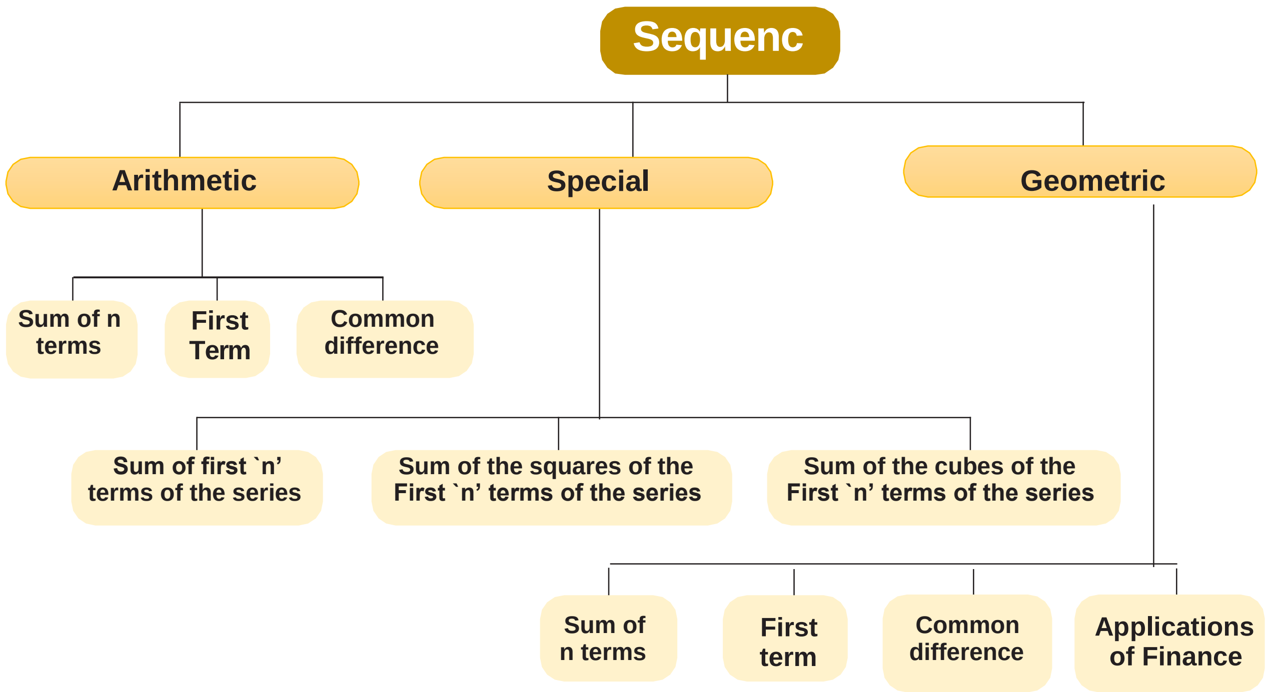
CHAPTER - 6
SEQUENCE AND SERIES-ARITHMETIC AND GEOMETRIC PROGRESSIONS



Sequence	An ordered collection of numbers $a_1, a_2, a_3, a_4,$ ...., a_n, ... is a sequence if according to some definite rule or law, there is a definite value of a_n, called the term or element of the sequence, corresponding to any value of the natural number An expression of the form $a_1 + a_2 + a_3 + \ldots + a_n +$ .... which is the sum of the elements of the sequence $\{a_n\}$ is called a *series*. If the series contains a finite number of elements, it is called a *finite series*, otherwise called an infinite series.
Arithmetic Progression	A sequence $a_1, a_2, a_3, \ldots, a_n$ is called an Arithmetic Progression (A.P.) when $\mathbf{a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1}}$. That means A. P. is a sequence in which each term is obtained by adding a constant d to the preceding term. This constant 'd' is called the

	common difference of the A.P. If 3 numbers a, b, c are in A.P., we say b – a = c – b or a + c = 2b; b is called the arithmetic mean between a and c. **N^{th} term (t_n) = a + (n – 1)** Where a = First Term D = Common difference= t_n- t_{n-1} Sum of 1st n natural or counting numbers

Sum of n terms of AP	$s=\frac{N}{2}[2a+(n-1)d]$
Sum of the first n terms	Sum of 1st n natural or counting numbers **S = n(n + 1)/2**
Sum of 1st n odd number	**$S = n^2$**
Sum of the Squares of the first, n natural numbers	n(n + 1) (2n + 1)

Geometric Progression (G.P)

If in a sequence of terms each term is constant multiple of the proceeding term, then the sequence is called a Geometric Progression (G.P).The constant multiplier is called the common ratio

$$\frac{Anyterm}{Precedingterm} = \frac{t_n}{t_{n-1}}$$

= ar^{n-1}/ar^{n-2} = r

Sum of first n terms of a GP	**$S_n = a (1 - r^n) / (1 - r)$ when r < 1** **$S_n=a(r^n-1)/(r-1)$when r>1**
Sum of infinite geometric series	**$S\infty$= a/(1-r) where 0< r<1**

Geometric mean

A.M. of a &b is=(a+b)/2

If a, b, c are in G.P we get b/a = c/b => b^2= ac, b is called the geometric mean between a and c

Question: 1

Find the 7th term of the A.P. 8, 5, 2, -1, -4,...

(a) 10
(b) -10
(c) 8
(d) -8

Answer: b

Explanation:

Here a = 8, d = 5 – 8 = -3
Now t_7 = 8 + (7-1) d
= 8 + (7 – 1) (-3)
= 8 + 6 (-3)
= 8 – 18
= -10

Question: 2

If 5th and 12th terms of an A.P. are 14 and 35 respectively, find the A.P.

(a) 2, 5, 8, 11, 14,………
(b) 2, 3, 8, 11, 12,……….
(c) 2, 3, 4, 11, 14 …
(d) 2, 5, 8, 1, 4,………..

Answer: a

Explanation:

Let a be the first term & d be the common difference of A.P.
t_5 = a + 4d = 14
t_{12} = a + 11d = 35
On solving the above two equations
7d = 21 =i.e. d = 3
And a = 14 – (4 × 3) = 14 – 12 = 2
Hence, the required A.P. is 2, 5, 8, 11, 14,……….

Question: 3

Divide 69 into three parts are in A.P. and are such that the product of the first two parts is 483.

(a) 21, 23, 25.
(b) 21, 22, 23,
(c) 22, 23, 25.
(d) 21, 22, 25.

Answer: a

Explanation:

Given that three parts are in A.P., let the three parts which are in A.P. be a – d, a, a + d……

Thus a – d + a + a + d = 69

Or 3a = 69

Or a = 23

So the three parts are 23 – d, 23, 23 + d

Since the product of first two parts is 483, therefore, we have 23 (23 – d) = 483

Or $23 - d = \frac{483}{23} = 21$

Or d = 23 – 21 = 2

Hence, the three parts which are in A.P. are 23 – 2 = 21, 23, 23 + 2 = 25

Hence the three parts are 21, 23, and 25

Question: 4

Find the arithmetic mean between 4 and 10.

(a) 5 (b) 7

(c) 10 (d) 3

Answer: b

Explanation:

We know that the A.M. of a & b is = (a+ b) / 2 Hence, The A.M. between 4 & 10 = (4 + 10) / 2 = 7

Question: 5

Find the G.P. series where 4^{th} term is 8 and 8^{th} term is 128/625

(a) 125, 50,20,9, (b) 125,50,20,10, …..

(c) 125, 5,20,8… (d) 125, 50,20,8 …

Answer: d

Explanation:

$t4 = ar^3 = 8$

T8 = 128/625 → ar^7 = 128/625

T8/T4 = 128/625 × 1/8

- → $ar^7/ar^3 = 16/625$
- → r^4 = 2^4/5^4
- → r = 2/5

$ar^3 = 8$

- → $a (2/5)^3 = 8$
- → a × 8/125 = 8
- → a = 125

Therefore, a = 125, ar = 125 × 2/5 = 50, ar^2 = 125 × 4/125 = 20…..

Or 125, 50, 20, 8… Forms a G.P.

Question: 6

Insert three geometric means between $\frac{1}{9}$ and 9

(a) $\frac{1}{9}, \frac{1}{3}$, 1, 3, 9
(b) $\frac{1}{8}, \frac{1}{5}$, 1, 3, 9
(c) $\frac{11}{9}, \frac{1}{3}$, 1, 3, 9
(d) $\frac{121}{9}, \frac{1}{3}$, 1, 3

Answer: a

Explanation:

G.P. Series $\frac{1}{9}$, --,--,--,--, 9

Here t1 = a = $\frac{1}{9}$

t5 = a.r^4 = 9

Now, t5 = $\frac{1}{9}$.r^4 = 9

= r^4 = 81

= $r^4 = 3^4$

= r = 3

t2 = ar = $\frac{1}{9} \times 3 = \frac{1}{3}$

t3 = $ar^2 = \frac{1}{9} \times 3^2 = 1$

t4 = ar3 = $\frac{1}{9} \times 3^3 = 3$

Thus the series $\frac{1}{9}, \frac{1}{3}$, 1, 3, 9

Question: 7

Find the sum of 1st term of G.P. series 1+2+4+8+……

(a) 155
(b) 255
(c) 185
(d) -822

Answer: b

Explanation:

Here a = 1, r = 2, n = 8

$S_n = a.\frac{(r^n-1)}{(r-1)}$ When r > 1

$S_8 = 1.\frac{(2^8-1)}{(2-1)}$

= 1 (256 – 1) = 255

Thus S_8 = 255

Question: 8

Find the sum of the series -2, 6, -18 …..7 terms?

(a) 1554
(b) -1094
(c) 1094
(d) -8223

Answer: b

Explanation:

Here a = -2, r = -3, n = 7

$S_n = a.\frac{(1-r^n)}{(1-r)}$When <1

$S_7 = (-2)\ \frac{[1-(-3)^7]}{[1-(-3)]}$

$= (-2)\frac{(1+2187)}{4}$

$= (-2)\frac{(2188)}{4}$

$S_7 = -1094$

Question: 9

In a G.P. the product of the 1st three terms 27/8. The middle term is

(a) $\frac{27}{8}$ (b) $\frac{3}{2}$

(c) $\frac{2}{9}$ (d) $\frac{8}{27}$

Answer: b

Explanation:

Let the three terms Of GP are$\frac{a}{r}$, a, ar

Now product of terms

$\frac{a}{r}\times a\times ar = \frac{27}{8}$

$a^3 = \frac{27}{8}$

$a^3 = (\frac{3}{2})^3$

$a = \frac{3}{2}$

Thus the middle term, $a = \frac{3}{2}$

Question: 10

If you save 1 paisa today, 2 paisa the next day and 4 paisa the succeeding day and so on, then your total savings in two weeks will be.

(a) Rs. 168.32 (b) Rs. 163.98

(c) Rs. 163.83 (d) None

Answer: c

Explanation:

Here the pattern of savings the G.P series 0.01, 0.02, 0.04 …

Here a = 0.01, r = 2, n = 14

$S_n = a.\frac{(r^n-1)}{(r-1)}$ When r > 1

$S_{14} = 0.01\frac{(2^{14}-1)}{(2-1)}$

$= 0.01\frac{(16384-1)}{1}$

$= 0.01\times 16383$

$S_{14} = 163.83$

Thus the total savings in 14 days would be Rs. 163.83.

Question: 11

The sum of the infinite G.P series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27}$.....

(a) 0.75 (b) 75
(c) 0.57 (d) 57

Answer: a

Explanation:

Here a = 1, r = $\left(\frac{-1}{3}\right)$

$S_\infty = \frac{a}{(1-r)} = \frac{1}{\left[1-\left(\frac{-1}{3}\right)\right]}$

= 1/[4/3]

= ¾

= 0.75

Question: 12

Find the 10th term of the A.P.: 2, 4, 6, ...

(a) 20 (b) 40
(c) 2 (d) 0.20

Answer: a

Explanation:

Here the first term (a) = 2 and common different d = 4 – 2 = 2

Using the formula $t_n = a + (n - 1) d$, we have

t_{10}= 2 + (10 -1) 2 = 2 +18 = 20

Hence, the 10^{th} term of the given A.P. is 20

Question: 13

The 10th term of an A.P. is -15 and 31st term is -57, find the 15th term

(a) -20 (b) 20
(c) -25 (d) 25

Answer: c

Explanation:

Let a be the first term and d be the common d be the common difference of the A.P. Then from the formula:

t_n = a + (n – 1) d, we have

t_{10} = a + (10 -1) d = a + 9d

t_{31} = a + (31 – 1) d = a + 30 d

We have,

a + 9d = -15 .... (1)

a + 30d = -57 .... (2)

Solve equations (1) and (2) to get the values of a and d. Subtracting (1) from (2), we have

21d = -57 + 15 = -42

-42 ÷ 21 = 2

Again from (1), a = -15 -9d = -15 – 9(-2) = -15 + 18 = 3

Now t_{15} = a + (15 -1) d

= 3 + 14 (-2) = -25

Question: 14

Which term of the A.P.: 5, 11, 17 ... is 119?

(a) n = 20 (b) n = 2

(c) n = 30 (d) n = 19

Answer: a

Explanation:

Here a = 5, d = 11-5 = 6

t_n = 119 we know that

t_n = a + (n – 1) d

? 119 = 5 + (n -1) x 6

$(n - 1) = \frac{119-5}{6} = 19$

n = 20, therefore, 119 is the 20th term of the given A.P.

Question: 15

Is 600 a term of the A. P.: 2, 9, 16, ...?

(a) yes (b) no

(c) not sure (d) none

Answer: b

Explanation:

Here, a = 2, and d = 9 – 2 = 7.

Let 600 be the nth term of the A.P. We have t_n = 2 + (n – 1) 7

According to the question

2 + (n – 1) 7 = 600

(n – 1) 7 = 598

Or $n = \frac{598}{7} + 1$ $n = 86\frac{3}{7}$

Since n is a fraction, it cannot be a term of the given A.P. Hence, 600 is not a term of the given A.P.

Question: 16

The common difference of an A.P. is 3 and the 15th term is 37. Find the first term.

(a) -5 (b) 5

(c) 42 (d) -42

Answer: a

Explanation:

Here d = 3, t_{15} = 37, and n = 15 Let the first term be a. we have

$t_n = a + (n-1) d$

$37 = a + (15 – 1) 3$

Or, $37 = a + 42$

$a = -5$

Thus, first term of the given A.P. is -5

Question: 17

Geometric mean G between two numbers a and b id

(a) ab
(b) ab^2
(c) a^2b
(d) $\sqrt{ab}$

Answer: d

Explanation:

If a single geometric mean 'G' is inserted between two given numbers 'a' and 'b', then G is known as the geometric mean between 'a' and 'b'.

G.M. = G = $G^2 = \sqrt{ab}$

Question: 18

If A and G are arithmetic and geometric mean respectively between two positive numbers a and b, then A (AM) < G (GM) is correct?

(a) yes
(b) no
(c) not sure
(d) none

Answer: b

Explanation:

We have

A.M. = A = $\frac{a+b}{2}$ and G.M. = G = $G^2 = \sqrt{ab}$

$A – G = \frac{a+b}{2} - \sqrt{ab}$

$= \frac{a+b-2\sqrt{ab}}{2}$

$= (\sqrt{a} - \sqrt{b})^2$

Root will be open automatically

$A – G > 0$

➔ $A > G$

Question: 19

Find the sum of the AP: 11, 17, 23, and 29... of first 10 terms.

(a) 380
(b) 568
(c) 960
(d) 593

Answer: a

Explanation:

=> nth term for the given AP = 5 + 6 n
=> First term = 5+ 6 = 11
=> Tenth term = 5 + 60 = 65
=> Sum of 10 terms of the AP = 0.5n (first term + last term) = 0.5 x 10 (11 + 65)
=> Sum of 10 terms of the AP = 5 x 76 = 380

Question: 20

Find the G. M. between $\frac{3}{2}$ and $\frac{27}{2}$

(a) $\frac{9}{2}$ (b) $\frac{2}{9}$
(c) $\frac{6}{3}$ (d) $\frac{3}{6}$

Answer: a

Explanation:

We know that if a is the G. M. between a and b, then
G = $\sqrt{ab}$
G. M. between $\frac{3}{2}$ and $\frac{27}{2}$ = $\sqrt{\frac{3}{2} \times \frac{27}{2}}$
= $\frac{9}{2}$

Question: 21

Insert three geometric means between 1 and 256.

(a) 4, 16, 64, (b) -4, 16, -64
(c) Both (d) None

Answer: c

Explanation:

Let G_1, G_2, G_3, be 3
GMS both 1, & 256
Then,
1, G_1, G_2, G_3, 256 will be in GP
Let common ratio be r
$\therefore$ G_1 = r
So r^4 = 256
r = $\pm$ 4
G_1 = $\pm$ 4
G_2 = $\pm$ 16
G_3 = $\pm$ 64

Question: 22

If 4, 36, 324 are in G.P. insert two more numbers in this progression so that it again forms a G.P.

(a) 12,108 (b) 14,180

(c) 16,120 (d) 12, 10

Answer: a

Explanation:

G. M. between 4 and 36 = $\sqrt{4 \times 36}$ = $\sqrt{144}$ = 12

G.M. between 36 and 324 = $\sqrt{36 \times 324}$ = 6 x 18 = 108

If we introduce 12 between 4 and 36 and 108 between 36 and 324, the numbers 4, 12, 36, 108, 324 form a G.P.

The two new numbers inserted are 12 and 108.

Question: 23

The distance travelled (in cm) by a simple pendulum in consecutive seconds are 16, 12, 9,... How much distance will it travel before coming to rest?

(a) 64 cm (b) 46 cm

(c) 1 am (d) none

Answer: a

Explanation:

The distance travelled by the pendulum in consecutive seconds are, 16, 12, 9 ... is an infinite geometric progression with the first term a = 16 and r = $\frac{12}{16} = \frac{3}{4} < 1$

Hence, using the formula S = $\frac{a}{1-r}$ we have

S = $\frac{16}{1-\frac{3}{4}}$ = $\frac{16}{\frac{1}{4}}$ = 64

Distance travelled by the pendulum is 64 cm.

Question: 24

Which term of the G.P.: 5, -10, 20, -40,... is 320?

(a) 7 (b) 6

(c) 3 (d) 12

Answer: a

Explanation:

In this case, a = 5; r = $\frac{-10}{5}$ = -2

Suppose that 320 is the n^{th} term of the G. P.

By the formulate = ar^{n-1}, we get

t = 5. $(-2)^{n-1}$, we get

320 = 5. $(-2)^{n-1}$ = 64 = $(2)^6 = (-2)^{n-1}$

n – 1 = 6

n = 7

Hence 320 is the 7^{th} term of the G.P.

Question: 25

If a, b, c is in G.P., then

(a) $a(b^2 + a^2) = c(b^2 + c^2)$ (b) $a(b^2 + a^2) = c(a^2 + b^2)$
(c) $b(b^2 + a^2) = c(b^2 + c^2)$ (d) None

Answer: b

Explanation:

If a, b, c is in to G.P. Then $b^2 = ac$

$b^2 (a - c) = ac (a-c)$

$b^2a - ac^2 = a^2c - b^2c$

$a (b^2 + c^2) = c (a^2 + b^2)$

Trick: Put a=1, b=2, c=4, and check the alternates.

Question: 26

The sum of infinity of the progression 9-3+1-$\frac{1}{3}$ + ... is

(a) 9 (b) 9/2
(c) 27/4 (d) 15/2

Answer: c

Explanation:

Infinite series 9-3+1-$\frac{1}{3}$ ...... ∝ is a G. P. with

a = 9, r = $\frac{-1}{3}$ \ $S_\propto = \frac{a}{1-r} = \frac{9}{1+\frac{1}{3}} = \frac{9\times3}{4} = \frac{27}{4}$

Question: 27

The product (32) $(32)^{1/6}$ $(32)^{1/36}$ .....To ∞ is.

(a) 16 (b) 32
(c) 64 (d) 0

Answer: c

Explanation:

(32) (32)1/6(32) 1/36 ...... ∞ = $(32)^{1+\frac{1}{6}+\frac{1}{36}+\cdots.\infty}$ = (32) $^{\left(1-\frac{1}{6}\right)}$

$(32)^{\frac{1}{5/6}} = (35)^{6/5} = 2^6 = 64$

Question: 28

Obtain the sum of all positive integers up to 1000, which are divisible by 5 and not divisible by 2.

(a) 10050 (b) 5050
(c) 5000 (d) 50000

Answer: d

Explanation:

The positive integers, which are divisible by 5, are 5, 10, 15,..., 1000

Out of these 10, 20, 30,.. 1000 are divisible by 2

Thus, we have to find the sum of the positive integers 5, 15, 25, ..., 995

If n is the number of terms in it the sequence then

995 = 5 + 10(n-1)
=> 1000 = 10n
Therefore, n = 100
Thus the sum of the series = (100/2) (5 + 995) = (50) (1000) = 50000.

Question: 29

If s is the sum of an infinite G.P., the first term a then the common ratio r given by

(a) $\frac{a-s}{s}$ (b) $\frac{s-a}{s}$
(c) $\frac{a}{1-s}$ (d) none

Answer: b

Explanation:

$S = \frac{a}{1-r}$
s-sr = a
-sr = a-s
$r=\frac{s-a}{s}$

Question: 30

If in an infinite G.P. first term is equal to the twice of the sum of the remaining terms, then its common ratio is

(a) 1 (b) 2
(c) 1/3 (d) -1/3

Answer: c

Explanation:

Given, $a=2\left(\frac{ar}{1-r}\right)$
1-r = 2r
$r = \frac{1}{3}$

Question: 31

If n geometric means between a and b be $G_1, G_2, \ldots. G_n$ and a geometric mean be G, then the true relation is

(a) $G_1, G_2, \ldots\ldots G_n = G$ (b) $G_1, G_2, \ldots\ldots G_n = G^{1/n}$
(c) $G_1, G_2, \ldots\ldots G_n = G^n$ (d) none

Answer: c

Explanation:

Here $G = (a\,b)^{1/2}$ and
$G_1 = ar^1, G_2 = ar^2, \ldots\ G_n = ar^n$. therefore
$G_1.G_2.G_3 \ldots.G_n = a^n r^{1+2+\cdots+n} = a^n r^{n(n+1)/2}$ But
$ar^{n+1} = b$

$r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Therefore, the required product is $a^n\left(\frac{b}{a}\right)^{\frac{1}{(n+1)}.n(n+1)/2}$

$= (ab)^{n/2}$

$= \left\{(ab)^{1/2}\right\}^n = G^n$

Note: It is a well-known fact.

Question: 32

7th term of the sequence$\sqrt{2}, \sqrt{10}, 5\sqrt{2}$ … is

(a) $125\sqrt{10}$
(b) $25\sqrt{2}$
(c) 125
(d) $125\sqrt{2}$

Answer: d

Explanation:

Given sequence is$\sqrt{2}$, $\sqrt{10}$, $5\sqrt{2}$**…..Common ratio**

r= $\sqrt{5}$, first term a =$\sqrt{2}$, then 7th term

t_7= $\sqrt{2}\left(\sqrt{5}\right)^{7-1}$ = $\sqrt{2}\left(\sqrt{5}\right)^{6}$ = $\sqrt{2}(5)^3$

$125\sqrt{2}$

Question: 33

If the first term of a G.P. be 5 and common ratio be -5, then which term is 3125?

(a) 6th
(b) 5th
(c) 7th
(d) 8th

Answer: b

Explanation:

Given that first term a=5 and common ratio r=-5. Suppose that nth term is 3125

Then ar^{n-1} = 3125

$5(-5)^{n-1} = \frac{5^5}{5}\ 5^4$

n – 1 = 4 = (n → 5)

Question: 34

The sums of n terms of three A.P.'s whose first term is 1 and common differences are 1, 2, 3 are S_1, S_2, S_3 respectively. The true relation is

(a) $S_1 + S_2 = S_3$
(b) $S_1 + S_3 = 2S_2$
(c) $S_1 + S_2 = 2S_3$
(d) none

Answer: b

Explanation:

We have $a_1 = a_2 = a_3 = 1$

$d_1 = 1, d_2 = 2, d_3 = 3$

Therefore, $S_1 = \frac{n}{2}(n+1)$....(i)

$S_2 = \frac{n}{2}(2n+1)$ ... (ii)

$S_3 = \frac{n}{2}(3n+1)$

... (iii) Adding (i) and (iii),

$S_1 + S_3 = \frac{n}{2}[(n+1) + (3n+1)] \rightarrow \frac{n}{2}[4n+2]$

$= 2\left[\frac{n}{2}(2n+1)\right] = 2S_2$

Hence correct relation $S_1 + S_3 = 2S_2$

Question: 35

What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

(a) 897 (b) 164,850

(c) 164,749 (d) 149,700

Answer: b

Explanation:

The smallest 3 digit number that will leave a remainder of 2 when divided by 3 is 101.

The next number that will leave a remainder of 2 when divided by 3 is 104, 107, ...

The largest 3 digit number that will leave a remainder of 2 when divided by 3 is 998.

So, it is an AP with the first term being 101 and the last term being 998 and common difference being 3.

Sum of an AP = $\frac{\text{First term+Last term}}{2} \times$ Number of term

We know that in an A.P., the nth term $a_n = a_1 + (n-1)*d$

In this case, therefore, 998 = 101 + (n – 1)*3

i.e. 897 = (n – 1)*3

Therefore n – 1 = 299

Or n = 300

Sum of the AP will therefore be $\frac{101+998}{2} \times 300 = 164{,}850$

Question: 36

What is the sum of the following series? -64, -66, -68,...., -100

(a) -1458 (b) -1558

(c) -1568 (d) -1664

Answer: b

Explanation:

The sequence is -64, -66, -68,....-100.

The given set of numbers are in an arithmetic progression

Key data: First term is -64. The common difference is -2. The last term is -100

Sum of the first n term is an AP = $\frac{n}{2}$[2a_1+ (n-1) d]

To compute the sum, we know the first term a_1 = -64 and the common difference d = -2

We do not know the number of terms n. Let us first compute the number of terms and then find the sum of the terms.

Step to compute number of terms of the sequence

$a_n = a_1 + (n – 1) d$

-100 = -64 + (n – 1)(- 2)

Therefore, n = 19.

Sum $S_n = \frac{19}{2}$[2(-64) + (919-1) (-2)]

$S_n = \frac{19}{2}$[-128-36]

S_n = 19 x (-82) = -1558

Question: 37

The sum of third and ninth term of an A.P. is 8. Find the sum of the first 11 terms of the progression.

(a) 44 (b) 22

(c) 19 (d) None of these

Answer: a

Explanation:

The third term t_3 = a + 2d

The ninth term t_9 = a + 8d

$t_3 + t_9$= 2a + 10d = 8

Sum of first 11 terms of an AP is given by

$S_{11} = \frac{11}{2}$[2a + 10d]

$S_{11} = \frac{11}{2}$[8] = 44

Question: 38

The sum of the three numbers in A.P is 21 and the product of the first and third number of the sequence is 45. What are the three numbers?

(a) 9, 7 and 5 (b) 3, 7, and 11

(c) Both A & B (d) None of these

Answer: a

Explanation:

Let the number are be a – d, a, a + d

Then a – d + a + a + d = 21

3a = 21

a=7

and (a – d) (a + d) = 45

$a^2 - d^2 = 45$
$d^2 = 4$
$d = \pm 2$
Hence, the number are 5, 7 and 9 when d = 2 and 9, 7 and 5 when d = -2. In both the cases numbers are the same.

Question: 39

If the first term of G.P. is 7, Its n^{th} term is 448 and sum of first n terms is 889, then find the fifth term of G. P.

(a) 112 (b) 110
(c) 62 (d) 39

Answer: a

Explanation:

Given a = 7 the first term $t_n = ar^{n-1} = 7(r)^{n-1} = 448$.
$7r^n = 448r$ ---- (1)
Also $S_n = \frac{a(r^n-1)}{r-1} = \frac{7(r^n-1)}{r-1}$
$889 = \frac{448r-7}{r-1}$ {value of r^n from (1)}
R = 2
Hence $T_5 = ar^4 = 7(2)^4 = 112$

Question: 40

If the third and fourth terms of arithmetic sequence are increased by 3 and 8 respectively. Then the first four terms form a geometric sequence. Find (i) the sum of the first four terms of A.P.

(a) 54 (b) 27
(c) 23 (d) 79

Answer: a

Explanation:

Sol. a,(a + d), (a + 2d), (a + 3d) in A.P.
a, a + d, (a + 2d + 3), (a + 3d + 8) are in G.P.
Hence a + d = ar
also $r = \frac{a+d}{a} = \frac{a+2d+3}{a+d} = \frac{a+3d+8}{a+2d+3}$
$\frac{d+3}{d} = \frac{d+5}{d+3}$
➔ $d^2 + 6d + 9 = d^2 + 5d$ → d = -9
$\frac{a-9}{a} = \frac{a-15}{a-9}$
➔ $a^2 - 18a + 81 = a^2 - 15a$ ➔ $3a = 8_1$ → a = 27
Hence A.P. is 27, 18, 9, 0,
Sum of the first four terms of AP = 54

Question: 41

Three positive numbers form a G.P. If the second term is increased by 8, the resulting sequence is an A.P. In turn, if we increase the last term of this A.P. by 6_4, we get a G.P. Find the three numbers.

(a) 4, 12, 36
(b) 4, 8, 16
(c) 5, 15, 20
(d) none

Answer: a

Question: 42

The sum of the first five terms of a geometric series is 18_9. The sum of the first six terms is
$3^{8\,1}$, and the sum of the first seven terms is $7^6 5$. What is the common ratio in the series?

(a) 3
(b) 2
(c) 6
(d) 56

Answer: b

Explanation:

Let the numbers be a, a r, a r^2 when r > 0
Hence a, (a r + 8), a r^2 in A.P. – (1)
Also a, (a r + 8), a r^2 + 6_4 in G.P. – (2)
→ (a r + 8 $)^2$ = a (a r^2 + 6_4) a = 4/4-r - (3)
Also (1) → 2(ar + 8) = (a + ar^2) →(1 – r$)^2$ = 16/a - (4)
From (3) and (4) r = 3 or -5 (rejected)
Hence a = 4 numbers are: 4, 12, 3^6
Explanation:
$S_5 = 18_9$; $S_6 = 3^{81;}$ $S_7 = 7^65$; $t_6 = S_6 – S_5 = 3^{81} – 18_9 = 19^2$
$t_7 = S_7 – S_6 = 7^65 - 3^{81} = 3^84$
Now common ratio = $\frac{t_7}{t_6} = \frac{384}{192} = 2$

Question: 43

Find the 3rd n th term for the AP: 11, 17, 23, 29,.....

(a) 23
(b) 17
(c) 11
(d) 6

Answer: a

Explanation:

Here, a = 11, d = 17-11 = 23 – 17 = 29 – 23 = 6
We know that nth term of an AP is a + (n – 1) d
=> nth term for the given AP = 11 + (n – 1)6
=> nth term for the given AP = 11 + (n – 1) 6
=> nth term for the given AP = 5 + 6n
We can verify the answer by putting values of ‘n’

=> n = a -> First term = 5 + 6 = 11
=> n = 2 -> Second term = 5 +12 = 17
=> n = 3 -> Third term = 5 + 18 = 23

Question: 44

The sum of three numbers in a GP is 26 and their product is 216. and the numbers.

(a) 2, 6 and 18
(b) 3, 7, and 11
(c) Both
(d) None of these

Answer: a

Explanation:

Let the numbers be $\frac{a}{r}$, a, ar.

=> $(\frac{a}{r})$ + a + ar = 26

=> a $\frac{(1+r+r^2)}{r}$= 26

Also, it is given that product = 216

=> $(\frac{a}{r}$) x (a) x (a r) = 216

=> a^3 = 216

=> a = 6

=> 6 $\frac{(1+r+r^2)}{r}$ = 26

=>$\frac{(1+r+r^2)}{r}$= = $\frac{26}{6}$= $\frac{13}{3}$

=> 3 + 3r + $3r^2$ = 13 r

=> 3 r^2 – 10r + 3 = 0

=> (r – 3) (r – $(\frac{1}{3})$) = 0

=> r = 3 or r = $\frac{1}{3}$

Thus, the required numbers are 2, 6 and 18.

Question: 45

A Sequence in which the ratio of two consecutive terms is always constant (1, 0) is called

(a) AP
(b) GP
(c) HP
(d) NP

Answer: b

Explanation:

A Sequence in which the ratio of two consecutive terms is always constant (1, 0) is called a Geometric progression (G.P.)

Question: 46

For the elements 4 and 6, verify

(a) $A \geq G \geq H$
(b) $A < G \geq H$
(c) $A > G \geq H$
(d) None

Answer: a

Explanation:

A = Arithmetic Mean = (4 + 6) / 2 = 5

G = Geometric Mean = $\sqrt{4 \times 6}$ = 4.8989

H = Harmonic Mean = (2 x 4 x 6) / (4 + 6) = 48 / 10 = 4.8

Therefore, $A \geq G \geq H$

Question: 47

A sequence of numbers is called?

(a) Geometric Progression
(b) Arithmetic progression (AP)
(c) Harmonic Progression
(d) All

Answer: d

Explanation:

Harmonic Progression (HP)

A sequence of numbers is called a harmonic progression if the reciprocal of the terms are in AP. In simple terms, a, b, c, d, e, f are in HP if 1/a, 1/b, 1/c, 1/d, 1/e, 1/f are in AP.

Arithmetic Progression (AP)

A sequence of numbers is called an arithmetic progression if the difference between any two consecutive terms is always same.

Geometric Progression (GP)

A sequence of numbers is called a geometric progression if the ratio of any two consecutive terms is always same.

Question: 48

An AP has 13 terms whose sum is 143. The third term is 5, then first term is :

(a) 4
(b) 7
(c) 9
(d) None of these

Answer: d

Explanation:

S (13) = 143

S (13) = (n/2)(2a+(n-1)d)

= (13/2) × (2a-12d)

= 13× (a+6d)

= 13a + 78d = 143 ------- (1)

Divide both sides by 13

a+6d=11 …… (1)

T (3) = a+2d=5 …… ……… (2)

Subtract (2) from (1)

4d=6

d=3/2
Substituted in any of the equations .....(am using 2)
a + 2(3/2) =5
a+ 3=5
a=2

Question: 49

The series $1^3 + 2^3 + 3^3 + \dots .. 20^3$ is equal to

a) 4410 b) 4410000
c) 44100 d) None of these

Answer: c

Explanation

$(n (n+1)/2)^2$
$(20(20+1)/2)^2$
44100.

PREPARE FOR WORST

Question 1

What is the sum of all 3 digit numbers that leave a remainder of `2' when divided by 3?

(a) 897 (b) 164,850
(c) 164,749 (d) 149,700

Question 2

A piece of equipment cost a certain factory Rs. 6, 00,000. If it depreciates in value , 15% the first year,13.5% the next year . 12% the third year , and so on , what will be its value at the end of 10 years , all percentages applying to the original cost

(a) 2,00,000 (b) 1,05,000
(c) 4,05,000 (d) 6,50,000

Question 3

If a rubber ball consistently bounces back 2⁄3 of the height from which it is dropped, what
Fraction of its original height wills the ball bounce after being dropped and bounced four times without being stopped?

(a) 16/81 (b) 16/27
(c) 4/9 (d) 37/81

Question 4

Find the sum of first 30 positive integer multiple of 6

Question 5

How many numbers are there between 200 and 800 which are divisible by both? 5 and 7?

Question 6

If (p + q)th term of an A.P is m and (p-q)tn term is n, then pth

(a) mn
(b) $\sqrt{mn}$
(c) $\frac{1}{2}(m-n)$
(d) $\frac{1}{2}(m+n)$

Question 7

If 7 times the 7th term of an A.P is equal to 11 times of its 11th term , then 18th term is

(a) 18
(b) 9
(c) 77
(d) 0

Question 8

There is a set of four numbers p,q,r and s respectively in such a manner that first three are in G.P. and the last three are in A.P with a difference of 6. If the first and the fourth numbers are the same find the value of P.

(a) 8
(b) 2
(c) -4
(d) -24

Question 9

An arithematic progression has 23 terms, the sum of the middle three terms of the arithematic progression is 270, and the sum of the last three terms of this Arithmetic progress is 1320. What is the 18^{th} term of this arithematic progression?

(a) 240
(b) 360
(c) 340
(d) 440

Question 10

Find the value of `a' given that the geometric mean between x and y is

(a) -2/3
(b) -1/4
(c) -3/2
(d) -7/6

Question 11

Sum of three numbers in GP with common ratio greater than 1 is 105 If the first two numbers are multiplied by 4 and the 3^{rd} number is multiplied by 3, then the resulting

Terms are in AP. What is the highest of the three numbers given?

(a) 60 (b) 50
(c) 30 (d) 45

Question 12

There are three terms x.y.z between 4&40 such that (i) their sum is 37 (ii) 4,x,y are consecutive terms of an A,P and (iii) y,z,40 are the consecutive terms of a G.P, Find the value of Z

(a) 20 (b) 10
(c) 12 (d) 15

Question 13

A tortoise walks 500 m in one day, the next day it walks 250 m, the next day 125, m and so on, what is the limiting distance which it could walk?

Question 14

In a geometric progression the sum of first 3X term of the series is S and the sum of first 2X terms of the series is 12s /133. If the sum of first X terms of the series is s/k, find the value of `k' it is given that the common difference of the gp is positive.

(a) 120 (b) 133
(c) 155 (d) 160

Question 15

In a infinite geometric progression with common ratios less than 1 the sum of any two consecutive terms is 8 times the sum of all the terms that follow. What is the ratio of any term and the sum of all the terms that follow it?

(a) 2 (b) -2
(c) -4 (d) Cannot be determined

Question 16

In an arithematic progression, the sum of the first 10 terms is half the sum of first 15 terms. Find the ratio of the sum of first 16 terms and first 21 terms of some AP.

(a) 7:11 (b) 6:10
(c) 12:17 (d) 8:13

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PAST EXAMINATION QUESTIONS:

MAY 2018

Question 1

The sum to m terms of the series 1 + 11 + 11 +1111 + ...Upto m terms is equal to:

(a) $\frac{1}{81}(10^{m+1} - 9m - 10)$

(b)) $\frac{1}{27}(10^{m+1} - 9m - 10)$

(c)) $(10^{m+1} - 9m - 10)$

(d) None

Answer: a

Explanation:

Given series:

1+11+111+ ....m term

$\frac{1}{9}$ [9 + 99 + 999 + ...m term]

$\frac{1}{9}$ (10 – 1)+(100 – 1) +(1000 – 1) + 1 ...... + m term]

$\frac{1}{9}\left[\frac{10.(10^{m-1})}{10-1} - m\right]$

$\frac{1}{9}\left[\frac{10^{m-1} - 10}{9} - m\right]$

$\frac{1}{9}\left[\frac{10.10^{m-1} - 10 - 9m}{9} - m\right]$

$\frac{1}{81}(10.10^{m-1} - 9m - 10)$

Question 2

A person pays Rs.975 in monthly installments; each installment is less than former by Rs.5. The amount

(a) 26 months

(b) 15 months

(c) both (a) & (b)

(d) 18 months

Answer: c

Explanation:

s_n = 975, a = 100, d = -5, n =?

$s_n = \frac{n}{2}(2a + (n-1)d)$

975=$\frac{n}{2}$[2 x 100+(n – 1)(-5)]

1950=n[200-5n+5]

1950=n[205-5n]

1950=205n-5n^2

5n^2-205n+1950 = 0

$5(n^2-41n+390) = 0$

$n^2 - 41n + 310 + 0$

n^2 -26n-15n+390=0

n(n-26)-15(n-26)=0

(n- 26) (n- 15)=0

If n-15=0 if n-26=0

N=15 n=26

The entire amount will be paid in 15 months

Question 3

If the sum of n terms of an AP is $3n^2$ – n and its common different is 6, then its term is:

(a) 3 (b) 2

(c) 4 (d) 1

Answer: b

Explanation:

Let s_n be the sum of n terms of an AP with first term a and common difference d.

Since $s_n = 3n^2$ - n and d = 6

➔ $S_n = \frac{n}{2}(2a + (n-1)d) = 3n^2 - n$

$= \frac{n}{2}(2a + (n-1)6) = 3n^2 - n$

$= n(a + (n-1)3) = 3n^2 - n$

$= (a + 3n - 3) = 3n - 1$

a = 2

Question 4

Insert two arithmetic means between 68 and 260.

(a) 132,196 (b) 130,194

(c) 70,258 (d) none

Answer: a

Explanation:

Let two A.M.'S between 68 and 260 are A_1, A_2

68, A_1, A_2:260

$d=\frac{b-a}{n+1}$

$d=\frac{260-68}{2+1} = \frac{192}{3} = 64$

$A_1 = a + d = 68 + 64 = 132$

$A_1 = a + 2d = 68 + 2 \times 64 = 196$

NOV 2018

Question:1

If the p^{th} term of an A.P. is 'q' and the q^{th} term is 'p', and then its r^{th} term is

(a) p+q-r (b) p+q+r

(c) p-q-r (d) p-q

Answer: a

Explanation:

Let 1st term of AP is 'a'

And common different is'd'

Given $T_p = q$

$a + (p-1)d = q$ ______ (i)

and $T_p = p$

$a+ (q-1)d = p$

$a+ qd - d = p$ ______ (ii)

Equation (i) and equation (ii)

$a+ pd-d = q$

$a+ qd -d = p$

$Pd - qd = q - p$

$d (p-q) = - (p-q)$

$d = -1$

Putting d = -1 in equation (i)

$a+ p(-1) - (-1) = q$

$a = (p + q - 1)$

Then, $T_r = a + (r - 1)d$

$= p + q - 1+ (r- 1)(-1)$

$= p+q-1-r+1$

$= p + q - r$

Question 2

The 3^{rd} term G.P. is $\frac{2}{3}$ and the 6^{th} term is $\frac{2}{81}$, term the 1^{st} term is

(a) 6 (b) $\frac{1}{3}$

(c) 9 (d) 2

Answer: a

Explanation:

Let 1st term of G.P. is 'a' and common ratio is 'r' then

Given $T_3 = \frac{2}{3}$ and $T_6 = \frac{2}{81}$

$ar^2 = \frac{2}{3}$ _____ (i)

$ar^5 = \frac{2}{81}$ ______ (ii)

Eq (2) / eq (1)

$\frac{ar^5}{ar^2} = \frac{\frac{2}{81}}{\frac{2}{3}}$

$r^3 = \frac{2}{81} \times \frac{3}{2} \rightarrow r^3 = \frac{1}{27} \rightarrow r = \frac{1}{3}$

Putting $r = \frac{1}{3}$ in equation (i)

$ar^2 = \frac{2}{3}$

$a = \left[\frac{1}{3}\right]^2 = \frac{2}{3} \rightarrow a \times \frac{1}{9} = \frac{2}{3}$

$a = \frac{2}{3} \times \frac{9}{1}$

a = 6

Question 3

The sum of the series -8,--6-4 …n terms is 52. The number of terms n is:

a) 11 b) 12

c) 13 d) 10

Answer: c

Explanation:

Given series

-8, -6, -4, …… n term

Let term (a) = -8

Common difference (d) = (-6) – (-8)

= -6+8

= 2

Sum of 'n ' term (S_n) = 52, n=?

We know that

$S_n = \frac{n}{2}(2a + (n-1)d)$

$52 = \frac{n}{2}[2 \times (-8) + (n-1)(2)]$

104 = n[2n-18]

$104 = 2n^2 - 18n$

$2n^2 - 18n - 104 = 0$

$n^2 - 9n - 52 = 0$

(n- 13) (n+ 4)=0

If n-13 → n = 13 and n + 4 = 0 → n = -4

Question 4

The value of K, for which the mean the term 7K+3,4K-5,2K+10 are in A.P., is

(a) 13 (b) -13

(c) 23 (d) -23

Answer: d

Explanation:

If 7K+3, 4K-5, 2K+10 are in A.P
Then,
(4K-5)- (7K+3) = (2K+10)-(4K-5)
4K-5-7K-3 = 2K+10-4K+5
-3K-8 = -2K+15
-8-15 = -2K+3K
-23 = K

MAY 2019

Question1

If y = 1+x+x^2+ …………∞ then x =

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$

(c) $\frac{y}{y+1}$ (d) $\frac{y}{y-3}$

Answer: a

Explanation:

y = 1+x+x^2+ ……….∞

is equivalent to GP = $\frac{a}{1-r}$

Y = $\frac{1}{1-x}$

1-x = $\frac{1}{y}$

1$\frac{1}{y}$ = x

$\frac{y-1}{y}$ = x

Question2

If 2 + 6 + 10 + 14 + 18 + ………. + x = 882 then the value of x

(a) 78 (b) 80

(c) 82 (d) 86

Answer: c

Explanation:

2 + 6 + 10 + 14 + 18 + ………… + x = 882

Sum of AP

$S_m = \frac{n}{2}[2a + (n-1) d]$

$S_m = \frac{n}{2}[a+1]$

882= $\frac{n}{2}$ [2 + x] ………. (1)

882= $\frac{n}{2}$ x 2[2+ (n-1)2]

$882 = n[2+2n+2]$
$882 = 2n^2$
$N^2 = 441$
$n = \sqrt{441}$
$n = 21$
Put n in eq 1
$882 = \frac{21}{2}[2 + x]$
$84 = 2 + x$
$X = 84 - 2 = 82$

Question 3

In a G.P, if the fourth term is '3' then the product of first seven terms is

(a) 3^5 (b) 3^7
(c) 3^6 (d) 3^8

Answer: b

Explanation:

Let first term be a and common ratio be r.
Then according to question
$ar^3 = 3$
Product of 1st 7 terms $(a)^7(r)^{21} = (ar^3)^7 = (3)^7$

Question 4

The ratio of sum of n terms of the two AP's is (n +1): (n – 1) then the ratio of their m^{th} terms is

(a) (m + 1) : 2m (b) (m + 1) : (m – 1)
(c) (2m – 1 : (m + 1) (d) m : (m – 1)

Answer: d

Explanation:

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']} = \frac{n+1}{n-1}$$

$$\frac{a+\frac{(n+1)d}{2}}{\frac{a'+(n-1)d\prime}{2}} = \frac{n+1}{n-1}$$

$T_n{}^{th} = a + (n-1)d$
$\frac{n-1}{1} = n - 1$
$n - 1 = 2n - 2$
$n = 2m - 2 + 1$
$n = 2m$
$n = 2m - 1$

$$\frac{2m}{2m-2} = \frac{2m}{2(m-1)} = \frac{m}{m-1}$$

Question 5

The sum of the series

0.5+0.55+0.555+ ……… to n terms is:

a) 5n/9+5/9[1-(*0.1)^n]
b) 5n/9-5/81[1-(0.1)^n]
c) 5n/81 +5/81[1-(0.1)^n]
d) 5n/9 +5/81 [1-(0.1)^n]

Answer: b

Explanation:

Given series 0.5 + 0.55 + 0.555…… n terms

we know that,

$0.1 + 0.1^2 + 0.1^3 + \ldots\ldots = \frac{0.1\,(1-0.1^n)}{0.9} = \frac{(1-0.1^n)}{9}$

→ 5(0.1 + 0.11 + 0.111 + ……..)

➔ $5\left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots\ldots\right)$

➔ $\frac{5}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots.\right)$

➔ $\frac{5}{9}\left(\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots..\right)$

➔ $\frac{5}{9}(1 + 1 + \cdots \text{ n terms}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots.\right)$

➔ $\frac{5}{9}\left(n - \frac{(1-0.1^n)}{9}\right)$

NOV 2019

Question 1

If $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in AP then a, b, c are in

a) AP
b) GP
c) HP
d) None

Answer: (c)

Explanation:

Given :

$\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in A.P.

Add 2 to each

$\frac{(b+c-a)}{a} + 2, \frac{(c+a-b)}{b} + 2, \frac{(a+b-c)}{c} + 2$

$= \frac{b+c-a+2a}{a}, \frac{c+a-b+2b}{b}, \frac{a+b-c+2c}{c}$

$= \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$

Now, divide by a+b + c

$= \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

We know, HP = $\frac{1}{A.P.}$

= a, b, c are in H.P.

∴ Option c i.e. H.P is the correct option,

Question 2

Sum upto infinity of series

$\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{25^2} +$

(a) 19/24 (b) 24/19

(c) 5/24 d) none

Answer: (a)

Explanation:

We know

$S\infty = \frac{a}{1-r}$, r < 1

Here, $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{25^2} + \ldots\ldots\ldots$

$\left(\frac{1}{2} + \frac{1}{3^3} + \frac{1}{2^5} + \ldots\ldots.\infty\right) + \frac{1}{2} + \frac{1}{3^3} + \frac{1}{2^5} + \ldots\ldots\ldots\infty)$

$\left\{a = \frac{1}{2}, r = \frac{1}{4} < 1\right\}; \left\{a = \frac{1}{2},\ r = \frac{1}{4}, 1\right\}$

$\left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right) + \left(\frac{\frac{1}{9}}{1-\frac{1}{9}}\right)$

$\frac{\frac{1}{2}}{\frac{3}{4}} + \frac{\frac{1}{9}}{\frac{8}{9}}$

$\frac{1}{2} \times \frac{4}{3} + \frac{1}{9} \times \frac{9}{8}$

$\frac{2}{3} + \frac{1}{8}$

$\frac{19}{24}$

Question 3

Sum the series $\frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \ldots\ldots\frac{1}{5^n}$

(a) $\frac{1}{4}\left[1 - \left(\frac{1}{5}\right)^n\right]$ (b) $\frac{1}{5}\left[1 - \left(\frac{1}{4}\right)^n\right]$

(c) both (d) none

Answer: (a)

Explanation:

Series $\frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \ldots\ldots\frac{1}{5^n}$

So, here a = $\frac{1}{5}$, r = $\frac{1}{5}$, $\frac{1}{5}$<1

Sn= $a\frac{(1-r^n)}{(1-r)}$, r < 1

Sn = $\frac{1}{5}\left[\frac{1-\left(\frac{1}{5}\right)^n}{1-\left(\frac{1}{5}\right)}\right]$

Sn = $\frac{1}{5} \times \frac{5}{4}\left[1-\left(\frac{1}{5}\right)^n\right]$

Sn = $\frac{1}{4}\left[1-\left(\frac{1}{5}\right)^n\right]$

Question 4

Find the no. of terms of the series 25, 5, 1………….$\frac{1}{3125}$

(a) 6 (b) 7
(c) 8 (d) 9

Answer: (c)

Explanation:

Here gives the series 25, 5, 1/5…..
Let the Total Number of Terms = n
First Term a = 25
Common ratio r = 1/5
Last Term a_n = $\frac{1}{3125}$
we have the formula
$a_n = ar^{n-1}$

➔ $\frac{1}{3125} = 25\left(\frac{1}{5}\right)^{n-1}$

➔ $\left(\frac{1}{5}\right)^5 = \left(\frac{1}{5}\right)^{n-3}$

➔ n – 3 = 5

➔ n = 8

Yes, 1/3125 is the 8^{th} term of the series.

Question 5

If the sum of five terms of AP is 75. Find the third term of the series.

(a) 35 (b) 30
(c) 15 (d) 20

Answer: (c)

Explanation:

We know

$S_n = \frac{n}{2}$[2a + (n-1) d]

n=5 S5 = 75

$S_5 = \frac{5}{2}$[2a+ (5-1) d]

$75 = \frac{5}{2}[2a + 4d]$

$15 = a + 2d$ -------Eq (1)

$T_3 = a + (3 - 1)\,d$

$T_3 = a + 2d$

------From Eq (1)

$T_3 = 15$

Question 6

If the AM and GM of the two numbers is 6.5 and 6 the no's are:

(a) 3 and 2 (b) 9 and 4

(c) 81 and 16 (d) None

Answer: (b)

Explanation:

Let the two nos.be 'a' and 'b'

AM = $\frac{a+b}{2}$; GM = $\sqrt{ab}$

$\sqrt{ab} = 6$

$\frac{a+b}{2} = 6.5$ On squaring

ab = 36 ----Equation (2)

a +b = 13

a = 13 – b ----Equation (1)

Put Eq (1) in Eq (2)

$b \times (13 - b) = 36$

$13b - b^2 = 36$

$b^2 - 13b + 36 = 0$

$b^2 - 9b - 4b + 36 = 0$

b (b-9) – 4 (b-9) = 0

b = 9 b = 4

a = 13 – 9 a = 13 - 4

a = 4 a = 9

So the two numbers are 4 and 9

Question: 7

If AM and HM for numbers are 5 and 3:2, respectively GM will be

a) 20 b. 16

c. 4 d. 5

Answer:(c)

Explanation:

We know that

$(GM)^2 = AM \times HM$

Here $(GM)^2 = 5 \times 3.2$
$(GM)^2 = 16$
(GM) = 4.

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Question 8

The 20th term of arithmetic progression whose 6th term is 38 and 10th term is 66 is_____

(a) 136 (b) 118
(c) 178 (d) 210

Answer: a

Explanation:

Let a and d be the first term and common difference of an AP
It is given that, 6th term $a_6 = 38$ and 10th term $a_{10} = 66$.
Therefore,
a + 5d = 38 ... (i)
a + 9d = 66 ... (ii)
Subtracting (i) from (ii), we have
4d = -28
d = 7
Substituting in (i), we have
a + 5(7) = 38
Hence, the 20th term is 136.

Question 9

Three numbers in G.P with their sum is 130 and their product is 27,000 are ______

(a) 90, 30, 10 (b) 10, 30, 90
(c) 10, 20, 30 (d) Both

Answer: d

Explanation:

Let the three number be $\frac{a}{r}$,a, ar

$$\frac{a}{r} + a + ar = 130$$

$$\frac{a}{r}.a.ar = 27000 \to a^3 = (30)^3$$

= 30

$$a\left[\frac{1 + r + r^r}{r}\right] = 130$$

$$\frac{1 + r + r^r}{r} = \frac{13}{3}$$

➔ $3r^r$ -10r +3

➔ r=3 or $\frac{1}{3}$

The numbers are 10, 30, and 90

Question 10

Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460

(a) 20, 23, 26 (b) 21, 23, 25

(c) 19, 23, 27 (d)22, 23, 24

Answer: a

Explanation:

Let the first term of the AP be 'a'

And the common difference be 'd'

Since 69 split into 3 parts such that they form an AP.

Let the three parts be (a - d), (a) and (a + d).

Therefore,

(a - d) + (a) + (a + d) = 69

3a = 69

a = 23

The product if two smaller parts = 460

So,

(a) × (a - d) = 460

23 × (23 - d) = 460

⇒ 529 - 23d = 460

⇒ - 23d = 460 - 529

⇒ - 23 d = - 69

⇒ d = 63/23

⇒ d = 3

Therefore,

The 3 parts are

23 - 3 = 20;

And 23 + 3 = 26

Hence the parts of the given AP are 20, 23, and 26

JAN 2021

Question 1

The nth term of the series 3 + 7 + 13 + 21 + 31 + ... is

(a) 4n - 1 (b) $n^2 + 2n$

(c) $n^2 + n + 1$ (d) $n^3 + 2$

Answer: c

Explanation:

3 + 7 + 13 + 21 + ...$a_{n-1} + a_n$---------(1)

3 + 7 + 13 + 21 + ...$a_{n-2} + a_{n-1} + a_n$ --------- (2)

Eq 1 – Eq 2

s-s = 3-0+ (7+3) + (13-7)+ ......+ ($a_{n-1} - a_{n-2} + (a_n - a_{n-1}) - a_n$

0= [3+4+6+....+a_{n-1}] $- a_n$

$a_n = 3 + [4 + 6 + 8 + ---+ a_{n-1}$ -------- (3)

Now 4+6+8+---- + a_{n-1} are in A.P.

First term a = 4, Common difference d = 2

Sum of n herm of AP = $\frac{n}{2}[2a + (n-1)d]$

=$4 + 6 + 8 \pm -- a_{n-1} = \frac{n-1}{2}[2 \times 4 + (n-1-1) \times 2]$

=$\left(\frac{n-1}{2}\right)[8 + 2n - 4]$

=$\frac{n-1}{2}(2n + 4)$

= (4+6+8+.......+a_{n-1} = (n-1) (n+2)

By Eq 3

a_n = 3+ [4+6+8+--- + a_{n-1}]

a_n = 3 + (n-1) (n+2)

= 3+ n^2 –n + 2n-2

a_n=n^2 + n+ 1

Question 2

The number of integers from 1 to 100 which are neither divisible by 3, nor by 5 nor by 7, is

(a) 67 (b) 55

(c) 45 (d) 33

Answer: c

Explanation:

Total No. – 100

divide by 3 = 100/3 = 33

divide by 5 = 100/5 = 20

divide by 7 = 100/7 = 14

= 33+20+14 = 67

3& 5 = 100/15=6

5& 7 = 100/35=2

7& 3 = 100/21=4

= 6+2+4 = 12
= 67-12 = 55
Total Divisible by 3,5&7 are 55
Total –divisible = not divisible
100-55 = 45

Question 3

In a geometric progression, the 3rd and 6th terms are respectively, 1 and -1/8. The first term (a) and common ratio are respectively.

(a) 4 and $\frac{1}{2}$ (b) 4 and $\frac{-1}{4}$
(c) 4 and $\frac{-1}{2}$ (d) 4 and $\frac{1}{4}$

Answer: c

Explanation:

By option c
a=4 & r = -1/2
Check 3rd GP
1/2 ×4 ====(4 time equals to) =1
checking 6th GP
1/2 ×4 =======(7 time equals to) =-0.125
=-1/8 = 0.125

JULY 2021

Question 1

The number of team of the series: 5 + 7 + 9 +…….. Must be taken so that the sum may be 480

(a) 20 (b) 10
(c) 15 (d) 25

Answer: Options (a)

Explanation:

5 + 7 + 9 -------
a = 5, d = 2, s = 480
S = $^{n}/_{2}$ (2a + n - 1) d
480 = $^{n}/_{2}$ (2 (5) + (n – 1) (2)
480 = $^{n}/_{2}$ (10 + 2n – 2)
480 = n (2n +8)
480 = $2n^2$ + 8n

$2n^2 + 8n - 480$

$2(n^2 + 4n - 480)$

⇨ $n^2 + 4n - 480$

$n^2 + 20n + 24n - 480$

$n(n - 20) + 24(n - 20)$

$n + 24 = 0$ $n - 20 = 0$

$n = -24$ $n = 20$

Question 2

The fifth term of an AP of n terms, whose sum is $n^2 - 2n$, is

(a) 5 (b) 7

(c) 8 (d) 15

Answer: Options (b)

Explanation:-

Given: Sum of n terms of an AP = $n^2 - 2n$.

To find: The fifth term =?

Sum of 'n' terms of an AP = $n^2 - 2n$

∴ Sum of 1st 5 terms

⇨ $s_5 = 5^2 - 2.(5)$

⇨ $25 - 10 = 15$

Similarly,

Now, sum of first 4 terms

$S_5 = 5^2 - 2.(5)$

$= 25 - 10 = 15$

Similarly,

Now, sum of first 4 terms

$S_4 = 5^2 - 2.(4)$

$= 16 - 8 = 8$

∴ The 5th term of an AP"

⇨ $t_5 = S_5 - S_4$ ...,(Using $T_n = S_n - S_{n-1}$)

$= 15 - 8$

$= 7$

So, option 2 is correct.

Question 3

The sum of three numbers in a geometric progression is 28. When 7, 2 and 1 are subtracted from the first, second and third numbers respectively, then the resulting numbers are in arithmetic progression. What is the sum of squares of the original three numbers?

(a) 510 (b) 456

(c) 400 (d) 336

Answer: Options (d)

DEC 2021

Question 1

If the sum and product of three number in G.P. are 7 and 8 respectively, then 4th term of the series is

(a) 6 (b) 4

(c) 8 (d) 16

Answer: d

Explanation:-

$t_n = ar^{n-1}$

Let the three terms of G.P. be a/r, a and ar respectively.

Since the product is 8, we have:

$$\frac{a}{r} \times a \times ar = 8$$

$a^3 = 8$

A = $(8)^{1/3} = 2$

Also, it is given that $\frac{a}{r} + a + ar = 7$

$$\frac{a + ar + ar^2}{r} = 7$$

$a+ar+ar^2 = 7r$

Putting the value of a=2 above, we get:

$2+2r+2r^2 = 7r$

$2r^2+2r-7r+2=0$

$2r^2-5r+2=0$

$2r^2-4r-r+2 = 0$

2r (r-2) - (r-2) = 0

(2r-1) (r-2) =0

So, either 2r-1 = 0- r= $\frac{1}{2}$

Or r-2 = 0 -r =2

Taking r= 1/2 , we have $t_4 = 2\left[\frac{1}{2}\right]^3 = 0.25$

Taking r= 2, we have $t_4 = 2\ (2)^3 = 16$

Since 0.25 is not in the options, option(d) is the answer.

Question 2

The sum of series 7+14+21+.... To 17th term is:

(a) 1071 (b) 971

(c) 1171 (d) 1271

Answer: a

Explanation:-

Clearly, this is an AP with a=7; d=14-7=7 ; n= 17

$S_n = \frac{n}{2}\{2a + (n-1) d\}$

$S_{17} = \frac{17}{2}\{(2\times7)+17-1) 7\} = 1,071$

Question 3

The sum of first n terms an AP is $3n^2 + 5n$. The series is:

(a) 8, 14, 20, 26, …… (b) 8, 22, 42, 68, …

(c) 22, 68, 114, …. (d) 8, 14, 28, 44, …

Answer:

Explanation:-

$S_1 = t_1 = a = 3(1)^2 + 5(1) = 3+5 = 8$

$S_2 = 3(2)^2 + 5(2) = 22$

$t_2 = 22-8 = 14$

$d = t_2 - t_1 = 14 - 8 = 6$

Question 4

The largest value of n which $\frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^n} < 0.998$is

(a) 9 (b) 6

(c) 7 (d) 8

Answer: d

Explanation:-

The given series is a GP with a= ½ ; r= ½

Since r<1, $S_n = a\left[\frac{1-r^n}{1-r}\right]$

Try the options,

Option (a) – 9

If n = 9

$S_9 = \frac{1}{2}\left(\frac{1-(1/2)^9}{1-(1/2)}\right) = 0.998046875$

Option (b) – 6

If n=6

$S_6 = \frac{1}{2}\left(\frac{1-(1/2)^6}{1-(1/2)}\right) = 0.984375$

Option (c) – 7

If n=6

$S_7 = \frac{1}{2}\left(\frac{1-(1/2)^7}{1-(1/2)}\right) = 0.9921875$

Option (d) – 8

If n=8

$S_8 = \frac{1}{2}\left(\frac{1-(1/2)^8}{1-(1/2)}\right)$=0.99609375

Clearly, option (d) is the answer as it is the largest value for which the sum of the series is less than 0.998.

JUNE 2022

Question 1

The n^{th} term of the series 9,7,5,………… and 15,12,9,…….are same. Find the n^{th} term?

(a) 7 (b) 8

(c) 9 (d) 10

Answer: Options (a)

Explanation:

Given Series

9, 7, 5……………………. n term

a = 9, d = 7 – 9 = - 2, n =n

T_n = a + (n -1) d

T_n = 9 + (n – 1)(-2)

= 9 – 2n +2

= 11 – 2n

and other series

= 15,12,9…………………n term

a = 15, d = 12 – 15 = -3, n = n

T_n = a + (n -1) d

= 15 + (n – 1)(-3)

= 15 – 3n + 2 = 18 – 3n

= 11 – 2n

Given n^{th} term of both series are equal

then 11 – 2n = 18 – 3n

3n – 2n = 18 – 11

n = 7

Question 2

Then sun of first 8 terms of a G.P is five times the sum of the first 4 terms. Find the common ratio?

(a) $\pm\sqrt{2}$ (b) 16

(c) $\pm\sqrt{20}$ (d) 4

Answer: Options (a)

Explanation:

Let 1st term of G.P = a

Common Ratio (r) = r

Given

$S_8 = 5\ S_4$

$$\frac{a(r^8-1)}{r-1} = 5\frac{a(r^4-1)}{r-1}$$

$r^8 - 1 = 5\ (r^4 - 1)$

$(r^4)^2 - (1)^2 = 5(r^4 - 1)$

$r^4 + 1 = 5$

$r^4 = 4$

$(r^2)^2 = (2)^2 \Rightarrow r = \pm\sqrt{2}$

Question 3

A person pays ₹ 975 in monthly instalments; each instalment is less then former by ₹ 5. The amount of 1st instalments is ₹100. In what time will the entire amount be paid?

(a) 26 months (b) 15 months

(c) Both (a) and (b) (d) 18 months

Answer: Options (a)

Explanation:

Given Sn = 975, 1st Installment (a) = 100, d = -5

Then series is

100, (100-5), (100 -5-5), ..n term

100, 95, 90, …… n term

$S_n = \frac{n}{2}[2a + (n-1)d]$

$975 = \frac{n}{2}[2 \times 100 + (n-1)(-5)]$

$975 \times 2 = n[200 - 5n + 5]$

$1950 = n\ (205 - 5n)$

$1950 = 205\ n - 5n^2$

$5n^2 - 205n + 1950 = 0$

or $n^2 - 41n + 390 = 0$

Solving this we get n = 15 or n = 26 (not valid)