rational number.

Solution:

(i) The given statement p is false.

By the definition of chord, it should intersect the circle at two distinct points

(ii) The given statement q is false.

The centre will not bisect that chord which is not the diameter of the circle.

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In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If we put $a = b = 1$, then we get

$x^2 + y^2 = 1$, which is an equation of a circle

Hence, circle is a particular case of an ellipse.

Therefore, statement r is true

(iv) $x > y$

By a rule of inequality

$$-x < -y$$

Hence, the given statement s is true

(v) 11 is a prime number

We know that, the square root of any prime number is an irrational number.

Therefore $\sqrt{11}$ is an irrational number

Hence, the given statement t is false.

Miscellaneous exercise

Question 1

Write the negation of the following statements:

(i) p: For every positive real number x, the number $x - 1$ is also positive.

(ii) q: All cats scratch.

(iii) r: For every real number x, either $x > 1$ or $x < 1$.

(iv) s: There exists a number x such that $0 < x < 1$.

Solution:

(i) The negation of statement p is given below

There exists a positive real number x, such that $x - 1$ is not positive

(ii) The negation of statement q is given below

There exists a cat which does not scratch.

(iii) The negation of statement r is given below

There exists a real number x, such that neither $x > 1$ nor $x < 1$

(iv) The negation of statement s is given below

There does not exist a number x, such that $0 < x < 1$

Question 2

State the converse and contra positive of each of the following statements:

(i) p: A positive integer is prime only if it has no divisors other than 1 and itself.

(ii) q: I go to a beach whenever it is a sunny day.

(iii) r: If it is hot outside, then you feel thirsty.

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Solution:

- (i) Statement p can be written in the form 'if then' is as follows
 If a positive integer is prime, then it has no divisors other than 1 and itself
 The converse of the statement is given below
 If a positive integer has no divisors other than 1 and itself, then it is prime.
 The contra positive of the statement is given below
 If a positive integer has divisors other than 1 and itself, then it is not prime.
- (ii) The given statement can be written as follows
 If it is a sunny day, then I go to a beach.
 The converse of the statement is given below
 If I go to a beach, then it is a sunny day.
 The contra positive of the statement is given below
 If I do not go to a beach, then it is not a sunny day.
- (iii) The converse of statement r is given below
 If you feel thirsty, then it is hot outside.
 The contra positive of statement r is given below
 If you do not feel thirsty, then it is not hot outside.

Question 3

Write each of the statement in the form "if p, then q".

- (i) p: It is necessary to have a password to log on to the server.
 (ii) q: There is traffic jam whenever it rains.
 (iii) r: You can access the website only if you pay a subscription fee.

Solution:

- (i) The statement p in the form 'if then' is as follows If you log on to the server, then you have a password.
 (ii) The statement q in the form 'if then' is as follows If it rains, then there is a traffic jam.
 (iii) The statement r in the form 'if then' is as follows
 If you can access the website, then you pay a subscription fee.

Question 4

Re write each of the following statements in the form "p if and only if q".

- (i) p: If you watch television, then your mind is free and if your mind is free, then you watch television.
 (ii) q: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly. (iii) r: If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular.

Solution:

- (i) You watch television if and only if your mind is free
 (ii) You get an A grade if and only if you do all the homework regularly

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(iii) A quadrilateral is equiangular if only if it is a rectangle

Question 5

Given below are two statements

p: 25 is a multiple of 5.

q: 25 is a multiple of 8.

Write the compound statements connecting these two statements with “And” and “Or”. In both cases check the validity of the compound statement.

Solution:

The compound statement with ‘And’ is as follows

25 is a multiple of 5 and 8

This is false statement because 25 is not a multiple of 8

The compound statement with ‘Or’ is as follows

25 is a multiple of 5 or 8

This is true statement because 25 is not a multiple of 8 but it is a multiple of 5

Question 6

Check the validity of the statements given below by the method given against it.

(i) p: The sum of an irrational number and a rational number is irrational (by contradiction method).

(ii) q: If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).

Solution:

(i) The given statement is as follows

p: The sum of an irrational number and a rational number is irrational.

Let us assume that the statement p is false. That is,

The sum of an irrational number and a rational number is rational.

Hence, $\sqrt{a} + \frac{b}{c} = \frac{d}{e}$ where \sqrt{a} is irrational and b, c, d, e are integers.

$$\therefore d/e - b/c = \sqrt{a}$$

But here, $d/e - b/c$ is a rational number and \sqrt{a} is an irrational number

This is a contradiction. Hence, our assumption is false.

\therefore The sum of an irrational number and a rational number is irrational.

Hence, the given statement is true.

(ii) The given statement q is as follows

If n is a real number with $n > 3$, then $n^2 > 9$

Let us assume that n is a real number with $n > 3$, but $n^2 > 9$ is not true

i.e. $n^2 < 9$

So, $n > 3$ and n is a real number

By squaring both sides, we get

$$n^2 > (3)^2$$

This implies that $n^2 > 9$ which is a contradiction, since we have assumed that $n^2 < 9$

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Therefore, the given statement is true i.e., if n is a real number with $n > 3$, then $n^2 > 9$

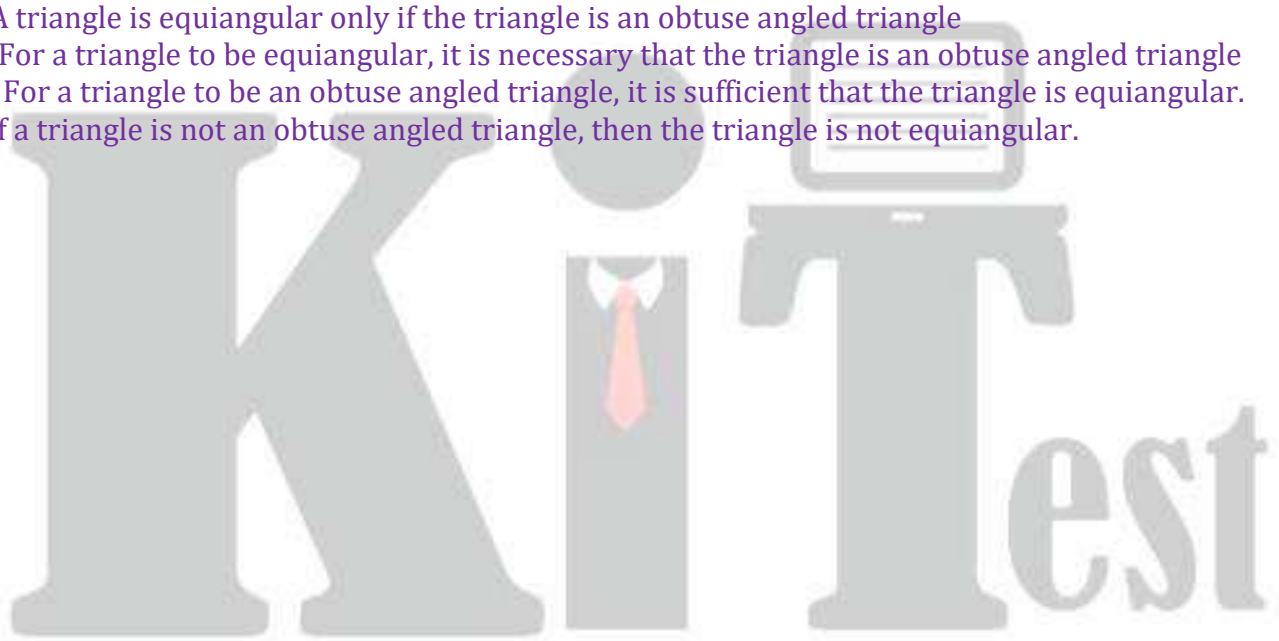
Question 7

Write the following statement in five different ways, conveying the same meaning. p: If triangle is equiangular, then it is an obtuse angled triangle.

Solution:

The given statement can be written in five different ways is given below

- (i) A triangle is equiangular implies that it is an obtuse angled triangle
- (ii) A triangle is equiangular only if the triangle is an obtuse angled triangle
- (iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse angled triangle
- (iv) For a triangle to be an obtuse angled triangle, it is sufficient that the triangle is equiangular.
- (v) If a triangle is not an obtuse angled triangle, then the triangle is not equiangular.



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