CHAPTER - 3
LINEAR INEQUALITIEs

INEQUALITIES

Inequalities are statements where two quantities are unequal but a relationship exists between them. These type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.

LINEAR INQUALITIES IN ONE VARIABLE AND THE SOLUTION SPACE

Any linear function that involves an inequality sign is a linear Inequality. It may be of one variable or, of more than one variable. simple example of linear inequalities are those of one variable only ; viz., $x > 0$, $x \leq 0$ etc.

SUMMARY OF GRAPHICAL METHOD

It involves:

i. Formulating the linear programming problem, i.e. expressing the objective function and constraints in the standardized format.

ii. Plotting the capacity constraints on the graph paper. For this purpose, normally two terminal points are required. This is done by presuming simultaneously that one of the constraints is zero. When constraints concern only one factor, then line will have only one origin point and it will run parallel to the other axis.

iii. Identifying feasible region and coordinates of corner points. Mostly it is done by breading the graph, but a point can be identified by solving simultaneous equation relating to two lines which intersect to form a point on graph.

iv. Testing the corner point which gives maximum profit. For this purpose, the coordinates relating to the corner point should put in objectives function and the optimal point should be as certained.

v. For decision – making purpose, sometimes, it is required to know whether optimal point leaves some resources unutilized. For this purpose, value of coordinates at the

		optimal point should be put with constraint to find out which constraints are not fully utilized.
	vi.	Linear inequalities in two variables may be solved easily by extending our knowledge of straight lines.

Question 1

On solving the inequalities 6x + y 2 18, x + 4y2 12, 2x + y 10, we get the following situation

(a) (0, 18), (12, 0), (4, 2), & (7, 6)
(b) (3, 0), (0, 3), 0, 0) and (7, 6)
(c) (5, 0), (0, 10), (4, 2), (7, 6)
(d) (0, 18), (12, 0), (4, 2), (0, 0) and (7, 6)

Answer: a

Explanation:

We draw the graph of 6x+y 218, x+4y 212, and 2x+y 210 in –the same plane. The solution set of system is that portion of the graphs of the given inequality which is represented by the intersection of the above three equations.

Question 2

Solve x +2 < 4

(a) $x<2$
(b) $x>2$
(c) $x\neq 2$
(d) $x<4$

Answer: a

Explanation:

We need to subtract 2 from both sides of the inequality.

$X+2<4$

$X<4-2$

$X<2$

Question 3

Solve the inequality 3 – 2x≥15

(a) $x\leq 6$
(b) $x\leq -6$
(c) $x>-6$
(d) $x>6$

Answer: b

Explanation:

We need to subtract 3 from both sides; then divide both sides by –2(remembering to change the direction of the inequality).

$=3-2x \geq 15$

$=-2x \geq 15-3$

$= -2x \geq 12$

$=x \leq \frac{12}{-2}$

$= x \leq -6$

Question 4

Solve -1 < 2x +3 < 6

(a) $-2<x<3/2$

(b) $2<x<23/2$

(c) $2<x<3/2$

(d) $-3<x<23/3$

Answer: a

Expectation:

$= -1<2x+3<6$

Subtract 3 from all 3 sides

$= -1-3<2x+3-3<6-3$

$= -4<2x<3$

Divide all sides by 2

$= -2<x<3/2$

Question 5

Solve $\frac{x}{2}>8$

(a) $x<8$

(b) $x>16$

(c) $x=8$

(d) $x=4$

Answer: b

Explanation:

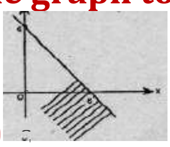
$= \frac{x}{2}>8$

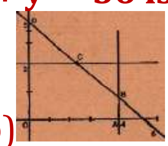
$=x>8\times2$

$= x>16$

Question 6

The graph to express the inequality x + y = 56 is:

(a) 

(b) 

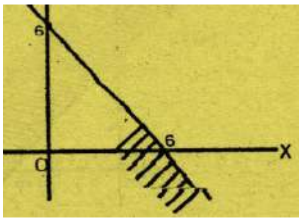
(c) Either a or b

(d) None of these

Answer: a

Explanation:

X + y = 56 is graphically represent by



Question 7

On the average, experienced person does 5 units of work while fresh one 3 units work daily but the employer have to maintain the output to at least 30 units work per day. The situation can be expressed as

(a) $5x + 3y = 30$ (b) $5x + 3y = 30$

(c) $5x + 3y = 30$ (d) None of these

Answer: b

Explanation:

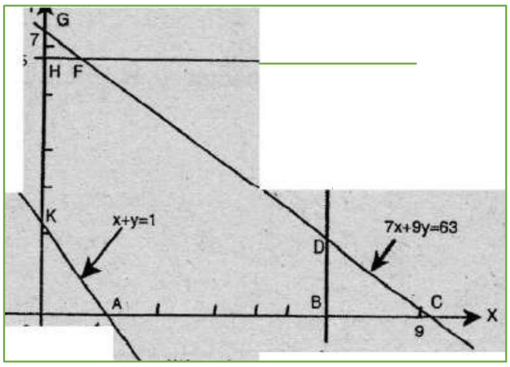
Let Experience Person x unit work per day

Fresh one = y unit work per day

So situation is $5x + 3y = 30$

Question 8

Common region of the inequalities is:



(a) BCDB and DEFD (b) Unbounded

(c) HFGH (d) ABDFHKA

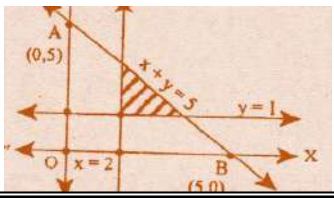
Answer: d

Explanation:

Common region of the inequalities is ABDFHKA

Question 9

The shaded region represents:



(a) x + y s 5, x : 1'.2, y :s; 1
(b) x + y: 1'. 5, x : 1 '.2 , y 1
(c) x + y s 5, X : 1!4, y : 1; ,1
(d) None of these

Answer: b

Explanation:

Region represented by the line x + y = 5 touch the coordinate axes at (5, 0) and (0, 5) since the shaded region lies below the line x + y = 5. Hence it is represented by the in equation x + y = 5

Question 10

A company produces two products A and B, each of which requires processing in two machines. The first machine can be used at most for 60hours, the second machine can be used at most for 40 hours. The product A requires 2 hours on machine one and one hour on machine one and two hours on machine two. Above situation is using linear inequalities?

(a) True
(b) False
(c) Partial
(d) None

Answer: a

Explanation:

Let the company produce, x number of product A and y number of product B.

As each of product A requires 2 hours in machine one and one hour in machine two, x number of product A requires 2x hours in machine one and x hours in machine two. Similarly, y number of product B requires y hours in machine one and 2y hours in machine two for 40 hours. Hence 2x + y cannot exceed 40. In other words,

$$2x + y = 60 \quad \text{and} \quad x + 2y = 40$$

Thus, the conditions can be expressed using linear inequalities.

Question 11

The inequalities $5x_1 + 4x_2 \geq 9$, $x_1 + x_2 \geq 3$, $x_1 \geq 0$ and $x_2 \geq 0$ is correct?

(a) True
(b) False
(c) Not sure
(d) None

Answer: a

Explanation:

We draw that straight lines 5× 1 + 4 × 2 = 9 and × 1 + x2 = 3.

Table for $5x_1 + 4x_2 = 9$

× 1	0	9/5
× 2	9/4	0

Table for $x_1 + x_2 = 3$

×1	0	3
×2	3	0

Now, if we take the point (4, 4), we find

5×1 + 4×2 * 9

i.e., 5.4 + 4.4 * 9

or, 36 * 9 (True)

x1+ x2 * 3

i.e., 4 +4 * 3

8 * 3 (True)

Hence (4, 4) is in the region which satisfies the inequalities

Question 12

Solve the inequality -2(x+3)<10

(a) x>-8 (b) x>16

(c) x>8 (d) x>-16

Answer: a

Explanation:

-2x-6<10-2x-6<10

-2x-6+6<10+6-2x-6+6<10+6

--2x<16-2x<16

-2x-2<16-2-2x-2>16-2

x>-8

Question 13

Solve the absolute value inequality $2|3x+9|<36$

(a) -9<x>3 (b) -9<x<3

(c) 9<x>3 (d) 9<x<3

Answer: b

Explanation:

$2|3x+9|<36$ $2|3x+9|2<36$

$|3x+9|<18|$

-18<3x+9

-18-9<3x

-27<3x

-9 <x

Question 14

Solve x + 2 < 4

(a) x< 1 (b) x>2

(c) x>-2 (d) x<2

Answer: d

Explanation:

We need to subtract 2 from both sides of the inequality.

X+2<4

X<4-2

X<2

Question 15

Solve $\frac{x}{2}$>4

(a) x<4 (b) x>8

(c) x>-4 (c) x<2

Answer: b

Explanation:

We need to multiply both sides of the inequality by 2.

$\frac{x}{2}>4$

x>4×2

x>8

Question 16

Solve the inequality $\frac{3}{2}(1 - x) > \frac{1}{4} - x$

(a) $x < \frac{5}{2}$ (b) x < 5

(c) $x < \frac{10}{2}$ (d) $x < \frac{5}{6}$

Answer: a

Explanation:

$\frac{3}{2}(1 - x) > \frac{1}{4} - x$

6-6x>1-4x

-6x+4x > 1-6

-2x > -5

$X < \frac{5}{2}$

Question 17

The solution of the inequality 8x + 6 < 12x + 14 is:

(a) (-2, 2) (b) (0, -2)

(c) (2,) (d) (-2,)

Answer: d

Explanation:

= 8x + 6 < 12x + 14

= 6 – 14 < 12x – 8x

= -8 < 4x

= x > -2

Question 18

Solve x-1 < 2x + 2 < 3x + 1

(a) (x>3 and x>1)
(b) (x>-3 and x<1)
(c) (x<-3 and x>1)
(d) (x>1)

Answer: d

Explanation:

We need to find the intersecting of the ''true'' values.
X -1<2x+2 and 2x+2<3x+1
x<2x+3 and 2x-<3x-1
x>-3 and x>1
The intersection of these 2 regions is x>1.

Question 19

Solve -2(x+4)>1 – 5x

(a) x<3
(b) x>3
(c) x≠3
(d) x = 3

Answer: b

Explanation:

-2(x+4)>1-5x
[-2x -8]1-5x
3x-8>1
3x>9
x>3

Question 20

Solve the inequality $|2x - 1| > 5$

(a) x<3
(b) x>3
(c) x≠3
(d) x = 3

Answer: b

Explanation:

Applying the relationships discussed earlier:
2x-1<5or 2x-1>5
Solving both inequalities, we get:

2x<5+1	or	2x>5+1
2x<-4	or	2x>6
X<-2	or	x>3

Question 21

Find all pair if consecutive even positive integers, both of the which are larger than 5 such that their sum is less than 23.

(a) (7,8),(7,3)and(2,3)
(b) (6,8),(8,10)and(10,12)
(c) (5,7),(7,9)and(2,6)
(d) (2,3),(4,5)and(3,1)

Answer: b

Explanation:

Let x and x+2 be two consecutive even positive integers.

Since both the integers are larger than 5. X>5x>5 ------ (1)

Also sum of two is less than 23

X+x+2<23

=>2x+x<23

Adding -2 to both sides

2x<23-2

2x<212

Dividing by 2 on both sides

$\frac{2x}{2} < 23 - 2$

$X < \frac{21}{2}$

X < 10.5

Step 2:

Since x is an even positive integer greater than 5 and less than 10.5 × can take value 6,8,10.

Thus the required pair of number is (6, 8), (8, 10) and (10, 12)

Hence B is the correct answer.

Question 22

The longest side of a triangle is three times the shortest side and third side is 2cmshortest than the longest side. If the perimeter of the triangle is at least 61cm. find the minimum length of the shortest side.

(a) 9cm
(b) 3cmm
(c) 5cm
(d) 5cm

Answer: a

Explanation:

Let the length of the shortest side be x cm

Length of the largest side is 3x cm

Length of the third side is 3x-2cm

Since the perimeter of the triangle is at least 61 cm, we get,

X+3x+3x-2≥61

7x-2≥ 61

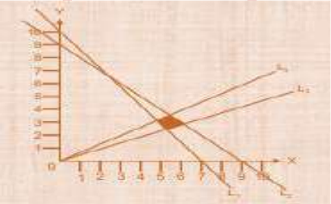
Adding 2 on both sides

$=> 7x \geq 61 + 2$

$7x \geq 63$
Dividing both sides by positive number 7
$\frac{7x}{7} \geq \frac{63}{7}$
$X \geq 9$
Step 2:
The minimum length of the shortest side is 9 cm.
Hence A is the correct answer.

Question 23
Solve the inequality: $2 \leq 3x - 4 \leq 5$
(a) [2, 8] (b) [4, 5]
(c) [3, 4] (d) [2, 3]
Answer: d
Explanation:
The given inequality is $2 \leq 3x - 4 \leq 5$
Adding +4+4 throughout the inequality $2+4 \leq 3x - 4 + 4 \leq 5 + 4$
$= > 6 \leq 3x \leq 9$
Dividing by positive number 3 throughout the inequality $= > 2 \leq x \leq 3$
$= > 2 \leq x \leq 3$
Step 2:
Thus all real number, which are greater than or equal to 2, and less than or equal to 3, are solutions to the given inequality.
The solution set is [2, 3]
Hence D is the correct answer.

Question 24
Graphs of in equations are drawn below:



L1: 5x+3y=30 **L2: x+ y = 9**
L3: Y=X/3 **L4: y=x/2**
The common region (Shaded part) shown in the diagram refers to the inequalities
(a) $5x+3y \leq 30$
$X + y \leq 9$
$Y \leq 1/2x$
$y \leq x/2$
$x \geq 0, y \geq 0$
(b) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq x/3$
$y \leq x/2$
$x \geq 0, y \geq 0$
(c) $5x+3y \geq 9$
(d) None of these

$X + y \geq 9$
$Y \leq x/3$
$y \geq x/2$
$x \geq 0, y \geq 0$

Answer: d

Explanation:

$5x + 3y > 30$
$X + y < 9$
$Y > 9$
$Y \leq x/2$
$X \geq 0; y \geq 0$

PAST EXAMINATION QUESTIONS:

MAY 2018

Question 1

The linear relationship between are variable in an inequality:

(a) ax+by≤c
(b) ax.by≤c
(c) axy+by≤c
(d) ax+bxy≤c

Answer: a

The linear relationship between two variables in an inequality ax+by≤c

NOV 2018

Question 1

On solving the inequalities 5x+y≤100, x+y≤60, x≥0, y≥, we get the following solutions:

(a) (0,0), (20, 0), (10, 50), & (0, 60)
(b) (0,0), (60,0), (10,50) & (0,60)
(c) (0,0), (20,0), (0,100), & (10,50)
(d) None

Answer: a

Explanation:

On solving the inequalities 5x+y≤100, x+y≤60, x+y≤60, x≥0, y≥, we get (0, 0), (20, 0) (10, 50) & (0, 60) all satisfied above inequalities

MAY 2019

Question 1

The solution set of the in equation x + 2 > 0 and 2x – 6 > 0 is

(a) (-2, ∞) (b) (3,∞)
(c) (-∞, -2) (d) (-∞, -3)

Answer: b

Explanation:

X + 2 > 0

X > -2

$2X - 6 > 0$

$2X > 6$

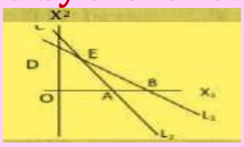
X > $\frac{6}{2}$

X > 3

X € (3,∞)

Questions 2

The common region represented by the following in equalities

$L_1 = X_1 + X_2 \leq 4$; $L_2 = 2X_1 + X_2 \geq 6$ 

(a) OABC (b) Outside of OAB
(c) △ BCE (d) △ ABE

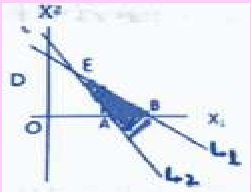
Answer: d

Explanation:

$= x_1 + x_2 \leq 4 - L_1$

$=2X_1 + X_2 \geq 6 - L_2$

△ABE



NOV 2019

Question 1

6x + y ≥ 18, x + 4y ≥ 12, 2x + y ≥ 10 on solving the inequalities; we get

(a) (0, 18), (12, 0), (4, 2), & (7, 6) (b) (3, 0), (0, 3), (4, 2), & (7, 6)
(c) (5, 0), (0, 10), (4, 2), & (7, 6) d) (0, 18), (12, 0), (4, 2), & (0, 0), and (7,6

Answer: (a)

We draw the graph of 6x + y ≥ 18, x + 4y ≥ 12 and 2x + y ≥ 10 in the same plane.

The solution set of system is that portion of the graphs of the given inequality which is

Represented by the intersection of the above three equations.

For this purpose, we replace, the inequalities respectively by

6x+y=18, x + 4y = 12 and 2x + y = 10

For 6x + y = 18, For x + y = 12

x	0	3
y	18	0

X	0	12
y	3	0

For 2x + y = 10

x	0	5
y	10	0

DEC - 2020

Question 1

If Y = x(x-1)(x-2) then dy/dx is

(a) -6x (b) $3x^2$ -6x+2

(c) 6x+4 (d) $3x^2$ -6x

Answer: b

Explanation:

$y = x(x-1)(x-2)$

$y = (x^3 - 2x^2 - x^2 + 2x)$

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 - x^2 + 2x)$

$\frac{dy}{dx} = 3x^2 - 4x - 2x + 2$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

Question 2

The average cost function of a good is 2Q+6+ Q/13 where Q is the quantity produced. The approx. cost at Q = 15 is___

(a) 42 (b) 36

(c) 66 (d) None of these

Answer: d

Explanation

Note: According to the given question the correct answers is Rs.553. There is no correct

JAN 2021

Question 1

The common region in the graph of the inequalities x + y ≤ 4, x – y ≤ 4, x ≥ 2, *is*.

(a) equilateral triangle (b) Isosceles triangle

(c) Quadrilateral (d) Square

Answer: b

Explanation:

common region in the graph of the inequalities x + y ≤ 4, x – y ≤ 4, x ≥ 2, *is* it made isosceles triangle

Question 2

If A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and A – 2B = $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A =

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$
(c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$
(d) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Answer: c

Explanation:

2(a+b) = 2 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = 2A + 2B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ – – – – – (1)

$A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ – – – –(2)

2A +2B+A-2B = $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

3A = $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

A= $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

Hence answer will be = $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$

Question 3

The matrix A = $\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -1 & 1 & -2 \end{bmatrix}$ is

(a) Symmetric
(b) Skew – symmetric
(c) Singular
(d) Non – Singular

Answer: c

Explanation:

A singular matrix is one which is non-invertible i.e. there is no multiplicative inverse, B, such that the original matrix A × B = I (Identity matrix) A matrix is singular if and only if its determinant is zero.

Question 4

The cost function of production is given by C(x) = $\frac{x^3}{2}$ - 15x² + 36x where x denotes thee number of items produced. The level of output for which marginal cost is minimum and the level of output for which the average cost is minimum are given by, respectively

(a) 10 and 15
(b) 10 and 12
(c) 12 and 15
(d) 15 and 10

Answer: a

Question 5

$\int_1^0 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$ds =

(a) $e\left(\frac{e}{2}-2\right)$ (b) e(e – 1)

(c) a (d) $e^2(e-1)$

Answer: a

JULY 2021

Question 1

If y = 4+9 sin 5x then which holds good?

(a) -5≤ y ≤ 13 (b) −4 ≤ y ≤ 8

(c) 0 < y < 1 (d) -5 < y < 5

Answer: Options (a)

DEC 2021

Question 1

Xyz Company has a policy for its recruitment as: it should not recruit more than eight men (x) to three women(y). How can this fact to be express in inequality?

(a) $3y \geq 8x$ (b) $3y \leq x/8$

(c) $8y \geq 3x$ (d) $8y \leq 3x$

Answer: c

Explanation:

As per the company's policy,

When y=3, x ≤ 8

It can also be written as:

When $\frac{y}{3} = 1$ -----Eq (1)

$\frac{x}{8} \leq 1$ ….. Eq (2)

Now, as per Eq 1, we have $\frac{y}{3} = 1$

It can also be written as 1=$\frac{y}{3}$... Eq 3

Substituting the value of 1= $\frac{y}{3}$ from eq (3) to Eq(2), we'll get:

$$\frac{x}{8} \leq \frac{y}{3}$$

3x≤ 8y

$8y \geq 3x$